Introduction to Diversity

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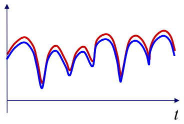
ECE4823

Motivation

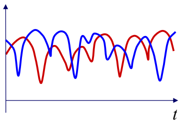
- If a fading radio signal is received through only one channel, then in a deep fade, the signal could be lost, and there is nothing that can be done
- Diversity is a way to protect against deep fades

Diversity: Choice in Fading

- The key: create multiple channels or *branches* that have uncorrelated fading



The fading of two highly correlated channels



Two channels with uncorrelated fading

Diversity 1

- Common assumption: signals that scatter off of different objects fade independently
- Diversity is created when these signals are separated in the receiver
- Examples:
    - RAKE receiver- - separates paths by delay [PATH DIVERSITY]
    - Multibeam antenna- - separates paths by angle [ANGLE DIVERSITY]

Diversity 2

- Another way to create diversity: change the relative phases of the multipath signals
- Examples:
    - Identical antennas, slightly different locations [SPACE DIVERSITY]
    - Same signal received on different RF carriers [FREQUENCY DIVERSITY]. Required carrier separation depends inversely on delay spread

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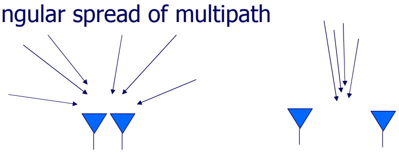
Diversity 3

- Still other ways to generate diversity:
    - dual polarized antennas [POLARIZATION DIVERSITY]
    - successive retransmission in a channel with Doppler spread [TIME DIVERSITY]. Time separation depends inversely on Doppler spread.

Antenna Separation for Spatial Diveristy

- The required separation distance between antennas for spatial diversity depends on the angular spread of multipath

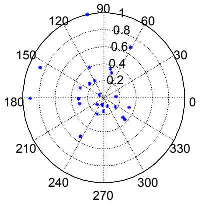


- Wide spread
- Spacing can be as small as 0.25 wavelength
- Typical for indoor channels

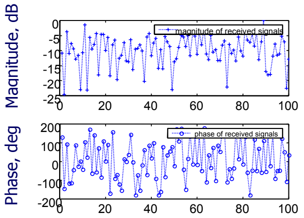
- Narrow spread
- Spacing may need to be as wide as 30 wavelengths
- Typical for tall base stations

Fading as a Function of Rx Position - Wide Angle Spread

Large angle spread implies large variation over short distance



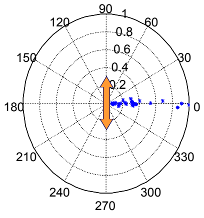
Relative amplitude and angle-of-arrival of paths



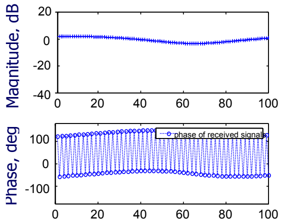
Distance in half-wavelengths

Fading as a Function of Rx Position - Narrow Angle Spread

Small angle spread implies slow variation over distance



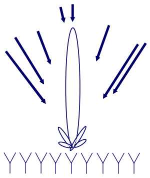
Relative amplitude and angle-of-arrival of paths
5 degree spread



Distance in half-wavelengths
(along ↕ direction)

Directional Antennas Limit Angular Spread

- Highly directional antennas or phased arrays make multipath seem to have a narrow angular spread
- If you want to use such antennas or arrays for diversity channels, they must be spaced widely apart



A phased array

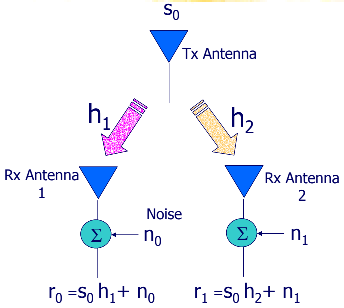
Diversity Combining

- Once you have created two or more diversity channels, what do you do with them?

Types of Diversity Combining

- Selection diversity
  - Pick the branch with highest signal power
- Maximal Ratio Combining (MRC)
  - Branches weighted prior to summing
  - MRC=Matched filter--maximizes SNR of desired signal in additive white Gaussian noise
  - Necessary to estimate branch gains
- Minimum Mean Squared Error (MMSE)
  - Same as MRC in absence of colored interference

Two-branch Spatial Diversity



$r_0 = s_0 h_1 + n_0$ $r_1 = s_0 h_2 + n_1$

Maximal Ratio Combining

The received signals:

$r_0 = s_0 h_1 + n_0$ $r_1 = s_0 h_2 + n_1$

The maximum likelihood decision statistic:

$d_0 = h_1^* r_0 + h_2^* r_1 = \left(|h_1|^2 + |h_2|^2\right) s_0 + h_1^* n_0 + h_2^* n_1$

SNR After Diversity Combining

$$\frac{E\{\text{signal pwr} \mid h_1, h_2\}}{E\{\text{noise pwr} \mid h_1, h_2\}} = \frac{\left(|h_1|^2 + |h_2|^2\right)^2 E\{|s_0|^2\}}{E\left\{\left(h_1^* n_0 + h_2^* n_1\right)^2\right\}} = \frac{\left(|h_1|^2 + |h_2|^2\right)^2 E\{|s_0|^2\}}{E\left\{\left(h_1^* n_0\right)^2 + \left(h_2^* n_1\right)^2\right\}}$$

$$= \frac{\left(|h_1|^2 + |h_2|^2\right)^2 E\{|s_0|^2\}}{E\left\{|h_1^*|^2 |n_0|^2 + |h_2^*|^2 |n_1|^2\right\}} = \frac{\left(|h_1|^2 + |h_2|^2\right)^2 E\{|s_0|^2\}}{E\left\{|h_1^*|^2 |n_0|^2\right\} + E\left\{|h_2^*|^2 |n_1|^2\right\}} = \frac{\left(|h_1|^2 + |h_2|^2\right)^2 E\{|s_0|^2\}}{\left(|h_1^*|^2 + |h_2^*|^2\right) E\{|n_1|^2\}}$$

$$= \left(|h_1|^2 + |h_2|^2\right) \frac{E\{|s_0|^2\}}{E\{|n_1|^2\}} = \frac{\left(|h_1|^2 + |h_2|^2\right)}{E\{|h_1|^2\}} \frac{E\{|h_1|^2\} E\{|s_0|^2\}}{E\{|n_1|^2\}} = \underbrace{\frac{\left(|h_1|^2 + |h_2|^2\right)}{E\{|h_1|^2\}}}_{\text{SNR Improvement Factor}} \underbrace{\frac{E\{|h_1|^2\} \mathcal{E}_b}{N_0 / 2}}_{\text{Avg SNR for 1 branch}}$$

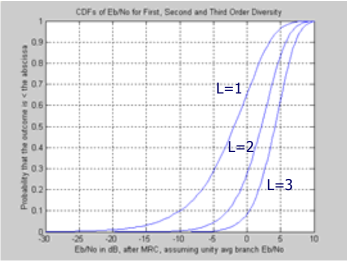
Received E_b/N_o with Diversity

- Let the received E_b/N_o without diversity on the ith branch be denoted γ_i
- In a flat-fading channel, γ_i is a random variable
- If L diversity branches have iid γ_i's, then the E_b/N_o after MRC diversity combining is

$$\gamma_{MRC} = \sum_{i=1}^{L} \gamma_i$$

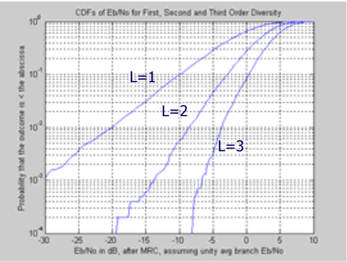
The CDFs of SNR Improvement Factor for MRC on the Linear Scale

- 10,000 trials of iid complex Gaussian channel
- Observe shift to the right as well as a more vertical orientation



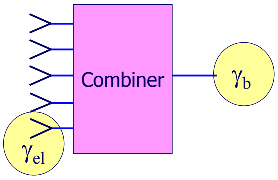
The CDFs of SNR Improvement Factor for MRC on the Log Scale

- Choppiness at bottom end is because I did only 10,000 trials



Performance Metrics 1

- SNR per bit after combining: $\gamma_b = E_b/N_o$
- SNR per bit before combining (per diversity branch): γ_{el}
- Both are random variables that depend on the fading state


Combiner

γ_b

γ_{el}

With MRC and K iid branches,

$$\overline{\gamma}_b = \overline{\gamma}_{el} K,$$

Where $\overline{\gamma}_{el} = E(\gamma_{el})$ is the average SNR per bit per branch and K is the number of branches

Performance Metrics 3

- The probability density function (pdf) of γ_b: $f_\Gamma(\gamma_b)$
- Probability of error for given γ_b: $P_e(\gamma_b) = \alpha \operatorname{erfc}\sqrt{\beta\gamma_b}$
- Probability of outage: $P_{out} = \int_0^{\gamma_{out}} f_\Gamma(\gamma_b)\,d\gamma_b$
- Average BER $\overline{P}_e = \int_0^{+\infty} P_e(\gamma_b) f_\Gamma(\gamma_b)\,d\gamma_b$

Performance Metrics 4

- When the average SNR per branch, $\overline{\gamma}_{el}$, is greater than 10 dB, $\overline{P}_e$ can be approximated as

$$\overline{P}_e \approx \left(\frac{1}{4\overline{\gamma}_{el}}\right)^K \binom{2K-1}{K}$$

for coherent BPSK. Therefore, error rate decreases inversely with no. of diversity branches.

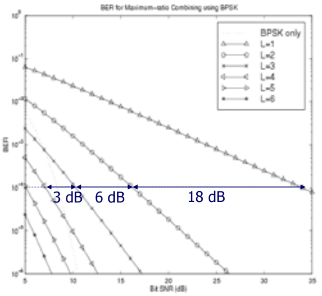
Diversity Gain

Diversity gain is the decrease in the required SNR per branch to achieve a desired average BER

iid Rayleigh fading on each diversity branch

Results shown for coherent BPSK

Improvement in Req'd SNR diminishes:
from 1 to 2: 18 dB
from 2 to 3: 6 dB
from 3 to 4: 3 dB



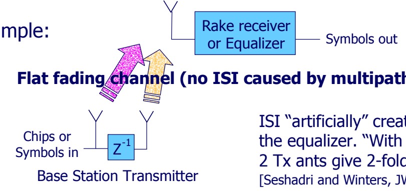
Antenna Gain

- The antenna gain is the ratio of the average SNR on the output of the array combiner to the average SNR per branch of the array

Transmit Diversity

- The use of multiple antenna elements at the transmitter to create diversity in the receiver

Example:


Mobile Receiver

Rake receiver or Equalizer

Symbols out

Flat fading channel (no ISI caused by multipath)

Chips or Symbols in

Z^{-1}

Base Station Transmitter

ISI "artificially" created for the equalizer. "With MLSE, 2 Tx ants give 2-fold diversity"
[Seshadri and Winters, JWIN'94]

Space-time Block Coding

- ⌘Less receiver complexity than Rake/equalizer
- ⌘Decoding gives MRC performance
- ⌘Requires no channel knowledge at Transmitter
  - ◸Alamouti's space-time block codes

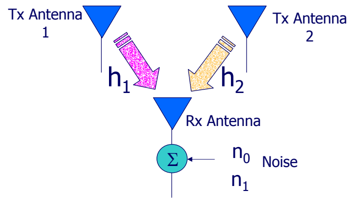
*can have two polarizations rather than two antennas

Alamouti's Scheme

Alamouti, IEEE JSAC, Oct 1998

Input at time t_0 : s_0 s_1

Input at time t_1 : $-s_1^*$ s_0^*



Output at time t_0 : $r_0 = s_0 h_1 + s_1 h_2 + n_0$

Output at time t_1 : $r_1 = s_0^* h_2 - s_1^* h_1 + n_1$

Combining Scheme

Alamouti, IEEE JSAC, Oct 1998

$$\text{Let } r = \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix}, \text{ then } r = Hs_{01} + n_{01}, \text{ where}$$

$$s_{01} = \begin{bmatrix} s_0 \\ s_1 \end{bmatrix}, \quad n_{01} = \begin{bmatrix} n_0 \\ n_1 \end{bmatrix}, \quad H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

The Key: Observe that H is a scaled unitary matrix:

$$H^H H = \left(|h_1|^2 + |h_2|^2\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Combining, cont'd.

$$\text{So, } \tilde{s}_{01} = \overbrace{H^H}^{\text{Simple linear combining (matched filtering)}} r = H^H (Hs_{01} + n_{01})$$

$$= \underbrace{\left(|h_1|^2 + |h_2|^2\right) s_{01}}_{\text{Diversity from two branches}} + \underbrace{H^H n_{01}}_{\text{Gaussian noise}}$$

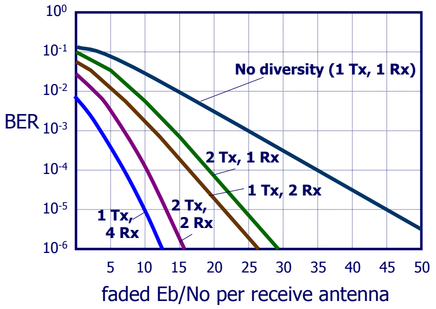
Combining, Concluded

Alamouti, IEEE JSAC, Oct 1998

- ⌘Observe that two time-slots are required for two symbols (code rate=1).
- ⌘If total transmit power is conserved, decrease diversity gain by $10\log_{10}N$
  - ◸This decrease is the "power-splitting loss"
  - ◸This loss is also suffered by the time-transmit diversity discussed prior to STBC

Average BER Results

Alamouti, IEEE JSAC, Oct 1998



Extensions to the Alamouti Scheme

- N>2 transmit antennas uses Q symbols and >Q time-slots, so code rate is not as good. Same benefit as N receive diversity branches, if total transmit power increases multiplies by N.
- Rate 1/2 codes exist for any N
- Rate 3/4 codes exist for N= 3 or 4
- For real-only symbols, rate 1 codes exist for N=2, 4, or 8, only.

Tarokh, et al., IEEE Trans Info Theory, July 1999

Notes

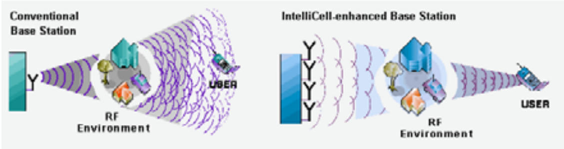
- If L multiple users all use same space-time code, synchronously, then the extra transmit antennas do NOT count as extra interferers from the point of view of cancellation [Naguib et al., 32nd Asilomar Conf, '98]..
- S-T coded systems are degraded by frequency selective channels, but can be used in concert with MIMO equalizers [Gong and Letaief, VTC '99].

Yet Another Kind of Transmit Diversity: Adaptive Array

- Suppose the transmitter knows the channel coefficients
- It can weight each transmit antenna's waveform with the conjugate of the respective channel coefficient
- These weights will have the effect of maximizing the SNR at the receive antenna output
- This approach has the benefit of no power splitting loss and no loss in code rate for more than two antennas. It's disadvantage is the requirement of channel knowledge at the transmitter

Adaptive Array Transmitter

- In TDD, slow fading systems, the channel coefficients can be learned on the uplink and their conjugates applied on the downlink. This condition can be satisfied in indoor WLANs



ArrayComm.com, 1999

Summary

- Receive and transmit diversity techniques have been discussed
- Diversity gain is the reduction in fade margin provided by diversity combining
- MRC transmit diversity, with the exception of adaptive arrays, give the same performance as receive diversity except for the power-splitting loss