Equalization

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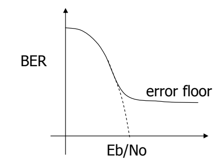
ECE 4823

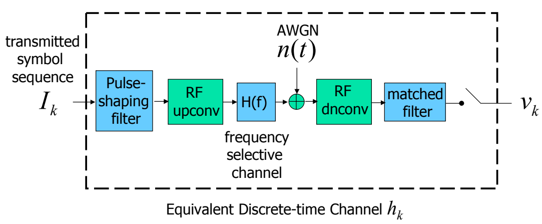
Motivation

- Frequency selective channels cause intersymbol interference (ISI), which creates a BER floor
- Equalizers mitigate ISI



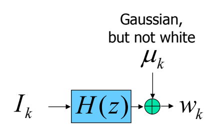
Model of the TX+Channel+RX

- Everything from where the symbols go in at the transmitter to where the samples come out at the receiver can be grouped into a discrete-time filter



Equivalent Discrete-time Channel Model

- Samples are at the symbol rate



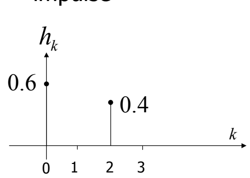
$$H(z) = \sum_{j=0}^{L} h_j Z^{-j}$$

Z transform

$$w_k = I_k * h_k + \mu_k$$

ISI

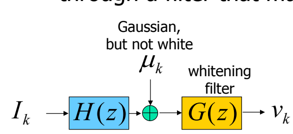
- There is ISI if the discrete equivalent channel impulse response is not a single delayed impulse



$$h_0 = 0.6$$
$$h_1 = 0$$
$$h_2 = 0.4$$
$$h_k = 0 \quad \text{for} \quad k \geq 2$$

Whitening Filter

- Because white noise is easy to analyze, it is customary to imagine passing the samples through a filter that makes the noise white



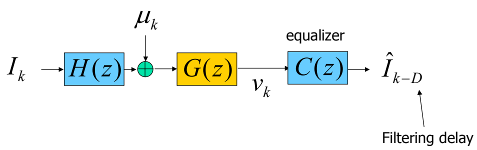
$$F(z) = H(z)G(z)$$

Gaussian, white

$$v_k = I_k * f_k + \eta_k$$

Equalizers

- The equalizer is a discrete-time filter that operates on the samples
- Its output is a sequence of estimates of delayed symbols



MMSE Equalizer

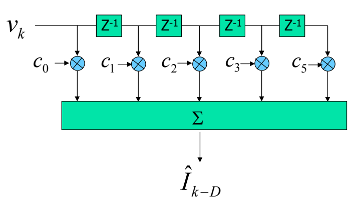
- The MMSE equalizer tries to minimize the mean squared error between its output and the appropriately delayed true symbol sequence

$$MSE = E\left\{|\varepsilon|^2\right\} = E\left\{\left|\hat{I}_{k-D} - I_{k-D}\right|^2\right\}$$

How much you delay your reference signal, is a design parameter

Linear Transversal Equalizer

- This type is implemented as a feedforward tapped delay line



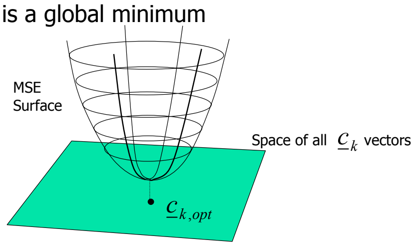
Vector Formulation

tap coefficients

$$\hat{I}_{k-D} = [c_0, c_1, \ldots, c_N] \begin{bmatrix} v_k \\ v_{k-1} \\ \vdots \\ v_{k-N} \end{bmatrix} = \underline{c}_k^T \underline{v}_k$$

MSE Performance Surface

- The MSE, as a function of all possible tap coefficient vectors, is a paraboloid
- There is a global minimum



Optimal Solution

- The Wiener-Hopf Solution
- The autocorrelation matrix Γ_v and cross-correlation vector r_{Dv} can be estimated from the received data, using a training sequence

$$\underline{c}_{k,opt} = \Gamma_v^{-1} r_{Dv}$$

$$\Gamma_v = E\left\{\underline{v}_k^{\,*} \underline{v}_k^T\right\}$$

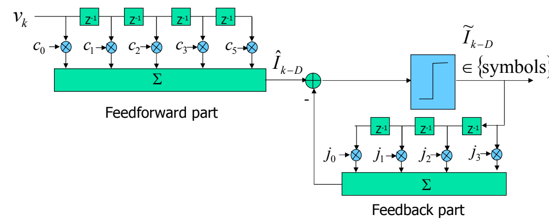
$$r_{Dv} = E\left\{I_{k-D} \underline{v}_k^*\right\}$$

A Problem in Multipath Channels

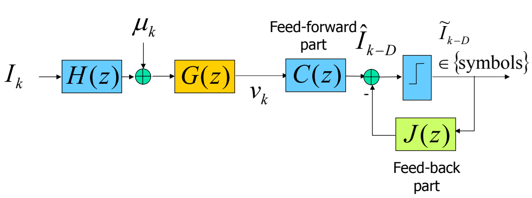
- The frequency selective channel is a feed-forward channel, which can be modeled by a tapped delay line filter
- One tapped-delay line filter cannot invert another another tapped-delay
- Therefore, the linear transversal equalizer does not perform well for multipath channels
- The decision feedback equalizer does a much better job

Decision Feedback Equalizer

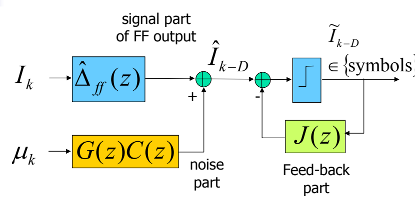
- The DFE has a feedforward part and a feedback part
- The feedback part feeds back detected symbols (the detector is the nonlinearity)



Lumping the FF and FB Parts



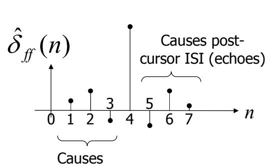
Output of Feed-forward Part



Impulse Response of Combined Channel+Whitener+FF Part

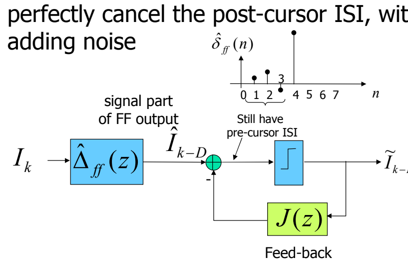
$$\hat{\delta}_{ff}(n) = Z^{-1}\{\hat{\Delta}_{ff}(z)\}$$

Example:
D=4



Post-cursor ISI

- Assuming perfect detection, the FB part can perfectly cancel the post-cursor ISI, without adding noise



Summary

- Equalizers attempt to make the discrete time-impulse response of the channel ideal
- Training sequences can be used to adapt equalizer weights
- Linear equalizers cannot get close to ideal for a typical multipath channel
- Decision-feedback equalizers can do a better job because they cancel post-cursor ISI