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On the Optimal Lifetime of Cooperative Routing for Multi-hop Wireless Sensor Networks

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Abstract—**Theoretically optimal performance and behavior of a routing protocol are very important because they can be used to guide the design of practical protocols. Motivated by the promising lifetime performance of an existing online cooperative transmission (CT) routing protocol for wireless sensor networks (WSNs), we formulate the lifetime optimization problem of cooperative routing using linear programming (LP), which requires considerations of CT's unique characteristics and sophisticated variable definitions. By using our LP, one can obtain the optimal lifetime of multi-hop WSNs that use cooperative routing, which can serve as a performance bound or a benchmark to compare to the lifetimes of existing routing protocols. Through the evaluations of our LP, we show the benefit of cooperative routing and usefulness of our LP formulation.**

I. INTRODUCTION

Cooperative transmission (CT) [1] is one way to improve the communication quality of single-antenna communication devices. A transmitting node that uses CT shares a data packet with neighboring single-antenna nodes, and then, the collection of these nodes transmits the packet to an intended receiver, thereby creating a virtual multiple-input-single-output (VMISO) system. The intended receiver can use physical-layer combining schemes to get diversity and array gains, which give CT a signal-to-noise ratio (SNR) advantage over the traditional single-input-single-output (SISO) case. CT's SNR advantage can be used to reduce the transmit powers of transmitting nodes, and this CT power-saving strategy has been shown by some authors to increase the lifetime of the network [2], [3], [4], [5]. However, when the circuit energy consumption of a node is comparable to its radiated energy, which is the case for popular sensor radios today [6], the power-saving strategy of CT cannot significantly reduce the overall energy consumption [7]. Therefore, CT's power saving alone is not always appropriate for maximizing the network lifetime.

Using CT's SNR advantage to extend the communication range in multi-hop wireless networks has also been studied [8], [9], [10], [11], [12]. The range-extension strategy of CT, which has been successfully demonstrated in [12], intentionally consumes more energy to extend the communication range, which provides more routing choices and reduced hops to the destination. In [11], we showed how the energy hole problem [13] in multi-hop wireless sensor networks (WSNs) can be mitigated by using both the range-extension and power-saving CT. Although the REACT protocol in [11] mainly utilizes the range extension CT, it tries to use the transmit power that is necessary, and therefore, it can benefit from both range extension and power saving of CT, thereby providing the full benefit of cooperative routing. Although the REACT protocol extends the network lifetime, it is not clear whether its lifetime performance is close to the optimal or not. Motivated by this fact, in this paper, we study the optimal lifetime of cooperative routing. We note that many CT-based routing works [2]-[4], [9] ignore the circuit energy consumption, which oversimplifies the problem or makes their approaches incorrect; in contrast, we try to correctly capture the energy cost of CT in our problem formulation.

For non-CT networks, the theoretical approach of the optimization of network lifetime when the transmit power control is possible is developed in [14]. The optimization problem that we want to solve for CT is more demanding because there are many more parameters in a VMISO link than there are in a SISO link. For instance, to specify a VMISO link, one must determine the number of cooperators and exactly which nodes will be the cooperators. By considering these different parameters of CT, we formulate the linear programming (LP) problem that successfully captures the lifetime optimality of cooperative routing. The LP solutions obtained from our formulation give the maximum achievable lifetime values of cooperative routing, and these values can be used as benchmarks to compare to the lifetimes of any cooperative routing protocol. We use this fact to compare the optimal lifetime of CT with that of non-CT and the lifetime of an existing cooperative routing protocol, REACT [11]. By doing so, we show not only the lifetime benefit of cooperative routing but also the usefulness of our LP formulation.

Our paper is organized as follows. We formulate the lifetime optimization problem in Section II. We evaluate the optimal lifetime performances of cooperative routing in Section III, and we compare those performances with the lifetime performances of the REACT protocol and optimal non-CT case. Finally, concluding remarks are offered in Section IV.

II. PROBLEM FORMULATION

We formulate the lifetime optimization problem for cooperative routing under the following assumptions. We consider a single-commodity[1] multi-hop wireless sensor network where sensed data is gathered at sink nodes, which are not energy constrained. Each node can be a source except for the sink

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[1] Here, "single commodity" means that, when there are multiple sink nodes, a source node needs only to send its data to one of the sink nodes (a "single" destination).

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nodes, and the data generated by a source node is forwarded to one of the sink nodes. We also adopt some of implicit assumptions of [14], such as collision is avoided so that no retransmission occurs, for our problem formulation.

We define the lifetime, T, of the network to be the time that the first node dies, as in [14], [15], [16]; we note that for energy balancing schemes such as REACT, sink isolation (i.e., complete energy hole formation) follows very soon after first node death. We let A be the set of all nodes in the network, S_i be the set of neighbors of Node i, D be the set of destination nodes, and E be the set of nodes that are not energy constrained. In our formulation, D is the set of sink nodes, and $D \subset E$. We define e_{ij}^{TX} as the required energy for Node i to transmit a data unit from Node i to Node j, e_{ji}^{RX} as the required energy for Node i to receive a data unit coming from Node j, Q_i as the information generation rate (data/time) of Node i, and E_i as the initial energy of Node i. Since e_{ji}^{RX} is just circuit energy consumption, we use e^{RX} for the receiving energy instead of e_{ji}^{RX}.

In the case of non-CT, the lifetime optimization problem can be formulated using LP as follows [14]:

$$\text{Maximize } T$$

$$\text{s.t. } n_{ij} \geq 0, \qquad \forall i \in A, \forall j \in S_i, \quad (1)$$

$$\sum_{j \in S_i} e_{ij}^{\mathrm{TX}} \cdot n_{ij} + \sum_{j : i \in S_j} e^{\mathrm{RX}} \cdot n_{ji} \leq E_i, \ \forall i \in A - E, \quad (2)$$

$$\sum_{j : i \in S_j} n_{ji} + T \cdot Q_i = \sum_{j \in S_i} n_{ij}, \qquad \forall i \in A - D, \quad (3)$$

where n_{ij} is the total number of data units transmitted from Node i to Node j until the lifetime T of the network, and $T \cdot Q_i$ is the number of data units generated by Node i during the lifetime of the network. (2) indicates the energy constraint condition of Node i, and (3) is the data conservation (flow conservation) condition of Node i (we are directly looking at the amount of data instead of flows). Instead of the terms "energy constraint condition" and "data conservation condition," we use the abbreviated terms Cond-EC and Cond-DC, respectively.

We use the following terms and conditions for the optimization problem of CT. A node, when it has data to be transmitted, can either do CT by cooperating with its selected neighbors or do non-CT by sending its data to one of its neighbors. Neighbors of a node are the ones that are within SISO (non-CT) communication range of the node, and cooperators of a node are the neighbors of the node that are selected by the node to do CT. If a node decides to do CT, it becomes a CT initiator (or just "initiator"), and it first sends its data to selected cooperators in the "CT sharing" or "multicast" phase. Next, the node (initiator) performs CT in the "CT phase" with the selected cooperators to send the data to the VMISO receiver through a VMISO link. In the CT phase, the initiator and its cooperators transmit data to the VMISO receiver using orthogonal channels. The cooperators, which receive multicast (CT sharing) messages from an initiator, do CT with the initiator. Nodes can adjust their transmit power. A VMISO link can be formed between the 'cooperating nodes' (this term is different from the word cooperators because it includes cooperators and the initiator) and any node. Note that the maximum number of cooperating nodes, denoted by $N_c^{\max}$, cannot exceed the maximum number of orthogonal diversity channels. The initiator can select up to $N_c^{\max} - 1$ cooperators.

In the remainder of this section, we introduce intermediate variables that provide a framework for our LP formulation. Then, we give LP formulation for CT.

A. Intermediate Variables for CT

In this section, we define a set of intermediate variables, which capture the different ways of transmitting and receiving data in a CT network. These ways must be distinguished because they correspond to different values of energy consumption. In the next section, these intermediate variables will be expressed in terms of LP variables.

In a CT network, data can arrive at a node in different ways. It can be received over a SISO link from either a non-CT transmitter or from an initiator in the multicast (CT sharing) phase. Alternatively, it can arrive from a VMISO link or be self-generated. We denote the numbers of incoming data units arriving to Node i in each of these four ways by I_i^{n}, I_i^{m}, I_i^{v}, and I_i^{g}, respectively. Similarly, data can leave a node different ways. It can be transmitted over a SISO link as either a non-CT transmission or from an initiator in the multicast (CT sharing) phase of CT. Alternatively, the data can be transmitted over a VMISO transmission by a node, which can be either an initiator or a cooperator (non-initiator). We denote the numbers of outgoing data units from Node i in each of these three categories by O_i^{n}, O_i^{m}, and O_i^{v}, respectively. Here, O_i^{v} encompasses two cases where (i) Node i initiates and does CT and (ii) Node i has received a multicast (CT sharing) message from its neighboring initiator and does CT, which corresponds to I_i^{m}. Note that if Node i initiates x CT instances, then Node i will do x multicasts ($O_i^{\mathrm{m}}=x$) and x CTs, and therefore, the number of outgoing CT (VMISO) data from Node i initiated by Node i is equal to O_i^{m}. Therefore, the following holds:

$$O_i^{\mathrm{v}} = O_i^{\mathrm{m}} + I_i^{\mathrm{m}}. \quad (4)$$

B. Optimization Problem Formulation for CT

In this section, we define LP variables for CT and formulate the optimization problem using the variables. In our LP formulation, as in [17], we explain our problem in terms of data packets, for convenience. However, the formulation can be explained in terms of bits or any other unit of data. For the remainder of this section, when necessary, we use the network in Fig. 1 where solid lines indicate SISO links and dashed lines indicate VMISO links.

We first discuss why new LP variables are required for CT. Unlike the non-CT network, the energy consumption of a VMISO link between the initiator and a VMISO receiver highly depends on the combination of cooperating nodes. That is, as discussed in [11], when transmit power control for CT is possible, the minimum required transmit power to successfully reach the VMISO receiver depends on the combination of cooperating

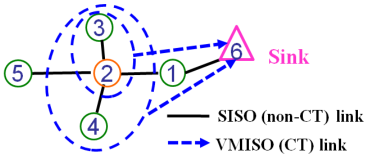

Fig. 1. A network example. Node 2 can form a VMISO link between itself and Node 6 (sink node) by cooperating with (i) Node 3 or (ii) Nodes 3 and 4. Node 3 can form a VMISO link to Node 6 by cooperating with Node 2.

nodes. Also, the energy consumption of the initiator for data sharing (multicast) depends on how it selects its cooperators (it only needs to reach all its cooperators). Therefore, we define the new LP variables for CT that can represent the number of CT sharing messages from Node i to Node j for each possible combination of cooperating nodes as follows.

We denote by $R_{i,v}^{N}$, the set of N-tuples, $(r_1, r_2, ..., r_N)$, where $r_m \in S_i$ $(1 \leq m \leq N)$ and each tuple indicates the combination of cooperators that can reach Node v (a VMISO receiver) directly by cooperating with the initiator Node i. For the example in Fig. 1, $R_{2,6}^{1}$={(3)}, $R_{3,6}^{1}$={(2)}, and $R_{2,6}^{2}$=$\{(3,4)\}$. Note that, given v, there can be $N_c^{\max} - 1$ different $R_{i,v}^{N}$'s for Node i (in other words, $1 \leq N \leq N_c^{\max} - 1$). Now, let $n_{i,v,j}^{N,k}$ be the number of CT sharing data from Node i to Node j that is related to the k-th tuple in $R_{i,v}^{N}$ denoted by $R_{i,v}^{N,k}$. For example, when $R_{2,6}^{2} = \{(3,4),(3,5)\}$, we can define variables $n_{2,6,3}^{2,1}$ and $n_{2,6,4}^{2,1}$ that are related to the first tuple (3,4), $R_{2,6}^{2,1}$, and variables $n_{2,6,3}^{2,2}$ and $n_{2,6,5}^{2,2}$ are related to the second tuple (3,5), $R_{2,6}^{2,2}$. In this example, even though $n_{2,6,3}^{2,1}$ and $n_{2,6,3}^{2,2}$ both correspond to SISO communication from Node 2 to Node 3, they are two distinct LP variables for CT. In addition to these variables, if we define $n_{i,j}^{\mathrm{nCT}}$ as the number of data units transmitted from Node i to Node j using non-CT ($n_{i,j}^{\mathrm{nCT}}$ matches with n_{ij} of the non-CT LP formulation), we also have the variable $n_{2,3}^{\mathrm{nCT}}$ for the link between Node 2 and Node 3. Therefore, unlike the non-CT case, which requires a single variable n_{ij} for one SISO link from i to j, the LP formulation for CT has multiple variables defined for one SISO link. With $n_{i,v,j}^{N,k}$, we can correctly form Cond-EC for CT.

Given $R_{i,v}^{N}$ and fixed k, the following equality should hold for $j_m \in R_{i,v}^{N,k}$ $(1 \leq m \leq N)$ because CT sharing messages from Node i are multicast:

$$n_{i,v,j_1}^{N,k} = n_{i,v,j_2}^{N,k} = ... = n_{i,v,j_N}^{N,k}. \tag{5}$$

For the example in Fig. 1, when Node 2 initiates one CT instance using the cooperating group of $R_{2,6}^{2,1}$ (Nodes 3-4), $n_{2,6,3}^{2,1}$=1 and $n_{2,6,4}^{2,1}$=1, and therefore, $n_{2,6,3}^{2,1}$=$n_{2,6,4}^{2,1}$.

Now, we express the variables I's and O's in terms of the LP variables. For fixed v and N, the total number of outgoing data packets from Node i for the data sharing of CT is $\left(\sum_{j \in S_i} \sum_{k: j \in R_{i,v}^{N,k}} n_{i,v,j}^{N,k}\right)/N$. The reason why the denominator N is required is because the data sharing is multicast so that $n_{i,v,j}^{N,k}$ for a fixed k cannot be considered as N different instances. Since $\left(\sum_{j \in S_i} \sum_{k: j \in R_{i,v}^{N,k}} n_{i,v,j}^{N,k}\right)/N$ is for fixed v and N, we get O_i^{m} as follows:

$$O_i^{\mathrm{m}} = \sum_{N=1}^{N_c^{\max}-1} \left\{ \left(\sum_{v: v \in A-i} \sum_{j \in S_i} \sum_{k: j \in R_{i,v}^{N,k}} n_{i,v,j}^{N,k} \right) / N \right\}. \tag{6}$$

Also,

$$I_i^{\mathrm{m}} = \sum_{N=1}^{N_c^{\max}-1} \left(\sum_{j: i \in S_j} \sum_{v: v \in A-j} \sum_{k: i \in R_{j,v}^{N,k}} n_{j,v,i}^{N,k} \right). \tag{7}$$

To formulate I_i^{v}, let us consider the case in Fig. 1 and assume that Node 2 transmits one packet to Node 6 by cooperating with Nodes 3 and 4. Each of three nodes (Nodes 2-4) sends the same packet to Node 6, and total three data packets are sent for this VMISO communication. However, these packets are combined to be one packet at the sink node. Note that when Node 2 initiates x CT instances, Node 6 receives x packets regardless of how Node 2 selects its cooperators. This means that I_i^{v} of some Node i (e.g., Node 6 in Fig. 1) can be obtained from the number of outgoing CT packets "initiated" by some Node h (e.g., Node 2 in Fig. 1). Since the number of outgoing CT packets "initiated" by Node h is equal to O_h^{m}, we can formulate I_i^{v} by (i) getting O_h^{m} for a fixed VMISO receiver i (using (6)) and (ii) using the fact that Node h can be any node except for destination nodes and Node i, which results in

$$I_i^{\mathrm{v}} = \sum_{h: h \in A-D-i} \sum_{N=1}^{N_c^{\max}-1} \left\{ \left(\sum_{j \in S_h} \sum_{k: j \in R_{h,i}^{N,k}} n_{h,i,j}^{N,k} \right) / N \right\}. \tag{8}$$

For I_i^{n}, I_i^{g}, and O_i^{n}, we can simply use (3) and get

$$I_i^{\mathrm{n}} = \sum_{j: i \in S_j} n_{j,i}^{\mathrm{nCT}}, \quad I_i^{\mathrm{g}} = T \cdot Q_i, \quad O_i^{\mathrm{n}} = \sum_{j \in S_i} n_{i,j}^{\mathrm{nCT}}. \tag{9}$$

Let us consider Cond-DC for CT. In the case of CT, all incoming packets of Node i are transmitted to receiving nodes using either VMISO links or SISO links. Note that O_i^{m} is counting the packets for CT sharing, which is just an intermediate step to do CT and not the transmission to the intended receiver. Because of this, the sum of all incoming data packets should be equal to O_i^{n}+O_i^{v} (not O_i^{n}+O_i^{m}+O_i^{v}), which leads to the following Cond-DC of Node i for CT: $I_i^{\mathrm{n}} + I_i^{\mathrm{m}} + I_i^{\mathrm{v}} + I_i^{\mathrm{g}} = O_i^{\mathrm{n}} + O_i^{\mathrm{v}}$, or, using (4),

$$I_i^{\mathrm{n}} + I_i^{\mathrm{v}} + I_i^{\mathrm{g}} = O_i^{\mathrm{n}} + O_i^{\mathrm{m}}. \tag{10}$$

(10) can be explained using the example in Fig. 1. Suppose that Nodes 2, 3, 4, and 5 each generate one packet, and Node 3 decides to do CT with Node 2. Node 2 has four incoming packets including its generated packet (I_2^{g}=1), two of which are non-CT receptions from Nodes 4 and 5 (I_2^{n}=2), and since one packet received from Node 3 (I_2^{m}=1) is already determined by Node 3 to do CT, Node 2 should use CT for that packet. For the remaining three packets, we assume that Node 2 decides to do CT for two packets and do non-CT for one packet (O_2^{n}=1). Therefore, total three packets are transmitted through VMISO links by Node 2 (O_2^{v}=3), and Node 2 sends two CT sharing packets (O_2^{m}=2) and has no VMISO reception (I_2^{v}=0). Using

I_2^{n}=2, I_2^{m}=1, I_2^{v}=0, I_2^{g}=1, O_2^{n}=1, O_2^{m}=2, and O_2^{v}=3, we can easily see that (10) holds for Node 2. Cond-DC of Node i for CT can be formed in terms of LP variables by substituting (6), (8), and (9) into (10).

Now, we formulate Cond-EC for CT. Note that the energy consumptions related to I_i^{n}, I_i^{m}, I_i^{v}, I_i^{g}, O_i^{n}, O_i^{m}, and O_i^{v} should all be considered for Cond-EC. The receiving energy consumption of SISO communication is e^{RX}, and it applies to I_i^{n} and I_i^{m}. The energy consumption of receiving CT packets depends on the number of cooperating nodes, N_c (=N+1). For example, when the orthogonal diversity channel is obtained by using different time slots, the VMISO receiver has to spend more energy for VMISO reception than SISO reception because it has to receive all N_c packets. Therefore, if we define the energy consumption of receiving a CT packet (a CT data unit in general) when n cooperating nodes are transmitting as e_n^{RX}, the energy consumption related to I_i^{v} is

$$\sum_{h:h\in A-D-i} \sum_{N=1}^{N_c^{\max}-1} \Big\{ \big(\sum_{j\in S_h} \sum_{k:j\in R_{h,i}^{N,k}} n_{h,i,j}^{N,k} \big) \cdot \frac{e_{N+1}^{\mathrm{RX}}}{N} \Big\}. \tag{11}$$

For the transmit energy consumption, we define $e_{i,v}^{N,k}$ as the required transmit energy for Node i to send a data unit to Node v (VMISO receiver) when all elements (nodes) of $R_{i,v}^{N,k}$ and Node i cooperate. Also, we denote the required transmit energy for Node i to multicast a data unit to all nodes that are elements of $R_{i,v}^{N,k}$ by $E_{i,v}^{N,k}$. O_i^{v} is related to CT packets that are either initiated by Node i (related to O_i^{m}) or initiated by neighbors of Node i that Node i needs to cooperate (related to I_i^{m}). Note that when CT is initiated by one of Node i's neighbors, j, Node i has to use $e_{j,v}^{N,k}$ because the energy cost for doing CT depends on how Node j selects the rest of cooperators. Hence, the energy consumption related to O_i^{v} is

$$\begin{aligned} &\sum_{N=1}^{N_c^{\max}-1} \Big\{ \sum_{v:v\in A-i} \sum_{j\in S_i} \sum_{k:j\in R_{i,v}^{N,k}} \Big(n_{i,v,j}^{N,k} \cdot e_{i,v}^{N,k} \Big) / N + \\ &\sum_{j:i\in S_j} \sum_{v:v\in A-j} \sum_{k:i\in R_{j,v}^{N,k}} \Big(n_{j,v,i}^{N,k} \cdot e_{j,v}^{N,k} \Big) \Big\}. \end{aligned} \tag{12}$$

e_{ij}^{TX} and $E_{i,v}^{N,k}$ are related to O_i^{n} and O_i^{m} respectively, and the energy consumption related to I_i^{g} can be expressed using the energy consumption for generating a data unit denoted by e_i^{gen} (which is mostly the energy consumption for sensing in the case of the sensor network and is ignored in [14] and [17]). Cond-EC for Node i states that the total energy consumption of Node i should be less than or equal to its initial energy, which leads to the following Cond-EC of Node i for CT:

$$\begin{aligned} &\sum_{j:i\in S_j} n_{j,i}^{\mathrm{nCT}} \cdot e^{\mathrm{RX}} + \sum_{j\in S_i} n_{i,j}^{\mathrm{nCT}} \cdot e_{ij}^{\mathrm{TX}} + (TQ_i) \cdot e_i^{\mathrm{gen}} + \\ &\sum_{N=1}^{N_c^{\max}-1} \Big\{ \sum_{j:i\in S_j} \sum_{v:v\in A-j} \sum_{k:i\in R_{j,v}^{N,k}} \Big(n_{j,v,i}^{N,k} \cdot (e_{j,v}^{N,k} + e^{\mathrm{RX}}) \Big) \\ &+ \sum_{v:v\in A-i} \sum_{j\in S_i} \sum_{k:j\in R_{i,v}^{N,k}} \Big(n_{i,v,j}^{N,k} \cdot (e_{i,v}^{N,k} + E_{i,v}^{N,k})/N \Big) + \\ &\sum_{h:h\in A-D-i} \Big(\big(\sum_{j\in S_h} \sum_{k:j\in R_{h,i}^{N,k}} n_{h,i,j}^{N,k} \big) \times e_{N+1}^{\mathrm{RX}}/N \Big) \Big\} \le E_i. \end{aligned} \tag{13}$$

In summary, the LP formulation for CT is

$$\begin{aligned} &\text{Maximize } T \\ &\text{s.t. } n_{i,j}^{\mathrm{nCT}} \ge 0, \quad \forall i \in A, \forall j \in S_i, && (14) \\ &\qquad (5) \ge 0, \quad \forall i, v \in A, i \ne v, R_{i,v}^{N} \ne \{\}, && (15) \\ &\qquad (13), \quad \forall i \in A - E, && (16) \\ &\qquad (10), \quad \forall i \in A - D. && (17) \end{aligned}$$

(16) and (17) become (2) and (3) respectively when LP variables related to CT ($n_{i,v,j}^{N,k}$'s) are ignored. Note that (5) reduces the number of required LP variables. Sink nodes require neither Cond-EC nor Cond-DC, and the number of packets that reach a sink node d can be calculated by I_d^{n}+I_d^{v}.

III. EVALUATION

In this section, we formulate and solve the LP equations derived in Section II for given network models, and we discuss the LP results.

A. Simulation Models and Parameters

We consider multi-hop networks having a sink node with no energy constraint. All nodes have the same maximum SISO transmission range of $d_{\mathrm{tx}}^{\max}$=20m, and a SISO link exists from Node i to Node j if Node j is within $d_{\mathrm{tx}}^{\max}$ from Node i. This makes SISO links deterministic. The LP solution can provide the optimal lifetime bound of cooperative routing, and, to see the usefulness of this bound more clearly, we also perform network simulations for REACT[2] in [11] and compare its lifetime performance with the optimal case. Since we compare the results of REACT with our LP results, we adopt some of the features of [11]. A VMISO link, as in [11], is formed between cooperating nodes and a sink, which makes v in $R_{i,v}^{N}$ one of the elements of D. In [11], the possibility of reaching a certain destination using CT (and the power control of CT as well) is determined through d_{req}=$(10^{G} \cdot \sum_{i=1}^{N_c} d_{i,v}^{-\alpha})^{-1/\alpha}$ where G is the diversity gain in dB, α is the path loss exponent, and $d_{i,v}$ is the distance between i-th cooperator and a VMISO receiver,

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[2] REACT first establishes a non-CT route (primary route) using an existing non-CT routing scheme. A node on the route triggers CT by comparing its residual energy with that of the next-hop node in its primary route. If the next-hop node has less energy, the node decides to do CT; it selects cooperators based on their residual energies and their distances to the sink node.

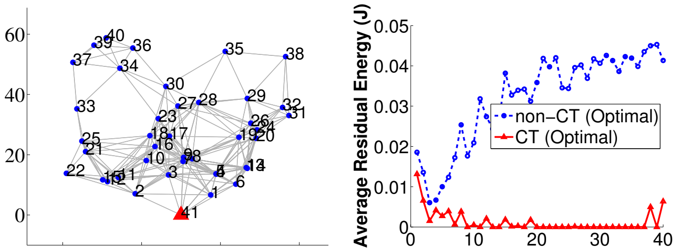
Node v. To be more specific, in order for cooperators to reach the destination, the following condition should hold:

$$d_{\text{req}} \leq d_{\text{tx}}^{\text{max}}. \tag{18}$$

We use d_{req} in [11] for creating the set $R_{i,v}^{N}$ for Node i. That is, for Node i and Node v, we first fix N and consider each combination of Node i's cooperators and calculate d_{req}, and if (18) holds, we add the combination to $R_{i,v}^{N}$. We do this procedure for all possible N's ($1 \leq N \leq N_c^{\text{max}} - 1$) and get the complete set of $R_{i,v}^{N}$. Based on $R_{i,v}^{N}$, the LP variables for CT are defined. We also use d_{req} to get $e_{i,v}^{N,k}$. That is, $e_{i,v}^{N,k}$ = 'circuit energy consumption' + 'radiated energy consumption required to reach d_{req}'. When calculating d_{req}, we use the path loss exponent of 2.9 and cooperative diversity gains in [18], obtained from assuming Rayleigh fading, log-normal shadowing with the shadowing standard deviation of 5dB, BPSK modulation, and a target bit error rate of 10^{-3}. We obtain the LP results by considering the *data packet* flows of the network, which makes the LP variables $n_{i,j}^{\text{nCT}}$ and $n_{i,v,j}^{N,k}$ the total number of "data packets" transmitted by Node i. We use the energy model in [13] and assume 128 bytes of data. We do not consider the energy consumption for generating data ($e_{i,\text{gen}}$=0), and $E_{i,v}^{N,k}$=max($e_{ij_1}^{\text{TX}}, e_{ij_2}^{\text{TX}}, ..., e_{ij_M}^{\text{TX}}$) where $j_1, j_2, ...j_M$ are the elements of $R_{i,v}^{N,k}$. As in [11], we measure the lifetime performance in terms of the number of packets that successfully reach sink nodes, which is widely used in the existing literature [15], [19], [11]. Note that using CT may require designing control packets to support CT, which is presented in [8]. In our evaluation, however, we ignore control packets for both CT and non-CT because there can be many possible ways to design them.

The orthogonal diversity channel is obtained by space-time-block-code (STBC) [20], and the maximum number of orthogonal channels is three (N_c^{max}=3). All nodes have the initial energy of 50mJ. Each node has an identical data generation rate, and we set Q_i=1. Note that the conditions and assumptions made so far are for "our" evaluation purpose. One can use different link definitions, physical-layer characteristics, energy-consumption models, traffic models, etc. to evaluate one's own particular situation using the LP formulation derived in Section II.

For REACT, we run network simulations to get the number of packets that reach the destination until the first node dies. Note that, in the case of LP, we form matrices and vectors, and the LP formulation can be solved using any LP solution method, whereas, in the network simulation, packets are sent from node to node following routing decisions, and we measure the lifetime in terms of the number of packets that the destination receives[3]. When simulating REACT, we consider two non-CT primary routing schemes: (i) Ad hoc On-demand Distance Vector (AODV) [21] and (ii) Capacity Maximization (CMAX) [15], which is an energy-aware routing scheme.



(a) One sample topology.

(b) Average residual energy result for 60m×60m network (20 sample trials).

Fig. 2. Evaluation of LP for 60m×60m networks.

We consider square-shaped networks with a single sink node located at the bottom center of the network. Four different sizes of network are considered: 50m×50m, 60m×60m, 70m×70m, and 80m×80m. There are 40 nodes in the network (excluding the sink node), and nodes are randomly deployed except for the sink node. One sample topology of 60m×60m networks is shown in Fig. 2a. For each size of network, 20 trials are performed (for both LP and the network simulations), and, in each trial, nodes are randomly relocated except for the sink node. The node ID of a node is assigned according to its proximity to the sink node (the node closer to the sink node gets lower node ID) in order to capture the energy hole problem more clearly.

B. Results

The LP solution can provide not only the lifetime but also the residual energy of each node after the network is dead, and the average residual energies (averaged over 20 trials) of nodes for the 60m×60m networks obtained from solving LP are shown in Fig. 2b. The energy hole problem of the optimal non-CT case is evident; the nodes close to the sink node (having low node IDs) tend to use up their energies while the nodes far away from the sink nodes have relatively large energy left. On the other hand, for the optimal cooperative routing case, the figure indicates that the energy consumption of nodes can be notably balanced.

The average lifetime performances (in terms of number of packets) of four different network sizes are shown in Fig. 3. Here, A, B, C, and D indicate optimal non-CT, optimal CT, REACT-AODV, and REACT-CMAX, respectively. Note that LP is used for A and B, and network simulation is used for C and D. The solid lines indicate the mean values and each dotted line is the outcome of one sample trial. When viewing the outcomes of individual trials, we observe that the *relative* performance of the different schemes are highly correlated, indicating the significant dependence of performance on topology, for any scheme. In other words, if Scheme A is better than Scheme B in one trial, A is very likely to be better than B in another trial. As can be seen from Fig. 3, the optimal lifetime performances

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[3] Comparing the optimal lifetimes obtained from LP with the lifetimes obtained from network simulations has been done for non-CT routing in [14], and, likewise, this is also possible for CT networks. Note that the optimal lifetime obtained from LP is an ideal upper bound, and therefore, the lifetime result of network simulations can never be higher than the optimal lifetime.

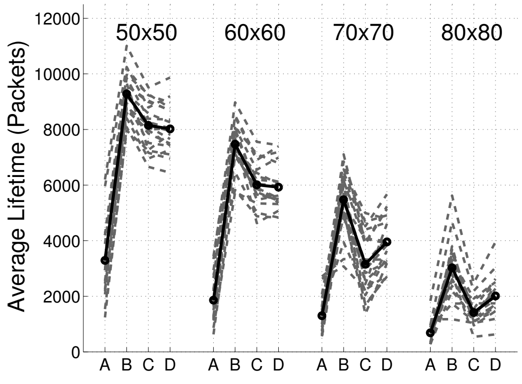

Fig. 3. Average lifetime performance of four different network sizes. A and B are the optimal cases of non-CT and CT, respectively, and C and D are the cases of REACT-AODV and REACT-CMAX, respectively.

of CT clearly outperform those of non-CT[4]. We can also see that REACT's performance is significantly higher than that of the optimal non-CT scheme, however, it is observed that the lifetime of REACT deviates more from the optimal value as the network size grows. The lifetime optimization of cooperative routing, which uses VMISO links, gets more complicated as the network size grows because the increased number of hops from the sink node means that each sensed data may go through more than one CT decision (including whether to do CT or non-CT and selecting cooperators); all decisions need to be optimally made in order to maximize the network lifetime, and, because of the suboptimal methods of REACT, the lifetime of REACT deviates from the optimal as the network gets larger. Note that the main motivation of formulating LP for cooperative routing is to determine whether a CT-based protocol behaves optimally or not, and, as we were able to find out that REACT does not always perform optimally, one can determine the optimality of one's own cooperative routing protocol using the LP formulation developed in Section II.

IV. CONCLUSION

In this paper, we studied the lifetime optimization of cooperative routing. We formulated the linear programming (LP) problem for cooperative routing that allows transmit power control and variable numbers of cooperators, which requires considerations of CT's unique characteristics and sophisticated variable definitions. By solving LP for cooperative routing, the optimal network lifetimes of cooperative routing were obtained and compared with those of the non-CT case to show the non-trivial lifetime improvement that CT can theoretically achieve. The optimal lifetime obtained from our LP formulation can be used as a reference value to determine the optimality of the existing cooperative routing methods, and we used it to determine the optimality of an online CT protocol.

Although we have not presented in this paper, the optimal value of the LP variables obtained by solving LP can provide how an optimal cooperative routing scheme should behave. An extensive investigation of what we can learn from the optimal routing behaviors has been performed and is intended for a future publication.

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[4] For 50m×50m, 60m×60m, 70m×70m, and 80m×80m, the average lifetime performances of A are 3292, 1861, 1301, and 686, respectively, and those of B are 9278, 7471, 5471, and 3024, respectively.