Single and Multiple Packet Reception in a Random Access OFDMA System

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Abstract— **This paper models a multi-channel stabilized S-Aloha system with capture. A capture model for single packet reception in a multi-carrier stabilized S-Aloha network is first proposed. This model can be effectively used for a single and a multiple antenna base station (BS). An analytical model for multi-packet reception is also proposed for multi-antenna BS. In this framework the BS can use a linear minimum mean square error (LMMSE) receiver. Two frame structures have been used to allow multiuser detection. It is shown that for a lightly loaded system there might not be enough advantage to use a multiple antenna system. It is also shown that in a finite user case the maximum attainable stabilized throughput reduces slightly with increasing number of users and very quickly reaches the infinite user case.**

Keywords— **OFDMA, stabilized S-Aloha, multichannel, capture effect, random access.**

I. INTRODUCTION

Random access channels (RACHs) such as slotted Aloha (S-Aloha) have been of interest because of their simplicity since they were first proposed in 1970s [1]. Many existing wireless systems use such channels not only for control signalling [2], [3], but also for data transfer [4], [6]. RACHs are used for bandwidth request and ranging in OFDMA-based WiMAX standard [3] and carries short messaging services (SMS) in WCDMA-based 3GPP standard [6]. In this paper, we develop a packet capture model for S-Aloha OFDMA for single and multiple packet reception by base stations with multiple antennas, and we use the model to evaluate throughput and delay of the system.

It is well known that S-Aloha systems are unstable when the arrival rates are above a certain threshold. Control mechanisms have been developed to stabilize both single [7] and multichannel [8]-[9] S-Aloha networks. The above mentioned schemes require only one-bit feedback per subchannel to the subscriber stations, informing them if the subchannels in the previous slot had a collision or not. The subscriber station update their retransmission probabilities based on this feedback. The control mechanism proposed in [7] and [8] stabilize the network as well as would have been possible with the perfect knowledge of how many nodes transmitted in each slot. Recent work by Bruvold, et al. in [10] uses this concept to propose contention policies that ensure a specified two-class-QoS as well as stability.

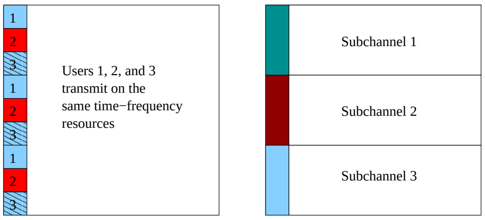
The classical S-Aloha systems are based on the assumption that a packet is successfully received only when there is no collision. However, it has been shown that the variation in the wireless channel conditions and advanced receiver processing enable successful packet reception even under collision [10]-[16]. This is called the capture effect.

The capture effect has also been used to model multi packet reception (MPR). MPR is achieved by using advanced signal processing. For example, [10] uses multi-user detection (MUD), assuming all the users transmit on common time and frequency resource, in a DS-CDMA framework. Another example is [14], which uses an adaptive array in a narrow band system. Orthogonal preambles are commonly used for MUD [3, 15]. The WiMAX standard uses orthogonal preambles in a mode called collaborative spatial multiplexing [3] wherein the interfering users transmit data on common sub-carriers but the pilots are on orthogonal sub-carriers. The authors in [15] consider a synchronous random access network where each colliding packet has a unique preamble that is orthogonal to the other preambles.

The purpose of this paper is to abstract an analytical model of an OFDMA system employing stabilized S-Aloha based RACH and MUD receiver processing. Only a simple abstraction of MUD is used: a signal is considered successfully received when the channels of all interfering signals can be estimated *and* the number of simultaneous transmissions is less the number of BS antennas. Such an abstraction gives the best possible performance using these techniques. The stabilized S-Aloha model is based on the work in [8]. A capture model for single packet reception in a multi-carrier stabilized S-Aloha network is first proposed. This model can be effectively used for both single and multiple antenna base stations. A multi-packet reception model is also proposed for multi-antenna base station. Two frame structures with orthogonal preambles, with designs similar to those in [3] and [15], have been used to allow multiuser detection. Finally, the performance of this system is analyzed in terms of its throughput and delay.

II. PERFORMANCE ANALYSIS OF STABILIZED MC-ALOHA WITH CAPTURE

The capture effect can be modeled in different ways depending upon the physical layer characteristics. Three physical layer variants are considered in this paper. The first type encompasses single packet reception under collision. The second and the third types model MPR. Single packet reception under collision can also be viewed as the capture model where only the packet with $Pr\{SINR > \gamma_{th}\}$ is received successfully. Such a system may have a single antenna or multiple antennas. It must, however, be noted that the above capture model implies that a packet can be successfully received in presence of one or more interferers. This effect is modeled by parameter M in this paper, which is the maximum number of collisions that allow at least one success. M can also be thought of as the number of antennas in a multiple antenna receiver. There is no explicit formulation of capture using SINR in this paper. In the case of MPR, a frame structure consisting of preamble and data is first defined. Two different preamble designs are considered to facilitate MPR. These form the basis of the other two capture models proposed in this paper. In the following sections $Pr\{D_k = d | T_k = t\}$ will be calculated for each of the proposed capture models. These capture models will then be used to compute network performance in terms of throughput and delay of stabilized multi-channel S-ALOHA network. Performance of the stabilized algorithm with capture is analyzed for both finite and infinite number of users.



(a) Preamble design − 1 (b) Preamble design − 2

Fig. 1. Frame structure for MUD

A. Single packet reception

1) Finite users: We need to compute d success given K users transmit in N channels, $P(d|K)$. Only one packet can be successfully received under collision. This occurs when the base station can estimate the channel of only the user whose SINR is maximum. In the general form we get the probability of success as

$$P(d|K) = \begin{pmatrix} N \\ d \end{pmatrix} \sum_{n=d}^{\min(K-d-1,Md)} \begin{pmatrix} K \\ n \end{pmatrix} \times \frac{E_s(d,n)E_f(N-d,K-n)}{N^K} \tag{1}$$

where $E_f(N,K)$ is the total number of ways in which none of the N channels are successful given a total of K users transmit. Similarly, $E_s(N,K)$ is the total number of ways in which all the N channels are successful given a total of K users transmit. Derivation of $E_f(N,K)$ and $E_s(N,K)$ is given in Appendix I.

2) Infinite users: In the infinite user case the new packet arrival is assumed to be Poisson distributed with mean arrival rate of λ. With the proposed capture model the throughput of an infinite user S-ALOHA network is

$$S = \sum_{n=1}^{M} np(n)p_s(n) \tag{2}$$

where $p(n)$ is poisson distribution with mean arrival rate λ and $p_s(n)$ is the success probability when n users transmit. For SISO/IC case $p_s(n) = 1/n$ Using (2), the value of λ for which the network is stable can be obtained.

B. Multi-packet reception

1) Frame Structure for multi-packet reception: A frame consists of preamble and data portions. The base station estimates the channel based on the preamble. These channel estimates are then used to compute the beam-forming weight for multi-packet reception (MPR). To estimate each user's channel correctly, the effect of interfering users' signal should be minimized. This is achieved by using orthogonal preambles for each user. Two preamble designs are used to achieve the orthogonality.

- The first design is similar to that used in the sounding zone of WiMAX standard [3]. The sub-carriers in each band are grouped into clusters consisting of non-overlapping sub-carriers. Figure 1a shows this frame structure. The sub-carriers in any cluster are evenly spaced, D ($\geq M$) sub-carriers apart, across the complete band. This enables the users to estimate the channel across the complete band by interpolation. Hence, the system can operate in a frequency selective channel. A user chooses one of the D clusters, with probability $1/D$, to transmit its preamble. All the users can use the same preamble sequence in this design. We assume that all the M packets will be successfully received if all the users in the same sub-channel occupy different set of sub-carriers in the preamble.
- The second preamble design is conceptually similar to that in [15]. All the users transmit their preamble on all the sub-carriers in their band. Figure 1b shows this frame structure. User orthogonality is attained in this case by choosing orthogonal preamble sequences. There can be multiple choices for these orthogonal sequences. One design is proposed in the following. Let s_k^l be the unity norm preamble symbol transmitted on the k^{th} sub-carrier by user l. An orthogonal sequence for user m is given as
$$s_k^m = s_k^l \exp(j2\pi k\tau_m/N)), \tag{3}$$
where, N is the total number of sub-carriers in the band and τ_m is an integer ≥ 0 that is different for each user.

In the case of MPR, described above, M is assumed to be governed by the number of antennas at the base station. Analytical model of success probabilities for MPR is provided in the following section.

2) Finite users: The analytical expression of $P(d|K)$ for preamble design 1 is difficult to compute and has not yet been obtained. Monte carlo simulations are performed to get the results for this design. However, $P(d|K)$ has been derived for preamble design 2 as given below. Total number of ways for d successful reception given K users transmit is given by

$$P(d|K) = \begin{pmatrix} K \\ d \end{pmatrix} \sum_{n_s=\lfloor \frac{d}{M} \rfloor + \mod(d,M)}^{\min(d,NM)} \begin{pmatrix} N \\ n_s \end{pmatrix} \times \frac{E_s(d,n_s)E_f(K-d,N-n_s)}{N^K}, \tag{4}$$

where $E_f(N,K)$ is defined in the same way as that for SPR and its expression is also the same given by (6) in Appendix I. $E_s(N,K)$, however, has a different expression and is derived in Appendix II. It is defined as the number of ways in which K users are successful and occupy n_s cells.

3) Performance analysis: The average throughput and delay expressions with capture are exactly the same as those without capture for the case of SPR. In the case of MPR the average throughput expression is slightly modified as follows

$$\bar{D} = \sum_{n=0}^{MN} nPr\{D_k = n\}. \tag{5}$$

Note that the difference between equations [8, Eq. 15] and (5) is in the upper limit of the summation, since with MPR a maximum of MN packets can be successfully received.

III. RESULTS

This section provides the throughput and delay results of finite user multi-channel S-ALOHA network with capture. Detailed results and discussion are presented for single packet reception scenario and MPR scenario is discussed only briefly. Detailed results and discussion for MPR will be presented in the full paper. Maximum stable throughput (MST) and behavior of average delay can be obtained easily using (2).

Fig. 2 shows the throughput per subchannel with and without capture for single packet reception as a function of p_g. As is well known, MST of classical S-Aloha is $1/e$. This is achieved when 20 users transmit in 4 subchannels. MST when capture is considered increases to $1.5/e$, this can be analytically obtained from (2) for infinite user case. One way to look at capture is assuming that the base station has M antennas and it can suppress up to $M-1$ interferers by using a MUD receiver. Hence, the result can also be interpreted as the performance gain seen by using multiple antennas. It is noted that when $p_g < 0.1$ there is no advantage of using a MUD receiver in terms of throughput. It is interesting to note that when there are only 10 users in the network, the MST

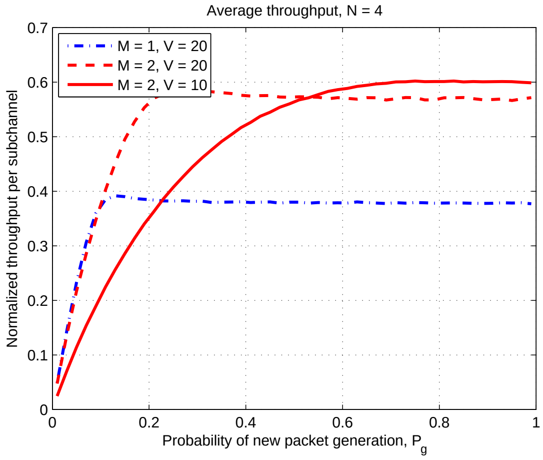


Fig. 2. Average throughput of multichannel S-ALOHA network with and without capture

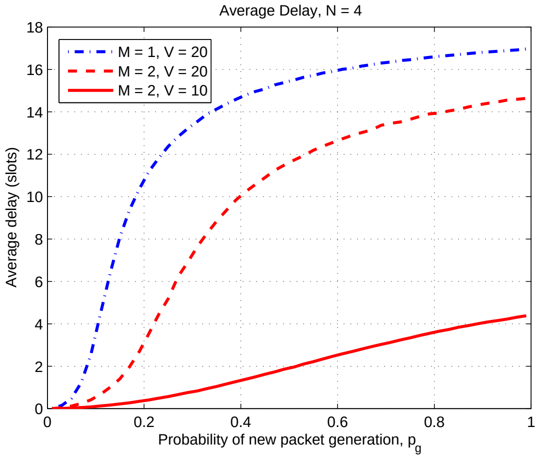


Fig. 3. Average delay of multichannel S-ALOHA network with and without capture

is slightly higher with capture. Fig. 3 gives average delay for the scenarios discussed above. We note that in the worst case, when operating at MST, the delay is reduced by up to 2 slots. The best delay benefit is obtained when $p_g = 0.2$. In this case, the average delay is reduced by about 6 slots because of capture.

Figs. 4 and 5 show stabilized throughput and average delay of a S-Aloha network with MPR using the two preamble designs. The legends scheme-1 and scheme-2 correspond to the first and the second preamble designs, respectively. In both the scenarios there are 20 users in the network. The complete band is divided into 4 subchannels and up to 2 users can be simultaneously received. For scheme-1, the decimation factor, $D = 2$; for scheme-2, the number of orthogonal preamble sequences, $N_t = 10$. Clearly, the scheme-2 will have a higher success probability because each sub-channel can have different set of 2 user scenarios. Hence, an improved MST compared to scheme-1. We observe that the MST in scheme-1 is $2/e$ where as that for scheme-2 is $1.5/e$. Note that MST remains almost the same with $p_g \leq 0.1$. Scheme-1 also has consistently lower average delay by about 1 slot. However, if may be noted that scheme-2 will not be able to extract any frequency diversity of the channel when coding is used where as scheme-1 will have this benefit. But this aspect has not been accounted for in the model presented here.

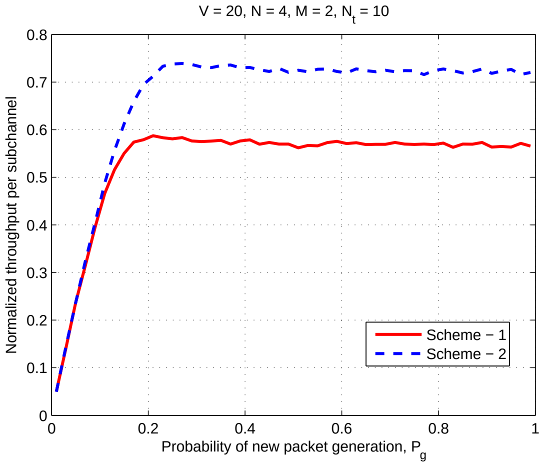


Fig. 4. Average throughput of multichannel S-Aloha with multiuser detection. Scheme - 1: Decimated preamble, Scheme - 2: Orthogonal training sequences

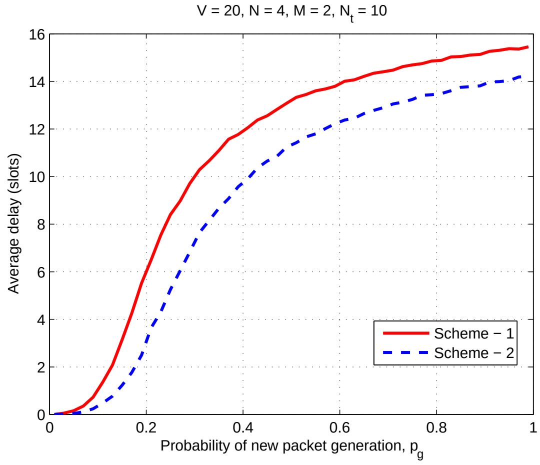


Fig. 5. Average delay of multichannel S-Aloha with multiuser detection

IV. CONCLUSION

We have modeled a multi-channel stabilized S-Aloha system with capture. Both single- and multi-packet reception is considered. For single-packet reception case, it is shown that for a lightly loaded system, i.e. when rate of new packet generation in the network is low, there might not be enough advantage to use a multiple antenna system. Also we note that in a finite user case the maximum attainable stabilized throughput reduces slightly with increasing number of users and very quickly reaches the infinite user case. The multi-packet reception framework is proposed for multi-antenna base station. Two preamble designs have been proposed to allow multiuser detection. The MST and average delay of networks employing the two types of preambles is compared.

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APPENDIX I

$E_f(N, K)$ can be easily obtained by observing that each of the N cells must have at least M users. Hence, rest of the $K - NM$ users can be placed arbitrarily in the N cells and none of the N cells would have any success. We obtain

$$\begin{aligned} E_f(N,K) &= \sum_{n_f=1}^{\lfloor \frac{K}{M+1} \rfloor} \left(\begin{array}{c} N \\ n_f \end{array} \right) \left(\begin{array}{c} K \\ n_f(M+1) \end{array} \right) \\ &\qquad \frac{(n_f(M+1))!}{(M+1)!^{n_f}} {n_f}^{K-n_f(M+1)} \end{aligned} \tag{6}$$

Next, the successful cells can be computed as

$$E_s(N,K) = \left(\begin{array}{c} K \\ N \end{array} \right) N!(N^K - E(N, K-N)) \tag{7}$$

where $E(N,K-N)$ is the total number of ways in which at least one channel has M or more users. This can be computed using the theorem on union of events as follows

$$E(N,K) = S_1 - S_2 \ldots + (-1)^{\nu+1} S_\nu \tag{8}$$

where S_ν is the total number of ways in which some ν cells have $> M$ users, it is given as

$$\begin{aligned} S_\nu = \left(\begin{array}{c} N \\ \nu \end{array} \right) \sum_{\mathbf{j}} \left(\begin{array}{c} K \\ \nu M + \mathbf{j}^T \mathbf{1} \end{array} \right) \times \\ \frac{\nu M + \mathbf{j}^T \mathbf{1}}{\prod_{k=1}^{\nu} (M + j_k)} (N-\nu)^{K-\nu M - \mathbf{j}^T \mathbf{1}} \end{aligned} \tag{9}$$

where $\nu = \lfloor \frac{K}{M+1} \rfloor$,

$$\begin{aligned} \sum_{\mathbf{j}} &= \sum_{j_1=1}^{K_1} \sum_{j_2}^{K_2} \sum_{j_\nu=1}^{K_\nu}, \quad \mathbf{j} = [j_1 \ j_2 \ \ldots \ j_\nu]^T, \\ \mathbf{1} &= \underbrace{[1 \ 1 \ldots \ 1]^T}_{\nu \text{ times}} \end{aligned}$$

and

$$\begin{aligned} K_1 &= K - \nu M - \nu + 1, \\ K_2 &= K - \nu M - j_1, \ \ldots, \\ K_\nu &= K - \nu M - j_1 \ldots - j_\nu \end{aligned}$$

The derivation of S_ν is as follows. We want to find the total number of events such that at least one cell has greater than M users. As mentioned earlier we use the property that

$$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2). \tag{10}$$

Let $A_{i_1}^{j}$ be number of events such that cell i_1 has $M + j$ users

$$A_{i_1}^{j} = \left(\begin{array}{c} K \\ M+j \end{array} \right) (N-1)^{K-M-j}. \tag{11}$$

Hence, the total number of ways in which cell i_1 has greater than M users

$$A_{i_1}^{j} = \sum_{j=1}^{K-M} \left(\begin{array}{c} K \\ M+j \end{array} \right) (N-1)^{K-M-j}. \tag{12}$$

The total number of ways in which some 1 cell has greater than M users is

$$S_1 = \left(\begin{array}{c} K \\ 1 \end{array} \right) \sum_{j=1}^{K-M} \left(\begin{array}{c} K \\ M+j \end{array} \right) (N-1)^{K-M-j}. \tag{13}$$

Next, we need to find the number of ways in which at least two cells have greater than M users. Let $A_{i_1,i_2}^{j_1,j_2} = A_{\mathbf{i}}^{\mathbf{j}}$ be the total number of events such that cell i_1 contains $M + j_1$ users and cell i_2 contains $M + j_2$ users

$$\begin{aligned} A_{\mathbf{i}}^{\mathbf{j}} = \left(\begin{array}{c} K \\ 2M + j_1 + j_2 \end{array} \right) \frac{2M + j_1 + j_2}{(M+j_1)!(M+j_2)!} \times \\ (N-2)^{K-2M-j_1-j_2}. \end{aligned} \tag{14}$$

Hence, the total number of ways in which some two cells have greater than M users is

$$\begin{aligned} S_2 = \left(\begin{array}{c} N \\ 2 \end{array} \right) \sum_{j_1, j_2} \left(\begin{array}{c} K \\ 2M + j_1 + j_2 \end{array} \right) \times \\ \frac{2M + j_1 + j_2}{(M+j_1)!(M+j_2)!} (N-2)^{K-2M-j_1-j_2}. \end{aligned} \tag{15}$$

Similarly, the result for ν cells can be obtained as given in (9).

APPENDIX II

The structure of $E_s(N,K)$ when using preamble design 2 is similar to that for SPR. The difference being that now the success also depends upon the preamble sequence overlap and LMMSE collision model. The expression is as given below

$$\begin{aligned} E_s^{MUD}(N,K) &= Pr(K\text{users choose orthogonal} \\ &\qquad \text{preambles}) E_s(N,K), \end{aligned} \tag{16}$$

where $E_s(N,K)$ is given by (7) and

$$\begin{aligned} & Pr(K\text{users choose orthogonal preambles}) = \\ & \qquad \frac{\sum_\alpha \sum_{j=1}^{n_s} \left(\begin{array}{c} N_t \\ \alpha_j \end{array} \right)}{\sum_\alpha \sum_{j=1}^{n_s} {N_t}^{\alpha_j}}, \end{aligned} \tag{17}$$

under the conditions that

$$\alpha_j \leq M, \forall j \ \text{ and } \sum_{j=1}^{n_s} \alpha_j = K. \tag{18}$$

N_t is the total number of orthogonal preamble sequences that the users can choose from, α_j is the number of user in cell j, α is the set of all α_j's that satisfy the constraints in (18).

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