A Simple Energy Model for the Harvesting and Leakage in a Supercapacitor

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Abstract—**The modeling of energy storage devices such as supercapacitors for wireless sensor networks is important for assessing performance of harvesting-aware routing protocols that could be key to a green energy future. In this paper, we present a circuit-based model (CBM), which is a good fit to the empirical data available in the existing literature. We compare it with a linear energy model (LEM) that is often used in literature, and show that the choice of models implies a significant difference in the throughput per node.**

I. Introduction

The network lifetime of energy-harvesting embedded systems that store their energies on rechargeable batteries (RBs) will be defined by the cycle life of the RB, because they can be discharged and recharged only 100s to 1000s of times before they fail to hold a charge [1]. Another type of storage medium, the supercapcitor (SC), offers an attractive alternative. SCs do not hold as much energy as the RBs, but can be discharged and recharged on the order of a million times [2]. This paper explores modeling an SC suitably for network simulation and analysis of such future systems.

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Network performance evaluations when the SC is one of the (or the only) storage media on a sensor node have all assumed that the harvested energy and the energy cost of relaying a packet are each proportional to the time in the particular state of harvesting or relaying, respectively [3], [4]. We refer to this as the linear energy model (LEM). However, some recent empirical characterizations of SC charging circuits do not quite agree with the LEM and indicate that the charging circuit is significantly more complicated than just a direct connection between an energy harvester and a SC [5]. When connected directly to an energy harvester, the SC pulls the voltage across the harvester to a value that is usually very far from the optimal operating point, thereby increasing the charging time of the SC due to the impedance mismatch [2]. In this paper, we propose a circuit-based model (CBM) for the SC, such that in each of the modes, the voltage across the SC is computed assuming that the SC is in a simple resistor-capacitor circuit with a constant current source. The current sources and resistances are different for each mode. We show that this model is a good fit to the empirical data of [5]–[6]. Furthermore, we compare the two models and

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show that the choice of models implies a significant difference in number of packets relayed by a single node.

II. Circuit-Based Model

A. System Models and Assumptions

The SC is assumed to leak and harvest energy continually. Packets are assumed to be equal-length and transmitted periodically at a rate of $\frac{1}{T_l+T_h}$. If the routing algorithm selects the node, the node relays the packet. The period of activity comprising route selection and relaying of the packet is assumed to be T_l seconds long. We define $t_k^+ = t_k + T_l$, where t_k defines the beginning of the k-th time slot. The remainder of the time slot until the arrival of the next packet is T_h seconds long. It follows that $t_{k+1} = t_k^+ + T_h$.

Our circuit-based model (CBM) is composed of the SC with Farad value C, and voltage $v(t)$, and the rest of the circuit is represented by its Norton Equivalent, which includes a constant current source, with current I and a parallel resistance, R. This circuit is modeled by a first order differential equation, and the solution or the voltage across the SC at any time t is given by: $v(t) = v(0^+)e^{\frac{-t}{\tau}} + IR(1 - e^{\frac{-t}{\tau}})$, where $\tau = RC$. We use plots from [1] and [5] to define I and R for each of the two modes: Charging and Discharging.

B. Charging Model for the Supercapacitor

In this mode, I and R in the above equation are replaced by source current, I_h, and the equivalent charging resistance, R_h. The energy harvester setup that is used to extract R_h is similar to the pulse frequency modulated (PFM) charging circuit used in [5]. By exploiting a buck configuration controlled to track the maximum power point of the photovoltaic cell, the energy harvester setup behaves like a voltage regulator in which the current delivered is dependent on the current provided by the solar cell. In the experiment, a single 4.7 F SC with a maximum operating voltage of 5.5 V is used; voltages are bounded between 0 V and 5 V, implying that $v(0^+) = 0$. The values of the I_h and R_h for this RC charging circuit are obtained by fitting the second term of the above equation to the 'measured points' from our experiment. This yields $R_h = 27.56\ \Omega$, $I_h = 0.18A$ and $\tau_h = 140s$. While the solution of the differential equation during the time $T_h : t_k^+ \rightarrow t_{k+1}$, when the SC voltage is given by $v(t_{k+1}) = v(t_k^+)\alpha_h + v_h(1 - \alpha_h)$, where $v_h = I_h R_h$, $\alpha_h = e^{\frac{-T_h}{\tau_h}}$, and $\tau_h = R_h C$.

C. Discharging Model for the Supercapacitor

To extract the SC leakage resistance, R_l, a maximum current consumption of a 100 mA is considered for a single 4.7 F SC. Using a plot of leakage power, P_{Leakage}, vs residual energy, E_{Residual}, our approach is to approximate the curve in this plot with a straight line, which we will show below is equivalent to modeling leakage power as the power dissipated through a resistor in parallel with an ideal capacitor. The leakage power is the power dissipated in R_l. We may express E_{Residual} and P_{Leakage} as $E_{\text{Residual}} = \frac{Cv(t)^2}{2}$ and $P_{\text{Leakage}} = \frac{v(t)^2}{R_l}$. Combining these expressions gives the linear equation $E_{\text{Residual}} = \frac{R_l C P_{\text{Leakage}}}{2}$. To decide where on the plot to fit the line, the minimum and maximum operating voltages of the SCs are considered, which corresponds to the minimum and maximum energies of SCs 2.35 J and 7.61 J, respectively. Setting the slope equal to $\frac{R_l C}{2}$ yields $R_l = 4478.1\ \Omega$. Because the energy harvesting and radio circuits are independent, the solution of the differential equation during the time $T_l : t_k \rightarrow t_k^+$ is given by: $v(t_k^+) = v(t_k)\alpha_l + v_l(1 - \alpha_l)$, where $v_l = -|I_l - I_h|(R_l \parallel R_h)$, $\alpha_l = e^{\frac{-T_l}{\tau_l}}$, and $\tau_l = (R_l \parallel R_h)C$. It is remarked that the values of R_l and R_h extracted using our experiments on a 4.7 F SC are very close (second decimal place) to what would be extracted from [1] and [5], respectively, using similar fitting techniques. We used supercapacitor from Cooper Bussman. Usually SCs have a limited maximum voltage (2.5 V or 5.5 V as in our case). The solar cell used was a 178×166 mm Polycrystalline Photovoltaic module with 7.6V as nominal voltage and 280 mA as nominal current. The maximum power point tracking (MPPT) converter featured a step down, and provided a suitable output voltage to charge the SC. Of course in the experiments, the light intensity was much less. The intensity was regulated during the experiments to ensure constant currents.

D. *Energy for the Circuit-Based Model*

The joint-voltage update equation for the time period $t_k \rightarrow t_{k+1}$ can be written as $v(t_{k+1}) = \beta_{hl} v(t_k) + \xi_{hl}$, where $\beta_{hl} = \alpha_h \alpha_l \approx 1$ and $\xi_{hl} \approx v_h(1-\alpha_h)$. Noting $E(t_{k+1}) = \frac{1}{2}C[v(t_{k+1})]^2$ and squaring both sides of the joint-voltage update equation, we write the energy update equation for $t_k \rightarrow t_{k+1}$ as $E(t_{k+1}) = \beta_{hl}^2 E(t_k) + C\xi_{hl}\alpha_h v(t_k) + \frac{1}{2}C\xi_{hl}^2$. It can be observed that in the above equation, the 2-nd term depends on the voltage. This means that the energy is a *nonlinear* update equation.

III. Linear Energy Model

In this section, the energy harvesting and leakage in a SC is modeled using what we refer to as an linear energy model (LEM). Following the same notations from the previous sections, for the time duration $t_k \rightarrow t_{k+1}$, the energy update equation is given by $E(t_{k+1}) = \alpha_l^2 E(t_k) - E_{\text{load}} + E_{\text{harvested}}$, where E_{load} is the energy consumed in routing a packet and $\alpha_l = e^{\frac{-T_l}{\tau_l}} = 1 - (8.9e^{-6}) \approx 1$ for 50% duty cycle, $\alpha_l^2 E(t_k) \approx E(t_k)$, and $E_{\text{harvested}} = \gamma(T_l + T_h)$ is the energy harvested with a rate of harvesting γ. One of the main differences between the CBM and LEM is that E_{load} is "independent" of the voltage (or energy) across the SC, and is fixed. So, our choice of E_{load} will determine the time range when the two energy models will have the same slope of residual energy versus time. In this paper, we choose E_{load} such that the slopes are equal at the start, when the SC is fully charged. From the energy update equation, it can be observed that the LEM is "decoupled" in the sense that the energy harvested and the energy consumed in routing a packet (the load energy) are simply added in the energy update equations. This approach is consistent with the "linear energy versus time" leakage models that have been assumed in the existing literature, exemplified by [8] and its references.

IV. Comparing the Two Energy Models

One of the goals of this whole analysis has been to model the charging-discharging in a SC accurately, and to determine the maximum number of packets sustained by a SC before its voltage drops, resulting in the SC energy falling below a threshold

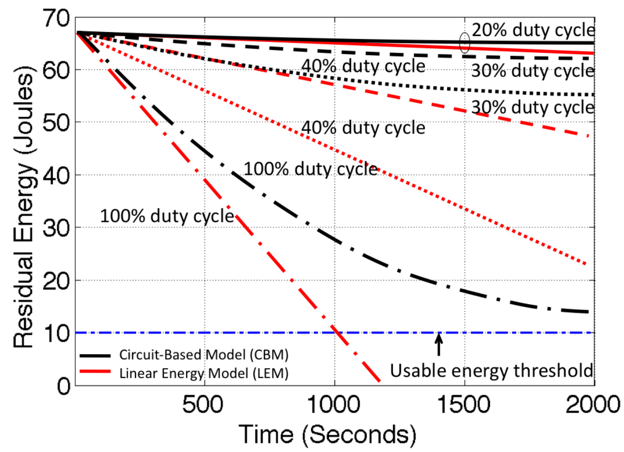

Fig. 1. The residual energy versus time for the packet rate = 1.

(i.e., minimum energy to route a packet), which would render the SC temporarily unusable. The duty cycle is defined as the ratio $\frac{T_l}{T_h+T_l}$. Both CBM and LEM model the leakage of the SC the same way, through the coefficient on the term $E(t_k)$ term, which makes leakage a function of the voltage drop across the SC. One difference between the LEM and CBM models is that in LEM, the harvesting and loading effects are respectively modeled by two additive constants in the update equation, while in CBM, there is only one constant term, which strictly depends on both harvesting and loading factors (v_h, v_l, α_h, and α_l). However, the most important difference is that CBM has a voltage-dependent term strictly depends on all of the same four factors as the constant term; this implies that also the voltage term cannot be said to be just the "harvesting" term or just the "loading" term. Simulations of residual energy also show significant differences between the two models. Fig. 1 is a plot of the residual energy on the SC for a single node versus time using the two energy models described above for different duty cycles, and the packet rate is chosen to be 1. In Fig. 1, duty cycles of 20%, 30%, 40% (adaptive scenarios for applications as in [6]), and 100%. Firstly, it can be seen that at any time, the residual energy on the SC increases as the duty cycle decreases. It is noted that high duty cycles correspond to high values of T_l, implying

$$m=\log\left\{\frac{\mathcal{T}'\big(1-\alpha_1\alpha_2^{(p-1)}\big)+E_{\text{load}}-\gamma_1-\alpha_2^{-(p-1)}\alpha_1\alpha_2^{(p-1)}\big(\sum_{i=0}^{p-2}\alpha_2^i\big)\gamma_2}{\alpha_2^{-(p-1)}E_{\max}\big(1-\alpha_1\alpha_2^{(p-1)}\big)+E_{\text{load}}-\gamma_1-\alpha_2^{-(p-1)}\big(\sum_{i=0}^{p-2}\alpha_2^i\big)\gamma_2}\right\}\left\{\log[\alpha_1\alpha_2^{(p-1)}]\right\}^{-1}. \quad (1)$$

more energy consumption due to longer packets. The red lines (or the linear-like curves [1]) denote the variation in the residual energy with time using the LEM, and the black curves denote the same using the CBM. The blue (or horizontal) dash-dot line indicates the "unusable energy hysteresis," or usable energy threshold, which renders the SC useless if the voltage drops below this value [1]. With increasing time, the residual energy computed using the linear energy model is always less than the energy computed using the more accurate CBM. As stated, this difference is because in the latter model, the load and the harvested energies are modeled as a function of the voltage drop across the SC at any time. Moreover, this difference becomes stark as the "harvesting only" period decreases.

V. Maximum Number of Packets Routed

In this section, we determine the number of packets that can be relayed by a node, when it is one of the p nodes in a cluster. An expression for the maximum number of bits that can be routed by a node before it's rendered unusable and needs recharging, i.e., when the voltage across the SC-parallel combination drops below a certain threshold $\mathcal{T}$ can be derived with the assumption that a node routes a packet once every p time slots.

A. Circuit-Based Model

Without loss of generality, we assume that the node transmits a packet during the first time slot, or $v_1 = I_h R_h = \sqrt{\frac{2E_{\max}}{C}}$. So, $v_{p+1} = v_1\hat{\beta} + \Delta V_p$, where $\hat{\beta} = \alpha_l \cdot \hat{\alpha}_h$, $\Delta V = v_h(1-\hat{\alpha}_h) + v_l(1-\alpha_l)\hat{\alpha}_h$, and $\hat{\alpha}_h = \exp\left\{\frac{-[T_h+(p-1)(T_l+T_h)]}{\tau_h}\right\}$. After the node has routed $n-1$ packets, i.e, at $k = (n-1)p+1$, we have $v_k = v_1\hat{\beta}^n + \left(\sum_{i=0}^{n-1}\hat{\beta}^i\right)\Delta V_p$. Let us also assume that after routing the n-th packet, the voltage across the SC-parallel combination drops below $\mathcal{T}$. Setting $v_k = \mathcal{T}$, we have $\mathcal{T} v_1\hat{\beta}^n + \underbrace{\left(\sum_{i=0}^{n-1}\hat{\beta}^i\right)}_{=\frac{1-\hat{\beta}^n}{1-\hat{\beta}}}\Delta V_p$, yielding $n = \log\left\{\frac{\mathcal{T}(1-\hat{\beta})-\Delta V_p}{v_1(1-\hat{\beta})-\Delta V_p}\right\}(\log\hat{\beta})^{-1}$, the maximum number of packets routed by the SC, and $(n-1)p+1$ is the maximum number of time slots for which the SC was 'useful.'

[1]The curves are not strictly linear because the α^2 coefficient is not exactly equal to one.

B. Linear Energy Model

Next, the maximum number of bits that can be routed by a single relay using a SC using the LEM is derived. $E_1 = \alpha_1 E_{\max} - E_{\text{load}} + \gamma_1$, where $\alpha_1 = \alpha_l^2$, and $\gamma_1 = \gamma T_h$, where γ is the energy harvested per unit time. If there are p nodes in the network, then every node routes a packet every $\frac{1}{p}$-th of the time. So, $E_2 = \alpha_2 E_1 + \gamma_2 = \alpha_1\alpha_2 E_{\max} - \alpha_2 E_{\text{load}} + \alpha_2\gamma_1 + \gamma_2$, where $\hat{\alpha}_l = e^{\frac{-(T_l+T_h)}{\tau_l}}$, $\alpha_2 = \hat{\alpha}_l^2$, and $\gamma_2 = \gamma(T_h + T_l)$. Now the first node routes a packet after p time slots.

$$\begin{aligned}E_{p+1} &= \underbrace{\alpha_1^2\alpha_2^{p-1}}_{=:A}E_{\max} - \underbrace{\left(\sum_{i=0}^{1}\alpha_1^i\alpha_2^{i(p-1)}\right)}_{=:B}E_{\text{load}} \\ &\quad + \underbrace{\left(\sum_{i=0}^{1}\alpha_1^i\alpha_2^{i(p-1)}\right)\gamma_1 + \alpha_1\left(\sum_{i=0}^{p-2}\alpha_2^i\right)\gamma_2}_{=:E_{\text{harvested}}}, \\ &= AE_{\max} - BE_{\text{load}} + E_{\text{harvested}}.\end{aligned}$$

From above, it can be seen that the energy harvested and the energy consumed in routing a packet (the load energy) are decoupled from the energy lost due to the SC in the energy update equations. For $E_{k'}$ at any $k' = (m-1)p + a,\ a \in [0, p-1]$:

$$E_{k'} = \Bigg[\alpha_1^m \alpha_2^{(m-1)(p-1)} E_{\max} - \left(\sum_{i=0}^{m-1} \alpha_1^i \alpha_2^{i(p-1)}\right) E_{\text{load}} + \left(\sum_{i=0}^{m-1} \alpha_1^i \alpha_2^{i(p-1)}\right)\gamma_1 + \left(\sum_{i=1}^{m} \alpha_1^i \alpha_2^{(i-1)(p-1)}\right) \left(\sum_{i=0}^{p-2} \alpha_2^i\right)\gamma_2\Bigg] \alpha_2^{(a-1)} + \sum_{i=0}^{a-2} \alpha_2^i \gamma_2.$$

Assuming that the residual energy stored in the SC-parallel combination drops below $\mathcal{T}'$ after routing the m-th packet, i.e., $a = 1$, setting $E_{k'} = \mathcal{T}'$, and simplifying yields (1).

Fig. 2 is a plot of the number of packets routed per node versus duty cycle, $\frac{T_l}{T_h+T_l}$ for a single relay node network. The solid and dash-dot lines correspond to the throughputs using the LEM and the CBM, respectively. Using both models, the number of packets routed per node decreased with increase in the duty cycles. While this decrease is exponential for the CBM, it is linear for the LEM. It can be observed that the choice of models implies a significant difference in number of packets relayed by a single node. Even when there is continual packet transmission, i.e., $\frac{T_l}{T_h+T_l} = 1$, there is a difference of more than 1000 packets routed using the two recursive energy models.

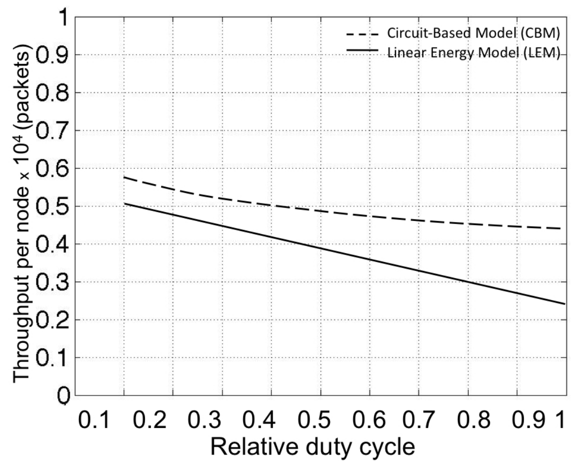

Fig. 2. The number of packets routed per node per second versus relative duty cycle, $\frac{T_l}{T_h+T_l}$ for $p = 1$ using CBM and LEM.

VI. Conclusions

A novel, simple analytical energy model for a SC that is consistent with the available empirical data is presented in this paper. Compared to the more traditional energy models, the harvested and load energy terms in the proposed model are dependent on the voltage across the SC, and this significantly impacts the network analysis (e.g., throughput per node). Extensions to this model such as accounting for a variable current load and module improvements for enhancing accuracy, and modeling energy harvesters using different environmental sources are some directions for future work.

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