Modeling and Analysis of CFO Pre-Synchronization for STBC-OFDM-based Multi-hop Cooperative Transmissions in WSNs

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Abstract – **In this paper, we model and analyze the carrier frequency offset (CFO) pre-synchronization for STBC-OFDM-based multi-hop cooperative transmissions in wireless sensor networks (WSNs). In our scenario, the nodes those can decode the message from the previous cluster correctly are selected as relays, and these relays will do STBC-OFDM encoding and CFO pre-synchronization autonomously before transmission. The CFO estimation at each relay is composed of coarse estimation based on preamble and residual CFOs estimation based on pilot symbols. We model the per-synchronization error for each relay, and apply a first-order dynamic system method is to get the closed forms of the variance, covariance and expected sample variance of the pre-synchronization errors of each hop. The convergent property of the expected sample variance indicates the feasibility of multi-hop transmissions. Numerical simulations validate our theoretical results of the error model and explore the effect of the number of relays to the synchronization performance.**

I. INTRODUCTION

In wireless sensor networks (WSNs), multi-hop cooperative transmission with distributed space-time coding is a good solution to increase reliability or extend the transmission range [1-3]. In cooperative transmission without central control, synchronization of multiple timing errors and carrier frequency offsets (CFOs) are the most challenging problems.

Orthogonal frequency division multiplexing (OFDM) is a popular modulation technique for wireless communications [4][5]. The cyclic prefix (CP) enables OFDM systems to tolerate signal delay differences caused by the multipath channel. So, combining OFDM with cooperative transmission can make the system robust to timing errors [6]. However, OFDM systems are sensitive to CFOs [4][5].

In some WSNs applications, the nodes are very chip, so that the DSP capability may not be very powerful and the oscillators are not very stable. The latter factor will cause large CFOs between nodes. So, for OFDM-based cooperative transmissions in WSNs, how to handle multiple large CFOs with a low complex approach is an important issue. This is the motivation of our work in this paper.

The existing frequency synchronization schemes for OFDM-based cooperative transmission can be classified into post-synchronization (post-synch), inter-channel interference (ICI) self-cancellation, and pre-synchronization (pre-synch). In the post-synch schemes [7-9], all the CFOs are estimated and compensated at the receiver, so the complexity is very high. The ICI self-cancellation scheme proposed by [10] is simple and effective, but its bandwidth efficiency is low, and it may not be applicable if the number of relays is larger than two. A pre-synch method is proposed by [3], in which the relay estimates the carrier frequency of the source in the "listening phase" and cooperates with the source during the "cooperation phase". Although this scheme is simple and effective, it is not suitable for multi-hop scenarios, where the source cannot reach the destination and no node acts as a leader.

Comparing these schemes, we think pre-synch is the best choice for WSNs because of its autonomy and low complexity. In [11], we proposed and analyzed a general pre-synch scheme for both timing errors and CFOs in cascaded distributed MIMO communications, assuming each relay transmits a preamble in an orthogonal channel, which is actually not practical in many applications. In this paper, we make the CFO pre-synch practical in the STBC-OFDM cluster-based multi-hop cooperative transmissions. In this structure, after receiving the message correctly, each relay in one cluster will choose different column of the STBC encoded matrix and do CFO pre-synch before transmitting the column. We do not address how to assign columns to relays in this paper. Different from [11], in this paper, the relay just does one coarse CFO estimation and correction based on the received preambles, so a residual CFO is left in each STBC branch. Based on the STBC structure, the received pilot subcarriers can be decoupled to multiple streams [3], so that the residual CFO in each branch can be estimated based on the corresponding pilot stream. Then, one final residual CFO is determined by a power-weighted combination of the estimated residual CFOs in all STBC branches. The summation of the final residual CFO estimation and the coarse CFO estimation is used in the CFO pre-synchronization. This paper models the pre-synch error for each relay in each cluster, and analyzes the pre-synch performance for each hop. Because of the autonomous operation of nodes in successive hops, we are concerned with *how the CFO pre-synch errors propagate from hop to hop*. We show that the relative CFO errors are statistically stable as a function of hop count. This conclusion implies that *the cooperative transmission with our simple pre-synch method might be practical for a sequence of clusters such as in a strip network or along a pre-designated route in a multi-hop network.* We assume the timing errors are within the tolerable range of the OFDM structure, and there are no Doppler shifts.

This paper is organized as follows. Section II gives the system model. Modeling and analysis of the CFO pre-synch errors are given in Section III. Numerical results are presented in Section IV. Section V discusses the possible future work, and the paper is concluded in Section VI.

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II. System Model

A. Network Structure

We consider the multi-hop cluster-based scenarios in Fig. 1. We assume the channel for each link is Rayleigh faded and independent of those for all other links. The carrier frequency of the source is f_s, and the carrier frequency of the k-th relay in the j-th cluster is $f_k^{(j)}$. In this paper, we assume the R relays are co-located in each cluster.

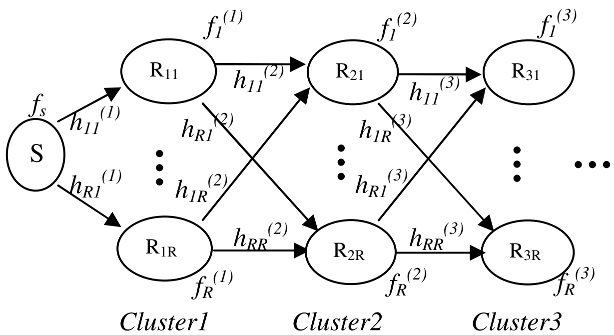

Fig. 1. Multi-hop Distributed MIMO Transmission Network

B. Transceiver Structure

Fig. 2 shows the block diagram of a relay transceiver. Our focus is the middle part in the frame. Choosing one column of the STBC encoded matrix is the simplest form of the distributed space-time coding scheme [12]. Because the pilot symbols are also STBC encoded at the transmitter, the "pilots decoupling" at receiver side is actually the STBC decoding for pilots [3]. Repeated training blocks ($s(n)$, n = 0, 1, …, ML-1, $E[|s(n)|^2] = \rho$) are used as the preamble for the coarse CFO estimation. We assume the CFO estimation for integer times of subcarrier spacing is perfect [13], so the pre-synch performance is only determined by the fractional CFO estimation. The length of each training block is L, and the number of training blocks is M. P pilot symbols ($s_p(n)$, n= 0,1,…, P-1) are equidistantly inserted in each OFDM symbol before STBC encoding for phase tracking and residual CFOs estimation. The FFT length is N, the CP length is N_g, and there are Q OFDM symbols in one packet.

III. Analysis of CFO Pre-Synchronization

A. Basic CFO Estimation Method

Because the CFO estimation algorithm is the same for each relay, we ignore the relay index k in this section. Suppose the received signal at a relay is

$$r(n) = hs(n)e^{j2\pi\Delta f n T_s} + w(n)\,, \tag{1}$$

where Δf is the CFO, h is the Rayleigh fading coefficient, and $w(n)$ is complex white noise with distribution $N_c(0, \sigma^2)$.
The basic estimator we choose is [4,14]

$$\Delta\hat{f} = \frac{angle(u)}{2\pi L T_s} \text{ or } \hat{v} = \frac{\Delta\hat{f}}{f_{samp}} = \frac{angle(u)}{2\pi L}\,, \tag{2}$$

where $u = \sum_{n=0}^{(M-1)L-1} r^*(n)r(n+L)$ and $\hat{v}$ is the estimated CFO normalized by the sampling frequency $f_{samp} = 1/T_s$. The acquisition range of estimator (2) is (-1/2L, 1/2L).

We choose this algorithm because: firstly, it's suitable for both coarse CFO estimation based on preamble and residual CFO estimation based on pilot vectors; secondly, diversity effect can be easily shown when orthogonal preambles are transmitted by different relays (see Section III.C.1).

B. Pre-Synch Errors for Relays in the First Cluster

B.1 Coarse CFO Estimation

Because the first hop is just simple point to point communication, the coarse CFO estimation is the same as (2). Here, we extend the analysis in [14], where the number of training blocks is two (M = 2), to the case when M > 2.

Following the similar procedure in [14], for each relay in the first cluster, the estimation error can be expressed as

$$\hat{v} - v \approx \frac{1}{2\pi L}\left[\frac{\sum_{n=0}^{(M-1)L-1} \mathrm{Im}\left[r^*(n)r(n+L)e^{-j2\pi vL}\right]}{\sum_{n=0}^{(M-1)L-1} \mathrm{Re}\left[r^*(n)r(n+L)e^{-j2\pi vL}\right]}\right]. \tag{3}$$

We define the received SNR as $SNR = |h|^2\rho / \sigma^2$. For normal receive SNR, we can have

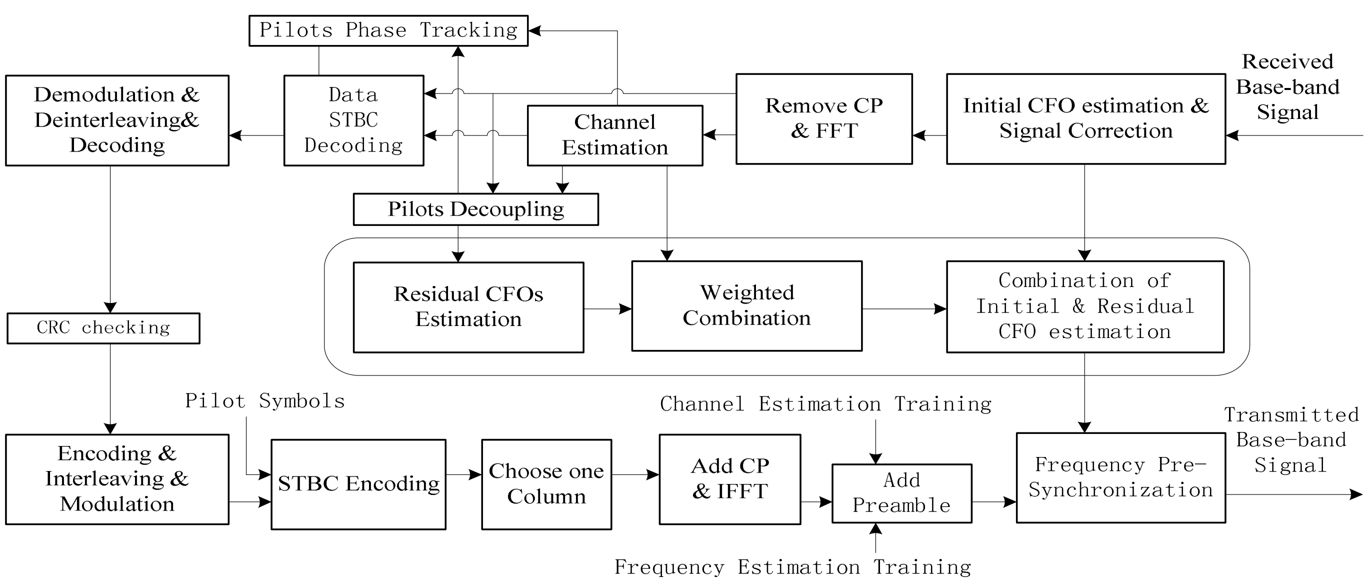

Fig. 2. Block diagram of the relay transceiver

$$\sum_{n=0}^{(M-1)L-1} \mathrm{Im}[r^*(n)r(n+L)e^{-j2\pi vL}] \approx \sum_{n=0}^{(M-1)L-1} \mathrm{Im}[h^* s^*(n)e^{-j2\pi v(n+L)}w(n+L) + hs(n)e^{j2\pi vn}w^*(n)] \quad (4)$$

and

$$\sum_{n=0}^{(M-1)L-1} \mathrm{Re}\left[r(n)r(n+L)e^{-j2\pi vL}\right] \approx \sum_{n=0}^{(M-1)L-1} |h|^2 |s(n)|^2 . \quad (5)$$

If define $\alpha(n) = hs(n)e^{j2\pi vn}w^*(n)$, we get

$$\sum_{n=0}^{(M-1)L-1} \mathrm{Im}[r^*(n)r(n+L)e^{-j2\pi vL}] = \mathrm{Im}\left[\sum_{l=0}^{L-1}[\alpha^*(L+l)+\alpha((M-1)L+l)]\right]. \quad (6)$$

Because $w(n) \sim N_c(0, \sigma^2)$, $\alpha^*(L+l)$ and $\alpha((M-1)L+l)$ are i.i.d random variables with distribution $N_c(0,\sigma^2|h|^2|s(n)|^2)$. Finally, for the k-th relay, we can get

$$\mathrm{var}(\hat{v}_{kc}^{(1)} - v_k^{(1)}) = \frac{1}{(2\pi L)^2} \frac{\sigma^2 L |h_{k1}^{(1)}|^2 \rho}{(M-1)^2 L^2 |h_{k1}^{(1)}|^4 \rho^2} = \frac{c_c}{SNR_k}, \quad (7)$$

in which $c_c = \frac{1}{(2\pi)^2}\frac{1}{(M-1)^2 L^3}$, $v_k^{(1)} = \frac{f_k^{(1)} - f_s}{f_{samp}}$, $\hat{v}_{kc}^{(1)}$ is the coarse estimation of $v_k^{(1)}$, and $v_k^{(1)} - \hat{v}_{kc}^{(1)} = v_{k\_resi}^{(1)}$ is the residual CFO. As long as $v_k^{(1)}$ is within the range (-1/2L, 1/2L), the variance (7) has nothing to do with the value of v_k.

B.2 Residual CFO Estimation

During the OFDM decoding, we get a pilot sequence $r_p(n)$, which contain Q pilot vectors, and each vector contains P pilots. These pilot vectors can also be seen as repeated training blocks in the frequency domain. So the residual CFO can be estimated as $\hat{v}_{k\_resi}^{(1)} = \frac{angle(u)}{2\pi N_s}$, where $u = \sum_{n=0}^{(Q-1)P-1} r_p^*(n) r_p(n+P)$ and $N_s = N + N_g$ is the length of one symbol.

Following the similar procedure as (3) ~ (7), we can easily get the variance of the residual CFO estimation as

$$\sigma_1^2 = \mathrm{var}(\hat{v}_{k\_resi}^{(1)} - v_{k\_resi}^{(1)}) = \frac{c_{resi}}{SNR_k}, \quad (8)$$

where $c_{resi} = \frac{1}{(2\pi)^2}\frac{1}{(Q-1)^2 N_s^2 P}$. Normally, Q is larger than M, N_s is much larger than L, and P is comparable to L, so (8) should be much smaller than (7), which means that the residual CFO estimation will improve the accuracy of the CFO pre-synch significantly.

Finally, $\hat{v}_k^{(1)} = \hat{v}_{kc}^{(1)} + \hat{v}_{k\_resi}^{(1)}$ is used for the CFO pre-synch at the k-th relay, so the pre-synch error is $e_k^{(1)} = v_k^{(1)} - \hat{v}_k^{(1)} = v_{k\_resi}^{(1)} - \hat{v}_{k\_resi}^{(1)}$, and the variance of the CFO pre-synch error is determined by (8).

C. Pre-Synch Errors for Relays in the Second Cluster

For the second hop, we assume the orthogonal preambles are transmitted from the relays in the first cluster ($s_r(n)$, $r = 1, 2, \ldots, R$, satisfying $\sum_{l=0}^{L-1} s_i(l)s_j^*(l) = 0, i \neq j$), and the receiver in the second cluster will only have one coarse CFO estimation based on the received composite preamble. Residual CFOs estimation is based on the decoupled pilot vectors for each STBC branch.

C.1 Coarse CFO Estimation

The received signal of the k-th receiver is

$$r_k(n) = \sum_{r=1}^{R} h_{kr}^{(2)} s_r(n) e^{-j2\pi v_{kr} n} + w_k(n), \quad (9)$$

in which $v_{k1}^{(2)} \approx v_{k2}^{(2)} \approx \ldots \approx v_{kR}^{(2)} \approx v_{ks}^{(2)}$ (the normalized CFO between the k-th relay in the second cluster and the source) because of the good CFO pre-synch in the first cluster. Then the coarse CFO estimation is $\hat{v}_{kc}^{(2)} = \frac{angle(u_k)}{2\pi L}$, where $u_k = \sum_{n=0}^{(M-1)L-1} r_k^*(n) r_k(n+L)$.

Following the similar procedure as (3) ~ (7) , and making use of the orthogonal condition, we can have $\sum_{n=0}^{(M-1)L-1} \mathrm{Re}\left[r_k^*(n)r_k(n+L)e^{-j2\pi v_{ks}^{(2)} L}\right] \approx \sum_{n=0}^{(M-1)L-1}\sum_{r=1}^{R} |h_{kr}^{(2)}|^2 |s_r(n)|^2$ and $\mathrm{Im}\left[\sum_{l=0}^{L-1}[\alpha^*(L+l)+\alpha((M-1)L+l)]\right] \sim N_c(0, \sigma^2 \sum_{l=0}^{L-1}\sum_{r=1}^{R} |s_r(l)|^2 |h_{kr}|^2)$.

So the variance of the coarse estimation can be calculated as

$$\mathrm{var}(\hat{v}_{kc}^{(2)} - v_{ks}^{(2)}) \approx c_c \Big/ \sum_{r=1}^{R} SNR_{kr}, \quad (10)$$

and $v_{ks}^{(2)} - \hat{v}_{kc}^{(2)} = v_{k\_resi}^{(2)}$ is the residual CFO relative to $v_{ks}^{(2)}$. It's clear that the factor $\sum SNR_{kr}$ in the denominator of (10) is the diversity gain given by the orthogonal preambles, which makes the coarse CFO estimation robust to the channel fading of links in the second hop. $\hat{v}_{kc}^{(2)}$ is then used to do coarse CFO correction to the received signal before converting the signal into the frequency domain, and the residual CFO is left in each STBC branch ($v_{kr\_resi}^{(2)}$, $r = 1, 2, \ldots, R$).

C.2 Residual CFO Estimation

Different from the SISO transmission in the first hop, in the second cluster, R residual CFOs exist in the R orthogonal branches, but only one value is needed for the CFO pre-synch. We need to combine these R residual CFO estimates to give one value for the final CFO pre-synch.

Assuming full rate space-time coding, after the pilots decoupling [3], a pilot sequence $r_{p\text{-}kr}(n)$ composed of Q/R repeated pilot vectors is collected for the r-th branch at the k-th relay. The interval between successive vectors is $R \cdot N_s$. and the SNR for each branch becomes $R \cdot SNR_{kr}$ due to the decoupling operation. According to (2), the residual CFO in the r-th branch can be estimated as $\hat{v}_{kr\_resi}^{(2)} = \frac{angle(u_{kr})}{2\pi R N_s}$, where $u_{kr} = \sum_{n=0}^{(Q/R-1)P-1} r_{p\text{-}kr}^*(n) r_{p\text{-}kr}(n+P)$. Then the variance of the estimation error (local error) in the r-th branch can be calculated as

$$\sigma_w^{\ 2} = \mathrm{var}(\hat{v}_{kr\_resi}^{\ (2)} - v_{kr\_resi}^{\ (2)}) = \frac{c_{resi-st}}{R \cdot SNR_{kr}}, \tag{11}$$

in which $c_{resi\text{-}st} = \frac{1}{(2\pi)^2} \frac{1}{(Q/R-1)^2 (RN_s)^2 P}$. Then, one final residual CFO estimate is produced by applying a power-based weighed combination to these R estimates as

$$\hat{v}_{k\_resi}^{\ (2)} = \sum_{r=1}^{R} \alpha_{kr}^{\ (2)} \hat{v}_{kr\_resi}^{\ (2)} = \frac{1}{2\pi R N_s} \sum_{r=1}^{R} \alpha_{kr}^{\ (2)} angle(u_{kr}), \tag{12}$$

in which $\underline{\alpha}_k^{\ (2)} = \left[\left|h_{k1}^{\ (2)}\right|^2, \left|h_{k2}^{\ (2)}\right|^2, ..., \left|h_{kR}^{\ (2)}\right|^2 \right] \Big/ \sum_{r=1}^{R} \left|h_{kr}^{\ (2)}\right|^2$. The coefficients $\underline{\alpha}_k^{\ (j)}$ can be calculated based on the estimated channel responses (see Fig. 2).

Because $\hat{v}_{kc}^{\ (2)} + v_{kr\_resi}^{\ (2)} = v_{ks}^{\ (2)} - e_r^{\ (1)}$, if define the local error as $w_{kr}^{\ (2)} = v_{kr\_resi}^{\ (2)} - \hat{v}_{kr\_resi}^{\ (2)}$, from (12) we can have

$$e_k^{\ (2)} = v_{ks}^{\ (2)} - \hat{v}_k^{\ (2)} = \sum_{r=1}^{R} \alpha_{kr}^{\ (2)} e_r^{\ (1)} + \sum_{r=1}^{R} \alpha_{kr}^{\ (2)} w_{kr}^{\ (2)}. \tag{13}$$

D. General Error Model for Relay Hops

Following the same procedure in subsection *C*, we can have the pre-synch error model for the *k*-th relay in the *j*-th cluster as

$$e_k^{\ (j)} = \sum_{r=1}^{R} \alpha_{kr}^{\ (j)} e_r^{\ (j-1)} + \sum_{r=1}^{R} \alpha_{kr}^{\ (j)} w_{kr}^{\ (j)}, \tag{14}$$

in which $\underline{\alpha}_k^{\ (j)} = \left[\left|h_{k1}^{\ (j)}\right|^2, \left|h_{k2}^{\ (j)}\right|^2, ..., \left|h_{kR}^{\ (j)}\right|^2 \right] \Big/ \sum_{r=1}^{R} \left|h_{kr}^{\ (j)}\right|^2$ and the local errors $w_{kr}^{(j)}$ can be taken as independent Normal distributed random variables.

From (14) we see that the total pre-synch error at a relay is composed of the pre-synch errors in the previous cluster and the local errors. The former part causes the coupling of the statistics of pre-synch errors for successive clusters (see subsection E). The variance of the local error in each STBC branch is determined by (11).

E. Statistics of the Pre-synch Errors

Because $w_{kr}^{(j)}$ ($k, r = 1, 2, 3 \ldots R, j = 1, 2, \ldots$) is the local error only caused by noise (see (6) and (7)), it is Gaussian distributed with zero mean. After the linear combination of (14), $e_k^{(j)}$ is also a Gaussian distributed random variable with zero mean. So we can define $\sigma_j^2 = \mathrm{E}[(e_k^{(j)})^2]$ as the variance of the pre-synch errors for the *j*-th cluster, $\gamma_j = \mathrm{E}[(e_k^{(j)} e_l^{(j)})]$ as the covariance within the *j*-th cluster, and $\sigma_{Sj}^{\ 2} = E\left[\frac{1}{R-1} \sum_{r=1}^{R} \left(e_r^{\ (j)} - \overline{e}^{\ (j)} \right)^2 \right]$ as the expected sample variance, where $\overline{e}^{\ (j)} = \frac{1}{R} \sum_{r=1}^{R} e_r^{\ (j)}$ is the sample mean.

Bringing (14) into the definitions of σ_j^2 and γ_j, and taking $h_{\mathrm{kr}}^{(j)}$ as Rayleigh faded random variables, we can get (proof is not given in this paper due to the limit of space)

$$\sigma_j^{\ 2} = \frac{2}{R+1} \sigma_{j-1}^{\ 2} + \frac{R-1}{R+1} \gamma_{j-1} + \frac{\sigma_w^{\ 2}}{R-1}, \tag{15}$$

and

$$\gamma_j = \frac{\sigma_{j-1}^{\ 2}}{R} + \frac{R-1}{R} \gamma_{j-1}. \tag{16}$$

(15) and (16) tell that the variances and covariances of pre-synch errors for successive clusters are coupled! So, if we define $x_j = \begin{bmatrix} \sigma_j^{\ 2} \\ \gamma_j \end{bmatrix}$, $A = \begin{bmatrix} \frac{2}{R+1} & \frac{R-1}{R+1} \\ \frac{1}{R} & \frac{R-1}{R} \end{bmatrix}$, and $b = \begin{bmatrix} \sigma^2 \\ 0 \end{bmatrix}$ ($\sigma^2 = \sigma_{\mathrm{w}}^2/(R\text{-}1)$), we can get a first-order dynamic system as

$$x_j = A x_{j-1} + b \ \ (j > 1), \ \ x_1 = \begin{bmatrix} \sigma_1^{\ 2} \\ 0 \end{bmatrix}. \tag{17}$$

Because $|I - A| = 0$, (17) does not have steady-state equilibrium. The other form of (17) is

$$x_j = A^{j-1} x_1 + \sum_{i=0}^{j-2} A^i b. \tag{18}$$

Solving (18), we get

$$x_j = \frac{1}{\beta - 1} \begin{bmatrix} \left(\beta\alpha^{j-1} - 1\right)\sigma_1^2 + \left(\beta\eta - j + 1\right)\sigma^2 \\ \left(\alpha^{j-1} - 1\right)\sigma_1^2 + \left(\eta - j + 1\right)\sigma^2 \end{bmatrix} \tag{19}$$

where $\alpha = \frac{R-1}{R(R+1)}$, $\beta = -\frac{R(R-1)}{R+1}$, and $\eta = \sum_{i=0}^{j-2} \alpha^i = \frac{1-\alpha^{j-1}}{1-\alpha}$.

For large *j*, we can have the approximated solution as

$$\begin{bmatrix} \sigma_j^{\ 2} \\ \gamma_j \end{bmatrix} \approx \frac{1}{1-\beta} \begin{bmatrix} \sigma^2 j + \sigma_1^{\ 2} - \frac{1+\beta-\alpha}{1-\alpha} \sigma^2 \\ \sigma^2 j + \sigma_1^{\ 2} - \frac{2-\alpha}{1-\alpha} \sigma^2 \end{bmatrix} \tag{20}$$

From (20) we see that, when the hop index *j* is large, both $\sigma_j^{\ 2}$ and γ_j are linearly proportional to *j*.

Then we can have the expected sample variance as [15]

$$\sigma_{Sj}^{\ 2} = \sigma_j^{\ 2} - \gamma_j = \alpha^{j-1} \sigma_1^{\ 2} + \eta \sigma^2. \tag{21}$$

When *j* increases, $\sigma_{Sj}^{\ 2}$ approaches to the steady state

$$\sigma_S^{\ 2} = \eta \sigma^2 = \frac{R(R+1)\sigma_w^{\ 2}}{(R-1)(R^2+1)} = \frac{c_{resi-st}(R+1)}{(R-1)(R^2+1) SNR_{kr}}. \tag{22}$$

The convergence of $\sigma_{Sj}^{\ 2}$ tells that, using our CFO pre-synch scheme in the STBC-OFDM systems, multi-hop cooperative transmission is feasible. (22) also tells that the convergence value of $\sigma_{Sj}^{\ 2}$ is determined by the number of relays in one cluster, the receive SNR for each branch and the constant $c_{\mathrm{resi\text{-}st}}$ (determined by the pilots structure in OFDM symbols), and is not related to the pre-synch errors in the first cluster ($\sigma_1^{\ 2}$ in (8)).

IV. Numerical Results

Simulations have been run to check the models and the analysis. In the first simulation, $R = 2$, $N = 512$, $N_g = 64$, $M = 9$, $L = 64$, $P = 32$, $Q = 16$, and we assume perfect channel estimations. Two columns of the 64-bit Hadamard matrix are chosen as the preambles for the two relays in each hop. All pilot symbols are set to be 1. Both the coarse CFO estimation based on the preambles and residual CFO estimation based on decoupled pilots are performed. Two-branch Alamouti STBC is performed, and the two columns of the coded matrix are transmitted by the two relays, respectively. Fig. 3 compares the variance, covariance and expected sample variance from simulations with the theoretical results (19) and (21) derived from model (14). The average SNR for each link is 25dB. We see that the two sets of results match very well, which proves that our model of the pre-synch errors and the analysis based on the model is correct. In the second simulation, we assume the space-time coding rate is 1, and examine the effect of R to the convergence value of expected sample variance (22). In Fig. 4, we see that larger R produces smaller convergence value of expected sample variance. This phenomenon can be taken as the diversity gain we get through the weighted combination (12).

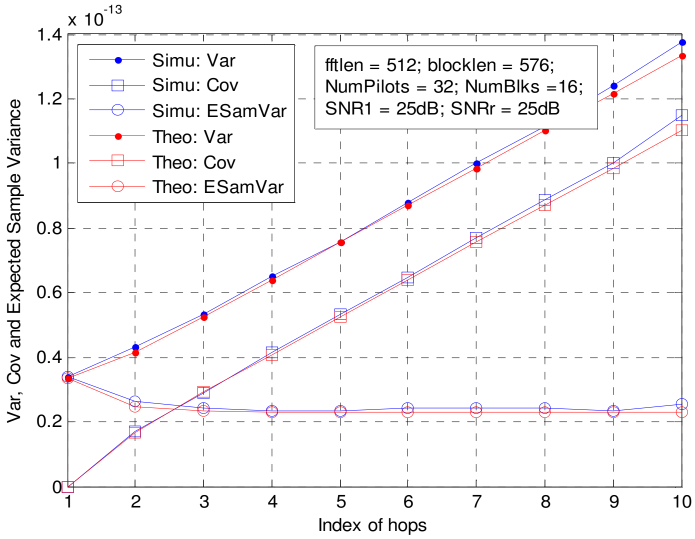

Fig. 3. CFO Pre-synch in 2-branch STBC-OFDM multi-hop CTs

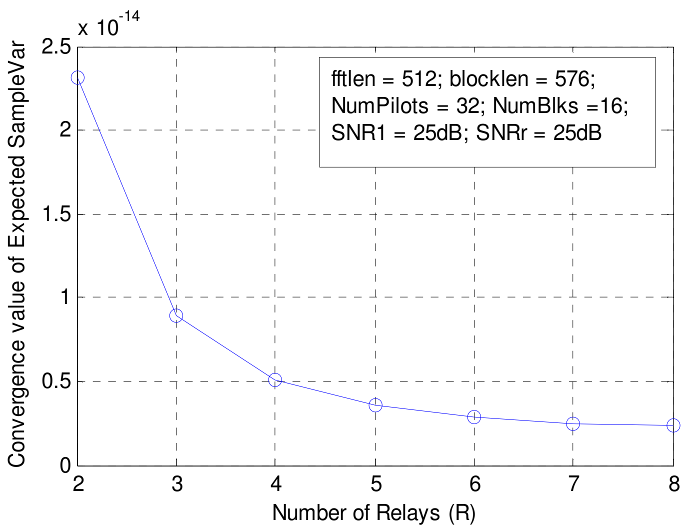

Fig. 4. Convergence Value of Expected Sample Variance for Different R

V. Future Work

Firstly, our analysis assumes the number of relays equals to the number of orthogonal branches. But how about the case where the number of relays is larger than that of orthogonal branches? Whether our pre-synch scheme works when multiple relays share the same branch is a question. Secondly, our CFO pre-synch scheme is proposed for multi-hop cluster-based unicast, whether this scheme works in other applications, e.g. cooperative broadcasting or multicast, is not clear. Thirdly, the error model we established has a general recursive form including the accumulated errors and the local errors, so it may be useful in other problems having such properties. Looking for such problems in telecommunications and other areas, such as distributed control or neural networks, and applying our analysis to the new problems will be a very valuable work.

VI. Conclusions

In this paper, we apply the CFO pre-synch scheme in STBC-OFDM-based multi-hop cooperative transmissions. Residual CFO estimation based on the decoupled pilots and the weighted combination is very effective in improving the CFO pre-synch accuracy. We establish a model of the pre-synch errors and apply a first-order dynamic system method to get the closed form of variance, covariance and the expected sample variance of the pre-synch errors. The convergence of expected sample variance indicates the feasibility of multiple hops. Numerical results validate our error model and the analysis based on the model, and reveal that larger number of relays will improve the per-synch performance. Because our model has a general recursive form including accumulated errors and local errors, our analysis may be valuable for other problems having the similar model structures.

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