# Stream Control for Interfering MIMO Links with Linear MMSE Receivers

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***Abstract*— Recent work has shown that the throughput performance of Wireless Local Area Networks (WLANs) can be significantly improved with the use of multiple-input and multiple-output (MIMO) links. This paper investigates the network throughput for different MIMO transmission schemes with linear MMSE receivers. Simulated throughput of two-link network topology suggests that stream control mechanism wherein an additional streams is added if it leads to an increment in the network throughput can improve the performance of interfering MIMO links, making them an attractive alternative to a TDMA fashion network. The performance of OL-MIMO can be further improved with optimal antenna selection. Our results further show that stream control renders significant throughput gain across a wide range of target bit-error rates (BER).**

***Index Terms*— MIMO, MMSE, BER, stream control, antenna selection, interference.**

## I. Introduction

The multiple-input-multiple-output (MIMO) point-to-point wireless links are well known to provide high data rates in a rich scattering environment through spatial multiplexing (SM) [1], [2]. The SM-MIMO has been adopted as a physical layer for the future generation wireless local area networks, IEEE 802.11n, and has also been the focus of research efforts involving its use to improve the throughput of ad-hoc networks [3]–[6]. In a networking context, one link can suffer from interference from other SM links if allowed to operate simultaneously. The multiple antennas provide additional degrees of freedom at the receiver which it can use to suppress the co-channel interference. Thus MIMO capable nodes can transmit simultaneously, utilizing the network resources efficiently and improving the overall network throughput performance.

The joint optimization problem for interfering MIMO links has been treated previously by several authors. For cellular systems, iterative methods were used to optimize the uplink in [7] and the downlink in [8]. In [9], Blum et al. show that under open-loop (OL) condition when the transmitter has no channel state information, capacities of interfering MIMO links can be increased when all transmit arrays transmit fewer than their maximum number of streams. In [6] and [10], the authors explore ways to control the relative closed-loop capacities (i.e. transmitter has channel knowledge) of interfering MIMO links. In [6], each link iteratively maximizes the closed-loop capacity of its whitened channel under power constraints that generally differ among nodes, and in [10], each link minimizes the interference it makes on its neighbors, subject to capacity constraints. In [11], Demirkol et al. considered closed-loop MIMO (CL-MIMO) systems (i.e. when each link does waterfilling over its whitened channel) and proposed a distributed stream control mechanism wherein an additional stream is added if it leads to an increment in the network throughput. The authors show that MIMO nodes operating under this strategy improve the overall network throughput compared to a time-division multiple access (TDMA) protocol, in which MIMO links operate in succession. In the sequel, we will refer to the transmission strategy with multiple antennas as SDMA, and its performance will be compared with the TDMA protocol.

The analysis presented in [6], [9]–[11] is of theoretical importance as it evaluates the network performance in terms of channel capacity assuming gaussian inputs. However, in practice, discrete signalling constellations are used and transceiver complexity is also rather limited. In this paper, we extend the analysis of [11] to investigate the impact of stream control on the throughput performance of interfering MIMO links, assuming M-QAM constellations and linear MMSE receiver processing. We also investigate a more practical scenario where interferer's channel information at the intended receiver is not available at the transmitter and show that this leads to substantial performance loss in severe interference even if stream control is employed. We further demonstrate that a middle-path approach of having a limited feedback channel (used to convey the set of selected transmit antennas) provides a trade-off between the feedback signaling load and the network throughput performance. Our simulation results show that the performance gap between CL- and OL-SDMA with limited feedback can be substantially abridged if optimal antenna selection is combined with stream control.

The organization of the rest of the paper is as follows. Section II present the system model used. In Section III, we review the precoder and MMSE decoder design assuming additive Gaussian noise with fixed interference. Section III also reviews the link adaptation mechanism. In Section V, simulation results are presented. Finally, Section VI summarizes the main conclusions.

## II. System Model

Consider a simple network as shown in Figure 1, consisting of two spatial multiplexing MIMO links where each link is subjected to co-channel interference from the other link. The average signal-to-interference ratio (SIR) varies linearly as $10\log(R/D)^n$ on a logarithmic scale, where $n$ denotes the path-loss exponent, $R$ and $D$ denote the receiver-transmitter separation

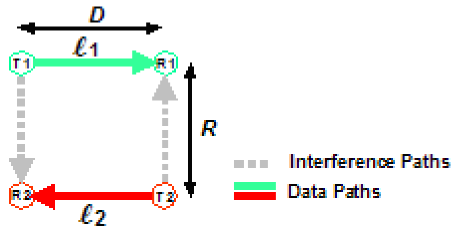



Fig. 1. A simple 2-link network with spatial multiplexing transmissions.

for the interfering and the desired link, respectively.

The transmitting nodes are equipped with $N_t$ antenna elements and receiver nodes use $N_r$ antennas. Each transmitter uses a linear precoder to improve the system performance. The received baseband vector corresponding to the $i^{th}$ link can be represented as

$$\mathbf{r}_i = \mathbf{H}_{ii}\mathbf{F}_i\mathbf{x}_i + \mathbf{H}_{ji}\mathbf{F}_j\mathbf{x}_j + \mathbf{n} \tag{1}$$

where $\mathbf{H}_{ij}$ denotes the channel gain matrix corresponding to the transmitter of the $j^{th}$ link and receiver of the $i^{th}$ link, $\mathbf{F}_i$ is the precoder matrix with complex entries, $\mathbf{x}_i$ is the transmit vector corresponding to the $i^{th}$ link and $\mathbf{n}$ is the additive white Gaussian noise (AWGN), having variance $N_o/2$ per real and imaginary dimension. The channel is assumed to be slowly varying, Rayleigh faded, and fixed for the duration of an entire burst. It is further assumed that

$$\mathrm{E}(\mathbf{n}\mathbf{n}^H) = N_o\mathbf{I}; \quad \mathrm{E}(\mathbf{x}_i\mathbf{n}^H) = \mathbf{0} \tag{2}$$

$$\mathrm{E}(\mathbf{x}_i\mathbf{x}_j^H) = \begin{cases} \mathbf{I} & \text{if } i = j \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{3}$$

where the subscript $H$ denotes the conjugate transpose and $E(.)$ denotes the expected value of its argument. Linear MMSE processing is assumed at the receivers.

## III. System Design

In this section, we briefly discuss the optimal precoder and decoder design with respect to system throughput for OL- and CL-SDMA systems. Our goal is to maximize the network throughput, which is defined as the sum of the link data rates.

### A. Precoder Design

*1) CL-SDMA System:* The traditional water-filling approach can be modified to accommodate fixed non-white interference at the receiver of a link by whitening the channel matrix [12]:

$$\widetilde{\mathbf{H}}_{ii} = (N_o\mathbf{I} + \mathbf{H}_{ji}\mathbf{F}_j\mathbf{F}_j^H\mathbf{H}_{ji}^H)^{-\frac{1}{2}}\mathbf{H}_{ii} \tag{4}$$

Let the singular-value decomposition (SVD) of the whitened channel matrix be denoted as $\widetilde{\mathbf{H}}_{ii} = \mathbf{U}_i\mathbf{\Sigma}_i\mathbf{V}_i^H$. The optimal precoder matrix for the $i^{th}$ link can be written as $\mathbf{F}_i = \mathbf{V}_i\mathbf{\Phi}_F$ where $\mathbf{\Phi}_F$ is a diagonal matrix whose $k^{th}$ non-zero diagonal element is given by [13]:

$$\alpha_k = \left[\left(\mu - \frac{1}{\sigma_k^2}\right)^+\right]^{\frac{1}{2}} \tag{5}$$

where $(.)^+$ indicates that only non-negative values are acceptable, $\sigma_k^2$ denotes the SVD values of the whitened channel, $P_T$ denotes the available transmit power, and $\mu$ is chosen such that $\sum \alpha_k^2 = P_T$. It may be noted that the capacity based precoder design is not necessarily optimal for finite complexity transceivers as will be explained in Section IV.

*2) OL-SDMA System:* For an OL-SDMA system, the best strategy is to allocate equal power to all transmit antennas [9]. In the presence of strong interference, not all transmit antennas are used, to avoid overloading the receiver. In this case, the optimal strategy is to excite as many as $N_r - N_{int}$ transmit antennas, where $N_{int}$ denotes the number of incident interfering streams on the intended receiver [14]. The precoder matrix in this case is:

$$\mathbf{F}_i = \sqrt{\frac{P_T}{N_r - N_{int}}}\mathrm{diag}\Big(\underbrace{1, 1, ..., 1}_{N_r - N_{int}}, \underbrace{0, 0, ..., 0}_{N_t - N_r + N_{int}}\Big) \tag{6}$$

where $diag(.)$ denotes the diagonal matrix formed by the elements of its vector argument. In case of transmit antenna selection, the transmit power correlation matrix is formed by reordering the diagonal elements in (6) based on antenna subset selection.

### B. Linear MMSE Decoder

Without loss of generality, let us consider the link $l_1$. And to keep the analysis general, let us drop the link specific subscripts. The symbol estimation error vector associated with a linear decoder $\mathbf{C}$ can be written as $\mathbf{e} = \mathbf{x} - \mathbf{C}\mathbf{r}$. The MMSE decoder requires that $\mathbf{C}$ be chosen such that the mean-squared error (MSE), $\mathrm{E}(||\mathbf{e}||^2)$, is minimized. The MSE can also be expressed as $\mathrm{tr}(\mathbf{R_e})$, where $\mathbf{R_e}$ is the error covariance matrix and $\mathrm{tr}(.)$ denotes the trace of its argument. Using simple matrix manipulations, $\mathbf{R_e}$ can be rewritten as,

$$\begin{aligned}\mathbf{R_e} &= (\mathbf{C} - \mathbf{F}^H\mathbf{H}^H\mathbf{R_r}^{-1})\mathbf{R_r}(\mathbf{C} - \mathbf{F}^H\mathbf{H}^H\mathbf{R_r}^{-1})^H \\ &\quad + \mathbf{I} - \mathbf{F}^H\mathbf{H}^H\mathbf{R_r}^{-1}\mathbf{H}\mathbf{F}\end{aligned} \tag{7}$$

where $\mathbf{R_r}$ is the receive covariance matrix. It is not difficult to show that the first term in (7) has non-negative diagonal entries. The MMSE solution, therefore, minimizes the contribution of the first term towards the MSE by choosing $\mathbf{C} = \mathbf{F}^H\mathbf{H}^H\mathbf{R_r}^{-1}$. Thus, the MSE for the $i^{th}$ data stream is given by

$$mse_i = (\mathbf{I} - \mathbf{F}^H\mathbf{H}^H\mathbf{R_r}^{-1}\mathbf{H}\mathbf{F})_{ii} \tag{8}$$

where $(.)_{ii}$ denotes the $i^{th}$ diagonal entry of its matrix argument. Now, the post-processing signal to interference and noise ratio (SINR) for the $i^{th}$ data stream can be obtained as $sinr_i = \frac{1}{mse_i} - 1$, [13].

### C. Rate Adaptation

In this subsection, we give a brief overview of rate adaptation for interfering MIMO links. We consider square as well as rectangular M-QAM modulation. The probability of bit error for the $i^{th}$ data stream modulated using $I \times J$ rectangular QAM can be linked to the SINR in (8) as [15]:

$$P_b(I, J) = \frac{1}{\log_2(I.J)}\left(\sum_{k=1}^{\log_2 I}\Psi_{I,J}(k) + \sum_{l=1}^{\log_2 J}\Psi_{J,I}(l)\right) \tag{9}$$

where the function $\Psi_{P,Q}$ can be computed as

$$\Psi_{P,Q}(n) = \frac{1}{P} \sum_{j=0}^{P(1-2^{-n})-1} (-1)^{\lfloor m \rfloor} \left(2^{n-1} - \lfloor m + 0.5 \rfloor\right) \times \text{erfc}\left((2j+1)\sqrt{\frac{3\log_2(P.Q.sinr)}{P^2+Q^2-2}}\right) \quad (10)$$

where $m = \frac{j.2^{n-1}}{P}$ and $\lfloor . \rfloor$ returns the greatest integer less than or equal to the argument. Now, the maximum rate supported by the $i^{th}$ data stream with SINR $= sinr_i$ can be determined as $R_i = \log_2(I_i \times J_i)$, where $\{I_i, J_i\}$ are chosen such that $M_i = I_i \times J_i$ is the largest constellation which meets the target BER requirement. The maximum possible rate $R_i$ is a *stair-case* function of the associated SINR and target BER. It is apparent that the MMSE decoder doesn't necessarily optimize the link throughput as it tries to minimize the sum $\sum \frac{1}{sinr_i}$ instead of maximizing the sum $\sum R_i$.

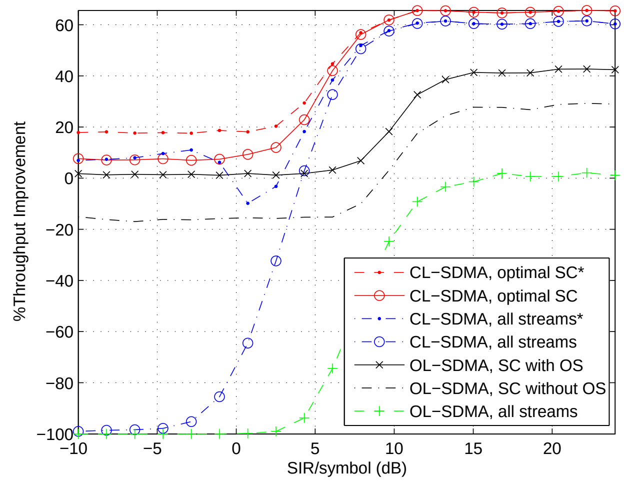
## IV. Simulation Results

In this section, we present average throughput simulation results for 2-link network as shown in Figure 1. We consider several SDMA schemes with optional features such as stream control, optimal selection, channel whitening and draw performance comparisons among them and CL-TDMA scheme. In [11], the authors demonstrated the usefulness of stream control for various SDMA techniques, and in [16], the authors further demonstrated the effectiveness of optimal selection aided stream control for OL-SDMA; both using channel capacity metric to quantify the network throughput. In this section, different from the past work, we assume linear MMSE receiver processing and evaluate the network throughput performance using the input signals drawn from both square and rectangular M-QAM constellations. For more detail about the stream control algorithm, the reader is referred to [10]- [11].

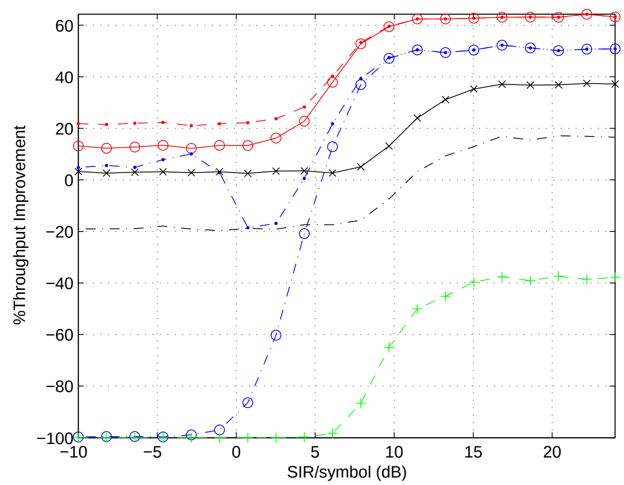
Each node is assumed to have 4 transmit/receive antennas. The results are generated using Monte Carlo simulation of 1000 channel trials. For the CL- and OL-SDMA results, the algorithm of [11] was used. We consider a fair energy transmission approach, which requires both TDMA and SDMA networks use equal transmit powers, to allow for a fair performance comparison. The noise-normalized transmit power is fixed at $P_T = 20$dB for the TDMA scheme. For SDMA scheme, the total transmit power is divided equally among all the transmitting nodes, i.e. $\acute{P_T} = P_T/2 = 17$dB for the 2-link network.

### A. Throughput Performance

Figure 2 shows the average percent throughput improvements of several SDMA schemes relative to CL-TDMA for a 2-link network as SIR is varied. The percentage improvement in throughput is computed as $(T_{SDMA} - T_{TDMA})/T_{TDMA} \times 100\%$, where $T_{SDMA}$ is the average network throughput with co-channel links, and $T_{TDMA}$ is the average single-link throughput. As a reference, the current 802.11 MAC enforces time multiplexing due to interference if $R/D < 2$. For $n = 3$, for example, $R/D < 2$ corresponds to SIR = 9dB. Therefore, if an SDMA




(a) Average BER $= 10^{-2}$




(b) Average BER $= 10^{-5}$

Fig. 2. Average improvement in the network throughput relative to CL-TDMA. SC stands for stream control and OS stands for optimal antenna selection. The subscript ()* indicates that whitened channel information is available at the transmitter. The legend in Fig. 2(a) also serves Fig. 2(b).

scheme has positive throughput improvement for SIR $< 9$dB, then a MAC that exploits SDMA, such as the one in [3], would outperform the 802.11 MAC.

Figure 2(a) shows SDMA throughput improvements for a target average BER $= 10^{-2}$. It is apparent that CL-SDMA with stream control yields the best performance as expected. For example, when the interference is strong (SIR $< 9$dB), CL-SDMA with stream control using the whitened channel information, offers an improvement of about $18\%$ over CL-TDMA. This gain can be explained as resulting from multiuser diversity because joint-optimization with stream control offers 8 channel modes to choose from rather than just 4 as in CL-TDMA. On the other hand, OL-SDMA without stream control gives extremely poor performance. This can be attributed to the fact that a linear decoder like MMSE is overwhelmed with

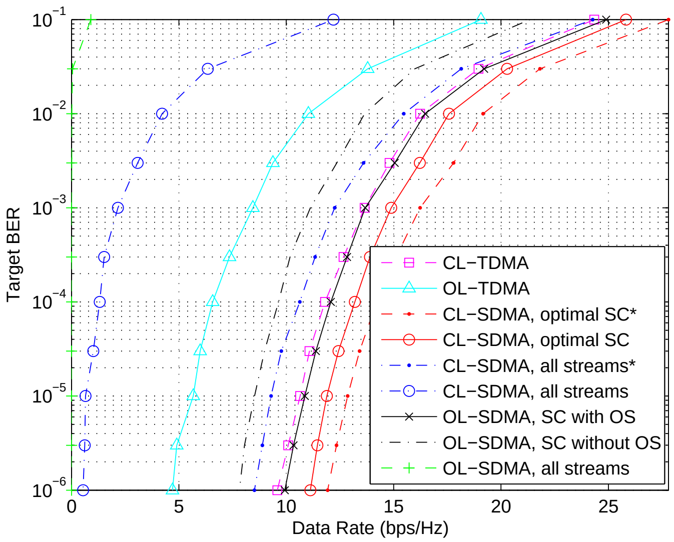
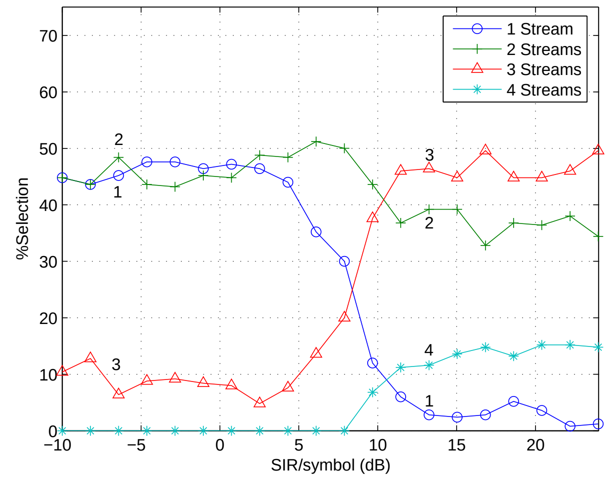
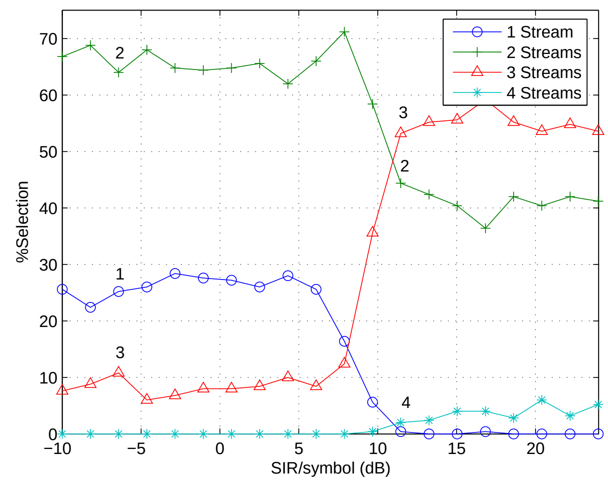


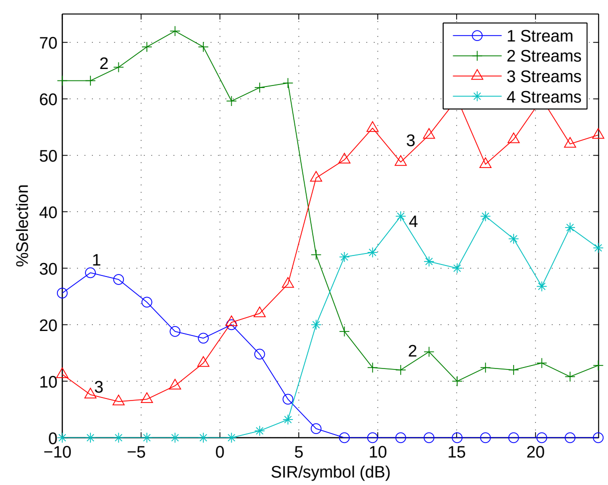

Fig. 3. Achievable bit rates for various MIMO schemes as a function of target BER for SIR = 0dB. The subscript ()* indicates that whitened channel information is available at the transmitter.




(a) OL-SDMA, stream control with deterministic antenna selection.




(b) OL-SDMA, stream control with optimal antenna selection.




(c) CL-SDMA, stream control using whitened channel information.

Fig. 4. Number of streams used by one link with different MIMO configurations for different SIR values. Target BER $= 10^{-5}$

interfering streams in the absence of stream control mechanism. When stream control is used with deterministic antenna selection, the performance of OL-SDMA is significantly improved but is still worse than that of CL-TDMA. However, optimal antenna selection aided stream control improves the throughput of OL-SDMA by almost $18\%$, enabling it to match the performance of CL-TDMA. Thus OL-SDMA systems using a limited amount of feedback information offers an attractive alternative to CL-TDMA systems which have significant signaling overhead. From Figure 2(a), we also observe that as the interference becomes weak, the performance of all SDMA schemes, barring OL-SDMA without stream control, observes significant improvement over CL-TDMA. We also note that the significance of whitened channel information at the transmitter reduces with reducing interference strength, as expected. It is also interesting to note that, contrary to the capacity results of [11], there is substantial gap between several SDMA schemes even in the high SIR zone.

Figure 2(b) shows the SDMA throughput improvements for a network with which permits far less packet errors, with average BER $= 10^{-5}$. The throughput curves for various SDMA schemes follow similar trends as in Figure 2(a). However, it can be seen that optimal antenna selection and stream control further improve the throughput performance of various SDMA schemes. On one hand CL-SDMA without stream control observes a fall in throughput, on the other hand CL-SDMA with stream control observes a further gain relative to CL-TDMA which now amounts to about $21\%$. For OL-SDMA, the gain due to optimal antenna selection aided stream control further increases to about $3\%$ in the high interference zone. It can also be observed that OL-SDMA systems without stream control always perform worse than CL-TDMA systems even when interference is negligible.

Figure 3 shows the achievable bit rates for different MIMO schemes employing an MMSE decoder as a function of target BER for fixed SIR = 0dB. The performance curves for CL-

SDMA with stream control using whitened channel information and OL-SDMA without stream control form the upper and lower bounds, respectively. The CL-SDMA scheme with stream control offers a gain of about $2.5$ bps/Hz relative to CL-TDMA over a wide range of target BER. The availability of whitened channel information improves the performance of stream control, rendering a gain of about $1.25$ bps/Hz. It is interesting to note that the relative gap between any two MIMO schemes is almost independent of the target BER. From Figure 3, we also observe that optimal selection for OL-SDMA with stream control improves its performance by about $2.5$ bps/Hz relative to deterministic selection. Infact, OL-SDMA with optimal selection, which requires a limited-feedback channel, matches the performance of CL-TDMA which requires full CSI at the transmitter. Thus it can be concluded that stream control along with optimal selection achieves a good tradeoff between performance required and feedback overhead.

### B. *Number of Streams and Stream Control*

Figure 4 illustrates the regulation of streams by different SDMA schemes as a function of SIR, assuming a target BER $= 10^{-5}$. A hundred channel trials are generated, and the link parameters are found using the stream control method. Figure 4(a) shows the relative frequency of the number of streams used by link $l_1$ for OL-SDMA with stream control but deterministic antenna selection. It is apparent that when interference is strong, SIR $< 9$dB, the link mostly uses one or two streams with about equal probability. This leads to receive diversity gain which is critical in satisfying the target BER. The use of optimal antenna selection along with stream control results in higher selection probability for two streams compared to one stream. This is because with optimal antenna selection, transmit diversity gain is also available and as the link can afford two streams more often. Thus optimal antenna selection aided stream control enables OL-SDMA to exploit spatial multiplexing better than the deterministic selection.

From 4(b), we also note that at high SIR, OL-SDMA with optimal selection aided stream control rarely chooses 4 streams, thus resulting in lower throughput compared to CL-SDMA with stream control, as shown in 2(b). Finally, Figure 4(c) shows the optimal stream control that could be achieved by CL-SDMA using whitened channel information. We observe that unlike OL-SDMA, both with and without optimal antenna selection, CL-SDMA rarely uses one or two streams when interference is relatively weak. It is important to note that the link throughput is dependent not only on the number of streams used, but also on the number of bits carried by each stream.

## V. CONCLUSION

We have analyzed the performance gains offered by the use of stream control for interfering MIMO links with linear MMSE receiver processing. Although, CL-SDMA with stream control using whitened channel information offers the best throughput, it has substantial overhead of providing the CSI to the transmit nodes. Our results indicate that OL-SDMA with optimal antenna selection is an attractive alternative to CL-TDMA as it incurs a minimal overhead of specifying the chosen antenna set.

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