Simple MMSE Interference Suppression for Real and Rate-1/2 Complex Orthogonal Space-time Block Codes

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***Abstract*—Orthogonal Space-Time Block Codes (OSTBCs) are known to provide transmit diversity gain with negligible decoding complexity. In this paper, we present a single-stage MMSE multi-user interference suppression technique that exploits the temporal and spatial structure of OSTBCs, leading to simple linear processing. In particular, we show that MMSE-based detection can be implemented for real OSTBCs as well as the derived rate-1/2 complex OSTBCs, using simple linear processing that avoids computationally intensive matrix inversions.**

***Index Terms*—STBC, interference suppression, MMSE, MIMO, multi-user detection.**

I. INTRODUCTION

SPACE-TIME Block Codes (STBCs) are well known to provide transmit diversity without requiring explicit channel feedback from the receiver [1], [2]. The temporal and spatial structure of certain STBCs, called orthogonal STBCs (OSTBCs), offer the additional advantage of maximum likelihood detection while requiring only linear processing at the receiver [2]. As a result, the OSTBCs are being widely adopted in various wireless communication standards such as the 3GPP cellular standard, WiMAX (IEEE 802.16), and IEEE 802.11n. This paper presents an interference cancellation scheme for OSTBCs with 3 and 4 antennas, which is optimal with respect to MMSE.

Despite the low data rates supported by various STBCs, they are attractive from a network point of view as they cause correlated interference which can be mitigated using only one additional antenna without sacrificing space-time diversity gains. In [3], Naguib et al. developed a minimum mean square error (MMSE) interference suppression technique for two co-channel users employing the Alamouti code [1]. It was shown that the impact of co-channel interference can be completely eliminated by adding one more receive antenna without sacrificing space-time diversity gains. This scheme was later extended to frequency-selective channels in [4]. In [5] authors extend the work of [3] to the interference cancellation (IC) and detection for users with 4 transmit antennas using quasi-orthogonal space-time block coding [6]. In another related work, Stamoulis et al. [7] presented a simple suboptimal linear processing scheme that achieves IC of two co-channel users employing any rate-1/2 complex STBC based on an orthogonal design. Like [3], [7] cancels the correlated STBC interference with the aid of additional receive antenna while preserving the diversity gains. However, the method is based on 2-stage receive processing and is not optimal in terms of mean square error (MSE).

In contrast to [7], the algorithm presented in this paper is single-stage and MMSE-optimal. In contrast to [3], the proposed algorithm treats unity rate real and derived rate-1/2 complex OSTBCs for 3 and 4 transmit antennas [2]. Like [3] and [7], the algorithm requires only one additional antenna to cancel an OSTBC interference. Some special algebraic properties of linear OSTBCs are exploited to give the algorithm low complexity.

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II. REVIEW OF OSTBC

In this section, we provide a brief overview of linear OSTBCs for frequency flat channels. Linear STBCs have a relatively simple structure, as the transmitted code matrix is linear in the real and imaginary parts of the data symbols. A generalized linear STBC code matrix can be represented as

$$\mathbf{X} = \sum_{i=1}^{n_s} s_i \mathbf{A}_i \tag{1}$$

where $\{s_i\}_{i=1}^{n_s}$ denotes the transmitted symbols and $\{\mathbf{A_i}\}_{i=1}^{n_s}$ are the $n_t \times n$ matrices representing the code structure, where n is the duration over which n_s symbols are transmitted using n_t antennas. From the definition of real orthogonal codes, it follows that

$$\mathbf{A}_i\mathbf{A}_j^T = \left\{ \begin{array}{cc} \mathbf{I}_n & \text{if } i=j \\ -\mathbf{A}_j\mathbf{A}_i^T & \text{otherwise} \end{array} \right\} \tag{2}$$

For a MIMO link with n_t transmit antennas and n_r receive antennas, the received space-time signal $\mathbf{Y}$ corresponding to $\mathbf{X}$ can be written as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{V}, \tag{3}$$

where $\mathbf{H}$ is the channel matrix and $\mathbf{V}$ denotes the complex additive white Gaussian noise matrix with zero mean and unity variance. The channel is assumed to be quasi-static flat Rayleigh faded. Assuming a single user, the received baseband signal can be represented using vector notation as

$$\mathbf{y} = \text{vec}(\mathbf{Y}) = \mathbf{F}_1\mathbf{s}_1 + \mathbf{v}, \tag{4}$$

where vec(.) denotes the standard vector representation of its argument, $\mathbf{v}$ = vec($\mathbf{V}$), and $\mathbf{F}_1$ is defined as

$$\mathbf{F}_1 = \left(\begin{array}{ccc} \text{vec}(\mathbf{HA}_1) & \dots & \text{vec}(\mathbf{HA}_{n_s}) \end{array} \right) \tag{5}$$

The vector representation for the received baseband signal in (4) can be expressed differently as $\mathbf{y}' = \mathbf{F}_1'\mathbf{s}_1 + \mathbf{v}'$, where the operator $(.)'$ is defined as $\chi' = \left(\begin{array}{c} \text{Re}(\chi) \\ \text{Im}(\chi) \end{array} \right)$.

III. REAL STBC AND IC OF TWO CO-CHANNEL USERS

In this section, we focus on the MMSE-based multiuser detection of two synchronous co-channel users, each employing the same real STBCs. We will show for n_t = 3 or 4, the linear receiver processing can be implemented without matrix inversions, and after the IC-stage, the symbols are recovered with space-time diversity gain.

Let us use subscripts 1 and 2 to differentiate between the corresponding channel matrices and symbol vectors of the two users. Thus the received baseband vector is given by

$$\mathbf{y}' = \mathbf{F}'_1\mathbf{s_1} + \mathbf{F}'_2\mathbf{s_2} + \mathbf{v}' = \mathbf{F}'\mathbf{s} + \mathbf{v}' \quad (6)$$

where $\mathbf{F}_i$'s are defined as in (5), $\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 \end{pmatrix}$, and $\mathbf{s} = \begin{pmatrix} \mathbf{s}_1^T & \mathbf{s}_2^T \end{pmatrix}^T$. Let us assume that the receiver employs the MMSE filter given by $\mathbf{C} = (\mathbf{F}'^T\mathbf{F}' + N_o\mathbf{I})^{-1}\mathbf{F}'^T$, where N_o denotes the noise variance. It is not difficult to see that $\mathbf{F}'^T\mathbf{F}' = \text{Re}(\mathbf{F}^H\mathbf{F})$. Exploiting the channel decoupling property of orthogonal STBCs, $\text{Re}(\mathbf{F}_i^H\mathbf{F}_i) = ||\mathbf{H}_i||^2\mathbf{I}$ [8, (7.4.1.4)], we can simplify $\mathbf{F}'^T\mathbf{F}'$ as

$$\mathbf{F}'^T\mathbf{F}' = \begin{pmatrix} ||\mathbf{H}_1||^2\mathbf{I} & \mathbf{R}_{12} \\ \mathbf{R}_{12}^T & ||\mathbf{H}_2||^2\mathbf{I} \end{pmatrix}, \quad (7)$$

where $\mathbf{R}_{12} = \text{Re}(\mathbf{F}_1^H\mathbf{F}_2)$ and $||.||$ denotes the Frobenius norm of the matrix argument. Now the matrix inversion component of MMSE filter can be simplified using (7) with the aid of block matrix inversion [9, (3.5.3.1)], as

$$(\mathbf{F}'^T\mathbf{F}' + N_o\mathbf{I})^{-1} = \begin{pmatrix} (||\mathbf{H}_2||^2 + N_o)\mathbf{\Gamma} & -\mathbf{R}_{12}\mathbf{\Gamma} \\ -\mathbf{\Gamma}\mathbf{R}_{12}^T & (||\mathbf{H}_1||^2 + N_o)\mathbf{\Gamma} \end{pmatrix}, \quad (8)$$

where

$$\mathbf{\Gamma} = \Big((||\mathbf{H}_1||^2 + N_o)(||\mathbf{H}_2||^2 + N_o)\mathbf{I} - \mathbf{R}_{12}\mathbf{R}_{12}^T\Big)^{-1}. \quad (9)$$

Next, by multiplying both sides of (8) on the right by $(\mathbf{F}'^T\mathbf{F}' + N_o\mathbf{I})$, substituting $\mathbf{F}'^T\mathbf{F}'$ with (7), and equating the $(2,1)^{th}$ sub-matrix element of the resulting product to $\mathbf{0}$, we get $\mathbf{R}_{12}^T\mathbf{\Gamma} = \mathbf{\Gamma}\mathbf{R}_{12}^T$. Exploiting the self-adjoint nature of $\mathbf{\Gamma}$, it can be further shown $\mathbf{\Gamma}$ also commutes with $\mathbf{R}_{12}$, i.e.,

$$\mathbf{R}_{12}\mathbf{\Gamma} = \mathbf{\Gamma}\mathbf{R}_{12}. \quad (10)$$

Using (8)-(10), the resulting MMSE filter can be expressed in a particularly useful form as

$$\mathbf{C} = \begin{pmatrix} \text{Re}(\mathbf{W}_{1,2}) & -\text{Im}(\mathbf{W}_{1,2}) \\ \text{Re}(\mathbf{W}_{2,1}) & -\text{Im}(\mathbf{W}_{2,1}) \end{pmatrix}, \quad (11)$$

where

$$\mathbf{W}_{i,j} = \mathbf{\Gamma}\Big((||\mathbf{H}_j||^2 + N_o)\mathbf{F}_i^H - \mathbf{R}_{ij}\mathbf{F}_j^H\Big). \quad (12)$$

It is interesting to note that the design matrices associated with the unity rate real orthogonal STBCs for n_t = 3 and 4 [2, (37)-(38)], have some unique properties in addition to (2), namely $\mathbf{A}_3\mathbf{A}_1^T = \mathbf{A}_4\mathbf{A}_2^T$ and $\mathbf{A}_3\mathbf{A}_2^T = -\mathbf{A}_4\mathbf{A}_1^T$ [8]. For such OSTBCs, it is not difficult to show that $\mathbf{R}_{12}\mathbf{R}_{12}^T = \alpha\mathbf{I}$, where

$$\alpha = \sum_{k=1}^{4}\big[\text{tr}\big(\text{Re}(\mathbf{H}_1^H\mathbf{H}_2\mathbf{A}_k\mathbf{A}_1^T)\big)\big]^2. \quad (13)$$

It may be noted that the product $\mathbf{A}_k\mathbf{A}_1^T$ is a sparse matrix and hence the computation cost associated with α is very insignificant. On an average, the computation of α involves n_rn_t multiplication operations.

After substituting the simplified expression for $\mathbf{R}_{12}\mathbf{R}_{12}^T$ into (9), the MMSE filter expression in (11) for real orthogonal codes with n_t = 3 or 4, can be simplified as

$$\mathbf{W}_{i,j} = \frac{(||\mathbf{H}_j||^2\mathbf{F}_i^H - \mathbf{R}_{ij}\mathbf{F}_j^H)}{(||\mathbf{H}_1||^2 + N_o)(||\mathbf{H}_2||^2 + N_o) - \alpha}, \quad (14)$$

where α is defined as in (13). Now the decoded streams for the two users can be easily obtained as

$$\mathbf{r} = \mathbf{C}\mathbf{y}' = \begin{pmatrix} \text{Re}(\mathbf{W}_{1,2}\mathbf{y}) \\ \text{Re}(\mathbf{W}_{2,1}\mathbf{y}) \end{pmatrix}, \quad (15)$$

where $\text{Re}(\mathbf{W}_{i,j}\mathbf{y})$ corresponds to the decoded stream for the i^{th} user. It may be noted that the expression $\mathbf{W}_{i,j}$ in (14) does not involve any matrix inversion. The computation of $\mathbf{W}_{i,j}$ with the aid of equation (14) only requires $3n_rn_t + 2nn_rn_s^2$ multiplication operations compared to $2n_rn_t + 3nn_rn_s^2 + n_s^2 + n_s^3$ needed if $\mathbf{W}_{i,j}$ is computed using (9) and (12). As a numerical example, for $n_t = 3$ and $n_r = 2$, computation of $\mathbf{W}_{i,j}$ in (12) and (14) requires 274 and 476 multiplication operations respectively. Thus, the proposed MMSE-based IC of two co-channel users with real STBCs can be achieved using simple linear processing.

In contrast to the presented algorithm, Stamoulis et al. [7] constrain the matrix filter to have identity blocks on the main diagonal and require that the off-diagonals remove all interference:

$$\mathbf{r}_i = \mathbf{y}_i - \mathbf{Z}_i\mathbf{y}_j, \quad (16)$$

where $i, j \in (1,2)$, $j \neq i$ and $\mathbf{r}_i$ indicates the post-IC recovered data stream corresponding to the i^{th} user. The matrix $\mathbf{Z}_i$ denotes the space-time filter for the i^{th} user as defined in [7, (11)], and $\mathbf{y}_i$ indicates the baseband signal corresponding to the i^{th} receive antenna. In other words, the i^{th} antenna is intended to receive only the i^{th} user; the other antenna is used only to estimate and subtract the interference that is in the i^{th} antenna. While this is a zero-forcing (ZF) solution, it does not have the most general (i.e. full matrix) formulation, and as we will show, it suffers a significant SNR degradation relative to the MMSE solution as well as the full-matrix ZF solution.

IV. IC WITH RATE-1/2 COMPLEX OSTBCS

Let us consider two co-channel users, each employing the same rate-1/2 complex orthogonal code. A rate-1/2 generalized complex orthogonal design can be constructed from a real orthogonal design $\mathbf{X}_{n_t}$ as $\mathbf{X}_{n_t}^C = \begin{pmatrix} \mathbf{X}_{n_t} & \text{conj}(\mathbf{X}_{n_t}) \end{pmatrix}$ and can also be represented as

$$\mathbf{X}_{n_t}^C = \sum_{i=1}^{n_s}\text{Re}(s_i)\begin{pmatrix} \mathbf{A}_i & \mathbf{A}_i \end{pmatrix} + j\text{Im}(s_i)\begin{pmatrix} \mathbf{A}_i & -\mathbf{A}_i \end{pmatrix}, \quad (17)$$

where $\mathbf{X}_{n_t}^C$ is the $n_t \times (2n)$ STBC matrix, $\{s_i\}_{i=1}^{n_s}$ denotes the transmitted complex symbols, $\{\mathbf{A_i}\}_{i=1}^{n_s}$ are the $n_t \times n$ matrices representing the code structure of parent real codes, and the operator conj(.) denotes the conjugate of its argument. Following the development similar to (3)-(6), and observing that n_s symbols are transmitted using n_t antennas in $2n$ timeslots, it is not difficult to show that the $2n \times 1$ vector representing the received baseband signal for the first link can be expressed as

$$\begin{pmatrix} \mathbf{y}_{1:n} \\ \mathbf{y}_{n+1:2n} \end{pmatrix}' = \mathbf{G}'\mathbf{s}' + \mathbf{v}', \tag{18}$$

where $\mathbf{G} = \begin{pmatrix} \mathbf{G}_1 & \mathbf{G}_2 \end{pmatrix}$, and $\mathbf{G}_k$'s are defined as

$$\mathbf{G}_k = \begin{pmatrix} \mathbf{F}_k & j\mathbf{F}_k \\ \mathbf{F}_k & -j\mathbf{F}_k \end{pmatrix}. \tag{19}$$

where the subscript $k \in (1, 2)$. Following the development similar to (6)-(13) and after some simplifications, the linear MMSE detector expression can be shown to be

$$\mathbf{C} = \frac{1}{2}\begin{pmatrix} \mathrm{Re}(\mathbf{W}_{1,2}) & \mathrm{Re}(\mathbf{W}_{1,2}) & -\mathrm{Im}(\mathbf{W}_{1,2}) & -\mathrm{Im}(\mathbf{W}_{1,2}) \\ \mathrm{Im}(\mathbf{W}_{1,2}) & -\mathrm{Im}(\mathbf{W}_{1,2}) & \mathrm{Re}(\mathbf{W}_{1,2}) & -\mathrm{Re}(\mathbf{W}_{1,2}) \\ \mathrm{Re}(\mathbf{W}_{2,1}) & \mathrm{Re}(\mathbf{W}_{2,1}) & -\mathrm{Im}(\mathbf{W}_{2,1}) & -\mathrm{Im}(\mathbf{W}_{2,1}) \\ \mathrm{Im}(\mathbf{W}_{2,1}) & -\mathrm{Im}(\mathbf{W}_{2,1}) & \mathrm{Re}(\mathbf{W}_{2,1}) & -\mathrm{Re}(\mathbf{W}_{2,1}) \end{pmatrix} \tag{20}$$

where $\mathbf{W}_{i,j}$ is defined in (12). After some simplifications, the decoded streams corresponding to the two users can be obtained as

$$\mathbf{r}' = \frac{1}{2}\begin{pmatrix} \mathrm{Re}(\mathbf{W}_{1,2}(\mathbf{y}_{1:n} + \mathbf{y}_{n+1:2n})) \\ \mathrm{Im}(\mathbf{W}_{1,2}(\mathbf{y}_{1:n} - \mathbf{y}_{n+1:2n})) \\ \mathrm{Re}(\mathbf{W}_{2,1}(\mathbf{y}_{1:n} + \mathbf{y}_{n+1:2n})) \\ \mathrm{Im}(\mathbf{W}_{2,1}(\mathbf{y}_{1:n} - \mathbf{y}_{n+1:2n})) \end{pmatrix} \tag{21}$$

or expressed differently as

$$\mathbf{r} = \frac{1}{2}\begin{pmatrix} \mathbf{W}_{1,2}\mathbf{y}_{1:n} + \mathrm{conj}(\mathbf{W}_{1,2}\mathbf{y}_{n+1:2n}) \\ \mathbf{W}_{2,1}\mathbf{y}_{1:n} + \mathrm{conj}(\mathbf{W}_{2,1}\mathbf{y}_{n+1:2n}) \end{pmatrix} \tag{22}$$

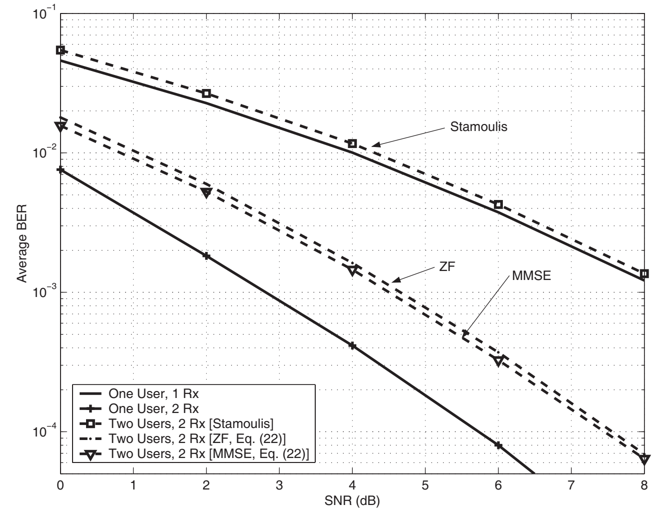
The above equation shows that the IC of two co-channel users employing the derived rate-1/2 complex OSTBCs can be implemented using the same MMSE filter designs, $\mathbf{W}_{i,j}$, as for the IC of two users employing real STBCs.

It may be noted that for $n_t = 3$ and 4 there exist higher rate complex OSTBCs than rate-1/2 codes considered in this paper. For example, there exist rate-3/4 codes called truncated Octonion and Octonion for $n_t = 3$ and 4, respectively [8, (7.4.8)-(7.4.10)]. We further note that the above IC simplification cannot be achieved using these codes as the corresponding matrices $\mathbf{R}_{12}\mathbf{R}_{12}^T$ are not diagonal as required to simplify (9).

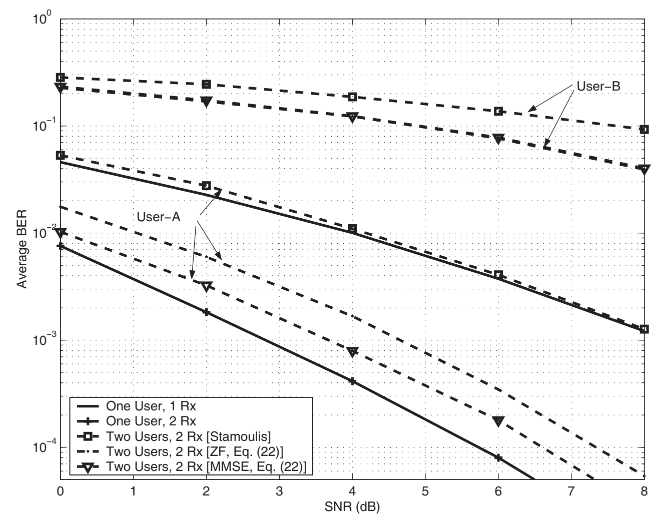
V. Simulation Examples

Figure 1 shows the average BER performance of the proposed interference suppression technique. The simulation results assume two co-channel users, A and B, each equipped with 3 transmit antennas and employing rate-1/2 complex OS-TBC [2, (37)]. All channels are assumed to be i.i.d Rayleigh fading channels. QPSK modulation is used and the receiver is assumed to have 2 antennas.

Fig. 1(a) shows the case where the two users have equal power. We also include the zero-forcing (ZF) solution, which can be obtained from the MMSE solution by setting $N_o = 0$. We observe that Stamoulis' IC technique for two users and two receive antennas delivers about the same ABER performance as if there were only one user and one receive antenna; this is consistent with the results in [4]. On the other hand, both the ZF and MMSE solutions have a significant SNR advantage over the Stamoulis' solution; for example, at ABER $= 10^{-3}$, the SNR advantage is approximately 3.5 dB. Therefore, although Stamoulis' solution separates the two users, it is not optimal with respect to the MSE criterion as it requires 2-stage receive processing and uses one receive antenna specifically to cancel out the interference from the other user.



(a) Equi-powered Users, $P_T(A) = P_T(B)$



(b) Power imbalance, $P_T(A) = 10P_T(B)$

Fig. 1. Average BER performance of User-A

Fig. 1(b) shows the case of a 10 dB power imbalance between the users. We see that for the higher-powered user, full matrix ZF results are about the same as in Fig. 1(a), but the MMSE case is about 1 dB better at ABER $= 10^{-3}$. It is well known that the lower-power user will suffer in a linear multi-user detector, and this is confirmed here for all three cases (the top three curves), although the full-matrix solutions still outperform Stamoulis et al.'s solution.

VI. Conclusion

A simple single-stage interference suppression method is proposed that exploits the algebraic structure of unity rate OSTBCs for 3 and 4 transmit antennas. The proposed method provides an MMSE-optimal interference suppression technique that is also low complexity because it avoids matrix inversion. The simulation results show the advantages of the proposed method.

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