# SNR Estimation for a Non-Coherent M-FSK Receiver in a Rayleigh Fading Environment

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***Abstract*—This paper deals with the problem of estimating average signal-to-noise ratio (SNR) for a communication system employing non-coherent M-ARY frequency shift keying (NCMFSK) over fading channels and white Gaussian noise (AWGN). We derive two estimators; one using maximum likelihood (ML) approach and other using the data statistics. Various scenarios have been taken into account including data-aided (DA), non-data aided (NDA) and joint estimation using both the data and pilot sequences. We also derive the Cramer-Rao bound (CRB) for the estimators. The results show that for a particular region of interest (e.g. high SNR or low SNR) and depending upon the availability of pilot sequence, a particular SNR estimation scheme is suitable.**

## I. Introduction

Estimates of signal-to-noise ratio (SNR) are used in many wireless receiver functions, including signal detection, power control algorithms and turbo decoding etc. The motivation for the study reported here is that SNR estimation is a way for a receiver to determine if it is near the edge of the decoding range of its source, and therefore, in a preferred location to participate in a cooperative transmission [1]. Furthermore, if the radios are energy constrained, e.g. if they are in a sensor network, non-coherent demodulations may be desired to reduce circuit consumption of energy. Therefore, in this paper, we consider the estimation of average SNR in an FSK non-coherent demodulator, over a Rayleigh fading channel.

Several authors have attacked the problem of estimating SNR for binary phase shift keying (BPSK) and frequency shift keying (FSK). For example, [2] compares a variety of techniques for SNR estimation in AWGN for M-PSK signals. Many approaches also include the channel effects such as multipath fading and address the issues of SNR estimation for fading channels for BPSK e.g. in [3-5]. FSK enables a power efficient transmitter and a simple receiver design that employs envelope detection. In [6], the authors have estimated the SNR for non-coherent binary FSK (NCBFSK) receiver, assuming unity noise power spectral density. In [7], the authors estimate the SNR for the NCBFSK system assuming no prior knowledge of noise. In this paper, we extend [7] to the M-ary FSK receiver. We derive two types of estimators of SNR, a maximum likelihood estimator (MLE) and an estimator that uses block statistics. We provide ML versions of partially data-aided (PDA), non-data aided (NDA), joint PDA-NDA, and fully data-aided (FDA) estimators for SNR. The PDA approach uses only the training sequence for estimation while the NDA approach does blind estimation using the entire sequence. The joint PDA-NDA uses all the information, operating blindly on the non-training part of the sequence. The FDA estimator uses the detected data as training sequence for SNR estimation and is reasonable in a multi-hop broadcast application, where every node must decode the entire message. The estimators derived in this paper are significantly different and lead to dramatically different results from those for slow (block) fading channels, which are addressed in [11].

The rest of the paper is organized as follows. In the next section, we describe the system model and the notations used for the MFSK system. Section III deals with the derivations of the SNR estimators, which includes three sub-cases for MLE and also the estimator using data statistics. Then we will derive the CRB, and in Section V, we will discuss the simulation results for various estimators and overall estimator performance in terms of mean-squared error and CRB. The paper then concludes in Section VI.

## II. System Model

Consider a communication system employing M-ARY FSK modulation, where each transmitted symbol is corrupted independently by fading and noise and the number of symbols in the constellation is $M = 2^n$, for a positive integer $n$. Even though we are targeting the non-coherent receiver, it is convenient to start with the assumption of a coherent FSK receiver. The resulting SNR estimates will happen to be in a form that can be computed in an NC-FSK receiver. Therefore, we will begin with the assumption, that the SNR estimation is done after the coherent matched filter. From [7], the received signal, for the MFSK case, is given as

$$\mathbf{x}_i = \mathbf{s}_i \alpha_i + \mathbf{n}_i, \tag{1}$$

where $i$ is the time index. Each of $\mathbf{x}_i$, $\mathbf{s}_i$, and $\mathbf{n}_i$ are real vectors with a dimension of $M \times 1$, and for $\mathbf{s}_i = [0, ..., 0, 1, 0, ..., 0]^T$, where 1 is in the $m_i$th position $(1 \leq m_i \leq M)$, where $m_i$ indicates the symbol transmitted at time $i$, and the other positions have a zero. For the sake of simplicity, we assume

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that the average symbol energy is unity so that the expected energy of the $ith$ received symbol is given as $E\left\{\alpha_i^2\right\} = S/2$, where $\alpha_i$ is a zero mean fading coefficient drawn from a Gaussian distribution. Similarly, the elements of $\mathbf{n}_i$ are also independent Gaussian random variables with zero mean and variance $N/2$, thus the signal-to-noise ratio (SNR) is given by $\gamma = S/N$. $\mathbf{s}_i$, $\alpha_i$, and $\mathbf{n}_i$ are assumed independent of each other. Our interest is to find the estimate of the average SNR using the observed data $\begin{bmatrix}\mathbf{x}_1^T & \mathbf{x}_2^T & \cdots & \mathbf{x}_k^T\end{bmatrix}^T$. For the estimation schemes considered, we assume that that there are $g$ pilot symbols and $l$ data symbols so that the total packet length is $k = g + l$. Throughout the paper, we assume perfect timing recovery at the receiver.

## III. Estimation Techniques

As mentioned previously, we will derive ML estimation for three cases, namely PDA, NDA and joint PDA-NDA. Another approach is using the statistics of observable data, which we call Estimation using Data Statistics (EDS).

### A. Partially Data Aided MLE

Without the loss of generality, the $g$ pilot symbols are set to $[1 \quad 0 \quad ... \quad 0]^T$. The received symbols from $M$ branches are denoted as $x_{m,i}$, where first index $m$ denotes the branch index where $m = 1, 2, ..., M$ and $i$ is the time index such that $i = 1, 2, ..., g$. The probability density functions (PDFs) of the received symbols $\mathbf{x}_i = [x_{1,i} \quad x_{2,i} \quad ... \quad x_{M,i}]^T$, are given as

$$p_{x_{1,i}}(x_{1,i}) = \frac{1}{\sqrt{\pi}\sqrt{S+N}} \exp\left(-\frac{x_{1,i}^2}{S+N}\right), \tag{2}$$

and

$$p_{x_{m,i}}(x_{m,i}) = \frac{1}{\sqrt{\pi}\sqrt{N}} \exp\left(-\frac{x_{m,i}^2}{N}\right), \tag{3}$$

for $m = 1, 2, ..., M$. The joint PDF of $\mathbf{x}_i$ is given as

$$p_{\mathbf{x}_i}(\mathbf{x}) = \frac{\pi^{-M/2}}{\sqrt{S+N}\ \sqrt[M-1]{N}} \exp\left(-\frac{x_1^2}{S+N} - \frac{1}{N}\sum_{m=2}^{M} x_m^2\right). \tag{4}$$

Thus the log-likelihood distribution of $g$ received symbols is given as

$$\Lambda_{\mathbf{x}_i}(\mathbf{x}; S, N) = -\, gM/2 \ln \pi - \frac{g}{2} \ln(S+N) - \frac{g(M-1)}{2} \ln N - \frac{1}{S+N}\left(\sum_{i=1}^{g} x_{1,i}^2\right) - \frac{1}{N}\sum_{m=2}^{M}\sum_{i=1}^{g} x_{m,i}^2. \tag{5}$$

To find the MLE of SNR $\hat{\gamma}$, we use the property that the ML estimate of the ratio of two parameters ($S$ and $N$ here), is the ratio of the individual ML estimates of the two parameters [8]. Thus using this property, the MLE of SNR can be written as

$$\hat{\gamma} = \frac{\hat{S}_{ML}}{\hat{N}_{ML}}. \tag{6}$$

Thus by differentiating $(5)$ with respect to $S$ and $N$, and setting the derivatives equal to zero results in

$$\hat{\gamma}_{DA} = \frac{(M-1)\sum_{i=1}^{g} x_{1,i}^2 - \sum_{m=2}^{M}\sum_{i=1}^{g} x_{m,i}^2}{\sum_{m=2}^{M}\sum_{i=1}^{g} x_{m,i}^2}. \tag{7}$$

### B. Non-Data Aided MLE

The PDF of the received symbol, given a 1 at the $nth$ position is expressed as

$$p_{\mathbf{x}_i}(\mathbf{x}|s_n = 1) = \frac{1}{\sqrt[M]{\pi}\sqrt{S+N}\ \sqrt[M-1]{N}} \exp\left(-\frac{x_{n,i}^2}{S+N} - \sum_{m=1, m\neq n}^{M} \frac{x_{m,i}^2}{N}\right). \tag{8}$$

Assuming equal prior probabilities of transmitted symbols, the PDF of the received symbols for M-FSK is given as

$$p_{\mathbf{x}_i}(\mathbf{x}) = \frac{M^{-1}\pi^{-M/2}}{\sqrt{S+N}\ \sqrt[M-1]{N}} \left[\underbrace{\exp\left(\frac{-x_1^2}{S+N} - \sum_{m=2}^{M}\frac{x_m^2}{N}\right)}_{a_1} + \underbrace{\exp\left(\frac{-x_2^2}{S+N} - \sum_{m\neq 2}^{M}\frac{x_m^2}{N}\right)}_{a_2} ... + \underbrace{\exp\left(\frac{-x_M^2}{S+N} - \sum_{m=1}^{M-1}\frac{x_m^2}{N}\right)}_{a_M}\right] \tag{9}$$

It can be seen from $(9)$, that the PDF contains sum of $M$ exponential terms, which are difficult to simplify into a product form. Thus taking log-likelihood and then differentials with respect to $S$ and $N$ will make the expression too complicated to solve. An idea to simplify the above equation is to apply an orthogonal transform on $\mathbf{a} = [a_1 \quad a_2 \quad ... \quad a_M]^T$, with elements as defined by the horizontal brackets in $(9)$, in order to make the expression simpler to solve. An appropriate method for this purpose is to apply Walsh coding [9] on these terms. Thus, we use

$$\mathbf{b} = \mathbf{H}\mathbf{a}, \tag{10}$$

where $\mathbf{b} = [b_1 \quad b_2 \quad ... \quad b_M]^T$ and $\mathbf{H}$ is the Hadamard matrix [9] of dimension $M \times M$. For BFSK, where $M = 2$, we use the Hadamard matrix of order 2, which is given as

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{11}$$

Thus using Sylvester's construction [9], we can make a Walsh matrix for any $M$. From $(10)$, $\mathbf{a}$ is given by the inverse transformation as $\mathbf{a} = \mathbf{H}^{-1}\mathbf{b}$, which simplifies the PDF to

$$p_{\mathbf{x}_i}(\mathbf{x}) = \frac{1}{M\ \sqrt[M]{\pi}\sqrt{S+N}\ \sqrt[M-1]{N}} \exp\left(-\frac{\sum_{m=1}^{M} x_{m,i}^2}{N}\right)\left[\sum_{m=1}^{M} \exp\left(-x_{m,i}^2\phi\right)\right], \tag{12}$$

where $\phi = \left[\frac{1}{S+N} - \frac{1}{N}\right]$. Thus the log-likelihood function for the k received symbols is given as

$$\Lambda_{\mathbf{x}_i}(\mathbf{x}) = -k\ln\left(M\pi^{M/2}\right) - \frac{k}{2}\ln(S+N) - \sum_{m=1}^{M}\sum_{i=1}^{k}\frac{x_{m,i}^2}{N} - \frac{k(M-1)}{2}\ln N + \sum_{i=1}^{k}\ln\left(\sum_{m=1}^{M}\exp\left(-x_{m,i}^2\phi\right)\right). \tag{13}$$

Taking partial derivative of (13) with respect to $S$ and $N$ results in

$$\frac{\partial\Lambda}{\partial S} = \frac{-k}{2(S+N)} + \frac{1}{(S+N)^2}\sum_{i=1}^{k}\frac{\sum_{m=1}^{M} x_{m,i}^2\exp\left(-x_{m,i}^2\phi\right)}{\sum_{m=1}^{M}\exp\left(-x_{m,i}^2\phi\right)}, \tag{14}$$

$$\frac{\partial\Lambda}{\partial N} = -\frac{k}{2(S+N)} - \frac{(M-1)k}{2N} + \sum_{m=1}^{M}\sum_{i=1}^{k}\frac{x_{m,i}^2}{N^2} + \left[\frac{1}{(S+N)^2} - \frac{1}{N^2}\right]\sum_{i=1}^{k}\frac{\sum_{m=1}^{M} x_{m,i}^2\exp\left(-x_{m,i}^2\phi\right)}{\sum_{m=1}^{M}\exp\left(-x_{m,i}^2\phi\right)}. \tag{15}$$

Putting the above equations equal to zero and solving them simultaneously gives us following result

$$\hat{S} + M\hat{N} = \frac{2}{k}\sum_{m=1}^{M}\sum_{i=1}^{k} x_{m,i}^2, \tag{16}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{k} x_{m,i}^2 - \frac{k(M-1)}{2}N = \sum_{i=1}^{k}\frac{\sum_{m=1}^{M} x_{m,i}^2\exp\left(-x_{m,i}^2\phi\right)}{\sum_{m=1}^{M}\exp\left(-x_{m,i}^2\phi\right)}. \tag{17}$$

The above forms of the equations will prohibit the closed form solutions for estimates of $S$ and $N$. Thus using high SNR approximations, the summation term is approximated as

$$\sum_{i=1}^{k}\frac{\sum_{m=1}^{M} x_{m,i}^2\exp\left(-x_{m,i}^2\phi\right)}{\sum_{m=1}^{M}\exp\left(-x_{m,i}^2\phi\right)} \approx \sum_{i=1}^{k}\max_{m=1,\ldots,M}\left(x_{m,i}^2\right), \tag{18}$$

and the estimate of noise power is given as

$$\hat{N} = \frac{2}{(M-1)k}\left[\sum_{m=1}^{M}\sum_{i=1}^{k} x_{m,i}^2 - \sum_{i=1}^{k}\max_{m=1,\ldots,M}\left(x_{m,i}^2\right)\right]. \tag{19}$$

Thus the estimate of signal to noise ratio is given by

$$\hat{\gamma}_{NDA} = \frac{-\sum_{m=1}^{M}\sum_{i=1}^{k} x_{m,i}^2 + M\sum_{i=1}^{k}\max_m\left(x_{m,i}^2\right)}{\sum_{m=1}^{M}\sum_{i=1}^{k} x_{m,i}^2 - \sum_{i=1}^{k}\max_m\left(x_{m,i}^2\right)}. \tag{20}$$

### C. Joint Estimation Using Pilot and Data Symbols

Consider $g$ pilot symbols and $l$ data symbols, so that the total packet is of length $k = g + l$. Assuming independent received symbols, the joint PDF is the product of PDFs resulting from pilot and data symbols. So we use (4) for

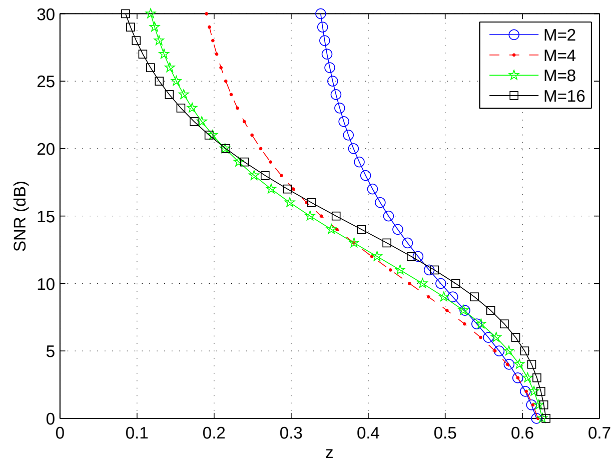


Fig. 1. Relationship of computed $z$ and $\gamma$ for different $M$

$i = 1, 2, ..., g$ and (12) for $i = g+1, ..., g+l = k$. Thus the log-likelihood function from the joint PDF is given as

$$\Lambda_{joint} = -\frac{k}{2}\ln(S+N) - \frac{k(M-1)}{2}\ln N - \frac{kM}{2}\ln\pi - \frac{1}{N}\sum_{m=2}^{M}\sum_{i=1}^{g} x_{m,i}^2 - \frac{1}{N}\sum_{m=1}^{M}\sum_{i=g+1}^{k} x_{m,i}^2 - \frac{1}{S+N}\sum_{i=1}^{g} x_{1,i}^2 + \sum_{i=g+1}^{k}\ln\left(\sum_{m=1}^{M}\exp\left(-x_{m,i}^2\phi\right)\right) \tag{21}$$

Using similar approximations as done in the previous section and taking partial derivatives with respect to $S$ and $N$ and setting them equal to zero result in the estimate of SNR as given in (22).

### D. EDS Approach

Define an $M \times M$ matrix $\mathbf{Z}$ which is determined by using the observed data as

$$\mathbf{Z} = (E|\mathbf{x}_i|)(E|\mathbf{x}_i|)^T\left(E\left\{\mathbf{x}_i\mathbf{x}_i^T\right\}\right)^{-1} \tag{23}$$

where $E|\mathbf{x}| = [E|x_1| \;\; E|x_2|\cdots \;\; E|x_M|]^T$. Since $\mathbf{x}_i = [x_{1,i} \;\; x_{2,i} \;\; \ldots \;\; x_{M,i}]^T$, where each of the elements of $\mathbf{x}$ are mutually uncorrelated, thus the autocorrelation of $\mathbf{x}$ is a diagonal matrix given as

$$E\left\{\mathbf{x}_i\mathbf{x}_i^T\right\} = \frac{1}{2M}(S+MN)\mathbf{I}_M, \tag{24}$$

where $\mathbf{I}_M$ is the identity matrix of dimension $M \times M$. This method proceeds from the observation that if we have a random variable X which follows a Normal distribution with zero mean and variance $\sigma^2$, the absolute value $|X|$ follows a half-Normal distribution which has a mean $\sqrt{2/\pi}\sigma$. Using this fact and that each element of $\mathbf{x}$ is identically distributed, we have

$$E|\zeta| = \frac{1}{M}\sqrt{\frac{1}{\pi}}\left[\sqrt{S+N} + (M-1)\sqrt{N}\right], \qquad \zeta \in \{\mathbf{x}\} \tag{25}$$

and

$$(E|\zeta|)^2 = \frac{N}{M^2\pi}\left[\gamma + 1 + (M-1)^2 + 2(M-1)\sqrt{1+\gamma}\right]. \tag{26}$$

Thus $\mathbf{Z}$ turns out to be a matrix with all elements being equal and is expressed as $z\mathbf{1}_M$, where $\mathbf{1}_M$ is an $M \times M$ matrix of all ones and $z$ is given as

$$z = \frac{(E|\zeta|)^2}{E\{\zeta^2\}}. \qquad \zeta \in \{\mathbf{x}\} \tag{27}$$

After putting in the values from (24) and (26), z is given as

$$z = \frac{2\left(\gamma + 1 + (M-1)^2 + 2(M-1)\sqrt{1+\gamma}\right)}{M\pi(M+\gamma)}. \tag{28}$$

where $\gamma = S/N$ is the signal to noise ratio. In practice, we replace the expectations in (23) with the corresponding block averages to compute the SNR estimate. From (28), the relationship between z and $\hat{\gamma}$ is given as

$$\begin{aligned}\hat{\gamma} =& \frac{1}{(-2+Mz\pi)^2}\left[(2-z\pi)\left(M^3 z\pi + 2M^2 - 4M\right)\right.\\ &\left. + 4\sqrt{\pi}\sqrt{(M-1)^3 M^2 z(2-z\pi)}\right].\end{aligned} \tag{29}$$

From Figure 1, it can be seen that, in the higher SNR regime, the EDS approach will suffer badly and will not give accurate estimates because all curves becomes steeper as the SNR rises. The same phenomenon can be seen at low SNR for higher $M$. But in general, the accuracy of SNR estimation becomes greater as we increase $M$, since the curves become more and more smooth as compared to small values of $M$. It should also be noted that the estimate of SNR resulting from EDS approach is similar as that of MLE, if the estimation is done using pilot sequence alone. Thus the EDS approach mentioned here will be treated as the non-data aided scheme for SNR estimation.

## IV. Cramer-Rao Lower Bound

In this section, we will derive the CRB for the SNR estimator. Although we could have different CRBs for all the different cases discussed in the previous section, the one that would really serve as a benchmark on the variance of all estimators is from the FDA case, which is same as PDA but uses all the information in the packet as a training sequence. Considering the unknown parameters as a vector i.e., $\theta = [S \quad N]^T$, the CRB for the SNR is given as [10]

$$CRB = \frac{\partial \mathbf{g}(\theta)}{\partial \theta}\mathbf{I}^{-1}(\theta)\frac{\partial \mathbf{g}(\theta)^T}{\partial \theta}, \tag{30}$$

where $\mathbf{g}(\theta) = \frac{S}{N}$ and the Jacobian of $\mathbf{g}(\theta)$ is given as

$$\mathbf{J_g}(\theta) = \begin{bmatrix} \frac{1}{N} & \frac{-S}{N^2} \end{bmatrix}, \tag{31}$$

and $\mathbf{I}(\theta)$ is the Fisher information matrix (FIM) given as

$$[\mathbf{I}(\theta)]_{ij} = -E\left[\frac{\partial^2 \Lambda}{\partial \theta_i \theta_j}\right]. \tag{32}$$

The FIM for the FDA estimator is given as

$$\mathbf{I}(\theta) = \begin{bmatrix} \frac{k}{2(S+N)^2} & \frac{k}{2(S+N)^2} \\ \frac{k}{2(S+N)^2} & \frac{k}{2(S+N)^2} + \frac{k(M-1)}{2N^2} \end{bmatrix}, \tag{33}$$

which gives the CRB from (30) as

$$CRB_{FDA} = \frac{2M}{k(M-1)}(1+\gamma)^2. \tag{34}$$

This bound has been plotted in Figures 3 and 4, which are further discussed below.

## V. Simulation Results

In this section, we compare the normalized mean squared error (normalized with respect to the square of the true value of the average SNR) of the estimators using simulations for different values of $M$ and for different packet lengths averaged over 10,000 trials. Figure 2 shows the NMSE vs. true SNR for the PDA estimator, with 1000 pilot symbols in the packet for increasing values of $M$. We observe that the estimates become more and more accurate as we have more and more branches with noise only. Thus increasing $M$ indirectly increases the number of samples, which gives lower NMSE. Although not shown in the figures, this behavior is found in all techniques discussed in Section III.

In Figure 3, the "long" packet is assumed to comprise 100 pilot symbols and 900 data symbols. We observe that the PDA NMSE is approximately constant over the entire SNR range. The NDA and *Joint* cases perform similarly because most of the packet is data and the NMSE is high in the low SNR region. This can be attributed to the approximation errors in the low SNR regime. The EDS method shows bad behavior at high SNR due to the steepness of curve from Figure 1. However, it performs better for low SNR estimation as compared to the NDA scheme because of the same approximation that we have done for the NDA in low SNR. For high SNR, the *Joint* estimation scheme works the best as expected. The crossing of the curves suggests that an adaptive mode of SNR estimation can also be derived consisting of estimation from the pilot only (PDA) or EDS during the low SNR while using the entire data packet for estimating high SNR values. In that case, the overall NMSE will remain minimum over a wide range of SNR values.

$$\hat{\gamma}_{joint} = \frac{\sum_{m=2}^{M}\sum_{i=1}^{g} x_{m,i}^2 + \sum_{m=1}^{M}\sum_{i=g+1}^{k} x_{m,i}^2 + M\sum_{i=g+1}^{k}\max_{m=1,\ldots,M}\left(x_{m,i}^2\right) + (M-1)\sum_{i=1}^{g} x_{1,i}^2}{\sum_{m=2}^{M}\sum_{i=1}^{g} x_{m,i}^2 + \sum_{m=1}^{M}\sum_{i=g+1}^{k} x_{m,i}^2 - \sum_{i=g+1}^{k}\max_{m=1,\ldots,M}\left(x_{m,i}^2\right)} \tag{22}$$

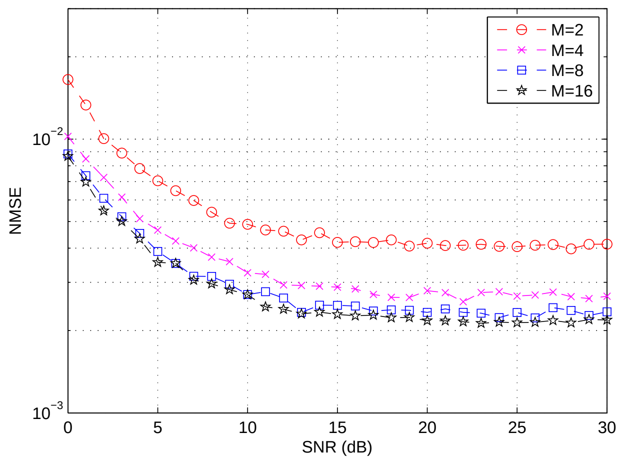


Fig. 2. Effect of increasing M on NMSE for k=1000

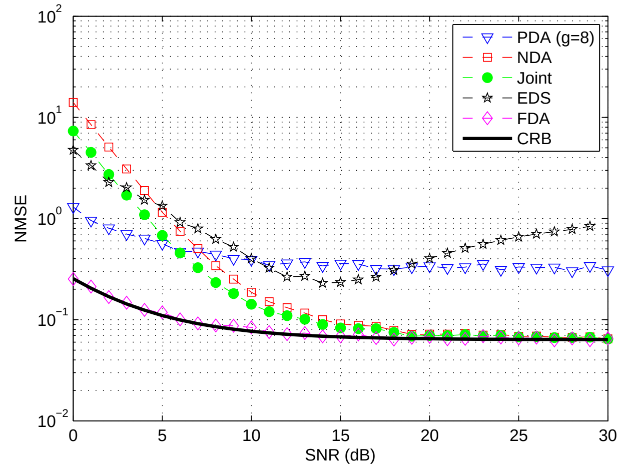


Fig. 4. NMSE for different schemes with M=8, k=36, (g=8)

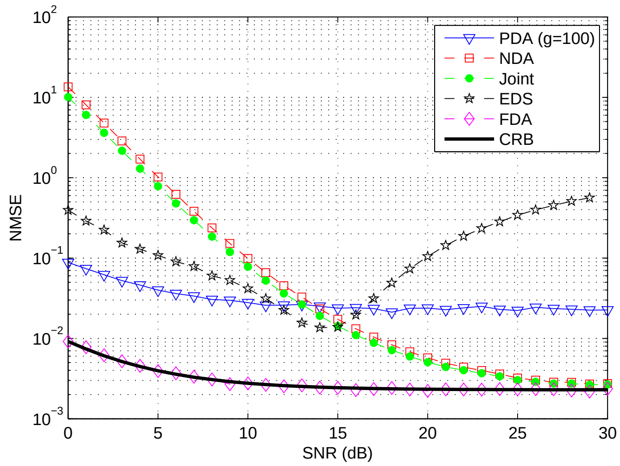


Fig. 3. NMSE for different schemes for M=8, k=1000 (g=100)

Figure 4 treats the short packet scenario, with 8 pilot symbols and 28 data symbols. The EDS approach does not perform well because of the limitations of the availability of data (approximation error of the ensemble averages with time averages for small data set is large). Thus for a short length packet and with the availability of pilot, the joint data estimation performs best. If the pilot is not available, then the NDA MLE also gives better performance.

Figures 3-4 include the results for the FDA approach (decision feedback), which utilizes the detected data, assuming no detection errors. It can be seen that the performance of the FDA estimator is enhanced significantly and it reaches the CRB. Using this approach, we gain two advantages: a larger data set and estimation using DA approach which has no approximation errors. The *no errors* assumption is good in the context of the Decode and Forward (DF) scenario, since passing the CRC check is a precondition for forwarding the packet [1].

## VI. Conclusion

We have derived the MLEs and CRB for SNR for an MFSK system assuming different degrees of data knowledge in a packet. It is thus concluded that different scenarios lead to different results based on packet length, availability of pilot sequence, and the region of SNR considered (low/high). If we cannot feed back detected symbols, (i.e., the FDA scheme), then an adaptive scheme is suggested. However, for the Decode and Forward cooperative relay applications, the FDA method gives good results.

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