Lifetime Optimization of Multi-hop Wireless Sensor Networks by Regulating the Frequency of Use of Cooperative Transmission

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Abstract—**Multi-hop wireless sensor networks with a single Sink are attractive from a cost point of view. When battery-driven, these networks suffer from the energy hole problem, where the nodes near the Sink die first because they must forward the packets of all the other nodes in the network. In this paper, by developing an analytical model, we show how Cooperative Transmission (CT), when it is used to extend the communication range, can balance the energy and avoid the energy hole. Using the analytical model, we derive the optimal rates of CT forwarding and the corresponding lifetime extensions compared to the non-CT case. We verify the accuracy of our analysis and effectiveness of our forwarding protocol through network simulations.**

I. Introduction

In wireless sensor networks (WSNs), the data collected from sensors is usually gathered in one or more base stations (Sinks), which are considered to have no energy constraint and sometimes unlimited resources. Because of this many-to-one or many-to-few characteristic, WSNs suffer from a so-called "energy hole" problem [1], which can be described as the situation when the nodes around the Sink consume relatively more energy and die early. When this energy hole is created, not a single node can reach the Sink, and the whole network becomes useless even though majority of the nodes still have enough energy to sense and send data. Because of this reason, the lifetime of the network is bounded by the lifetime of the nodes close to the Sink.

In the case of the multi-hop network, it has been shown that the energy hole problem cannot be overcome when nodes are uniformly distributed [2], and solving the energy hole problem by deploying the node non-uniformly is introduced [3], [4]. In this paper, however, we analytically show that balancing the energy is possible while maintaining the uniformity when Cooperative Transmission (CT) is used for the range extension. CT [5] is a mixture of communication protocol and physical layer combining scheme that can create a Multiple-Input-Single-Output (MISO) system virtually by utilizing separated antennas in multiple communication devices. Through cooperative diversity and simple aggregation of transmit power (array gain), CT can have an SNR advantage over non-CT schemes, and this advantage can be used to save transmit power, increase data rate, or extend communication range. The range extension strategy, in particular, is exploited in this paper. To achieve our goal, we devise a unique analytical model for avoiding the energy hole using CT, which has not been developed in the existing literature. The analytical model provides the average probability (rate) of doing CT for nodes in the network, and we provide a CT forwarding (routing) strategy that uses this feature. Our CT forwarding scheme is unique and simple because, unlike existing energy-efficient routing protocols [6], [7], [8], [9] that utilize the residual energy information (which needs to be updated from time to time), it only uses the level information to balance the energy consumption of the nodes and extend the network lifetime. Also, unlike the works of the energy-efficient routing protocols that rely on the simulation results to figure out how much lifetime can be extended [6]-[9], we derive the amount of the expected extended lifetime when our CT method is used through our analytical model.

Our paper is organized as follows. Section II introduces the models and assumptions that are used in this paper. Developing and introducing CT method through analysis are provided in Section III. The simulation results using our CT algorithm are presented in Section IV, and the concluding remark is offered in Section V.

II. Assumptions and Definitions

We make the following assumptions. The network is a many-to-one wireless sensor network that has a Sink with unlimited energy and resources so that it can communicate with any of the nodes directly even without CT. A node can successfully decode-and-forward a packet without an error when its received signal-to-noise ratio (SNR) is greater than or equal to a modulation-dependent threshold. Every node has equal energy at the beginning. The nodes all use equal transmit power for transmission, and therefore have the same maximum transmission range, $d_{\text{tx}}^{\max}$. We also assume that any node within this range $d_{\text{tx}}^{\max}$ of the transmitter will have high enough received SNR to decode the data without an error.

For cooperative routing, a non-CT route is formed first (*primary route* [10]). Non-CT routing is based on the shortest

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TABLE I
DIVERSITY GAIN AND RANGE EXTENSION (BPSK. BER = 10^{-3})

N_c	2	3	4
$G(N_c)$ (dB)	7.5	9.23	9.98
d_{ext}	$2.3d_{\text{link}}$	$3.04d_{\text{link}}$	$3.56d_{\text{link}}$

(minimum) hop for both CT and non-CT. Note that energy-aware routing protocols [6], [7], [8] cannot mitigate the energy hole problem for many-to-one network especially when the transmit power is fixed because the situation that all packets must be handled by nodes one-hop away from the Sink does not change. This lets us focus on the shortest-hop routing for our analysis, which is used in other papers that analyze the energy hole problem [3], [11].

Our CT method utilizes the extended range obtained by CT and tries to form a virtual MISO (vMISO) link between cooperators and the Sink. We assume that the orthogonal diversity channel of CT is obtained by time. The amount of the extended range of CT when the transmit power is fixed is provided in [10]. When N_c cooperators are using the same transmit power, P_{TX}, array and diversity gains are added to the received SNR, and the extended range of CT denoted as d_{ext} is

$$d_{\text{ext}} = d_{\text{link}} \cdot (N_c \cdot 10^{G(N_c)/10})^{1/\alpha}, \quad (1)$$

where d_{link} is the non-CT range when P_{TX} is used, $G(N_c)$ is the diversity gain in dB, and α is the path loss exponent. In this paper, $d_{\text{link}} = d_{\text{tx}}^{\max}$ because the transmit power is fixed.

For the physical layer, we assume a slowly varying (remains same for the entire virtual MISO transmission) Rayleigh fading channel and a log-normal shadowing with the shadowing standard deviation of 5dB and the path loss exponent of 2.9 (values from [12]). The modulation scheme is assumed to be BPSK. To get the cooperative diversity gain, $G(N_c)$, in this environment, we obtain array gain and cooperative diversity gain using Monte Carlo simulation for $N_c = 1$ to 4 with target BER of 10^{-3}, and then, we get the pure cooperative diversity gain by subtracting the array gain part ($10\log_{10} N_c$) according to the number of cooperators. The value of the cooperative diversity gain with varying numbers of cooperator (N_c) is given in Table I along with the value for d_{ext} in (1).

We define C_i as the area of a circle with its center at the location of the Sink and having a radius of $i \times d_{\text{tx}}^{\max}$, where $i \geq 1$ and $C_0 = 0$ as shown in Fig. 1. The network outer boundary is C_L, a circle with radius $L \cdot d_{\text{tx}}^{\max}$, with its center at the Sink. Let $A_i = C_i - C_{i-1}$ be the area between two circle boundaries. We also refer to this A_i as "i-th level" or "Level i". As in [11], we assume that the nodes in adjacent A_i's can communicate with each other. We also assume that the nodes in A_{i-1} equally share the traffic coming from A_i, which is the case when the nodes are uniformly deployed or the node density is high [11]. Any node in the network except for the Sink can be a source at random.

To define the lifetime of the network, we adopt the notion of *tasks* that is used in [11]. As in [11], one task involves transmitting a sensed data packet all the way to the Sink. The number of successfully performed tasks matches with the number of packets that successfully reach the Sink, and this value is widely used as the lifetime of the network [6], [9], [13]. Therefore, the number of tasks that can be performed before the network is completely disconnected will be considered as the lifetime of the network in this paper. Here, connectivity is determined by the number of nodes that can reach the Sink. We assume that tasks are well scheduled so that each task can be performed one after the other, incurring no collision.

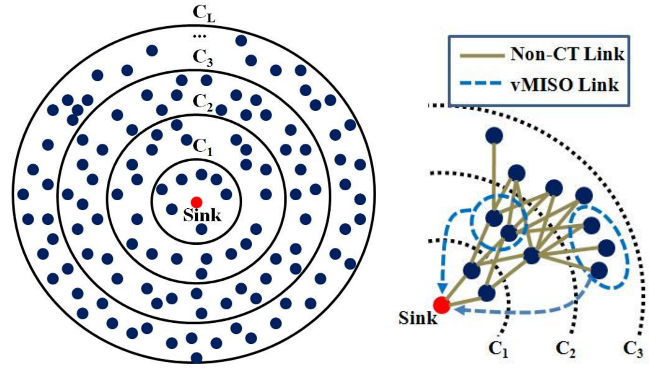


Fig. 1. Illustrations of our circular network (left) and vMISO (right). The illustration on the right shows non-CT links between adjacent levels and the extended vMISO link (by using CT) between cooperators and the Sink.

Following [11], we define the following variables. Let T denote the number of tasks performed during the lifetime of the network and n_i be the number of the nodes in the area A_i. Since the nodes in A_1 need to take care of all the tasks in the network, the expected number of tasks per node in A_1 is $\frac{T}{n_1}$. The nodes in A_i ($i \geq 2$) must relay all the tasks originating from A_k ($k > i$), but do not have to relay the ones from A_j ($j < i$). The total number of tasks originating from the nodes in A_i can be expressed as $\frac{n_i}{n}T$ where n is the number of the nodes in the entire network ($n = \sum_{i=1}^{L} n_i$). Therefore, the total number of the tasks, T_i, that nodes in A_i should handle is $T_i = T - T\left(\sum_{k=1}^{i-1} n_k/n\right)$. Then, the expected number of tasks per node in A_i is $\frac{T_i}{n_i}$. Note that $T_1 = T$, which means that the nodes in A_1 should handle all the tasks till the lifetime of the network, and the network lifetime is limited by the lifetime of the nodes in A_1.

We define E_i as the total energy consumed by a node in A_i. Also, E_{TX} and E_{RX} denote the energy consumptions for transmitting and receiving a packet (task) respectively, and E_{relay} denotes the energy consumption for relaying (E_{relay} = E_{TX} + E_{RX}). Since we are assuming a fixed transmit power, E_{TX} is fixed, and we define a fixed positive variable η as $\eta = E_{\text{RX}}/E_{\text{TX}}$. Then, E_{relay}=$(1+\eta)E_{\text{TX}}$.

III. THE ANALYSIS AND DERIVATION OF CT FORWARDING PROTOCOL

The main idea of our CT forwarding strategy is to balance the load of the nodes by using the range extension strategy of CT, which is illustrated in Fig. 1. If the nodes in A_2 use CT and the range extension when CT is used is more than twice the non-CT range ($d_{\text{ext}} \geq 2d_{\text{link}}$), then the nodes in A_2 can reduce the burden of the nodes in A_1 by communicating directly to the Sink using CT. In general, if the nodes in A_i use CT and if the range extended by CT is more than i times of the non-CT range ($d_{\text{ext}} \geq i \cdot d_{\text{link}}$), the nodes in A_j ($j < i$) can reduce their burdens. When CT is done by the nodes in M level(s) (starting from A_2 to A_{M+1}), it will be referred to as "M-Level" CT. M can be thought of as the maximum number of levels being "hopped over" by CT.

Here, we focus on a circular-shaped network (Fig. 1) and uniform distribution of nodes to argue that balancing the energy consumption is possible for this environment, which is shown to be impossible when non-CT is used [2]. Because of its relative simplicity, we first investigate the 1-Level CT ($M = 1$), in which CT is done only by the nodes in A_2.

Let T_c be the number of tasks performed during the lifetime of the network when CT is used, and let $N_{c,i}$ be the minimum number of the cooperating nodes in A_i required to directly reach the Sink ($i \geq 2$). Note that $N_{c,i}$ depends on CT's ability to extend the non-CT range, and $N_{c,i}$ should be large enough to satisfy $d_{\text{ext}} \geq i \cdot d_{\text{link}}$. Now, let's assume that the nodes in A_2 will do CT for $T_{c,2}$ tasks (throughout the paper, the number of tasks for which the nodes in A_i do CT will be denoted as $T_{c,i}$). As previously discussed, without cooperation, the nodes in A_1 are involved in all tasks, but now that $T_{c,2}$ tasks are handled by the nodes in A_2, the nodes in A_1 take care of only $T_c - T_{c,2}$ tasks. Therefore, a node in A_1 is involved in handling $(T_c - T_{c,2})/n_1$ tasks. Note that when a node is a source, it only consumes transmitting energy, E_{TX}, whereas, when a node is a relay, it consumes E_{relay}. Since each node in the network generates T_c/n tasks on average till the lifetime ends, this is the average amount of "source" tasks generated by a node in A_1. Therefore, the expected number of tasks that a node in A_1 has to relay is $(T_c - T_{c,2})/n_1$ - T_c/n, and the amount of energy that a node in A_1 consumes, E_1, is

$$E_1 = \left(\frac{T_c - T_{c,2}}{n_1} - \frac{T_c}{n}\right) \cdot E_{\text{relay}} + \frac{T_c}{n} E_{\text{TX}}. \tag{2}$$

Next, we obtain E_2, the total energy consumed by a node in A_2. Whether using CT or not, the actual number of tasks that nodes in A_2 need to take of remains the same (which is T_2). However, when CT is used, the energy consumption of the nodes in A_2 per task will increase because cooperation involves the usage of more than one node. To account for the additional energy required for cooperation, we define the number of *virtual tasks* to be the total number of extra transmissions caused by CT. Virtual tasks do not add to the total number of actual tasks (number of packets reached the Sink), but are required to take the additional energy consumption of CT into account. When handling tasks in Level 2, one of the nodes in Level 2 is always used regardless of doing CT or not. Therefore, when $N_{c,2}$ nodes are cooperating, one of these nodes cannot be considered as facing the additional tasks because that node is used even in the case of non-CT. Because of this reason, the additional tasks caused by CT (virtual tasks) can be expressed as $(N_{c,2} - 1) \times T_{c,2}$. Therefore, the nodes in A_2 should deal with $T_2 + (N_{c,2} - 1) \times T_{c,2}$ tasks, and the expected number of tasks per node in A_2 when CT is used is $(T_2 + (N_{c,2} - 1) \times T_{c,2})/n_2$. The average amount of source tasks generated by a node in A_2 is T_c/n, and therefore, we get E_2 as follows:

$$\begin{aligned} E_2 = \quad & \left(\frac{T_c - \frac{n_1}{n}T_c + (N_{c,2} - 1)T_{c,2}}{n_2} - \frac{T_c}{n}\right) \cdot E_{\text{relay}} \\ & + \frac{T_c}{n} E_{\text{TX}}. \end{aligned} \tag{3}$$

When $E_2 > E_1$, the nodes in A_2 die first, and $E_2 < E_1$ creates the energy hole in A_1. Therefore, it is clear that $E_2 = E_1$ will balance the energy consumption so that the lifetime of the network can be optimally increased (the increased lifetime through our method is discussed later). Before solving $E_2 = E_1$, it is helpful to reduce the number of variables. Note that $A_i = \pi(i \cdot d_{\text{tx}}^{\max})^2 - \pi((i-1) \cdot d_{\text{tx}}^{\max})^2 = \pi(d_{\text{tx}}^{\max})^2(2i-1)$. If we define the node density as ρ, the uniformity of node distribution gives $\rho = n_1/\{\pi(d_{\text{tx}}^{\max})^2\} = n_i/\{\pi(d_{\text{tx}}^{\max})^2(2i-1)\}$. Then, n_i and n can be expressed using n_1 as

$$\begin{aligned} & n_i = (2i-1)n_1, \\ & n = \textstyle\sum_{i=1}^{L} n_i = \sum_{i=1}^{L}(2i-1)n_1 = L^2 \cdot n_1. \end{aligned} \tag{4}$$

By solving $E_2 = E_1$ using (2), (3) and (4), the following relationship can be obtained:

$$T_{c,2} = \frac{2L^2 + 1}{L^2(N_{c,2} + 2)} T_c. \tag{5}$$

Note that if $N_{c,2}$ goes to infinity, $T_{c,2}$ becomes zero, which means that if too many cooperators are required, it is too expensive to do CT (so CT task is not used), and our CT model becomes a non-CT model.

In (5), L can be obtained when the network size is fixed, and $N_{c,2}$ is determined based on the amount of the extended range when CT is used. But still, T_c remains unknown. Observe that the number of the tasks that the nodes in A_2 will face is $T_2 = (1 - n_1/n)T_c$, which can be rewritten as $(1 - 1/L^2)T_c$ by using (4). Among these tasks, $T_{c,2}$ tasks (which is (5)) should be selected to cooperate, and, by dividing (5) by $(1 - 1/L^2)T_c$, we get the fraction of the tasks in A_2 that should do CT, which is

$$\frac{T_{c,2}}{T_2} = \frac{2L^2 + 1}{(L^2 - 1)(N_{c,2} + 2)}. \tag{6}$$

This motivates us to use CT with probability equal to the value in (6). That is, the nodes in A_2 choose to cooperate with the probability in (6) forcing $T_{c,2}$ to satisfy (5). If we define $\text{Prob}_{j,i}$ as the probability of doing CT for a node in A_i when the j-Level CT scheme is used, then (6) is denoted as $\text{Prob}_{1,2}$. It is not hard to prove that $0 < \text{Prob}_{1,2} < 1$ for $L \geq 2$ and $N_{c,2} \geq 2$. By regulating the CT usage based on $\text{Prob}_{1,2}$, we can achieve E_1=E_2, which is shown to be impossible when non-CT is used [2].

Now, we derive the extension of the lifetime, which is the increase in the number of tasks that can be performed when CT is used. Let us denote T_{nc} as the number of tasks performed during the lifetime of the network when non-CT is used. When T_{nc} tasks are performed for non-CT, the nodes in A_1 form an energy hole. However, in the case of the CT, the nodes in A_1 handle $T_{c,2}$ less tasks than non-CT, which means CT can perform "at least" $T_{c,2}$ more tasks than non-CT until the energy hole is generated. The reason why the increased lifetime (tasks) is not exactly $T_{c,2}$ is because $T_{c,2}$ tasks are "relay" burdens, and a relaying consumes more energy than just transmitting a packet (when the node is a source). The increased number of tasks by using CT, which will be denoted as T_x, has the following relationship with T_c:

$$T_c = T_{nc} + T_x. \tag{7}$$

To get the relationship between T_c and T_{nc}, we first express T_x in terms of T_c as follows. The additional T_x tasks are generated by all nodes in the network, and therefore, each node in A_1 will generate T_x/n tasks on average, and the expected number of tasks generated by nodes in A_1 is $n_1 \cdot T_x/n$ (out of T_x tasks). The rest $T_x - n_1 \cdot T_x/n$ tasks are the expected number of tasks relayed by the nodes in A_1. Therefore, the average energy consumption of the nodes in A_1 to carry out additional T_x tasks can be expressed as

$$\left(T_x - \frac{n_1 \cdot T_x}{n}\right) \cdot E_{\text{relay}} + \frac{n_1 \cdot T_x}{n} E_{\text{TX}}. \tag{8}$$

Note that $T_{c,2}$ is the number of tasks that the nodes in A_1 would have relayed if CT is not used, and therefore, the energy consumption for relaying $T_{c,2}$ tasks should be equal to (8). Solving $T_{c,2} \cdot E_{\text{relay}}$ = (8) gives

$$T_x = \frac{E_{\text{relay}}}{E_{\text{TX}} + (L^2-1)E_{\text{relay}}} \times \frac{2L^2+1}{(N_{c,2}+2)} T_c, \tag{9}$$

where we have used (4) and (5).

The relationship between T_c and T_{nc} is obtained by substituting (9) into (7), which yields

$$\frac{T_c}{T_{nc}} = \frac{(N_{c,2}+2)((L^2-1)\eta + L^2)}{N_{c,2}L^2 - 1 + (N_{c,2}L^2 - N_{c,2} - 3)\eta}, \tag{10}$$

where $E_{\text{relay}} = (1+\eta)E_{\text{TX}}$ is used. (10) is the lifetime extension factor when CT is used, and it is not hard to show that (10) is always larger than 1. Therefore, CT always results in performing more tasks than non-CT.

According to Table I, N_c=2 gives d_{ext}=$2.3 d_{\text{link}}$, which means that, with 2 cooperating nodes, the range extension is more than twice. Therefore, $N_{c,2}$ = 2, and if we substitute this value to (10), we get $\{4(\eta+1)L^2 - 4\eta\}/\{2L^2(1+\eta) - 5\eta - 1\}$, which is a monotone decreasing function of L (≥ 2) and converges to 2 as L goes to infinity. This implies that the lifetime extension is more than twice regardless of the size of the network (L).

So far, we developed 1-Level CT in which CT is done only by the nodes in A_2. For the nodes in A_k ($k \geq 3$), it is also possible that they can reach the Sink directly with more cooperating nodes. Consider the 2-Level CT in which the nodes in A_2 and A_3 do CT for $T_{c,2}$ and $T_{c,3}$ tasks, respectively. The burden of the nodes in both A_1 and A_2 is reduced by $T_{c,3}$ and the expected number of tasks per node in A_1 when CT is used is $(T_c - T_{c,2} - T_{c,3})/n_1$. Likewise, the expected number of tasks per node in A_2 is $\{T_c(1 - n_1/n) + (N_{c,2}-1)T_{c,2} - T_{c,3}\}/n_2$. For the nodes in A_3, the actual number of tasks handled by the nodes is $T_c(1 - n_1/n - n_2/n)$, but we need to add the virtual tasks, $(N_{c,3}-1)T_{c,3}$, to get the energy value. By considering the fact that each node in the network generates T_c/n messages on average, we get E_i's as follows:

$$\begin{aligned}
E_1 = & \left(\frac{T_c - T_{c,2} - T_{c,3}}{n_1} - \frac{T_c}{n}\right) \cdot E_{\text{relay}} + \frac{T_c}{n} E_{\text{TX}}, \\
E_2 = & \left\{\frac{T_c(1-n_1/n) + (N_{c,2}-1)T_{c,2} - T_{c,3}}{n_2}\right. \\
& \left. -\frac{T_c}{n}\right\} \cdot E_{\text{relay}} + \frac{T_c}{n} E_{\text{TX}}, \\
E_3 = & \left\{\frac{T_c(1-n_1/n-n_2/n) + (N_{c,3}-1)T_{c,3}}{n_3}\right. \\
& \left. -\frac{T_c}{n}\right\} \cdot E_{\text{relay}} + \frac{T_c}{n} E_{\text{TX}}.
\end{aligned} \tag{11}$$

If we set $E_1 = E_2 = E_3$, the following results are obtained:

$$\begin{aligned}
T_{c,2} &= \frac{(2L^2+1)N_{c,3} - 4}{A \cdot L^2} T_c, \\
T_{c,3} &= \frac{4(L^2+1)N_{c,2} - 2L^2 + 3}{A \cdot L^2} T_c,
\end{aligned} \tag{12}$$

where $A = N_{c,3}N_{c,2} + 2N_{c,3} + 4N_{c,2} - 2$.

To obtain the probability of doing CT for the nodes in A_2 and A_3 ($\text{Prob}_{2,2}$ and $\text{Prob}_{2,3}$), we need to know how many tasks are handled by those nodes. For the nodes in A_3, $(1 - n_1/n - n_2/n)T_c$ tasks ($= T_3$) are handled, and $\text{Prob}_{2,3}$ is obtained by dividing $T_{c,3}$ in (12) by this value. For the nodes in A_2, $(1 - n_1/n)T_c - T_{c,3}$ tasks are handled ($T_{c,3}$ can be expressed in terms of T_c using (12)), and $\text{Prob}_{2,2}$ can be calculated by dividing $T_{c,2}$ in (12) by this value. The resulting $\text{Prob}_{2,2}$ and $\text{Prob}_{2,3}$ are as follows:

$$\begin{aligned}
\text{Prob}_{2,2} &= \frac{(2L^2+1)N_{c,3} - 4}{N_{c,3}(N_{c,2}+2)(L^2-1) - (8N_{c,2}+1)}, \\
\text{Prob}_{2,3} &= \frac{4(L^2+1)N_{c,2} - 2L^2 + 3}{A \cdot (L^2 - 4)}.
\end{aligned} \tag{13}$$

Now, let's consider the lifetime extension. The nodes in A_1 have $T_{c,2} + T_{c,3}$ less "relay" burdens when 2-Level CT is used (compared to non-CT). Therefore, to get the lifetime extension for 2-Level CT, we solve $(T_{c,2} + T_{c,3}) \cdot E_{\text{relay}}$ = (8) and get T_x in terms of T_c. By substituting the obtained T_x to (7) and using $E_{\text{relay}} = (1+\eta)E_{\text{TX}}$, we get

$$\frac{T_c}{T_{nc}} = \frac{\{1 + (L^2-1)(\eta+1)\} \cdot A}{A + B(\eta+1)}, \tag{14}$$

where $B = (L^2-1)N_{c,3}N_{c,2} - 3N_{c,3} - 8N_{c,2} + 3$.

From Table I, we see that 3 cooperators (N_c=3) are sufficient to reach the Sink from Level 3, and therefore, $N_{c,3}$=3. Substituting $N_{c,3}$=3 and $N_{c,2}$=2 to (14) gives $9 \cdot \{(\eta+1)L^2 - \eta\}/\{3L^2(1+\eta) - 14\eta - 5\}$, which is again a monotone decreasing function of L (≥ 3) and shows that the lifetime extension is more than three times when 2-Level CT is used.

TABLE II
GENERALIZED PROCEDURE FOR M-LEVEL CT.

1) Generate M+1 linear equations, E_k (k=1,2,...,M+1), using the following formula.

$$E_k = \left\{ \frac{T_c\left(1-\sum_{i=1}^{k-1}\frac{n_i}{n}\right)+(N_{c,k}-1)T_{c,k}-\sum_{i=k+1}^{M+1}T_{c,i}}{n_k} - \frac{T_c}{n}\right\}\cdot E_{\text{relay}}+\frac{T_c}{n}E_{\text{TX}}.$$

Here, $N_{c,1}=0$ and $T_{c,1}=0$.

2) Solve M+1 E_k's for $T_{c,i}$ (i=2,3,...,M+1) using the condition $E_1=E_2=...=E_{M+1}$.

3) Calculate the number of tasks, $T(i)$ (i=2,...,M+1), handled by the nodes in A_i as follows.

$$T(i)=T_c\left(1-\sum_{j=1}^{i-1}\frac{n_j}{n}\right)-\sum_{j=i+1}^{M+1}T_{c,j}.$$

4) Using obtained $T_{c,i}$'s and $T(i)$'s, $\text{Prob}_{M,i}$ is calculated as $\text{Prob}_{M,i}=T_{c,i}/T(i)$.

5) Get T_x in terms of T_c by substituting $T_{c,i}$'s to the following equation.

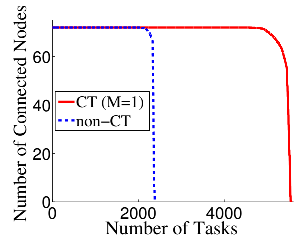
$$T_x=\frac{\sum_{i=2}^{M+1}T_{c,i}}{\left(1-\frac{n_1}{n}\right)\cdot E_{\text{relay}}+\frac{n_1}{n}E_{\text{TX}}}E_{\text{relay}}.$$

6) The extended lifetime is obtained by substituting T_x to $T_c=T_{nc}+T_x$ and solving it for T_c.

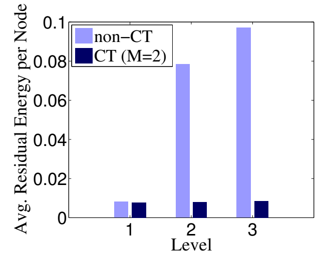
Instead of providing the results of M-Level CT for all levels, we provide the generalized procedure for getting probabilities and extended lifetime for M-Level CT in Table II. Next, we discuss our CT forwarding protocol.

The CT forwarding scheme that we have devised based on our analysis can be implemented on top of any existing cross-layer framework designed to use cooperative transmission, and such a cross-layer framework is well introduced in [10], which enables a node to gather cooperators and do CT. Also, as in [10], when a source node wants to send a data, it first establishes a non-CT primary route to the destination, which is identical to the operation of conventional non-CT routing protocols. This makes it possible for a node to do non-CT (i.e. forwarding data to the next hop without doing CT) when the node decides not to do CT based on the probability $\text{Prob}_{M,i}$.

L is fixed when the network topology is determined, and $\text{Prob}_{M,i}$'s can be obtained since M (M-Level CT) and $N_{c,i}$'s can be decided in advance. Each node can save $\text{Prob}_{M,i}$'s, and then be deployed. Once deployed, nodes need to figure out which level they are in (number of hops away from the Sink), which can be done when the network is initialized. After the initialization phase, whenever any node in A_i ($2\leq i\leq M+1$) has a task to perform (i.e., transmitting or relaying the message), the node chooses to cooperate with the probability $\text{Prob}_{M,i}$ where i is the level the node is in. When the node decides to do CT, it cooperates with neighboring cooperators that are in the same level. If the node decides not to cooperate, the node performs conventional non-CT routing (i.e., it routes the packet to the next hop).



(a) Connectivity vs. task graph. M=1 and L=3.



(b) The average residual energy (in Joules) per node for each level when the first node is dead. M=2 and L=3.

Fig. 2. Simulation results.

IV. SIMULATION

In this section, we check the validity of the probabilities ($\text{Prob}_{j,i}$) and show example values of the extended lifetimes derived in Section III through our network simulations. In simulations, we maintain the network and physical-layer models in Section II. The maximum non-CT communication range of the node is set to 40 ($d_{\text{tx}}^{\max}=40$), and $N_{c,2}$=2 and $N_{c,3}$=3 are are used following Table I. E_{TX}=0.092mJ and E_{RX}=0.277mJ are used ($\eta\approx 3$) following the model in [1] and assuming 256 bytes of data, and we choose the initial energy of each node to be 100mJ. We consider circular-shaped networks with deterministic uniform distribution. L=2,3,4, and 5 are considered and the number of nodes are 32, 72, 128, and 200 respectively, which is the case of n_1=8 in (4). The shortest-hop routing is used for both non-CT and CT (primary route in the case of CT). The simulation is done till no node can reach the Sink. We evaluate how the connectivity changes as the number of tasks increases. The sources are randomly selected, and we repeat the tests 25 times and average the data.

Fig. 2a shows the number of connected nodes versus the number of tasks performed when 1-Level CT with $\text{Prob}_{1,2}$=0.59 (obtained from (6)) is used for L=3 network. Note that T_c and T_{nc} are the number of tasks that can be performed before the network is "completely" disconnected. Since Fig. 2a is showing the averaged values, the point where the connectivity goes below 1 is regarded as T_c and T_{nc}. These points are 2375 for non-CT and 5545 for CT, and the lifetime extension factor is 2.34, which is close to the theoretical value 2.36 obtained from (10). The slight mismatch between the theoretical extension value and the actual extension value is inevitable because i) when source nodes are selected randomly, there is no guarantee that each node evenly generates and receives the exact expected number of tasks used in the derivation, and ii) in the ideal (theoretical) situation, all nodes should die at the same time, but in reality, some nodes should die first, and this early death causes a change in the network shape, and thereby a deviation from the derivation.

TABLE III
THE SIMULATION RESULTS FOR 1-LEVEL AND 2-LEVEL CT.

M-Level CT	L	$Prob_{1,2}$	LDL Extension - Theoretical	LDL Extension - Simulation (T_c/T_{nc})	FDL Extension - Simulation
1-Level CT	L=2	0.75	3.25	3.21 (8669 / 2698)	3.28
	L=3	0.59	2.36	2.34 (5545 / 2375)	2.26
	L=4	0.55	2.18	2.17 (4953 / 2287)	2.06
	L=5	0.53	2.11	2.10 (4702 / 2240)	2.01
		$Prob_{2,2}$ / $Prob_{2,3}$			
2-Level CT	L=3	0.67 / 0.72	4.87	4.81 (11430 / 2375)	4.83
	L=4	0.58 / 0.50	3.79	3.75 (8572 / 2287)	3.64
	L=5	0.55 / 0.43	3.45	3.44 (7704 / 2240)	3.24

In addition to T_c and T_{nc}, we provide the values when the first node dies, which is a widely used definition of the network lifetime. In order not to be confused with the terms, this definition of lifetime will be referred to as "first death lifetime (FDL)", and T_c and T_{nc} are "last death lifetimes (LDLs)". FDL is measured by getting the points when the connectivity falls below 'maximum connectivity-1'. The simulation results of 1-Level and 2-Level CT for all L's (L=2,3,4, and 5) are summarized in Table III, which clearly shows the similarities between theoretical and simulation values. Note that we don't have theoretical FDL extension values, and only the simulated values are provided in Table III.

To see if the energy consumption is actually balanced for CT, we provide Fig. 2b, which shows the average residual energy per node (in Joules) for each level right after one node is dead when non-CT and 2-Level CT are used for L=3. As can be seen from the figure, CT well balances the energy consumption compared to non-CT. Fig. 2b shows that non-CT leaves a huge amount of energy unused explaining why CT is so effective.

To see if the non-CT energy-aware routing scheme can make difference, one of the most efficient online energy-aware routing protocols, CMAX [6] is simulated, and we get LDL (T_{nc}) values of 2707, 2376, 2289, and 2251 for L=2, 3, 4, and 5 respectively, which is not notably different from T_{nc} values in Table III. This is not surprising because the energy-aware routing cannot reduce the burden of the nodes in A_1 and avoid the energy hole problem.

V. CONCLUSION

In this paper, we have presented a unique analytical model for avoiding the energy hole using the range extension of CT. We have calculated the probability (rate) of doing CT, which indicates how often CT should be performed, and we derived the amount of the lifetime extension that can be achieved by CT. Based on the analytical model, a CT forwarding method that only requires the level information obtained during the network initialization period is developed, and our analysis and CT method are verified by simulations. By using our CT method, it is possible to balance the energy consumption between the nodes in different levels and avoid the energy hole even with the uniform distribution of nodes. Also, our proposed CT method simplifies the operation of WSNs because it does not require residual energy information, which distinguishes our method from existing lifetime maximization routing approaches.

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