A Spatial Multiplexing Beamforming Scheme for Multicasting to Cooperators

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***Abstract*—This paper addresses the transmissions of a multi-antenna Base Station (BS), when communicating simultaneously with groups of radios in a multi-hop network of single-antenna radios. Each group is supposed to receive a distinct data stream, such that all members of the group receive the same packet at the same time. The intention is that each group will perform cooperative transmission in the next time slot or slots. We focus on just the first hop, in which the BS chooses the group members and beamforms to them, so that their individual signal to interference and noise ratios all exceed a minimum threshold. The problem we address, therefore, is a combination of spatial multiplexing and multicasting. Our approach does zero forcing on selected nodes and then checks that an adequate number of nodes in each group have at least the minimum signal-to-noise-and-interference ratio (SINR). Using simulation, we compare our approach to others and we consider the effects of errors in transmit-side channel state information.**

***Keywords*—Cooperative Transmission, Multicast, Spatial Multiplexing.**

I. INTRODUCTION

Cooperative transmission (CT) can extend the range of a transmission and reduce the number of hops in a multi-hop wireless network without requiring high transmit power per node [1][2]. CT is well known to provide diversity gain, which can greatly improve link reliability [3]. Typically, the node that intends to start a CT must first share data with cooperators, which costs extra time slots [4]. In this paper, we assume that a multi-antenna base station (BS) services a multi-hop network, composed of single-antenna radios. This type of network could represent a wireless sensor network with a multi-antenna sink, or a mobile ad-hoc network, with one MIMO-enabled mobile gateway node. This paper investigates how such a multi-antenna BS can spatially multiplex two or more multi-casts, such that each multi-cast excites multiple nodes that will cooperate in the next hop.

Some previous works [5] propose several downlink beamforming schemes without taking multi-cast into account and by regarding each user as an integral MIMO node, which is impractical in some applications where each user has strict constraints on platform size and complexity. Other works [6], by treating all nodes in the cluster equally, focus on the overall SINR performance of the cluster, which may result in not any nodes reaching a specified SINR. In this paper, we propose a downlink beamforming scheme to optimize the number of nodes that meet our SINR system requirement. Thus, besides spatial multiplexing gain, diversity gain can be achieved through opportunistically selecting cooperators.

This paper is organized as follows. The network model will be shown in the next section. Section III gives the selection criteria and describes the beamforming scheme. Section IV presents the simulation results and the conclusion is given in Section V.

II. NETWORK MODEL

We consider a multi-hop wireless network with one multi-antenna base station. The network objective is for the BS to initiate the transmission of two or more data streams, intended for two or more particular destinations, respectively, such that the destinations are generally many hops away from the BS. To enhance reliability and reduce the number of hops, the nodes in the network will form sequences of cooperating clusters, as shown in Figure 1. Such clusters can be defined by higher layer protocols, such as OLAROAD [7] or OLACRA [8], which will instruct nodes to decode and forward only the stream they are supposed to get and ignore the other stream even if the SINR of other stream is high enough. In this paper, we will focus on the downlink between base station and first-hop clusters. In this topology, clusters receiving in the first hop are assumed to be equidistant to the base station and formed by certain number of co-located nodes.

Suppose there are K receive clusters, each consisting of several single-antenna nodes. Here, a "cluster" is a group of *candidates* for cooperation (specified by the higher layer protocol as a "cooperative route" [9]). For example, $K = 2$ in Figure 1, where the first hop clusters are indicated by the dashed curves and the nodes selected for cooperation are the filled circles. Let the base station possess N transmit antennas, and let M_i be the number of nodes in receive cluster i. The objective is for the base station to simultaneously send K data packets, one packet to each cluster, so that at least two nodes per cluster can decode the packet. Shortly after decoding, these nodes are expected to relay their packets using concurrent CT

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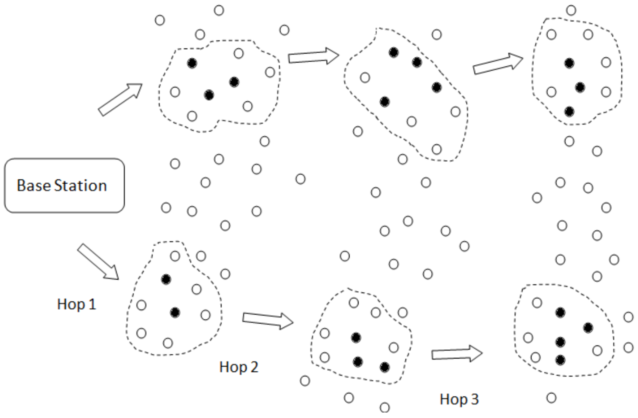

Fig. 1. Multi-hop system structure (black nodes decode desired data correctly and make CT)

[2] or some other form of CT. Let $\mathbf{x}$ denote the transmitted signal vector, given as

$$\mathbf{x} = \sum_{i=1}^{K} \mathbf{w}_i s_i, \tag{1}$$

where s_i denotes the complex scalar data intended for cluster i such that $E(|s|^2) = 1$, and $\mathbf{w}_i$ is its $N \times 1$ beamforming vector with the constraint $\|\mathbf{w}_i\|^2 = 1$.

Let the Rayleigh flat fading channel gains between the BS antennas and the antennas of the nodes in the ith cluster be expressed as the $M_i \times N$ matrix $\mathbf{H}^i$. Let each entry h_{pq}^i in $\mathbf{H}^i$ represent the channel state between the qth antenna at the base station and pth node in cluster i, and assume $E(\left|h_{pq}^i\right|^2) = 1$. $\mathbf{h}_j^i$ denotes the $1 \times N$ channel vector of node j in cluster i.

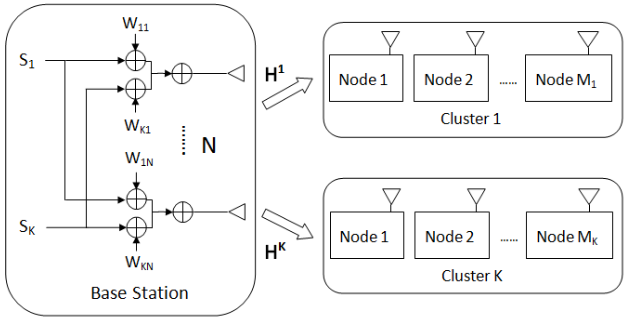

Fig. 2. Block diagram of spatial multiplexing multi-cast downlink system

As shown in Figure 2, the signal vector received at cluster i is given as

$$\mathbf{y}_i = \underbrace{\mathbf{H}^i\mathbf{w}_i s_i}_{\substack{Desired\\ Signal}} + \underbrace{\sum_{j=1,j\neq i}^{K} \mathbf{H}^i\mathbf{w}_j s_j}_{CCI} + \underbrace{\mathbf{n}_i}_{AWGN}, \tag{2}$$

where the summation in the 2nd term represents the co-channel interference (CCI), and the j^{th} element of $\mathbf{n}_i$, n_j^i, is complex additive white Gaussian noise with the distribution $C\mathrm{N}(0, \sigma^2)$. If the weights are independent of the channels, then the received average signal-to-noise ratio (SNR) at cluster i node j is

$$\gamma_j^i = \frac{E(\left|\mathbf{h}_j^i\mathbf{w}_i s_i\right|^2)}{E(|n_0|^2)} = 1/\sigma^2. \tag{3}$$

Let the entries of the fading channel matrices, $\mathbf{H}^i$, be mutually independent. Assume the matrices are known at the base station by sending downlink training pilots and by feeding back the channel estimates to the BS. The impact of imperfect channel estimation is mentioned in [10]. Let the elements of the channel state information (CSI) error matrix, $\Delta\mathbf{H}$, follow the Gaussian distribution $C\mathrm{N}(0, \delta^2)$, where the variance δ^2 reflects the estimation quality. The channel matrix used in the downlink beamforming process is given as

$$\mathbf{H}_{est} = \mathbf{H} + \Delta\mathbf{H}, \tag{4}$$

where $\mathbf{H}$ is the true channel matrix.

III. BEAMFORMING SCHEME

The beamforming objective is to find at least two nodes in each cluster such that the SINR of the selected nodes will be above a decoding threshold. Therefore our approach has a similarity to multi-user diversity in that the selected nodes are those that have favorable SINRs. The selection of nodes in different clusters must be done jointly, since the beams formed for different sets of nodes in a given cluster will result in different levels of interference on nodes in another cluster. We introduce the process to achieve the above goal by controlling the CCI to get the maximum number of CCI-free nodes, the optimization procedure to get the beamforming vectors and the system outage probability.

A. Co-channel Interference Control

Given the CSI, the SINR of node j in cluster i is given as

$$SINR_j^i = \frac{\overbrace{\left|\mathbf{h}_j^i\mathbf{w}_i\right|^2}^{\text{DesiredSignal}}}{\underbrace{\sigma^2}_{AWGN} + \underbrace{\sum_{k=1,k\neq i}^{K}\left|\mathbf{h}_j^i\mathbf{w}_k\right|^2}_{Interference}}. \tag{5}$$

The interference can be cancelled by zero-forcing [11]. If the node j shown in Equation (5) needs to be CCI-free, all $K-1$ beamforming vectors for the other clusters (except $\mathbf{w}_i$) are required to lie in the space orthogonal to $\mathbf{h}_j^i$ with the constraints

$$\left|\mathbf{h}_j^i\mathbf{w}_k\right|^2 = 0,\ \|\mathbf{w}_k\|^2 = 1, \forall k \in \{1, 2 \ldots i-1, i+1 \ldots K\}. \tag{6}$$

The beamforming vector $\mathbf{w}_i$ should be in the intersection of orthogonal spaces of CCI-free nodes' channel vectors in all clusters except cluster i. In symbols, $\mathbf{w}_i$ needs to satisfy

$\sum\limits_{k=1,k\neq i}^{K}\sum\limits_{m=1}^{M_k}\left|\mathbf{h}_m^k\mathbf{w}_i\right| = 0$ and $\|\mathbf{w}_i\|^2 = 1$. For example, suppose $K = 2$ and $M_1 = M_2 = 2$. Then $\mathbf{w}_i$ must satisfy a total of 3 constraints, implying the need for at least 3 BS antennas. The maximum possible number of CCI-free nodes depends on the number of transmit antennas. Since there are N transmit antennas, the beamforming vector has N degrees of freedom. With the normalization constraint $\|\mathbf{w}_i\|^2 = 1$, each complex beamforming vector can be in the intersection of at most $N-1$ complex channel vectors' orthogonal spaces. Thus, the maximum number of CCI-free nodes in the system should be $K \times \left\lfloor \frac{N-1}{K-1} \right\rfloor$.

B. CCI-free Nodes Selection Procedure

The selection procedure should indicate which nodes can reach a high enough SINR in each cluster according to the CSI. Each cluster should have the same number of CCI-free nodes so as to maximize the minimum number of CCI-free nodes in all clusters. This implies that the minimum number of CCI-free nodes in all clusters should be $m = \left\lfloor \frac{(N-1)}{K-1} \right\rfloor$. In this downlink scheme, we say the transmission fails if any of the K clusters can't have enough nodes to make CT in the next hop. Therefore, although some clusters may have more CCI-free nodes, the system performance is determined by the worst multi-cast branch. So we focus on m CCI-free nodes in each cluster during the analysis, which also meets the maximum number constraint $K \times \left\lfloor \frac{N-1}{K-1} \right\rfloor$, and these nodes are more likely to reach the SINR threshold compared with others. The CCI-free nodes selection matrix $\mathbf{S}_n$ is given as

$$\mathbf{S}_n = \begin{pmatrix} \mathbf{a}_{11} & \cdots & \mathbf{a}_{1m} \\ \vdots & \ddots & \vdots \\ \mathbf{a}_{K1} & \cdots & \mathbf{a}_{Km} \end{pmatrix} \tag{7}$$

$$\mathbf{a}_{ij} \in \{\mathbf{h}_1^i, \mathbf{h}_2^i ... \mathbf{h}_{M_i}^i\}, \forall i \in \{1, 2...K\}, j \in \{1, 2...m\},$$

where the entries in row i of $\mathbf{S}_n$ represent the channel vectors of nodes selected to have no CCI in cluster i. Each cluster will have m CCI-free nodes and, for all K clusters, the total number of different combinations L is given as

$$L = \prod_{k=1}^{K} \begin{pmatrix} M_k \\ m \end{pmatrix}, \tag{8}$$

therefore $n \in \{1, 2...L\}$.

Once a certain $\mathbf{S}_n$ is given, we can get all the beamforming vectors, $\mathbf{w}_k, k \in \{1, 2...K\}$, by solving the equation set (6) for all k and have a $N \times K$ beamforming matrix $\mathbf{W}_n = (\mathbf{w}_1 ... \mathbf{w}_K)$. Finally, we will have L beamforming matrices as candidates. Since the previous step concerns only the cancelling of co-channel interference, the process of selection among all L beamforming matrices focuses on the optimization of desired signal power of these CCI-free nodes, which can help them to gain enough high SNR.

The optimal beamforming matrix can maximize the minimum number of nodes that can decode intended data correctly in each cluster and the total number of these nodes in all K clusters. C_i represents the number of nodes that meet the requirement $SINR_j^i \geq \gamma_{threshold}$ in cluster i. We follow this procedure because in each cluster, the more nodes that decode the desired data, the more reliable is the CT in the next hop. The optimization process is given as

$$\begin{array}{l} \mathbf{W}^o = \arg \max\limits_{\mathbf{W} \in \mathbf{W}_s} \{\sum\limits_{i=1}^{K} C_i\} \\ \mathbf{W}_s = \{\mathbf{W} | \arg \max\limits_{\mathbf{W} \in \mathbf{W}_p} \{\min\limits_{i}(C_i)\}\} \\ \mathbf{W}_p = \{\mathbf{W}_1 ... \mathbf{W}_L\} \end{array} . \tag{9}$$

C. Outage Analysis

In this section, we derive upper and lower bounds on the system outage probability. The system outage probability O_{system} is the probability that in any of the K receive clusters, the number of nodes reaching SINR threshold is less than required number C. The cluster outage probability O_i shows the probability that in one cluster, the number of nodes reaching SINR threshold is less than C, and, as long as the number of CCI-free nodes in each cluster remains the same and as the cluster sizes are the same, the cluster outage probability will be the same for all clusters.

Since the fading channel follows the Rayleigh distribution, the output signal power follows the exponential distribution and the outage probability P for a single node is

$$\begin{array}{l} P = \int\limits_0^{\alpha} \frac{\exp(-x/\theta)}{\theta} dx, (x \geq 0) \\ \alpha = 10^{[(\gamma_{threshold} - SNR)_{dB}/10]} \end{array} . \tag{10}$$

Let X be the number of qualified nodes. Suppose there are M nodes, then we have $E(X) = (1-P)M$ and since the PDF of X is positive, according to the Markov's inequality,

$$\frac{E(X)}{\beta} \geq P(X > \beta), 1 - \frac{E(X)}{\beta} \leq P(X < \beta), \tag{11}$$

we can have the system outage lower bound with the requirement of minimum number of successful nodes $\beta = 2$, given as

$$\begin{array}{l} O_{system\_lower\_bound} = \max(O_i) \\ O_i \geq 1 - [\frac{M_i}{2} \times (1-P)] \end{array} . \tag{12}$$

To get the upper bound, we note that if different sets of elements in a cluster have at least one common element, then the outage events for these two sets cannot be independent. With the cluster outage upper bound given in the form of union bound, the system outage upper bound is shown as

TABLE I
SIMULATION PARAMETERS

Parameter	**Symbol**	**Value**
Transmit Antenna	N	5
Number of Nodes in Cluster	M_i	$2 \sim 7$
Number of Clusters	K	3
Decoding SINR Threshold	$\gamma_{threshold}$	12 dB
Required Number of Qualified Nodes	C	2
Fading Channel	$\mathbf{H}^i$	Rayleigh
Estimation Error	$\mathbf{H}_{err}$	Normal
Normalized Error Standard Deviation	δ	$0.01 \sim 0.3$

$$O_{system\_upper\_bound} = \min(1, \sum_{i=1}^{K} O_i)$$

$$O_i \le \binom{M_i}{m} \left[\sum_{j=0}^{C-1} \binom{m}{j} P^{(m-j)}(1-P)^j\right] \cdot \qquad (13)$$

$$m = \left\lfloor \frac{N-1}{K-1} \right\rfloor \ge C, i \in \{1,2...K\}$$

If the channel estimation error is taken into account as Equation (4), the estimated SINR using flawed CSI and flawed weight vectors of node j in cluster i, $SINR^i_{j,est}$, is given as

$$SINR^i_{j,est} = \frac{\left|\mathbf{h}^i_{j,est}\mathbf{w}^o_{i,est}\right|^2}{\sigma^2 + \sum\limits_{k=1,k\neq i}^{K}\left|\mathbf{h}^i_{j,est}\mathbf{w}^o_{k,est}\right|^2} \qquad (14)$$

$$\mathbf{h}^i_{j,est} = \mathbf{h}^i_j + \Delta\mathbf{h}^i_j,$$

where $\Delta\mathbf{h}^i_j$ stands for the estimation error vector. The actual SINR using true CSI with flawed weight vectors is the same as Equation (14), with $\mathbf{h}^i_{j,est}$ replaced with $\mathbf{h}^i_j$.

The SINR then will not only depend on received SNR and the impact of extra interference causes degradation on the overall performance. In the practical scenario, the perfect channel estimation case provides the upper bound of system performance with all given parameters.

IV. SIMULATION RESULT

In this section, we simulate our proposed beamforming method, and compare it with other approaches and with the bounds of the previous section. Table 1 shows the parameters used in the simulation. The numbers of nodes in all clusters are same in the simulation and results are averaged over 3000 channel realizations for outage probability curves. As shown in Table 1, the number of transmit antennas is 5, cluster number is 3 and the required number of nodes to reach the threshold is 2. Thus, at most 2 CCI-free nodes can be selected in each cluster and the maximum number of CCI-free nodes in the system is 6.

Figure 3 shows the simulation results of outage probability and their corresponding bounds with different numbers of nodes per cluster. We observe that the more nodes there are in the clusters, the steeper is the outage curve, indicating larger diversity gain.

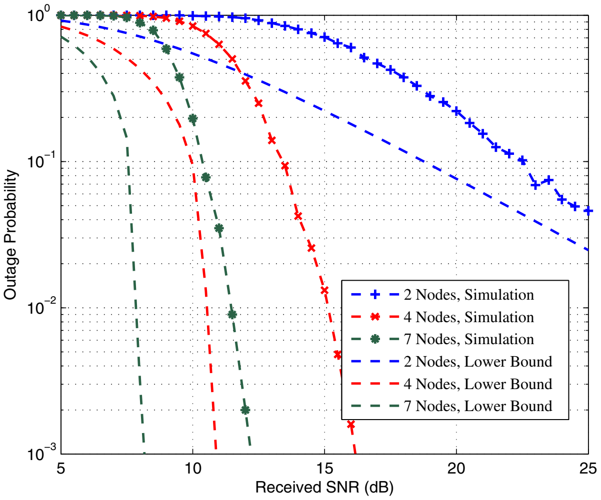

Fig. 3. Simulated system outage probabilities with different numbers of nodes in each cluster. No channel estimation error. $\gamma_{threshold} = 12dB$

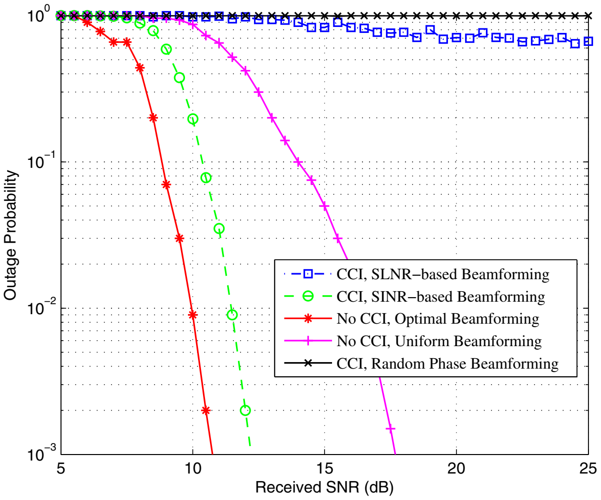

Fig. 4. System outage probability of different beamforming schemes. 7 nodes in each cluster. No channel estimation error. $\gamma_{threshold} = 12dB$

Since the desired signal power is affected by the Rayleigh fading channel, the possibility to find the combination of 6 CCI-nodes with strong desired signal power is larger when there are more nodes in each cluster. The theoretical lower bound and upper bound are also related to the number of nodes in each cluster. With more nodes, the bounds are looser due to the larger diversity gain.

Figure 4 compares the system outage probabilities of our proposed SINR-based beamforming scheme to the Signal to Leakage and Noise Ratio (SLNR) based one of [6] as well as to several special cases. In the "No-CCI" special case, the interference part is removed in Equation (5) prior to optimization. With no co-channel interference, the only goal of the optimal beamforming scheme is to maximize desired signal power and its performance should be much better than any others'. The No-CCI uniform beamforming special case sets each beamformer weight to $1/\sqrt{N}$. Therefore, it shows the performance of receive selection diversity, when two nodes are selected from seven, in the absence of CCI. The result shows that the performance of our SINR-based beamforming scheme is comparable with No-CCI case by taking advantage of the diversity gain. Since the optimization of desired signal power follows zero forcing in our scheme, its performance is worse than the optimal beamforming scheme, but better than doing no optimized beamforming as the uniform beamforming scheme. In random phase beamforming scheme, the beamformer weights have equal magnitude and independent random phases. It is very hard for enough number of nodes to gain high SINR with random phase beamforming vectors. Thus, such beamforming scheme is not feasible under this scenario.

The decoding threshold is determined by the modulation scheme and system bit error rate (BER) requirement and higher decoding threshold leads to larger outage probability with given system quality requirement.

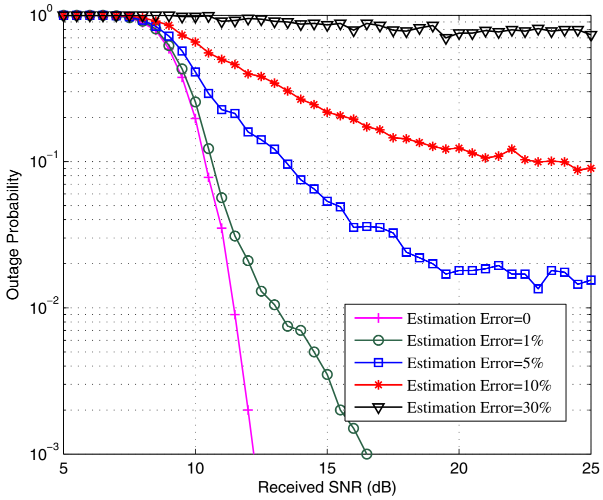

Fig. 5. System outage probability at channel estimation error (7 nodes in each cluster. $\gamma_{threshold} = 12dB$)

Estimation error degrades the system performance, as shown in Figure 5. The error percentages in this simulation are normalized by average channel gain. The impact of estimation error is negligible compared with perfect channel estimation case when the channel estimation error is small enough and the extra co-channel interference is still tolerable. However, the system outage probability increases rapidly when the estimation error percentages is larger than 5 percent, which can be used as guideline in the channel estimation phase.

V. CONCLUSION

In this paper, we proposed a spatial multiplexing downlink beamforming scheme for cooperative multi-cast system. We show that using an SINR-based beamforming scheme to suppress co-channel interference is very effective and makes full use of both spatial multiplexing gain and diversity gain. The proposed scheme far outperformed other schemes for this CT objective, however, its high complexity is a concern. With relatively high average received SNR and good channel estimation, this SINR-based spatial multiplexing multicast downlink beamforming scheme can ensure that for each cluster in the system, enough nodes can decode the intended data correctly and simultaneously in one time slot and make cooperative transmission in the next hop. In future work, we will seek lower complexity versions of this scheme.

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