TRANSMIT/RECEIVE ANTENNA SELECTION FOR MIMO SYSTEMS TO IMPROVE ERROR PERFORMANCE OF LINEAR RECEIVERS

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ABSTRACT

Recent work has shown that multiple antenna transceivers can be used to enhance the data-rates and coverage of wireless links as compared to single antenna transceivers. However, using multiple antennas requires costly hardware and is computationally intensive. Antenna selection could be done at the transmitter and/or the receiver to overcome these disadvantages while incurring minimal loss in performance. Considerable work has been done to provide the solution for antenna selection that maximizes channel capacity. However, for finite complexity receivers this solution is not optimal. In this paper, we present optimal antenna selection solutions to minimize the error rates of linear receivers. Several greedy algorithms are developed to improve the error performance of linear receivers. The proposed algorithms have low implementation complexity and offer near optimal error performance.

1. INTRODUCTION

A multiple-input-multiple-output (MIMO) capable physical layer can greatly improve the throughput and range of a wireless link. This fact has led to a concerted research effort in exploring the throughput gains possible with MIMO capable nodes in an ad-hoc network [1]-[4]. While the initial phase of research focused mainly on the capacity aspects of a MIMO link, of late research efforts are being made to study the error performance of MIMO links.

In [1], the authors show that a MIMO capable physical layer allows WLAN nodes to operate co-channel resulting in higher network throughput than if they operated under a CSMA/CA protocol such as 802.11 MAC. In a multiuser environment it is important to avoid overloading receiver node with more number of data streams than the number of available antenna elements [5]. As such, the transmitting nodes are often required to use only a subset of transmit antennas so that the receiver can use additional degrees of freedom to suppress the interfering streams. Link error performance can be significantly improved by choosing optimal set of transmit antennas. Optimal antenna selection requires a search through all possible combinations and chooses the subset of antennas that minimizes the error associated with the link. However, for large number of available antennas an optimal search becomes computationally intensive and is often not feasible in practice owing to power and delay constraints. This motivates the need for efficient antenna selection algorithms with reasonable power requirements and minimum latency.

In the past, many researchers have addressed the problem of antenna selection for MIMO systems. In [6]-[11], the focus has been to choose transmit or receive antennas that maximize the Shannon capacity for MIMO channels. Shannon capacity is an upper bound on achievable channel capacity and is a source of inspiration to design yet better systems to achieve it. However, practical wireless systems with restricted receiver complexity cannot achieve Shannon capacity. Thus antenna selection solutions that optimize the channel capacity for arbitrary complexity receivers are unlikely to achieve optimum error performance for practical receivers that use zero-forcing (ZF) or minimum-mean-square-error (MMSE) processing.

In [12], the authors consider a MMSE receiver and develop receive antenna selection algorithms to maximize the channel capacity, which again need not be optimal as far as error performance is concerned. In fact, as we will see in the paper, such schemes perform only slightly better than deterministic antenna selection. Therefore, it is essential to model antenna selection problem with the aim of minimizing the error rate of a link, taking receiver processing into consideration. Unfortunately, only a handful of papers [13]-[15] have appeared in literature that address this issue. In [14], Gore et al. proposed transmit antenna selection strategy for ZF receivers to mitigate the effect of transmit antenna correlation. The suggested algorithm pre-determines the subset of antennas to use, exploiting apriori knowledge of transmit correlation matrix at the transmitter. However, the

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proposed algorithm fails to exploit the transmit diversity gain that could be leveraged by selecting transmit antennas that exploit the current state of the fading channel. In [15] Berenguer et al. presented a transmit antenna selection algorithm for ZF receivers and later in [16] extended it to lattice-reduction-aided (LRA) ZF receivers, which have additional complexity, compared to traditional ZF receivers. The authors also proposed an approximate selection rule for LRA-MMSE receivers based on the minimization of maximum mean square error [Eq (24), 16]. The analysis in [14]-[16] is limited to single user and in [14]-[15] antenna selection is considered only at the transmitter. Moreover, no exact analysis is provided for the MMSE receiver, which is known to have much superior performance than the ZF receiver at low and moderate SNR [17].

In this paper, we present a unified framework to study the problem of transmit and receive antenna selection for both ZF and MMSE receivers. Although non-linear receivers such as successive interference cancellation (SIC) achieve better error rates than linear receivers, the latter have lower decoding complexity and latency compared to the former [17]. Several algorithms are developed to improve the error performance of linear receivers by means of transmit/receive antenna selection. Our algorithms have lower implementation complexity while offering near optimal error performance. Furthermore, these algorithms are applicable to the multi-user interference case, as is the case with ad-hoc networks. Also, the performance of these algorithms is least influenced by the statistical properties of the channel matrix and is satisfactory even for correlated channels.

2. BACKGROUND

2.1 MIMO System Model

The system under consideration is an ad-hoc network with K links (pairs of transmit and receive nodes) where each link is subjected to co-channel interference from the remaining K-1 links. The transmitting nodes are equipped with N_t antenna elements while receiver nodes use N_r antennas. It is assumed that each transmitting node chooses a subset of L_t antennas from N_t available antennas to improve the BER performance of the desired link. The received baseband vector corresponding to the i^{th} link is given by

$$\mathbf{r}_i = \mathbf{H}_{i,i}\mathbf{a}_i + \sum_{j \neq i}^{K} \mathbf{H}_{i,j}\mathbf{a}_j + \mathbf{n} \qquad (1)$$

where $\mathbf{H}_{i,j}$ denotes the $N_r \times L_t$ channel matrix corresponding to the transmitter of the j^{th} link and the receiver of the i^{th} link, $\mathbf{a}_i$ denotes the transmit vector for the i^{th} link and $\mathbf{n}$ is the additive white noise. We assume that the noise components are uncorrelated with complex variance N_O. We also assume that the inputs are chosen from same unit-energy constellation and are uncorrelated so that $\mathbf{E}[\mathbf{a}_i\mathbf{a}_i^H] = \mathbf{I}$, where $\mathbf{E}[.]$ denotes the expected value of its argument. We further assume that perfect CSI is available to the receiver and an error-free limited-feedback channel exists in case of transmit antenna selection.

2.2 Antenna Selection for MMSE/ZF Receivers

Let us consider the 1^{st} link of the network for which the receive vector is given by

$$\mathbf{r} = \tilde{\mathbf{H}}\tilde{\mathbf{a}} + \mathbf{n} \qquad (2)$$

where $\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \ldots & \mathbf{H}_{1,K} \end{bmatrix}$ and $\tilde{\mathbf{a}}$ denotes the stacked column vector formed by the transmit vectors $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \ldots & \mathbf{a}_K \end{bmatrix}$. If the receiver employs an MMSE linear detector, it chooses a filter $\mathbf{C}$ so as to minimize the mean-square error, $MSE = \mathbf{E}\left[\| \mathbf{C}\mathbf{r} - \tilde{\mathbf{a}} \|^2 \right]$, where $\| . \|$ denotes the Euclidean norm of the vector argument. The MSE can also be expressed as $MSE = \text{tr}(\mathbf{W})$, where $\text{tr}(.)$ denotes the trace of its argument and

$$\mathbf{W} = \mathbf{E}\left[(\mathbf{C}\mathbf{r} - \tilde{\mathbf{a}})(\mathbf{C}\mathbf{r} - \tilde{\mathbf{a}})^H \right] \qquad (3)$$

Now (3) can be rewritten as [17]

$$\begin{aligned} \mathbf{W} = (\mathbf{C} - \tilde{\mathbf{H}}^H\mathbf{R}_r^{-1})\mathbf{R}_r(\mathbf{C} - \tilde{\mathbf{H}}^H\mathbf{R}_r^{-1})^H \\ + \mathbf{I} - \tilde{\mathbf{H}}^H\mathbf{R}_r^{-1}\tilde{\mathbf{H}} \end{aligned} \qquad (4)$$

where $\mathbf{R}_r = \tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + N_O\mathbf{I}$. Since only the first term in (4) depends on $\mathbf{C}$, MMSE solution chooses $\mathbf{C} = \tilde{\mathbf{H}}^H\mathbf{R}_r^{-1}$ to make it zero. The term $\text{tr}(\mathbf{I} - \tilde{\mathbf{H}}^H\mathbf{R}_r^{-1}\tilde{\mathbf{H}})$ represents the MSE error for all detected symbols including those from interfering links. Optimal antenna selection must choose the transmit antennas so as to minimize the mean-square error associated with the desired link which is

$$\begin{aligned} \text{MSE} &= \sum_{i=1}^{L_t} (\mathbf{I} - \tilde{\mathbf{H}}^H\mathbf{R}_r^{-1}\tilde{\mathbf{H}})_{ii} \\ &= \text{tr}(\mathbf{I} - \mathbf{H}_{1,1}^H\mathbf{R}_r^{-1}\mathbf{H}_{1,1}) \end{aligned} \qquad (5)$$

Observing that MSE is minimized when $\text{tr}(\mathbf{H}_{1,1}\mathbf{R}_r^{-1}\mathbf{H}_{1,1})$ is maximized and using the trace identity $\text{tr}(\mathbf{XY}) = \text{tr}(\mathbf{YX})$, optimal antenna selection solution can be obtained as

$$\begin{aligned} \mathbf{S}_{MMSE} &= \underset{\mathbf{H}_{1,1} \in \mathbf{H}_{N_t}}{\arg\max}\, \text{tr}(\mathbf{H}_{1,1}\mathbf{H}_{1,1}^H\mathbf{R}_r^{-1}) \\ &\triangleq \underset{\mathbf{H}_{1,1} \in \mathbf{H}_{N_t}}{\arg\max}\, \text{tr}\left[\mathbf{H}_{1,1}\mathbf{H}_{1,1}^H \left(\mathbf{I} + \frac{1}{N_O}\sum_{i=1}^{K} \mathbf{H}_{i,1}\mathbf{H}_{i,1}^H \right)^{-1} \right] \end{aligned} \qquad (6)$$

where $\mathbf{S}_{MMSE}$ denotes the subset of selected antennas and $\mathbf{H}_{N_t}$ denotes the $N_r \times N_t$ channel matrix for the 1st link. To select the antennas in the optimal way, the trace involving matrix inversion has to be computed for $\binom{N_t}{L_t}$ possible combinations. This could be computationally burdensome when the number of antenna combinations is large. In the following section, we present several reduced complexity antenna selection algorithms. Looking at (4), it is evident that as $N_O \to 0$, $\mathbf{C} \to \tilde{\mathbf{H}}^H (\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H)^{-1}$, which is the Zero-Forcing filter. Thus, the optimal antenna selection solution for ZF receiver can be expressed as

$$\mathbf{S}_{ZF} = \arg\max_{\mathbf{H}_{1,1} \in \mathbf{H}_{N_t}} \lim_{N_O \to 0} \mathrm{tr}\left[\mathbf{H}_{1,1}\mathbf{H}_{1,1}^H \times \left(\mathbf{I} + \frac{1}{N_O}\sum_{i=1}^{K}\mathbf{H}_{i,1}\mathbf{H}_{i,1}^H\right)^{-1}\right] \quad (7)$$

Equations (6)-(7) present a unified framework for optimal antenna selection solution for MMSE and ZF receivers. All the algorithms presented in this paper for MMSE receiver can also be applied to ZF receiver by choosing $N_O \to 0$.

3. TRANSMIT ANTENNA SELECTION

In order to make transmit antenna selection, instead of computing the trace in (6)-(7) for all possible combinations of antennas as in the optimal selection procedure, we present two suboptimal but efficient antenna selection algorithms. Both greedy algorithms try to maximize the trace at each intermediate step. An antenna is selected/removed so that doing so leads to maximum increase in trace.

3.1 Algorithm-A: Incremental Antenna Selection

The algorithm begins with an empty set of selected antennas and selects one antenna in each step. After *n* steps, there are *n* selected antennas and the corresponding channel matrix is denoted by $\mathbf{H}_n$. The matrix $\mathbf{H}_n$ consists of the columns of $\mathbf{H}_{N_t}$ in the same order as they appear in $\mathbf{H}_{N_t}$. We will denote the k^{th} column of $\mathbf{H}_{N_t}$ by $\mathbf{h}_k$ and will use the following notations

$$\mathbf{B}_n = \left(\mathbf{I} + \frac{1}{N_O}\left\{\mathbf{H}_n\mathbf{H}_n^H + \sum_{j=2}^{K}\mathbf{H}_{1,j}\mathbf{H}_{1,j}^H\right\}\right)^{-1} \quad (8)$$

$$\Delta_n = \mathrm{tr}(\mathbf{H}_n\mathbf{H}_n^H\mathbf{B}_n) \quad (9)$$

where the subscript *n* denotes the n^{th} step. Now if in the $(n+1)^{\text{st}}$ step, transmit antenna corresponding to the k^{th} column of $\mathbf{H}_{N_t}$ is selected, then $\mathbf{h}_k$ is inserted in proper position in $\mathbf{H}_n$ to obtain the channel matrix $\mathbf{H}_{n+1}$. The matrices in (8) are updated as

$$\mathbf{H}_{n+1}\mathbf{H}_{n+1}^H = \mathbf{H}_n\mathbf{H}_n^H + \mathbf{h}_k\mathbf{h}_k^H \quad (10)$$

$$\mathbf{B}_{n+1} = \mathbf{B}_n - \frac{1}{N_O + \mathbf{h}_k^H\mathbf{B}_n\mathbf{h}_k}\mathbf{B}_n\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n \quad (11)$$

where (11) follows as a direct consequence of the matrix inversion lemma [(3.5.2.2), 18]. Using (10) and (11), the updated trace can be computed as

$$\begin{aligned}\Delta_{n+1} &= \mathrm{tr}[\mathbf{H}_{n+1}\mathbf{H}_{n+1}^H\mathbf{B}_{n+1}] \\ &= \Delta_n - \frac{\mathrm{tr}[\mathbf{H}_n\mathbf{H}_n^H\mathbf{B}_n\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n] + \mathrm{tr}[\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n]}{N_O + \mathbf{h}_k^H\mathbf{B}_n\mathbf{h}_k} \\ &\quad + \mathrm{tr}[\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n]\end{aligned} \quad (12)$$

Since $\mathbf{B}_n$ is a Hermitian matrix, we can use the identity $\mathrm{tr}(\mathbf{XY}) = \mathrm{tr}(\mathbf{YX})$ to further simplify (12) as

$$\mathrm{tr}[\mathbf{H}_n\mathbf{H}_n^H\mathbf{B}_n\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n] = \| \mathbf{h}_k{}^H\mathbf{B}_n\mathbf{H}_n \|^2 \quad (13)$$

$$\mathrm{tr}[\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n] = (\mathbf{h}_k^H\mathbf{B}_n\mathbf{h}_k)^2 \quad (14)$$

$$\mathrm{tr}[\mathbf{h}_k\mathbf{h}_k^H\mathbf{B}_n] = \mathbf{h}_k^H\mathbf{B}_n\mathbf{h}_k \quad (15)$$

Finally, substituting (13)-(15) in (12) and after some algebraic simplifications, we can express the change in the trace after (n+1) st step as

$$\delta_{n+1,k} = \Delta_{n+1} - \Delta_n = \frac{N_O\mathbf{h}_k^H\boldsymbol{\beta}_{n,k} - \| \boldsymbol{\beta}_{n,k}^H\mathbf{H}_n \|^2}{N_O + \mathbf{h}_k^H\boldsymbol{\beta}_{n,k}} \quad (16)$$

where $\boldsymbol{\beta}_{n,k} = \mathbf{B}_n\mathbf{h}_k$, and $\delta_{n+1,k}$ represents the increase in the trace if k^{th} transmit antenna is selected in the $(n+1)^{\text{st}}$ step. Since our objective is to maximize the trace, we select the antenna whose contribution to the trace is the maximum. Thus the selection criteria is

$$P = \arg\max_k \delta_{n+1,k} \quad (17)$$

To reduce the number of computations, $\boldsymbol{\beta}_{n+1,k}$ can be computed recursively as

$$\begin{aligned}\boldsymbol{\beta}_{n+1,k} &= \mathbf{B}_{n+1}\mathbf{h}_k \\ &= \mathbf{B}_n\mathbf{h}_k - \frac{\mathbf{B}_n\mathbf{h}_P\mathbf{h}_P^H\mathbf{B}_n\mathbf{h}_k}{N_O + \mathbf{h}_P^H\mathbf{B}_n\mathbf{h}_P} \\ &= \boldsymbol{\beta}_{n,k} - \boldsymbol{\beta}_{n,P}\frac{\mathbf{h}_P^H\boldsymbol{\beta}_{n,k}}{N_O + \mathbf{h}_P^H\boldsymbol{\beta}_{n,P}}\end{aligned} \quad (18)$$

assuming that $\mathbf{h}_P$ is selected in the n^{th} step. The proposed algorithm is summarized in Table 1 with the right column showing order of complexity of each step.

Table 1 Incremental Transmit Antenna Selection, Algorithm-A

Algorithm-A $(N_t, N_r, L_t, N_O, K, \mathbf{H}_{N_t}, \mathbf{H}_{1,2}, ..., \mathbf{H}_{1,K})$	*Complexity*
$S := \{1, 2, ..., N_t\}$	
$\mathbf{H} := \mathbf{O}_{N_r \times 1}$	
IF $K == 1$	
$\mathbf{B} := \mathbf{I}_{N_r}$	
ELSE	
$\mathbf{B} := \left(\mathbf{I}_{N_r} + \frac{1}{N_O} \sum_{j=2}^{K} \mathbf{H}_{1,j} \mathbf{H}_{1,j}^H \right)^{-1}$	$O(KN_r^2 L_t + N_r^3)$
END	
FOR $\forall\ k \in S$	
$\boldsymbol{\beta}_k := \mathbf{B}\mathbf{h}_k$	$O(N_r^2 N_t)$
END	
FOR $n := 1$ to L_t	
FOR $\forall\ k \in S$	
$\delta_k := \frac{N_O \mathbf{h}_k^H \boldsymbol{\beta}_k - \Vert \boldsymbol{\beta}_k^H \mathbf{H} \Vert^2}{N_O + \mathbf{h}_k^H \boldsymbol{\beta}_k}$	$O(N_r N_t L_t^2)$
END	
$P := \arg\max_k \delta_k$	$O(N_t L_t)$
$S := S - \{P\}$	
$\mathbf{H} := [\mathbf{H},\ \mathbf{h}_P]$	
FOR $\forall\ k \in S$	
$\boldsymbol{\beta}_k := \boldsymbol{\beta}_k - \boldsymbol{\beta}_P \frac{\mathbf{h}_P^H \boldsymbol{\beta}_k}{N_O + \mathbf{h}_P^H \boldsymbol{\beta}_P}$	$O(N_r N_t L_t)$
END	
END	
RETURN $\{1, 2, ..., N_t\} - S$	

3.2 Algorithm-B: Decremental Antenna Selection

As the name suggests, the algorithm begins with full set of selected antennas and removes one antenna in each step. Thus at end of n^{th} step, n antennas are rejected and $\mathbf{H}_{N_t - n}$ represents the channel matrix corresponding to remaining N_t-n antennas. The matrix $\mathbf{B}_n$ is defined as

$$\mathbf{B}_n = \left(\mathbf{I} + \frac{1}{N_O} \left\{ \mathbf{H}_{M-n} \mathbf{H}_{M-n}^H + \sum_{j=2}^{K} \mathbf{H}_{1,j} \mathbf{H}_{1,j}^H \right\} \right)^{-1} \quad (19)$$

If in the $(n+1)^{st}$ step, transmit antenna corresponding to k^{th} column of $\mathbf{H}_{N_t}$ is removed, then $\mathbf{B}_{n+1}$ is given by

$$\mathbf{B}_{n+1} = \mathbf{B}_n + \frac{1}{N_O - \mathbf{h}_k^H \mathbf{B}_n \mathbf{h}_k} \mathbf{B}_n \mathbf{h}_k \mathbf{h}_k^H \mathbf{B}_n \quad (20)$$

where (20) is obtained using matrix inversion lemma

Table 2 Decremental Transmit Antenna Selection, Algorithm-B

Algorithm-B $(N_t, N_r, L_t, N_O, K, \mathbf{H}_{N_t}, \mathbf{H}_{1,2}, ..., \mathbf{H}_{1,K})$	*Complexity*
$S := \{1, 2, ..., N_t\}$	
$\mathbf{H} := \mathbf{H}_{N_t}$	
$\mathbf{G} := \mathbf{H}_{N_t} \mathbf{H}_{N_t}^H + \sum_{j=2}^{K} \mathbf{H}_{1,j} \mathbf{H}_{1,j}^H$	$O(KN_r^2 L_t + N_r^2 N_t)$
$\mathbf{B} := \left(\mathbf{I}_{N_r} + \frac{1}{N_O} \mathbf{G} \right)^{-1}$	$O(N_r^3)$
FOR $\forall\ k \in S$	
$\boldsymbol{\beta}_k := \mathbf{B}\mathbf{h}_k$	$O(N_r^2 N_t)$
END	
FOR $n := 1$ to N_t - L_t	
FOR $\forall\ k \in S$	
$\delta_k := \frac{\Vert \boldsymbol{\beta}_k^H H \Vert^2 - N_O \mathbf{h}_k^H \boldsymbol{\beta}_k}{N_O - \mathbf{h}_k^H \boldsymbol{\beta}_k}$	$O(N_r N_t^3)$
END	
$P := \arg\max_k \delta_k$	$O(N_t^2)$
$S := S - \{P\}$	
$\mathbf{H} := \mathbf{H} - \{\ \mathbf{h}_P\}$	
FOR $\forall\ k \in S$	
$\boldsymbol{\beta}_k := \boldsymbol{\beta}_k + \boldsymbol{\beta}_P \frac{\mathbf{h}_P^H \boldsymbol{\beta}_k}{N_O - \mathbf{h}_P^H \boldsymbol{\beta}_P}$	$O(N_r N_t^2)$
END	
END	
RETURN S	

[(3.5.2.2), 18]. Following the development similar to (8)-(16), the change in trace can be expressed as

$$\delta_{n+1,k} = \Delta_{n+1} - \Delta_n = \frac{\Vert \boldsymbol{\beta}_{n,k}^H \mathbf{H}_n \Vert^2 - N_O \mathbf{h}_k^H \boldsymbol{\beta}_{n,k}}{N_O - \mathbf{h}_k^H \boldsymbol{\beta}_{n,k}} \quad (21)$$

where $\boldsymbol{\beta}_{n,k}$ is defined as in (16). Now the elimination criteria is to remove antenna whose removal leads to maximum increase in trace, i.e,

$$P = \arg\max_k \delta_{n+1,k} \quad (22)$$

It is not difficult to show that the vectors $\boldsymbol{\beta}_{n+1,k}$ can be updated as

$$\boldsymbol{\beta}_{n+1,k} = \boldsymbol{\beta}_{n,k} + \boldsymbol{\beta}_{n,P} \frac{\mathbf{h}_P^H \boldsymbol{\beta}_{n,k}}{N_O - \mathbf{h}_P^H \boldsymbol{\beta}_{n,P}} \quad (23)$$

The algorithm along with the complexity analysis is outlined in Table 2. Let us compare the complexity of the two algorithms noting that $KL_t \le N_r$ as the number of streams falling on a receiver node cannot exceed number of receiver antennas. For single user case, Algorithm-A doesn't require matrix inversion and has the lower complexity of $O(\max\{N_r, L_t^2\}N_r N_t)$ compared to $O(N_r N_t^2 L_t)$ of transmit selection algorithm for LRA-ZF in [Table.1, 15]. For multiple users, the complexity of Algorithm-A increases to $O(\max\{N_t L_t^2, N_r^2, N_r N_t\}N_r)$. On the other hand, Algorithm-B always requires matrix inversion and has a complexity order of $O(\max\{N_t^3, N_r^2\}N_r)$. Thus incremental selection algorithm has significantly lower complexity compared to decremental selection algorithm.

When only one antenna is selected, Algorithm-A always finds the optimal solution as it requires just one step. Whereas when N_t-1 transmit antennas are selected, Algorithm-B yields the optimal solution. It is intuitive to think that if Algorithm-A takes fewer steps compared to Algorithm-B, then it is more likely to yield optimal solution than Algorithm-B and vice versa.

4. RECEIVE ANTENNA SELECTION

Receive antenna selection is as important as transmit antenna selection and could be very effective for interference suppression. In this section we will outline the antenna selection procedure at the receiver. In order to exploit the analysis presented in the previous section, we rewrite the matrix $\mathbf{W}$ in (4) using [(3.5.2.3), 18] as

$$\mathbf{W} = (\mathbf{C} - \widetilde{\mathbf{H}}^H \mathbf{R}_r^{-1})\mathbf{R}_r(\mathbf{C} - \widetilde{\mathbf{H}}^H \mathbf{R}_r^{-1})^H + (\mathbf{I} + 1/N_O \widetilde{\mathbf{H}}^H \widetilde{\mathbf{H}})^{-1} \quad (24)$$

Now the optimal solution for receive antenna selection can be obtained as

$$\mathbf{S}_{MMSE} = \arg\min_{\mathbf{F}\in\widetilde{\mathbf{H}}} \sum_{i=1}^{N_t} \left[\left(\mathbf{I} + \frac{1}{N_O}\mathbf{F}^H \mathbf{F} \right)^{-1} \right]_{ii} \quad (25)$$

where $\mathbf{S}_{MMSE}$ denotes the subset of selected receive antennas for MMSE receiver and $\mathbf{F}$ denotes the $L_r \times (KN_t)$ channel matrix obtained by selecting L_r rows of $\widetilde{\mathbf{H}}$. Unlike the development of the transmit antenna selection solution, the expression in (25) cannot be further simplified for the multi-user case. To overcome this, we will consider following approximate solution instead,

$$\hat{\mathbf{S}}_{MMSE} = \arg\min_{\mathbf{F}\in\widetilde{\mathbf{H}}} \operatorname{tr}\left[\left(\mathbf{I} + \frac{1}{N_O}\mathbf{F}^H \mathbf{F} \right)^{-1} \right] \quad (26)$$

The above solution is optimal when there are no interferers and works reasonably well when all symbols including interfering ones have comparable power. As argued earlier, antenna selection for ZF receiver can be implemented by letting $N_O \to 0$ in (26). In the following subsections, we present reduced complexity receive antenna selection algorithms.

4.1 Algorithm-C: Incremental Antenna Selection

The algorithm works in a similar fashion as the incremental transmit antenna selection algorithm presented in the previous section. In each step, we select the row vector that leads to minimal increase in the trace given by (26). Let us use the following notations

$$\mathbf{B}_n = \left(\mathbf{I} + \frac{1}{N_O}\widetilde{\mathbf{H}}_n^H \widetilde{\mathbf{H}}_n \right)^{-1} \quad (27)$$

$$\Delta_n = \operatorname{tr}(\mathbf{B}_n) \quad (28)$$

where $\mathbf{B}_0 = \mathbf{I}$ is the initial condition for the algorithm. Now if in the $(n+1)^{st}$ step, the k^{th} row of $\widetilde{\mathbf{H}}$, $\mathbf{r}_k$, is selected then $\mathbf{B}_{n+1}$ can be updated using matrix inversion lemma [(3.5.2.2), 18] as

$$\mathbf{B}_{n+1} = \mathbf{B}_n - \frac{1}{N_O + \mathbf{r}_k \mathbf{B}_n \mathbf{r}_k^H}\mathbf{B}_n \mathbf{r}_k^H \mathbf{r}_k \mathbf{B}_n \quad (29)$$

Thus the change in trace after $(n+1)^{st}$ step can be easily computed as

$$\delta_{n+1,k} = \Delta_{n+1} - \Delta_n = -\frac{\|\boldsymbol{\alpha}_{n,k}\|^2}{N_O + \boldsymbol{\alpha}_{n,k}\mathbf{r}_k^H} \quad (30)$$

where $\boldsymbol{\alpha}_{n,k} = \mathbf{r}_k \mathbf{B}_n$. The selection rule is to choose the antenna that leads to minimum increase in trace, i.e,

$$P = \min \delta_{n+1,k} \quad (31)$$

The vectors $\boldsymbol{\alpha}_{n+1,k}$ can be recursively computed as

$$\boldsymbol{\alpha}_{n+1,k} = \boldsymbol{\alpha}_{n,k} - \boldsymbol{\alpha}_{n,P}\frac{\boldsymbol{\alpha}_{n,k}\mathbf{r}_P^H}{N_O + \boldsymbol{\alpha}_{n,P}\mathbf{r}_P^H} \quad (32)$$

The algorithm is summarized in Table 3 with the right column showing the complexity corresponding to each part of the algorithm.

4.2 Algorithm-D: Decremental Antenna Selection

The algorithm is quite similar to Algorithm-B presented in the previous section. For the sake of brevity we omit the development and present the implementation in Table 4. From Table 3 and 4, it is apparent that Algorithm-C has significantly lower complexity, $O(KN_t N_r L_r)$, compared to $O(\max\{K^2 N_t^2, KN_t N_r, N_r^2\}KN_t)$ for Algorithm-D. Observing that $KN_t \le N_r$ is always satisfied to avoid

Table 3 Incremental Receive Antenna Selection, Algorithm-C

Algorithm-C $(N_t, N_r, L_r, N_O, K, \widetilde{\mathbf{H}})$	
$S := \{1, 2, ..., N_r\}$	*Complexity*
$\mathbf{B} := \mathbf{I}_{KN_t}$	
FOR $\forall\ k \in S$	
$\boldsymbol{\alpha}_k := \mathbf{r}_k$	
END	
FOR $n := 1$ to L_r	
FOR $\forall\ k \in S$	
$\delta_k := \dfrac{-\lVert \boldsymbol{\alpha}_k \rVert^2}{N_O + \boldsymbol{\alpha}_k \mathbf{r}_k^H}$	$O(KN_tN_rL_r)$
END	
$P := \arg\min_k \delta_k$	$O(N_rL_r)$
$S := S - \{P\}$	
FOR $\forall\ k \in S$	
$\boldsymbol{\alpha}_k := \boldsymbol{\alpha}_k - \boldsymbol{\alpha}_P \dfrac{\boldsymbol{\alpha}_k \mathbf{r}_P^H}{N_O + \boldsymbol{\alpha}_P \mathbf{r}_P^H}$	$O(KN_tN_rL_r)$
END	
END	
RETURN $\{1, 2, ..., N_r\} - S$	

overloading the receiver, complexity order of Algorithm-D can be expressed as $O(KN_tN_r^2)$. The performance relationship between the two algorithms is same as between Algorithm-A and Algorithm-B.

5. SIMULATION RESULTS

In this section we validate the antenna selection algorithms developed in previous sections and compare them against some of the existing antenna selection techniques [12]-[15],[16]. We also consider the norm-based selection (NBS) approach where transmit/receive antennas corresponding to the columns of the channel matrix with largest Euclidean norms are selected. We consider an independent and identically distributed (i.i.d) Rayleigh fading channel unless specified otherwise. All the simulations results are generated using Monte Carlo simulation of 10^6 channel realizations. Throughout our simulations, we use 16-QAM modulation.

Figure 1 compares the error performance of MMSE receiver for various transmit antenna selection algorithms for a single-link network. We observe that when only one transmit antenna is selected, NBS and Algorithm-A are equivalent and optimal, as expected. Whereas when N_t-1 transmit antennas are selected, Algorithm-B yields optimal performance. In fact, incremental antenna

Table 4 Decremental Receive Antenna Selection, Algorithm-D

Algorithm-D $(N_t, N_r, L_r, N_O, K, \widetilde{\mathbf{H}})$	
$S := \{1, 2, ..., N_r\}$	*Complexity*
$\mathbf{B} := \left(\mathbf{I}_{KN_t} + \dfrac{1}{N_O}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)^{-1}$	$O(K^3N_t^3)$
FOR $\forall\ k \in S$	
$\boldsymbol{\alpha}_k := \mathbf{r}_k\mathbf{B}$	$O(K^2N_rN_t^2)$
END	
FOR $n := 1$ to N_r - L_r	
FOR $\forall\ k \in S$	
$\delta_k := \dfrac{\lVert \boldsymbol{\alpha}_k \rVert^2}{N_O - \boldsymbol{\alpha}_k \mathbf{r}_k^H}$	$O(KN_tN_r^2)$
END	
$P := \arg\min_k \delta_k$	$O(N_r^2)$
$S := S - \{P\}$	
FOR $\forall\ k \in S$	
$\boldsymbol{\alpha}_k := \boldsymbol{\alpha}_k + \boldsymbol{\alpha}_P \dfrac{\boldsymbol{\alpha}_k \mathbf{r}_P^H}{N_O - \boldsymbol{\alpha}_P \mathbf{r}_P^H}$	$O(KN_tN_r^2)$
END	
END	
RETURN S	

selection is close to optimal for $L_t < N_t/2$ whereas decremental antenna selection is desirable for $L_t > N_t/2$.

One can also observe that as number of selected transmit antennas increases, the error rate of deterministic selection deteriorates less rapidly compared to various antenna selection algorithms. This is because of the fact that the diversity gain associated with deterministic selection is only from receive diversity, which reduces by one as one more transmit antenna is excited. Whereas transmit antenna selection algorithms help achieve transmit diversity in addition to receive diversity and the diversity gain is $(N_t$-$L_t)(N_r$-$L_t)$, which decreases more rapidly with increasing L_t. Hence, as L_t increases selection algorithms suffer more compared to deterministic/non-optimal selection strategies.

Figure 2 evaluates the performance of various sub-optimal selection approaches in presence of co-channel interference. We consider a 2-link network with each node having 6 antennas. Each transmit node chooses 3 antennas to transmit. We notice that both deterministic selection and NBS selection BER curves are flat and no diversity gain is rendered. On the other hand, sub-optimal selection algorithms A and B are able to exploit transmit diversity better and provide reasonably good BER performance. It is evident that the incremental selection algorithm performs better than the decremental selection

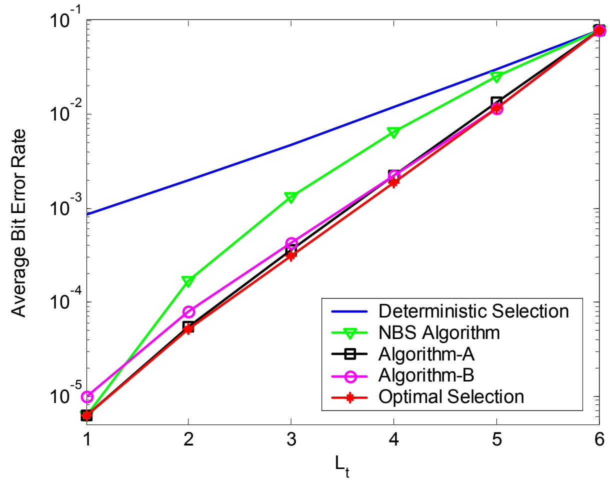
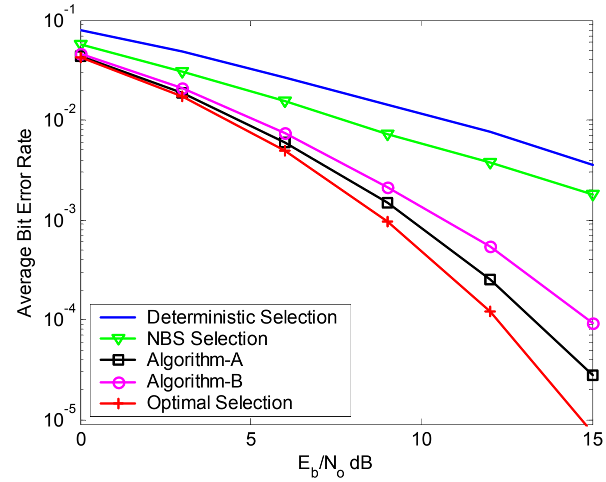


Figure 1 MMSE error rate L_t at 5dB SNR/bit for (6,6) MIMO in a Rayleigh faded channel.

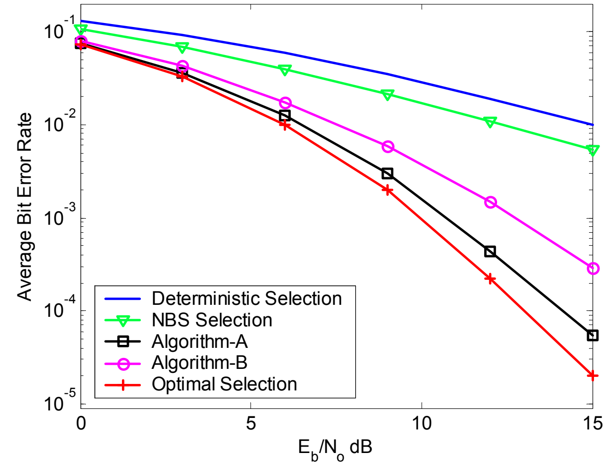
algorithm both and the performance gap widens as interference becomes strong. The robustness of the algorithms developed in this paper is highlighted by the observation that even in presence of strong interference, there is only minimal degradation in error performance.

Figure 3 compares the performance of our algorithms against Gore's Algorithm [14] and Berenguer's algorithm [15] for ZF receiver in the presence of transmit correlation and no correlation at the receiver. The transmit correlation matrix, $\mathbf{R}_t$, for the four transmit antennas used in the simulations is same as in [Eq. (15), 14]. Gore's algorithm can be described as an optimal-deterministic selection. It selects transmit antennas to minimize the error probability of the worst stream; the selected antennas are determined by the correlation matrix and are 1, 2, 4 in this case. Obviously it isn't an optimal strategy because all the streams and not just the worst stream determine the error performance of a linear receiver.

From Figure 3, we observe that although Gore's algorithm performs almost 3 dB better than non-optimal deterministic selection, which selects {1, 2, 3}, it still leaves room for improvement as it fails to exploit the transmit diversity that can be achieved by choosing optimal set of transmit antennas at any given stage. For example, when the channel is in deep fade, it is better to choose two strongly correlated streams and an uncorrelated stream in deep fade rather than all three uncorrelated streams with two being in deep fade. We also note that Berenguer's transmit antenna selection algorithm for ZF receiver performs very close to Algorithm-A but is still 1 dB worse than Algorithm-B, which is optimal in this case. This example again highlights the strengths of Algorithm-A/B, which always give near optimal performance regardless of the channel matrix's statistical properties.



(a) SIR = 10 dB



(b) SIR = 0 dB

Figure 2 MMSE error rate for (6,6) MIMO in a Rayleigh faded channel with 2 links each using 3 transmit antennas.

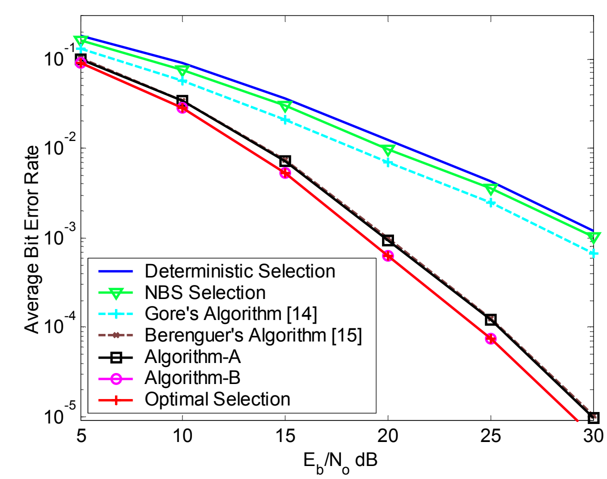


Figure 3 ZF error rate versus the SNR for (4,4) MIMO in a Rayleigh faded channel with transmit correlation. Three transmit antennas are selected.

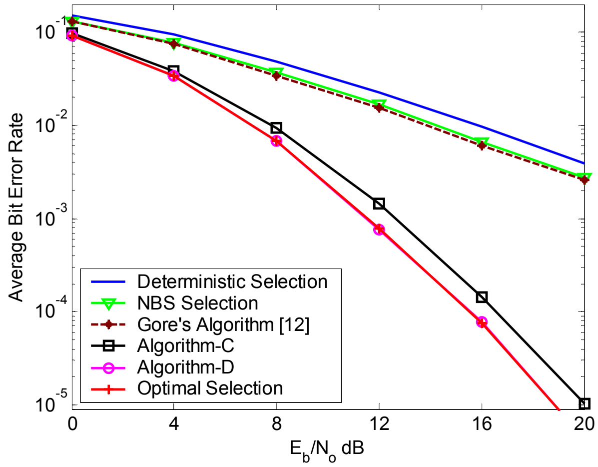


Figure 4 MMSE error rate for (6,6) MIMO in a Rayleigh faded channel. Four receive antennas are selected.

Figure 4 illustrates the error performance of various receive antenna selection algorithms for the MMSE receiver. We notice that Gore's algorithm [12] is only slightly better than the NBS algorithm and is way off the optimal solution, which is not surprising considering the fact that the algorithm aims to optimize the capacity instead of error rate. As evident, sub-optimal algorithms C/D yield near-optimal performances. In particular, decremental selection algorithm performs better than the incremental algorithm as 4 receive antennas are selected out of 6 antennas.

6. CONCLUSION

In this paper, we have presented new computationally efficient algorithms for transmit and receive antenna selection for MMSE and ZF receivers. The proposed algorithms are found to be near optimal both in presence of interference and transmit/receive correlation. Incremental selection algorithms have lower complexity compared to decremental selection algorithms and are thus more attractive.

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