Pilot Assisted SNR Estimation in a Non-Coherent M-FSK Receiver with a Carrier Frequency Offset

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***Abstract*—Pilot-assisted estimation of signal-to-noise ratio (SNR) is considered for an orthogonal non-coherent M-ary FSK receiver having a carrier frequency offset (CFO) at the front end. The received signal is also corrupted by symbol-rate Rayleigh fading and additive white Gaussian noise. A two-step estimation procedure is designed by observing received data in the receiver. In the first step, an asymptotically unbiased estimate of CFO is obtained using a moments based approach. This estimate is then used to derive a maximum-likelihood estimator for the SNR. The Cramer Rao Bound for SNR has been derived to compare the overall estimator performance in terms of its variance.**

I. Introduction

Signal-to-noise ratio (SNR) is an important performance metric in a communication system. In addition to its use in many receiver functions, the estimates of SNR can be used by a receiver to determine its range and connectivity strength from its source, which may help the receiver in deciding about helper selection [1], [2] in cooperative communications, which is the main motivation behind this study. Furthermore, if the radios are battery-assisted and energy-constrained, simple modulation schemes are required to reduce the circuit consumption of energy. A non-coherent frequency shift keying (NCFSK) modulation is not only power efficient at the transmitter side, but is relatively simple at the receiver side owing to the envelope detection. Therefore, SNR estimation for non-coherent FSK receiver finds its applications in many wireless communications areas. However, the carrier frequency offset (CFO) present at the receiver side not only increases the bit error rate but also prevents accurate estimation of received SNR. For an orthogonal FSK receiver, the CFO affects the orthogonality of receiver branches and sophisticated signal processing algorithms should be designed to cater for the correlation between the receiver branches introduced by CFO. Therefore, in this paper, we deal with the SNR estimation for a non-coherent M-ary FSK receiver with a CFO, where CFO is treated as a nuisance parameter.

Communication systems frequently use training sequences before actual data transmission to help receiver in correct signal detection. Many approaches that employ NCFSK receivers use training symbols to perform various receiver functions, like channel estimation and start of packet synchronization, e.g., [3]. Also the problem of CFO is highly non-linear in nature and many other approaches can be found in literature that try to estimate the CFO using training sequences, e.g., [4] and [5]. In many cooperative communication applications, the estimator can use the detected data as training sequence for SNR estimation and is reasonable in a multi-hop broadcast application [6], where every node must decode the entire message; the detected data are all assumed to be correct regardless of the value of SNR. Therefore, in this paper, we will estimate the SNR for an NCMFSK receiver as well as the carrier frequency offset with the help of a training sequence.

Several authors have attacked the problem of estimating the SNR for the perfectly synchronized receiver under Rayleigh fading conditions. For example, [7] considers the SNR estimation for binary FSK having knowledge of noise power spectral density, while [8] treats a more general case of SNR estimation for M-ary FSK receiver without any prior knowledge of noise power. In [9], the authors estimate the SNR for a non-coherent binary FSK receiver with a CFO, however, the analysis is only limited to binary case and the approach in [9] cannot be generalized to M-FSK SNR estimation in presence of CFO. In this paper, we design a two-step procedure for SNR estimation in M-FSK receiver. We estimate the CFO first using a moment based estimator and then use this estimate of CFO to design a maximum likelihood estimator for SNR. We will also derive the Cramer Rao bound (CRB) for the estimator and show the overall SNR estimator performance with respect to this bound.

The rest of the paper is organized as follows. Section II describes the system model while Section III treats the derivations of the CFO and SNR estimators. Section IV derives the CRB for these estimators and in Section V, we will discuss the simulation results for various estimators. The paper then concludes in Section VI.

II. System Model

Consider a Rayleigh fading communication system employing M-FSK modulation, where a block of data with g symbols undergoes symbol-rate fading. The received signal observed at the receiver end is given as

$$r(t) = \sqrt{E_s}\alpha(t)\exp\left(-j2\pi(f_c + f_m + \Delta f)t + \phi\right) + n(t), \quad 0 \le t \le T, \quad m \in \{1, 2, ..., M\}, \tag{1}$$

where E_s is the signal power, T is the symbol time, f_c is the carrier frequency, and f_m is the FSK frequency corresponding to the message signal. Note that for noncoherently detected FSK the orthogonality condition specifies that the minimum frequency spacing should be $1/T$. The shift in the carrier frequency at the receiver is denoted by Δf and ϕ is the unknown carrier phase. Without the loss of generality we will set $\phi = 0$, since we are dealing with the non-coherent receiver. The additive white Gaussian noise is denoted as $n(t)$ and $\alpha(t)$ is the Rayleigh fading envelope. Thus, the integrator output, matched to the transmitted signal S_m is given as

$$v_m = \int_0^T r(t)\sqrt{E_s}\exp\left(j2\pi(f_c + f_m)t\right)dt. \quad (2)$$

Since we are using pilot symbols for estimation purpose, therefore, without the loss of generality, we assume that all the transmitted symbols are identical, and correspond to frequency f_1. Therefore, we get the the signal part, after simplifying the above equation, as

$$v_{ms} = P\alpha\left[\frac{1-\exp\left(-j2\pi\Delta fT\right)}{j2\pi(m-1)+j2\pi\Delta fT}\right], \quad m \in \{1, ..., M\}, \quad (3)$$

where $P = \frac{E_s T}{2}$. For the sake of simplicity, we assume that the average symbol energy is unity, i.e., $P = 1$, and thus the matched filter output for the M-FSK receiver having a frequency carrier shift of Δf is given as

$$x_m = \alpha\left[\frac{1-\exp\left(-j2\pi\rho\right)}{j2\pi(m-1)+j2\pi\rho}\right] + n_m, \quad m \in \{1, 2, ..., M\}, \quad (4)$$

where $\rho = \Delta fT$ is the normalized frequency error and the CFO factors in all M branches are given as

$$A_m = \frac{1-\exp\left(-j2\pi\rho\right)}{j2\pi(m-1)+j2\pi\rho}, \quad m \in \{1, 2, ..., M\}. \quad (5)$$

The received symbols from M branches are denoted by $x_m = \alpha A_m + n_m$, where the index m denotes the branch index; $m = \{1, 2, ...M\}$. The channel gain, α, and the noise element, n_m, are zero mean complex Gaussian random variables with variance of $S/2$ and $N/2$ per real dimension, respectively. Thus the received data is represented as $\mathbf{X} = [x_1\ x_2\ ...\ x_M]^T$, where T denotes the matrix transpose operation. Since we assumed that the average symbol energy is unity, the expected energy of the received symbol is given as S and hence the signal-to-noise ratio is given by $\gamma = \frac{S}{N}$. Our interest is to find the estimate of the average SNR using the observed data, but it can be seen that the factor A_m in (5) will reduce the signal power if $\rho \neq 0$. From (4), it can be seen that the signal power is reduced by the CFO factor in the first branch (x_1) that contains the signal and there is also a signal spill which introduces some part of signal power in the rest of the branches. Because of the signal leakage in all of the receiver branches, the branches no longer remain orthogonal and hence there exists a correlation between the outputs.

From (4), as α and n are zero-mean complex Gaussian, therefore, the probability density function (PDF) of the received data from all the branches remain complex Gaussian. However, because of the non-linear nature of CFO factors, which are given as

$$|A_m|^2 = \frac{\sin^2(\pi\rho)}{\left(\pi(m-1+\rho)\right)^2}, \quad (6)$$

the conventional maximum likelihood (ML) estimator cannot be applied to get closed form estimates of three unknown parameters simultaneously. Although numerical methods can be applied but they will increase the complexity of the receiver. Hence, we propose a two step procedure. In the first step, a method of moments (MoM) estimator is designed to compute the estimates of CFO and the noise power. It will be shown that these estimates are unbiased and gives accurate estimation specifically at high SNR. However, the estimate of the signal power is highly biased at high SNR and this method cannot be used for SNR estimation. However, an ML estimator is then designed that takes knowledge of the CFO and noise power from MoM and compute high reliable estimates of signal power. Together, both estimators give efficient estimates of CFO and the SNR.

III. Design of Estimators

In the following subsections, we estimate ρ using a moments based approach.Then we will show how the estimate of CFO can help in estimating the SNR using the conventional ML method.

A. Method of Moments Approach

From (5), it can be noticed that the signal spill in the receiver branches is relative to the first branch and because of this relativity two consecutive branches are sufficient to estimate CFO. Therefore, we select the first two branches with received data x_1 and x_2. In this case, the first order statistics obtained from both the branches are as follows

$$\begin{bmatrix} \mathbb{E}|x_1|^2 \\ \mathbb{E}|x_2|^2 \end{bmatrix} = \begin{bmatrix} |A_1|^2 S + N \\ |A_2|^2 S + N \end{bmatrix} := \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad (7)$$

and the first cross-moment is given as

$$\mathbb{E}\left\{|x_1|^2|x_2|^2\right\} = 2|A_1|^2|A_2|^2S^2 + SN(|A_1|^2 + |A_2|^2) + N^2. \quad (8)$$

Note that in an NCFSK receiver, the $|x_m|^2$ $\forall m$ are available after the square law detector. In practice, we replace the ensemble averages in previous equations with those of time averages, i.e., $\mathbb{E}\left\{|x|^2\right\} \approx \frac{1}{g}\sum_{i=1}^{g}|x_i|^2$. We also let $\tilde{A}_1 = |A_1|^2 S$, $\tilde{A}_2 = |A_2|^2 S$, and $\mathbb{E}\left\{|x_1|^2|x_2|^2\right\} := z_3$, then Equations (7) and (8) can be solved simultaneously to get the estimate of the noise power, N, as

$$\hat{N} = \frac{1}{2}\left(z_1 + z_2 - \sqrt{z_1^2 - 6z_1z_2 + z_2^2 + 4z_3}\right), \quad (9)$$

and the estimates of $\tilde{A}_1$ and $\tilde{A}_2$ are given as $\hat{\tilde{A}}_1 = z_1 - \hat{N}$ and $\hat{\tilde{A}}_2 = z_2 - \hat{N}$. Finally, we get the estimates of ρ and then the signal power, as given by following equations,

$$\hat{\rho} = \frac{\hat{\tilde{A}}_2 + \sqrt{\hat{\tilde{A}}_1\hat{\tilde{A}}_2}}{\hat{\tilde{A}}_1 - \hat{\tilde{A}}_2}, \quad (10)$$

$$\hat{S} = \frac{\hat{\hat{A}}_1}{\hat{A}_1} = \hat{A}_1 \left(\frac{\pi \hat{\rho}}{\sin(\pi \hat{\rho})} \right)^2 . \tag{11}$$

The above procedure can be repeated for each y_i, where $y_i = \left[\ |x_{2i-1}|^2 \quad |x_{2i}|^2\ \right]^T$, $i = \{1, 2, ..., M/2\}$, and the estimates of noise power are averaged over these $M/2$ epochs since the lower branches contain mostly noise. If we define $\zeta(\theta) := \frac{1}{n} \sum_{i=1}^{n} (\hat{\theta}_i - \theta)^2$ as the sample variance of the error of estimator for parameter θ, where $\theta \in \left\{ \tilde{A}_1, \tilde{A}_2, \rho \right\}$, then a plot of ζ is shown for various values of SNR in top plot of Fig. 1. It can be seen that though $\zeta(\tilde{A}_1)$ and $\zeta(\tilde{A}_2)$ increases with increasing SNR, the variance of error for ρ decreases. Thus the estimator of ρ is anticipated to give better performance as SNR increases. However, since $\zeta(\tilde{A}_1)$ increases with increasing SNR, the SNR estimate resulting from (11) will show a high mean squared error in the high SNR region. We can write $\hat{\rho} = f(\mathbf{z})$, where $\mathbf{z} = [z_1\ z_2\ z_3]^T$, and the non-linear function f is given as

$$f(\mathbf{z}) = \frac{z_2 - z_1 + 2\sqrt{z_3 - z_1 z_2} + \sqrt{z_1^2 - 6 z_1 z_2 + z_2^2 + 4 z_3}}{2(z_1 - z_2)}. \tag{12}$$

Let $\boldsymbol{\mu} = [\mu_1\ \mu_2\ \mu_3]^T$ be the vector of true moments, then a second order Taylor series expansion of f about $\boldsymbol{\mu}$ yields $\hat{\rho} \approx \rho + \mathbf{v}^T(\mathbf{z} - \boldsymbol{\mu}) + 1/2(\mathbf{z} - \boldsymbol{\mu})^T \mathbf{H}(\mathbf{z} - \boldsymbol{\mu})$, where $\mathbf{v} = \bigtriangledown f|_{\mathbf{z} - \boldsymbol{\mu}}$ and $\mathbf{H}$ is the Hessian matrix of f. Taking expectation on both sides yields

$$Bias(\hat{\rho}) \approx \frac{1}{2g} tr(\mathbf{HC}). \tag{13}$$

In the above equation, tr represents the trace of a matrix and $\mathbf{C}$ is the symmetric covariance matrix of $\boldsymbol{\mu}$, where $c_{ii} = \left\{ \mu_1^2,\ \mu_2^2,\ 2\mu_3^2 + 4\mu_1\mu_2 \left(\mu_3 - \mu_1\mu_2\right) \right\}$, $i \in \{1, 2, 3\}$, $c_{21} = \mu_3 - \mu_1\mu_2$, and $c_{3i} = \mu_i(c_{21} + \mu_3)$, $i \in \{1, 2\}$. The final expression of bias from (13) can thus be obtained, which is given at the bottom of this page and where $P = \mu_1 - \mu_2$, $Q = c_{21}$, and $R = \mu_1^2 - 6\mu_1\mu_2 + \mu_2^2 + 4\mu_3$. The bottom plot of Fig. 1 shows this approximate theoretical bias of the CFO estimator and it can be seen that the estimator shows a very low bias and is asymptotically zero for $SNR \geq 7dB$.

B. Maximum Likelihood Estimation

Each received data symbol x_m $\forall m = \{1, 2, ...M\}$ is a complex number and can be written as

$$x_m = (\alpha_c + j\alpha_s)(A_{c_m} + jA_{s_m}) + (n_{c_m} + jn_{s_m}), \tag{14}$$

where a complex number β is written as $\beta = \beta_c + j\beta_s$, with $j = \sqrt{-1}$. Then

$$x_m = x_{c_m} + jx_{s_m}, \tag{15}$$

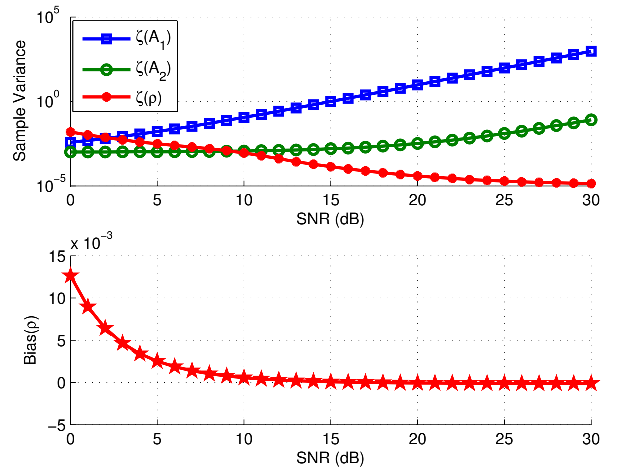

Fig. 1. Sample variance of error for different parameters and bias of CFO estimator; g=1000

where $x_{c_m} \triangleq \alpha_c A_{c_m} - \alpha_s A_{s_m} + n_{c_m}$ and $x_{s_m} \triangleq \alpha_c A_{s_m} + \alpha_s A_{c_m} + n_{s_m}$. Thus we can define zero mean real Gaussian random vectors X_c and X_s given by

$$\mathbf{X}_c \triangleq \begin{bmatrix} x_{c_1} \\ \vdots \\ x_{c_M} \end{bmatrix}, \quad \mathbf{X}_s \triangleq \begin{bmatrix} x_{s_1} \\ \vdots \\ x_{s_M} \end{bmatrix} \tag{16}$$

with covariance matrices $\mathbf{K}_{cc}$ and $\mathbf{K}_{ss}$ and cross-covariance matrix $\mathbf{K}_{cs}$, that is

$$\mathbb{E}\left[\mathbf{X}_c \mathbf{X}_c^T\right] = \mathbf{K}_{cc}, \mathbb{E}\left[\mathbf{X}_s \mathbf{X}_s^T\right] = \mathbf{K}_{ss}, \mathbb{E}\left[\mathbf{X}_c \mathbf{X}_s^T\right] = \mathbf{K}_{cs}, \tag{17}$$

such that the generating vector $\mathbf{X} = \mathbf{X}_c + j\mathbf{X}_s$ is a proper random vector with vanishing psuedocovariance matrix [10]. Thus,

$$\mathbb{E}\left[\left(\mathbf{X}_c + j\mathbf{X}_s\right) \left(\mathbf{X}_c + j\mathbf{X}_s\right)^T \right] = \mathbf{0}, \tag{18}$$

which implies

$$\mathbf{K}_{cc} = \mathbf{K}_{ss}, \quad \mathbf{K}_{cs}^T = -\mathbf{K}_{cs}. \tag{19}$$

Note that $\forall\ j, k = \{1, 2, ..., M\}$

$$\mathbb{E}[x_{c_j}^2] = (\mathbf{K}_{cc})_{jj} = \frac{1}{2} \left(S|A_j|^2 + N \right), \tag{20}$$

$$\mathbb{E}[x_{c_j} x_{c_k}] = (\mathbf{K}_{cc})_{jk} = \frac{S}{2} \left(A_{c_j} A_{c_k} + A_{s_j} A_{s_k} \right), \tag{21}$$

$$\begin{aligned} Bias(\hat{\rho}) \approx & \frac{1}{2gP^3\sqrt{Q}R^{3/2}} \left[-2P^4\sqrt{Q}\left(2Q + R\right) + P^2 \left\{ -8Q^{3/2}\left(R + \mu_1^2 - 4\mu_1\mu_2 + \mu_2^2\right) + R^{3/2}\left(-7Q + \mu_1\mu_2\right) \right. \right. \\ & \left. + \sqrt{Q}\left(\mu_1^2\left(R - \mu_1^2 + 6\mu_1\mu_2 - 17\mu_2^2\right) - 4\mu_3\left(R + \mu_1^2 - 6\mu_1\mu_2 + 2\mu_3\right)\right) \right\} + 2\sqrt{Q}\left\{ -4\left(QR\right)^{3/2} \right. \\ & \left. \left. + \left(2\sqrt{Q}R^{3/2} + R^2\right)\left(\mu_1^2 + \mu_2^2\right) + 2Q\left(\mu_1^4 + 44\mu_1\mu_2\mu_3 - 6\mu_1^2\left(3\mu_2^2 + \mu_3\right) + \left(\mu_2^2 - 8\mu_3\right)\left(\mu_2^2 + 2\mu_3\right)\right) \right\} \right] \end{aligned}$$

and

$$\mathbb{E}[x_{c_j}x_{s_k}] = (\mathbf{K}_{cs})_{jk} = \begin{cases} \frac{S}{2}\left(A_{c_j}A_{s_k} - A_{s_j}A_{c_k}\right) & j \neq k \\ 0 & j = k \end{cases} \quad (22)$$

However, from (5),

$$A_{c_m} = \frac{\sin(2\pi\rho)}{2\pi(m-1+\rho)}, \quad A_{s_m} = -\frac{1-\cos 2\pi\rho}{2\pi(m-1+\rho)}. \quad (23)$$

Hence for any $j, k \in \{1, ..., M\}$ such that $j \neq k$,

$$A_{c_j}A_{s_k} - A_{s_j}A_{c_k} = 0, \quad (24)$$

and thus $\mathbf{K}_{cs} = 0$. The joint PDF of $\mathbf{X}_c$ and $\mathbf{X}_s$ can, therefore, be written as

$$p_{\mathbf{X}_c,\mathbf{X}_s}(\mathbf{x}_c, \mathbf{x}_s) = \frac{1}{(2\pi)^M\sqrt{\det(\mathbf{K})}} \exp\left\{-\frac{1}{2}\begin{bmatrix} \mathbf{x}_c^T & \mathbf{x}_s^T \end{bmatrix}\mathbf{K}^{-1}\begin{bmatrix} \mathbf{x}_c \\ \mathbf{x}_s \end{bmatrix}\right\}, \quad (25)$$

where $\mathbf{K}$ is a non-singular matrix and it can be shown that

$$\mathbf{K}^{-1} = \begin{bmatrix} \mathbf{K}_{cc}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{cc}^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix}, \quad (26)$$

where we use $\mathbf{K}_{cc}^{-1} := \mathbf{D}$ for notational simplicity and

$$\sqrt{\det(\mathbf{K})} = (\det(\mathbf{D}))^{-1} = \frac{N^{M-1}}{M^2}\left[N + S\sum_{m=1}^{M}|A_m|^2\right]. \quad (27)$$

We can therefore write (25) as

$$p_{\mathbf{X}_c,\mathbf{X}_s}(\mathbf{x}_c, \mathbf{x}_s) = \frac{\exp\left\{-\frac{1}{2}\left(\mathbf{x}_c^T\mathbf{D}\mathbf{x}_c + \mathbf{x}_s^T\mathbf{D}\mathbf{x}_s\right)\right\}}{(2\pi)^M\sqrt{\det(\mathbf{K})}}. \quad (28)$$

For a packet length consisting of g symbols which are all independent, the joint PDF of the data is the product of the marginal PDFs and this likelihood is given as

$$L(\mathbf{X}_1, ..., \mathbf{X}_g; S, N) = (2\pi)^{-Mg}(\det \mathbf{K})^{-g/2} \exp\left\{-\frac{1}{2}\left[tr\left(\mathbf{D}\sum_{i=1}^{g}x_{c_{m,i}}x_{c_{m,i}}^T + \mathbf{D}\sum_{i=1}^{g}x_{s_{m,i}}x_{s_{m,i}}^T\right)\right]\right\}. \quad (29)$$

In (29), the real part of the ith received symbol is denoted as $x_{c_{m,i}}$ while the imaginary part as $x_{s_{m,i}}$; the first sub-index m denotes the branch index where $\{m = 1, 2, ..., M\}$ and the second sub-index i is the time index where $i = \{1, 2, ..., g\}$; g being the packet length. The log-likelihood is given as

$$\Lambda(\mathbf{X}_i; S|N, \rho) = c - g\log(\det\sqrt{\mathbf{K}}) - \frac{1}{2}\left[tr\left(\mathbf{D}\sum_{i=1}^{g}x_{c_{m,i}}x_{c_{m,i}}^T\right) + tr\left(\mathbf{D}\sum_{i=1}^{g}x_{s_{m,i}}x_{s_{m,i}}^T\right)\right], \quad (30)$$

where the constant term is lumped into c, which does not depends upon S. Now for the ML estimation, we have to take the derivative of the log-likelihood with respect to S. Let's consider each term one by one.

$$-g\log(\det\sqrt{\mathbf{K}}) = c_1 - g\log\left(N + S\sum_{m=1}^{M}|A_m|^2\right), \quad (31)$$

where c_1 is some other constant. Taking the derivative with respect to S yields

$$\frac{\partial}{\partial S}\left\{-g\log(\det\sqrt{\mathbf{K}})\right\} = \frac{-g\sum_{m=1}^{M}|A_m|^2}{N + S\sum_{m=1}^{M}|A_m|^2}. \quad (32)$$

The derivative of the term involving the real part of the received signal is given as

$$\frac{\partial}{\partial S}\left\{tr\left(\mathbf{D}\sum_{i=1}^{g}x_{c_{m,i}}x_{c_{m,i}}^T\right)\right\} = \sum_{i=1}^{g}\sum_{j=1}^{M}\sum_{k=1}^{M}d'_{jk}x_{c_{j,i}}x_{c_{k,i}}, \quad (33)$$

where

$$d'_{jk} = \frac{-2\left(A_{c_j}A_{c_k} + A_{s_j}A_{s_k}\right)}{\left(N + S\sum_{m=1}^{M}|A_m|^2\right)^2} = \frac{-2A_jA_k^*}{\left(N + S\sum_{m=1}^{M}|A_m|^2\right)^2}, \quad (34)$$

where $*$ denotes the conjugate of a complex number. For the term involving the imaginary part of the signal, the expressions remain the same as in (33) with x_c replaced with x_s. Therefore, combining the Equations (32) and (33) and setting the result of $\frac{\partial}{\partial S}\Lambda(\mathbf{X}_i; S|N, \rho) = 0$ gives the estimate of the signal power as

$$\hat{S} = \frac{1}{\Psi}\left(\frac{\sum_{i=1}^{g}\sum_{j=1}^{M}\sum_{k=1}^{M}A_jA_k^*\Re\left\{x_{j,i}x_{k,i}^*\right\}}{g\Psi} - \hat{N}\right), \quad (35)$$

where $\Psi := \sum_{m=1}^{M}|A_m|^2$ and $\Re$ denotes the real part of a complex number.

IV. Cramer Rao Bound

In this section, we will derive the CRB for the estimator of signal power derived in the previous section. The CRB, which is the benchmark on the variance of an estimator, is a function of the Fisher Information, $F(S)$, and is given as $CRB = 1/F(S)$, where $F(S)$ is given as $F(S) = -\mathbb{E}\left[\frac{\partial^2\Lambda}{\partial S^2}\right]$. It should be noticed that the CRB is a function of signal power S, since we assume that ρ and noise power are known in the maximum likelihood estimation. The second derivative of the log-likelihood function is given as

$$\frac{\partial^2\Lambda}{\partial S^2} = \frac{g\Psi^2}{(N + S\Psi)^2} - \frac{2\Psi\sum_i\sum_j\sum_k A_jA_k^*\Re\left\{x_{j,i}x_{k,i}^*\right\}}{(N + S\Psi)^3} \quad (36)$$

Taking expectation of the above equation with respect to data X will not change the first term, however, the expected value of the second term can be written as

$$\frac{g\Psi}{(N + S\Psi)^3}\mathbb{E}\left[\sum_{m=1}^{M}|A_m|^2|X_m|^2 + 2\sum_{j=1}^{M-1}\sum_{k>j}^{M}A_jA_k^*\Re\left\{x_jx_k^*\right\}\right] \quad (37)$$

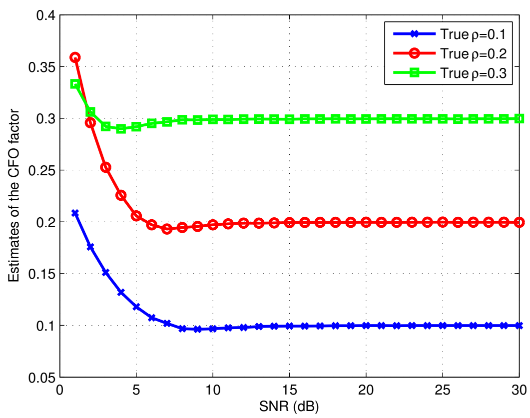

Fig. 2. Estimation of ρ by MoM estimator for $g = 1000$.

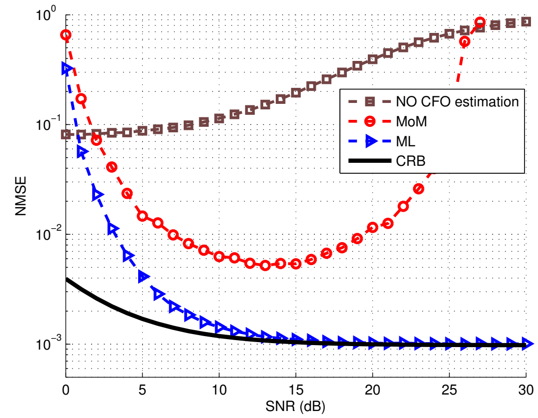

Fig. 3. NMSE plot for different SNR estimators; $M = 8$, $g = 1000$.

where these expectations can be evaluated using (20) and (21). Thus the Fisher Information of the signal to noise ratio given noise power N and CFO ρ is given as

$$F(\gamma|N,\rho) = \frac{g\Psi}{N^2\left(1+\gamma\Psi\right)^3} \left[\Psi + \gamma\left(\sum_{j=1}^{M}\sum_{k=1}^{M}(A_jA_k^*)^2 - \sum_{j=1}^{M-1}\sum_{k>j}^{M}|A_j|^2|A_k|^2\right)\right] \tag{38}$$

V. Simulation Results

In this section, we compare the normalized mean squared error (NMSE) (normalized with respect to the square of the true value of the SNR) of the estimators using simulations for a packet length, $g = 1000$, averaged over 20,000 trials. Fig. 2 shows the estimates the CFO factor, ρ, using the MoM approach described in Section III-A. It can be seen that the estimate of CFO is highly accurate for $SNR \geq 7dB$ for different values of CFO factors.

Fig. 3 shows the NMSE for the SNR estimation resulting from both the MoM and the ML estimators for $M = 8$. In the *No CFO estimation* case, the SNR is estimated, without estimating the CFO, from the ML algorithm derived in [8]. We observe that as the SNR increases, the leakage in the lower branches increases and consequently the error in the estimation increases. It can also be noticed that the MoM estimator works well for the SNR estimation at low values of SNR. However, it suffers in the high SNR region due to a higher bias for high values of signal power in this range. The same cup shaped curve of MoM estimator can also be seen in [8]. From the MoM estimator, we know that the estimate of ρ is quite accurate and this estimate also depends on the noise estimate from Equations (9)–(10), hence we can use these estimates to derive the ML algorithm and the resulting curve in Fig. 3 shows that the NMSE is quite small if we use the estimates from the MoM estimator. The CRB from (38) is also plotted to compare the performance of the estimators. The high error in the low SNR region of ML estimator is due to the bad estimates of ρ in MoM. However, the ML estimator shows good results at high SNR.

VI. Conclusion

In this paper, we have studied the problem of estimating the SNR for a non-coherent M-FSK receiver in the presence of a CFO. We have derived the data aided estimators for the Rayleigh fading channel and have derived method of moments estimator and the maximum likelihood estimator. Concerning the difficulty of the CFO estimation problem (in terms of its non-linearity), we have shown that the estimates of CFO can be found using the MoM estimator, which can then be used to derive the ML estimators, which give accurate estimates of the SNR at various high and low SNR scenarios.

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