Robust Detection of Number of Sources Using the Transformed Rotational Matrix

by

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Abstract

We present an accurate, robust, and computationally efficient number-of-source detection method, which utilizes the variance of transformed rotational submatrix (VTRS) as the criterion. Unlike the traditional methods that use the eigenvalues, the VTRS detector determines the number of sources by exploiting the eigenvectors of two subarrays. This method applies to arbitrary subarray geometries. In addition, no subjective threshold or extra parameter is required. In conjunction with spatial smoothing, this method can also detect coherent signals. One novelty of the VTRS method is that not only can it detect the number of sources, it can also indicate the quality of measured data and determine the optimum smoothing array size for parameter estimation. Because of the low computational complexity, the VTRS number detector is suitable for adaptive or real-time applications.

Keywords – Channel characterization, number of sources, enumeration, multiple source localization, smoothing array size, array signal processing, delay estimation, direction of arrival estimation.

1. Introduction

As shown in Table 1, the number-of-sources detection methods, or the enumeration techniques, can be divided into two main categories: those that treat this problem as a pure detection problem, and those that treat it as a combined detection-estimation problem. Compared to the pure detection methods, the combined detection-estimation methods provide better performance, but they also have high computational complexity. This is true when maximum likelihood (ML) estimation [1,2] is used, which requires a multi-dimensional search. Therefore, the combined detection-estimation approach is not suitable for real-time applications. According to the different mathematical criteria used in the methods, Categories I and II can be further classified into four and two groups, respectively. Next, we briefly introduce the various methods of each group.

Table 1. Categories of number detection methods.

	Detection		Combined Detection-Estimation	
	Subjective	Objective	Subjective	Objective
Information theoretic criterion	[3]	[4,6,5,7,20]		[1,2,15,16]
Eigenvector		[12]		
Data-Based		[13,19]	[10]	[11]
Threshold	[14]			
Root-finding			[8]	

Category I: Pure Detection

(1) Information theoretic criterion: Statistical hypothesis (SH) [3], Akaike's information criterion (AIC) [4,5], and minimum description length (MDL) [6,5] are the three most popular methods that detect the number of sources based on information theoretic criteria. The methods in this group detect the number by counting the multiplicity of the smallest eigenvalues of the correlation matrix. SH determines the number according to the log-likelihood function followed by a subjective threshold. MDL and AIC eliminate the requirement of this threshold by adding a "degrees of freedom" term after the log-likelihood function. Xu [7] modified the degrees of freedom to make MDL and AIC suitable for the applications when forward-backward smoothing is applied. There are many ideal assumptions in the deduction of the criterion, which have made it fail in many practical environments, such as under-water [8], sea-surface [9], and multipath measurements in urban area [10] and indoor office [11]. The assumptions include that the noise must be sphere-like and uncorrelated between any two sensors, and the number of snapshots is large enough to obtain an accurate correlation matrix.

(2) Eigenvector-based: Instead of using eigenvalues, the rank of the matrix composed of the eigenvectors can be used for the determination of number of sources. In [12], Di and Tian examine the rank of the matrix formed by appended subarrays, which are derived from the

correlation matrix and eigenvectors. The rank increases with the increase of number of subarrays and stabilizes when the rank is equal to the number of sources. Like the information criterion, this method also assumes that the noises of the sensors are mutually uncorrelated, and the noise variance is a known value, the latter of which is usually unavailable in practice. One feature of this method is that it can handle both non-coherent as well as fully coherent signals. It is interesting to notice that the collection of the subarrays is similar to the spatial smoothing, another way to deal with coherent signal sources by summing the correlation matrices of many similar subarrays containing the same signal subspace.

(3) Data-based: Similar to method in (2) above, Di [13] provides another way to detect the number of sources by stabilizing the rank. Instead of the eigenvectors, they use the correlation matrix of the received data of the sensors. Its performance and drawback are similar to eigenvector-based method. Krim and Cozzens [19] proposed a data-based enumeration technique, which also uses rank stabilization to detect the number. However, Krim and Cozzens used a different approach, which applies MDL on the prediction errors of a linear model. One potential problem is that the calculation of the error needs a singular value decomposition (SVD) of the accumulated data matrix. The data matrix becomes very large when the number of snapshots or subarrays is large. In that case, the pre-whitening process required in this method also requires intensive calculation.

(4) Threshold: Chen [14] showed a method that detects the number by setting an upper bound on the values of the eigenvalues. Because this bound is determined by an adjustable parameter, its performance is better than MDL at low SNR and better than AIC at high SNR. However, the decision of the value of this parameter depends on *a priori* information, such as the probability density function (PDF) of false alarm, PDF of eigenvalues, SNR level, etc. In many applications some of the information is not available, in which case the parameter must be subjectively selected based on empirical decision.

Category II: Combined Detection-Estimation

(1) Information theoretic criterion: Wax and Ziskind [1] proposed a method that simultaneously solves the detection of number of sources and multiple sources localization problems. The detection is based on MDL algorithm, while the localization is optimized by ML estimation. With this approach fully coherent signals can also be handled. Wax improved the performance via the ML estimator derived by Bohme [2]. Wax further proposed a solution that is applicable to arbitrary array geometry and the condition when unknown noise with arbitrary covariance matrix is present [15]. Another approach derived the number and parameters based on Bayesian predictive densities (BPD) and marginal Bayesian estimator [16]. A common drawback of these methods is the knowledge of the array manifold is required. In many cases this information is not available, or there exist errors in the estimated array manifold, which will distort the detection and estimation results.

(2) Data-based: Kuchar [10] presents a method that determines the number of sources by selecting the one that minimizes the data estimation error (DEE), the difference between the received data and the reconstructed data, which is derived from the estimates obtained by Estimating the Signal Parameter via Rotation Invariance Technique (ESPRIT) algorithm [17]. The drawback of this method is DEE decreases with the increase of assumed number of sources because of the increase of degrees of freedom. Therefore, like the SH method, this method also needs a subjective decision on the selection of local minimum of DEE. The residual estimation error (REE) [11] method eliminates the requirement of this subjective decision by using part of the sensor data to recover the signal and calculating the residual error of the data on the complementary part of the data. Through this approach, the number is determined by global minimum of REE [11].

(3) Root-finding: Silverstein [18] showed that if the assumed number of sources is correct in ESPRIT, the roots must be on the unit circle, and the roots caused by overestimation tend to deviate from the circle. No specific criterion is provided to determine the number of sources. Kotanchek [8] made use of similar property in another estimation method, generalized eigenvalues utilizing signal subspace eigenvectors (GEESE), to detect the number according to the deviation of the roots from the unit circle. However, a subjective threshold must be decided in initial detection, and some other follow-up steps are necessary to track if the initial detection is appropriate.

In this paper, we provide a robust number detector, which belongs to eigenvector-based group in Category I. This method exploits the property of the variance of the rotational submatrix (VTRS). This matrix should be a zero matrix in the noiseless condition. The rotational matrix is critical to the ESPRIT estimator. Like ESPRIT, the VTRS number detector is robust, insensitive to the perturbation of sensor, computationally efficient, and applicable to arbitrary array geometries. The only requirement is that two subarrays must be identical. Other than that, the sensors in each subarray are not required to be identical. In addition, the VTRS detector also provides a good indication for the quality of the channel or measured data. Furthermore, our detector can also be used as a criterion for the selection of smoothing array size, which is a trade-off between decorrelation effect and high resolution.

The paper is organized as follows. Section II introduce the general signal model for the array signal processing and describe the details of the VTRS method. Section III demonstrates the simulation results. Finally a brief conclusion is provided.

2. VTRS Number Detection Method

The general signal model for multiple source localization is

$$\mathbf{Y} = \mathbf{A}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $\mathbf{Y}$ and $\mathbf{n} \in \mathbf{C}_{N\times 1}$ is the received signal and noise of N sensors, $\mathbf{s} \in \mathbf{C}_{L\times 1}$ is the signal sources, and $\mathbf{A} \in \mathbf{C}_{N\times L}$ is the steering matrix (also known as array manifold). $\mathbf{A} = [\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_L]$, where $\mathbf{a}_i$ is the steering vector corresponding to the i^{th} signal source. The sensors of the array are not necessarily antennas. When the array is composed of antennas, the steering vector represents the directions of arrival (DOA) of the signals. When the array comprises samples on frequency domain, the steering vector reflects delays of the source signals. Therefore, the technique for multiple source localization can actually be applied to many fields, like delay estimation and frequency component retrieval.

The correlation matrix of the received signal is

$$\mathbf{R} = \mathbf{A}\mathbf{S}\mathbf{A}^{\dagger} + \mathbf{R}_{\mathbf{n}}, \quad (2)$$

where $\mathbf{S}$ is the signal correlation matrix, $\mathbf{R}_{\mathbf{n}}$ is the noise covariance matrix, and the superscript † stands for complex conjugate transpose. ESPRIT exploits the rotational invariance property between two identical subarrays to derive the estimates. Accordingly, the geometry of each subarray could be arbitrary, and the sensors of each subarray can have different gains. Here, we assume the first subarray is derived from the first N-1 sensors, and the second subarray is composed of the final N-1 sensors. The results, of course, can be generalized to the case where two subarrays are not overlapped.

The steering matrices of the first and second subarrays are $\mathbf{A}_{\mathbf{x}}$ and $\mathbf{A}_{\mathbf{y}}$, respectively. Then,

$$\mathbf{A}_{\mathbf{z}} = \begin{bmatrix} \mathbf{A}_{\mathbf{x}} \\ \mathbf{A}_{\mathbf{y}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\mathbf{x}} \\ \mathbf{A}_{\mathbf{x}}\mathbf{\Phi} \end{bmatrix}, \quad (3)$$

where $\mathbf{\Phi} = diag\{1 \quad e^{j\beta} \quad \cdots \quad e^{j(N-1)\beta}\}$ is the rotational matrix. The definition of β depends on the parameter of interest. The eigenvectors of $\mathbf{R}$ are

$$\mathbf{E} = [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \cdots \quad \mathbf{e}_N] = [\mathbf{E}_{\mathbf{s}} \quad \mathbf{E}_{\mathbf{n}}], \quad (4)$$

where

$$\mathbf{E}_{\mathbf{s}} = [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \cdots \quad \mathbf{e}_L], \text{ and} \quad (5)$$

$$\mathbf{E}_{\mathbf{n}} = [\mathbf{e}_{L+1} \quad \mathbf{e}_{L+2} \quad \cdots \quad \mathbf{e}_N] \quad (6)$$

The columns of $\mathbf{E}_{\mathbf{s}}$, which are the eigenvectors corresponding to the L largest eigenvalues, span the signal subspace, while the columns of $\mathbf{E}_{\mathbf{n}}$ span the noise subspace. Since both $\mathbf{E}_{\mathbf{s}}$ and $\mathbf{A}$ span the same subspace, there exist a non-singular matrix $\mathbf{T}$, such that

$$\mathbf{E}_{\mathbf{s}} = \mathbf{A}\mathbf{T} \quad (7)$$

Applying (7) to (3) we obtain

$$\mathbf{A}_{\mathbf{z}}\mathbf{T} = \begin{bmatrix} \mathbf{A}_{\mathbf{x}}\mathbf{T} \\ \mathbf{A}_{\mathbf{y}}\mathbf{T} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\mathbf{x}}\mathbf{T} \\ \mathbf{A}_{\mathbf{x}}\mathbf{\Phi}\mathbf{T} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{\mathbf{sx}} \\ \mathbf{E}_{\mathbf{sy}} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{\mathbf{sx}} \\ \mathbf{E}_{\mathbf{sx}}\mathbf{\Psi}_{\mathbf{s}} \end{bmatrix}, \quad (8)$$

where $\mathbf{\Psi}_{\mathbf{s}} = \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T}$ is the transformed rotational matrix of the signals, $\mathbf{E}_{\mathbf{sx}}$ and $\mathbf{E}_{\mathbf{sy}} \in \mathbf{C}_{(N-1)\times L}$ are the signal eigenvectors of two subarrays. Suppose $\mathbf{E}_{\mathbf{x}}$ and $\mathbf{E}_{\mathbf{y}} \in \mathbf{C}_{(N-1)\times N}$ are the first N-1 rows and last N-1 rows of $\mathbf{E}_{\mathbf{s}}$, respectively, and assume the noise is not present. Then

$$\mathbf{E}_{\mathbf{y}} = [\mathbf{E}_{\mathbf{sy}} \quad \mathbf{E}_{\mathbf{ny}}] = [\mathbf{E}_{\mathbf{sx}} \quad \mathbf{E}_{\mathbf{nx}}] \begin{bmatrix} \mathbf{\Psi}_{\mathbf{s}} & \mathbf{\Gamma}_1 \\ \mathbf{0} & \mathbf{\Gamma}_2 \end{bmatrix} = \mathbf{E}_{\mathbf{x}}\mathbf{\Psi}, \quad (9)$$

where $\mathbf{\Gamma}_1 \in \mathbf{C}_{L\times(N-L)}$, $\mathbf{\Gamma}_2 \in \mathbf{C}_{(N-L-1)\times(N-L)}$, $\mathbf{E}_{\mathbf{x}} = [\mathbf{E}_{\mathbf{sx}} \quad \mathbf{E}_{\mathbf{nx}}]$, and

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{\Psi}_{\mathbf{s}} & \mathbf{\Gamma}_1 \\ \mathbf{0} & \mathbf{\Gamma}_2 \end{bmatrix}. \quad (10)$$

Define

$$\mathbf{\Delta}_k = \begin{bmatrix} \psi_{k+1,1} & \psi_{k+1,2} & \cdots & \psi_{k+1,k} \\ \psi_{k+2,1} & \psi_{k+2,2} & \cdots & \psi_{k+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{N-1,1} & \psi_{N-1,2} & \cdots & \psi_{N-1,k} \end{bmatrix}. \quad (11)$$

When $k = L$, $\mathbf{\Delta}_k = \mathbf{0}$, the zero submatrix of the transformed rotational matrix $\mathbf{\Psi}$. When $k \neq L$, $\mathbf{\Delta}_k$ contains nonzero components. Therefore, we may select the number that minimizes the variance $\rho(k)$ of the components of $\mathbf{\Delta}_k$ as the detected number of sources when the noise is present, i.e.,

$$\hat{L} = \min_k \rho(k) = \frac{\|\mathbf{\Delta}_k\|^2}{(N-k-1)k}, \quad k = 1,\ 2,\ \cdots,\ N\text{-}2 \quad (12)$$

where $\|\cdot\|$ is the Frobenius norm. When the noise is present, the estimate of $\mathbf{\Psi}$ can be solved based on least square (LS) or total least square (TLS) criterion. Notice that $\|\mathbf{\Delta}_{k+1}\|^2$ can be derived from $\|\mathbf{\Delta}_k\|^2$ and reduce the computational cost as shown below.

$$\|\mathbf{\Delta}_{k+1}\|^2 = \|\mathbf{\Delta}_k\|^2 - \|\text{first row of } \mathbf{\Delta}_k\|^2 + \|\text{last column of } \mathbf{\Delta}_{k+1}\|^2 \quad (13)$$

The computational cost can be further reduced by replacing $\mathbf{E}_{\mathbf{x}}$ and $\mathbf{E}_{\mathbf{y}}$ by the first L_T columns of $\mathbf{E}_{\mathbf{x}}$ and $\mathbf{E}_{\mathbf{y}}$ provided the threshold L_T is larger than L. Besides, replacing the Frobenius norm with the summation of the absolute values of the components of $\mathbf{\Delta}_k$ can also reduce the computational cost, but the difference is negligible.

Like multi-dimensional ESPRIT [21], the VTRS method can be easily generalized to multi-dimensional detection by summing the calculated variances of each dimension. Given the dimension M, the number of sources is detected by

$$\hat{L} = \min_k \sum_{D=1}^{M} \rho_D(k)\,. \tag{14}$$

Notice that VTRS detector, or the variance, $\rho(k)$, is the variance of the estimation error of eigenvectors, which indicates how far the estimated signal subspace is deviated from real signal subspace and the value is relative to the SNR of the channel. It is also an indication about the quality of the final estimation results. From this viewpoint, the smoothing array size that achieves minimum variance $\rho(k)$ is expected to obtain optimum estimation. Therefore, the size N of the smoothing array can be determined by

$$N = \min_m \rho\left(\hat{L}_m\right), \tag{15}$$

where m is the smoothing subarray size, and $\hat{L}_m$ is the detected number of sources with subarray size m.

The steps of the VTRS method are summarized below.

Step 1: Calculate the correlation matrix $\mathbf{R}$.

Step 2: Derive the eigenvectors $\mathbf{E}_\mathbf{s}$ of $\mathbf{R}$.

Step 3: Solve $\mathbf{E}_\mathbf{y} = \mathbf{E}_\mathbf{x}\boldsymbol{\Psi}$ based on LS or TLS criterion.

Step 4: Calculate (12) or (14) efficiently by (13) and obtain the detected number of sources.

Since *Steps 1* and *2* are originally required for parameter estimation, the additional time used for VTRS detector is *Steps 3* and *4*. However, if the first $\hat{L}\times\hat{L}$ submatrix of estimated $\boldsymbol{\Psi}$ in *Step* 3 is used as the estimate of $\boldsymbol{\Psi}_\mathbf{s}$, in that case only *Step 4* is dedicated to the detector, which turns out to be a one-step detector.

The performance of VTRS can be further enhanced using the method below. In (10), $\boldsymbol{\Psi}$ is derived from the entire eigenvectors. However, using a reduced number of eigenvectors can produce reduced transformed rotational matrix, which also contains the zero submatrix provided that the number of eigenvectors is larger than the real number of signals. This reduced matrix can also be used to detect the number of signals based on VTRS criterion. Therefore, using multiple reduced rotational matrices, we can obtain multiple detection results. The final decision can be derived based on the majority of all detectors.

3. Simulation Results

Three sets of simulations are provided to demonstrate the performance of VTRS detector. In Set 1, we compare VTRS and MDL using the signals with close delays when the frequency response is flat. Set 2 shows the robustness of VTRS detector by using the sensor with non-flat frequency response. In Set 3, we show how accurate the VTRS is in determining the optimum smoothing array size.

In all simulations, there are L = 3 signal sources impinging the antenna at delay 3.3*ns*, 7.4*ns*, 11.8*n* unless otherwise specified. The center frequency of the signal is 5.8 GHz, and the bandwidth is 1 GHz. The number of frequency samples is 40. The smoothing array size in Simulations 1 and 2 is 20. The signal snapshots are Gaussian distributed and independent. The detection error rate is an average over 500 trials of simulations.

Set 1: Flat frequency response

In this simulation set, the antenna has flat frequency response. In Figure 1, we compare MDL and VTRS with 1 and 100 snapshots for just the three signal sources.

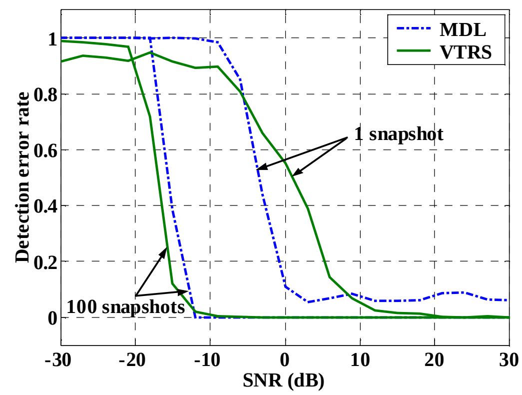


Figure 1: The performance of MDL and VTRS with 3 sources and flat frequency response.

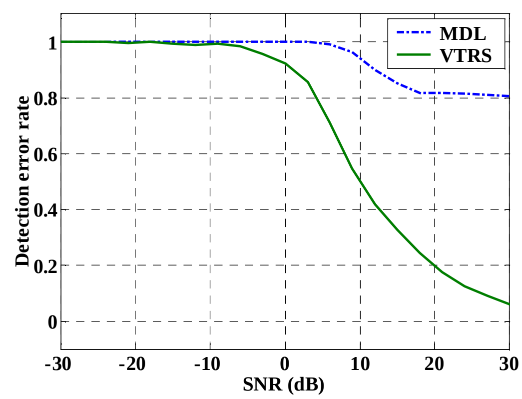


Figure 2: The performance of MDL and VTRS when the number of signals is large.

For the 1 snapshot case, smoothing is required to obtain multiple snapshots for the calculation of correlation matrix. Since the frequency response is flat, the signals of each smoothing frequency subarray have an identical signal subspace. We observe that the two methods yield similar trends in the error rate detection. For the 1 snapshot case, MDL outperforms VTRS at moderate SNR, while

VTRS outperforms MDL slightly at high SNR. When the number of snapshots is increased to 100, VTRS shows better performance than MDL in the low SNR range.

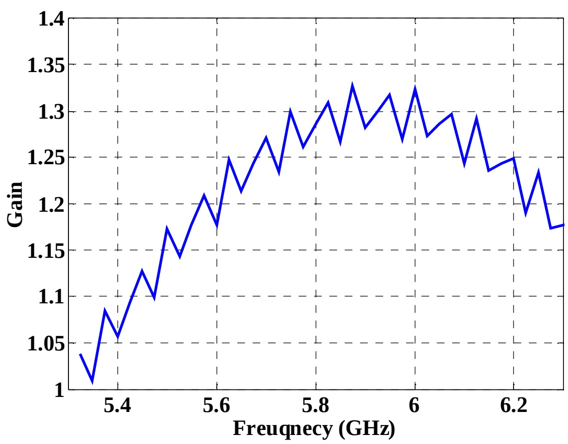
Next, we compare their performances when the number of signals is large. In this simulation, there are L = 16 signals with delays 2 to 32 *ns* with uniform spacing of 2 *ns*. In this situation, VTRS outperforms MDL from middle to high SNR.

Set 2*: *Non-flat frequency response

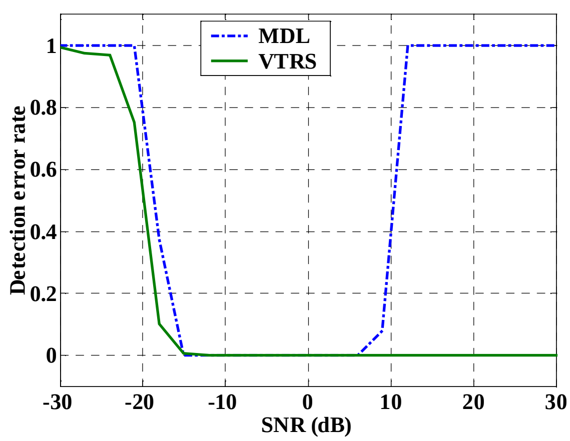
We next consider the performances of MDL and VTRS methods when the frequency response of the antenna is not flat, which is usually true, especially when the bandwidth is wide. In this case, the signal subspaces of all smoothing subarrays are no longer identical. As shown in Figure 3(a), the non-flat frequency response is generated by

$$G(n) = 1+ 0.3\sin(0.02\pi n) + 0.03\sin(0.8\pi n),$$

where n =1, 2, …, 40 is the index of 40 frequency samples.



(a) Non-flat antenna frequency response.



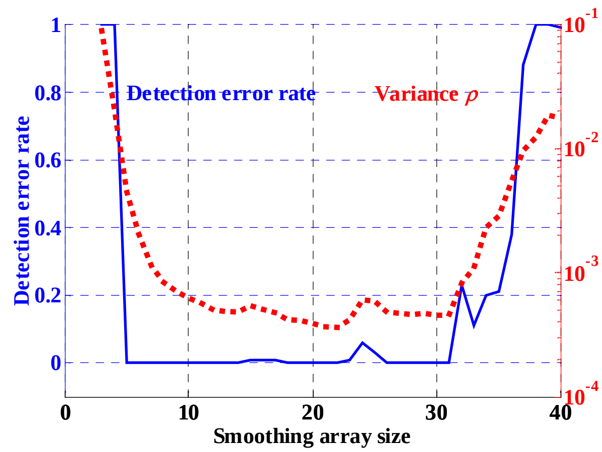
(b) MDL vs. VTRS

Figure 3: Results of Simulation 2: (a) Non-flat frequency response. (b) Performance of MDL and VTRS.

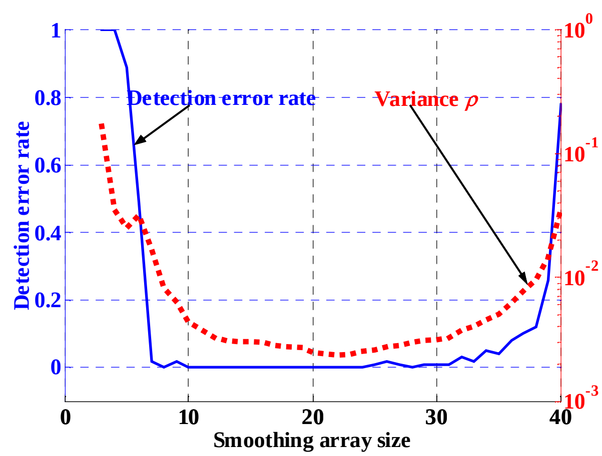
As expected, MDL fails at low and high SNR. Why the detection of MDL is correct from –15 to 5 dB is because it underestimates the number at low SNR and overestimate at high SNR, which results in a "transition state" at middle SNR range. On the other hand, VTRS shows the robustness in this condition at high SNR. Although the signal subspace is distorted due to the non-flat frequency response, the rotational matrix between two subarrays still provides accurate information for either detection or estimation.

Set 3*: *VTRS variance

This section shows how the detector function, the variance $\rho(k)$, can be used to indicate the deviation of the measured signal subspace from the real signal subspace, and determines the optimum smoothing array size when the number of snapshots is low. In this simulation, the smoothing array size is changed from 3 to 39.



(a) 1 snapshot



(b) 100 snapshots

Figure 4: Results of Simulation 3: Selection of smoothing array size. (a) 1 snapshot (b) 100 snapshots.

In Figure 4(a), the frequency response is flat, the SNR = 15 dB, and the number of snapshots is 1. Using the variance as the criterion, the optimum smoothing array size is 22, which also minimizes the detection error. In Figure 4(b), the frequency response is also flat, the SNR = -10 dB, and the number of snapshot is 100. The detection error is 0 when the array size minimizes the variance. Therefore, VTRS is a good indication for the selection of smoothing array size. Since the computation of VTRS detector is very efficient, the smoothing array size can be changed adaptively in the real-time application.

4. Conclusions

We present a novel VTRS number detector with the features of high accuracy, robustness, and applicability to arbitrary arrays and the condition when the noise has arbitrary covariance matrix. The VTRS number detector is versatile because aside from accurately detecting the number of sources, it can also indicate the quality of the channel (SNR) and determine the optimum size of the smoothing array to obtain the best estimation of signal subspace. The advantage of low computational complexity makes the VTRS detector a good candidate for practical applications that need real-time or adaptive high-resolution estimation.

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