Stream Control in Networks with Interfering MIMO Links

by

M. Fatih Demirkol and Mary Ann Ingram

{demirkol, mai}@ece.gatech.edu

School of Electrical and Computer Engineering
Georgia Institute of Technology

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M. Fatih Demirkol and Mary Ann Ingram
School of ECE, Georgia Institute of Technology, Atlanta, GA 30332-0250, USA.
{demirkol, maryann.ingram}@ece.gatech.edu

***Abstract*— A distributed algorithm, which exploits channel state information (CSI) at the transmitter, is presented for determining the maximum number of independent data streams for each transmitting node in a network of interfering multiple-input-multiple-output (MIMO) links. Simulated throughputs for two simple network topologies show that the algorithm yields nearly optimal stream control. These closed-loop MIMO throughputs are compared to those of open-loop MIMO, with and without optimal stream control, and to the throughput when the links operate in a non-interfering, TDMA fashion.**

I. Introduction

Multiple-input multiple-output (MIMO), or array-to-array, links are well known to provide extremely high spectral efficiency in rich multi-path environments through multiple spatial channels without increasing the system bandwidth [1–3].

Traditionally, if two wireless links would cause excessive interference on each other, they are assigned to different channels, e.g. they could be time-multiplexed. If the links are both MIMO, however, the spatial filtering capabilities at the receivers and perhaps also at the transmitters enable the MIMO links to operate co-channel with a higher network throughput than if they operated in a TDMA fashion [4], [5].

Some previous works treat joint optimization of interfering MIMO links. For cellular systems, iterative methods were used to optimize the uplink in [6] and the downlink in [7]. In [4] and [5], the authors explore ways to control the relative closed-loop capacities (i.e. transmitter has channel knowledge) of interfering MIMO links. In [4], each link iteratively maximizes the closed-loop capacity of its whitened channel under power constraints that generally differ among nodes, and in [5], each link minimizes the interference it makes on its neighbors, subject to capacity constraints. In [8], Blum et al. show that, under certain conditions, the open-loop (i.e. when the transmitter has no specific channel state information) capacities of interfering MIMO links can be increased when all transmit arrays transmit fewer than their maximum number of streams.

In closed-loop MIMO (CL-MIMO), each stream corresponds to a channel mode. As is well-known, the water-filling solution for CL-MIMO does not necessarily excite all of the possible channel modes. Therefore, one might think that when water-filling is done iteratively for interfering MIMO links, the solution at convergence would automatically find the best number of streams to optimize network throughput. However, this is not always the case, as was shown in [5] and with more elaboration, here. We attribute this to the fact that in the algorithms of [4], [5], all links are trying to maximize their own capacities at the same time; network throughput is not the cost function being optimized.

Since global numerical optimization of all antenna weights to optimize network throughput is computationally prohibitive, we consider a suboptimal solution in this paper. Specifically, we modify the power-control algorithm of [4] to include a limit on the number of streams transmitted by each transmitting node. First, we simulate network throughput as a function of this limit for both open- and closed-loop MIMO links for a couple of simple topologies. Second, we propose a distributed algorithm that automatically determines the limit, assuming a transmitter node knows the distance to the nearest unintended receiver. Obtaining this distance may be practical in some outdoor applications of ad hoc networks because the nodes could have global positioning systems (GPS). We show through simulation for two example topologies that stream control is very important and that the new stream control algorithm yields nearly optimal throughputs.

However, further improvements can be obtained by combining stream control with adaptive spatial filtering at the transmitter, under the assumption that the transmit nodes have perfect channel state information (CSI). In particular, we show that the throughput advantage of CL-MIMO over open-loop MIMO (OL-MIMO), both with stream control, can be as high as 40%, depending on network topology.

The rest of this paper is organized as follows. Section II presents the channel model used. In Section III, we review the MIMO water-filling solution assuming additive Gaussian noise with fixed interference. Section IV reviews the two iterative link adaptation algorithms and discusses the effect of stream control. In Section V, a simple condition to limit the number of streams is introduced, and results are presented. Section VI concludes the paper.

II. Channel Model

We consider MIMO links in ad hoc networks, where there is no central control of data transmissions, and all nodes have similar characteristics. Both ends of a link are expected to be located in environments with similar scattering properties.

Therefore, we utilize a Rayleigh fading model, and do not distinguish between uplink and downlink. We assume a quasi-static environment, where the channel stays fixed for the duration of an entire burst. The separation between elements of the arrays at the transmitter and receiver are assumed to be large enough for the fades to be independent. Each entry of the channel is a zero-mean, iid Gaussian random variable that has independent real and imaginary parts with equal variance. Equivalently, the entries of the channel matrix have uniformly distributed phase and Rayleigh magnitude.

The channel gain depends on the distance, D, between the nodes. The mean-square value of each element of the channel matrix is equal to $1/D^n$, where n is the path-loss exponent. In the simulations, $n = 3$ is used.

III. MIMO Channel Capacity

In a flat-fading MIMO system, the channel capacity is obtained by maximizing the mutual information between the transmitted and received signal vectors, as

$$C_{ij} = \max_{\mathbf{P}_{ij}} \log_2 \left|\mathbf{I} + \mathbf{H}_{ij}\mathbf{P}_{ij}\mathbf{H}'_{ij}\right|, \tag{1}$$

where the subscripts i and j denote the transmitting and receiving nodes, $\mathbf{P}$ is the transmitter signal vector correlation matrix, or the power allocation matrix, normalized by the received noise power on a single receive antenna element. $\mathbf{H}$ is the complex channel matrix. Each element $\mathbf{H}(a, b)$ of the channel matrix gives the complex channel gain from antenna element b of the transmitter to antenna element a of the receiver.

In OL-MIMO, no channel information is used at the transmitter and each antenna element transmits a different data stream with equal power. Specifically, setting $\mathbf{P} = (P_T/M)\mathbf{I}$, gives the best result [1], [2]. Here, P_T is the total transmitted power, normalized by the additive noise power; and M is the number of transmit antennas.

In CL-MIMO, channel-dependent matrix transformations in both the transmitter and receiver decompose the matrix channel into a collection of uncoupled parallel channels or "channel modes." The output of each transmit antenna is a linear combination of the signals associated with independent data streams.

Let the singular-value decomposition of $\mathbf{H_{ij}}$ be denoted as $\mathbf{H}_{ij} = \mathbf{U}_{ij}\mathbf{S}_{ij}\mathbf{V}'_{ij}$ and the eigenvalue decomposition of $\mathbf{P_{ij}}$ as $\mathbf{P}_{ij} = \mathbf{D}_{ij}\mathbf{\Sigma}_{ij}\mathbf{D}'_{ij}$. Furthermore, let $\alpha_{ij}^{(k)}$ and $\lambda_{ij}^{(k)}$, $k = 1, ...K$ be the non-zero eigenvalues of $\mathbf{P_{ij}}$ and $\mathbf{H}_{ij}\mathbf{H}'_{ij}$, respectively. With the choice of $\mathbf{D}_{ij} = \mathbf{V}_{ij}$, the expression for the capacity becomes

$$C_{ij} = \max_{\alpha_{ij}^{(k)}} \sum_{k=1}^{K} \log_2 \left(1 + \lambda_{ij}^{(k)}\alpha_{ij}^{(k)}\right). \tag{2}$$

With a total transmitted power of P_T, the classical water-filling solution

$$\alpha_{ij}^{(k)} = \left[\mu_{ij} - \frac{1}{\lambda_{ij}^{(k)}}\right]^+, \tag{3}$$

maximizes the sum in (2) where $[\cdot]^+$ indicates that only non-negative values are acceptable, and μ_{ij} is chosen so that $\sum_{k=1}^{K} \alpha_{ij}^{(k)} = P_T$.

The water-filling approach can be modified to accommodate fixed interference at the receiver of a link (represented by a covariance matrix, $\mathbf{R}$) by "whitening the channel matrix" first. Applying a spatial whitening transform to the channel yields

$$\widetilde{\mathbf{H}} = [\mathbf{I} + \mathbf{R}]^{-1/2}\,\mathbf{H}, \tag{4}$$

which reduces the capacity relation to the simple form in (1), with a substitution of $\mathbf{H} \rightarrow \widetilde{\mathbf{H}}$ [9].

IV. Distributed Optimization of Interfering Link Parameters

In a network with multiple interfering links, the interference correlation matrix seen by each receiver array varies with the transmitter correlation matrices of the interfering nodes. The whitened channel matrix, $\widetilde{\mathbf{H}}$, for a given link is a function of the interference, $\mathbf{R}$. The transmission strategy, in turn, is dependent on the whitened channel matrix. As a result, a change in the power allocation matrix of one link induces a change in the optimum power allocation matrix of the other co-channel links. Therefore, the optimum transmission strategies and the capacities of interfering co-channel links are mutually dependent, and cannot be calculated directly.

Two distributed iterative methods that jointly optimize interfering link parameters were introduced earlier [4], [5]. With total transmit powers or target capacities as control parameters, these methods allow us to manage each link's capacity. At each iteration, every transmitter-receiver pair optimizes its link capacity for the measured interference at the receiver. With the first method [4], each link's transmission strategy is determined according to the water-filling solution given in (3) for the current spatially whitened channel, and for its respective given total transmitted power. With the second [5], the links try to obtain a target capacity, C_T, while the transmitters minimize the interference caused. The total transmit power of each link is distributed among the channel modes as

$$\alpha^{(k)} = \left[\frac{\mu}{a_k + \nu} - \frac{1}{\widetilde{\lambda}^{(k)}}\right]^+, \tag{5}$$

where μ and ν are constants chosen such that the target capacity and maximum transmit power constraints are satisfied, and a_k is the projected channel mode gain along the interference path [5].

Both methods are distributed; each link determines its own transmission strategy based on the channel conditions. The cost functions considered are not based on the total network throughput, but on the capacities of the individual links. Although these iterative methods allow multiple co-channel CL-MIMO links to operate simultaneously, they do not maximize the total network throughput.

The key to achieving a large capacity for an isolated MIMO link is to partition a single high signal-to-noise ratio

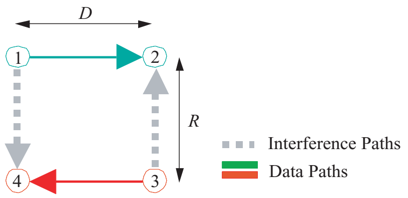


Fig. 1. A simple network with 4 nodes forming two interfering links.

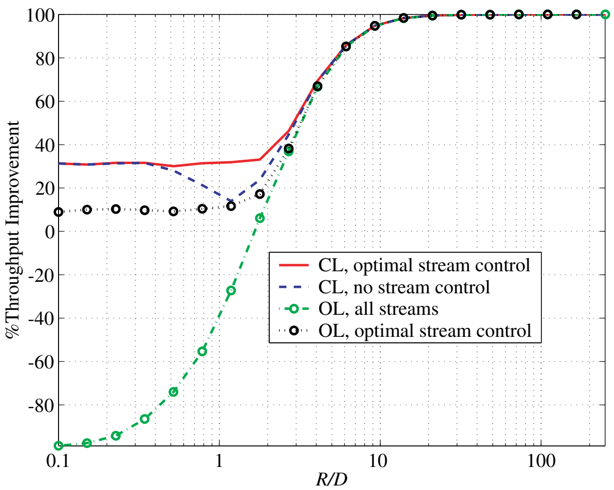


Fig. 2. Throughput improvement with respect to TDMA.

(SNR) channel into many lower SNR sub-channels. In OL-MIMO links, partitioning is over the transmit antennas, and in CL-MIMO links, it is over channel modes. However, when multiple interfering links are considered, partitioning into the maximum number of sub-channels can create too many uncorrelated interference streams. For independent flat-Rayleigh fading wireless systems, the data streams can be resolved effectively if the number of data and interference streams is less than or equal to the number of receive antennas [8], [10]. As a result, limiting the number of independent streams transmitted may yield better performance [8]. This means it might not be the best approach to utilize all channel modes in CL-MIMO links or all transmit antennas in OL-MIMO links.

Both capacity allocation algorithms can be modified to limit the number of channel modes used at each link based on the number of transmitting nodes in the neighborhood. In the next section we analyze the network throughput and propose an algorithm to limit the number of streams for each MIMO link.

V. Throughput Improvement and Stream Control

Allowing multiple MIMO links to transmit data simultaneously improves spectral efficiency over TDMA. For a set of co-channel links, the total network throughput, or the sum of the capacities of the links, depends on the number of independent interference streams and their respective strengths.

We shall consider the example topology of Figure 1, assuming the noise-normalized total transmit power of each link is set to 20dB, and each node is assumed to have 4 antennas. In Figure 1, D is the distance between a transmitter and its receiver in a link, and R is the distance between the links. Figure 2 shows the throughput improvement of several spatial multiplexing schemes with respect to TDMA for the two-link network of Figure 1 as R/D is varied. The capacities of the interfering links are calculated using the power-controlled iterative method described in Section IV. The percentage improvement is calculated as

$$\frac{T - T_{TDMA}}{T_{TDMA}} \times 100\%,$$

where T is the throughput with co-channel links, and T_{TDMA} is the average single-link capacity without interference.

The curves with stream control are found by trying all possible combinations of numbers of streams at each link and selecting the combination with the highest throughput. All schemes give similar results for high R/D or weak interference. On the other hand, at high interference-to-noise ratio (INR), or when R/D is small, CL-MIMO schemes give higher improvements. For $R/D \approx 1$, the effect of control on the number of transmitted streams is significant.

The fact that the two CL methods differ significantly in the range of approximately $0.5 < R/D < 3$ led us to examine the statistics on the number of streams. Figures 3(a) and (b) show histograms of the number of streams used by l_{12}, that is the link between nodes 1 and 2, over 100 channel trials. 20 different values of R/D are taken on the logarithmic scale from 0.1 to 250. The links use excessive number of streams without stream control in the middle region. For example, when $R/D = 1.786$, l_{12} used 3 streams in 80 channel trials with no stream control, while with optimal stream control it used 3 streams in only 26 trials, and 2 streams in 68 trials. When $R \ll D$, the interference is strong enough that water-filling naturally blocks some of the channel modes. On the other hand, when $R \gg D$, all dimensions can be used despite the interference.

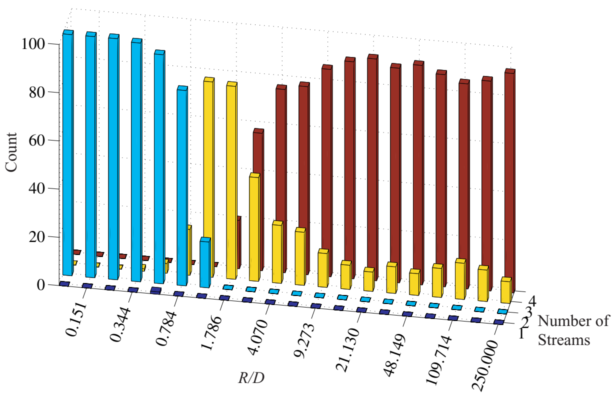
A. Stream Control Algorithm

To build a criterion to control the number of streams for the two link network in Figure 1, we first rewrite the link capacity in (2) as

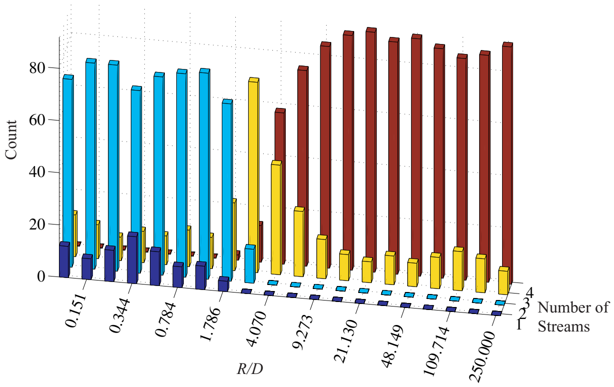
$$C_{ij} = \sum_{k=1}^{K} \log_2 \left[\mu_{ij}\lambda_{ij}^{(k)}\right]^+. \tag{6}$$

We look at a special case where all spatial dimensions are filled with data and interference streams, yet node 3 tries to increase its number of streams from L to $L+1$. For instance, assuming that each node has 4 antenna elements, each link could be transmitting 2 streams, and node 3 could try to add a third stream.

We shall assume that the additional interference stream is spatially aligned with the strongest channel mode of l_{12}.



(a) No stream control.



(b) Optimal stream control.

Fig. 3. Histograms of the number of streams of one the two links with different CL-MIMO configurations for different R/D values. 20 values of R/D are selected on logarithmic scale from 0.1 to 250.

Equivalently, we assume that our strongest channel mode is degraded, and the other modes remain unchanged. This is a worst case interference assumption, as proven in [11], and illustrated in Figure 4 and Table I.

Figure 4 shows four sets of channel mode gains, with each set corresponding to a certain number, J, of independent interference streams. Each interference stream has the same average power as the desired signal at the receiver. Because each node has 4 antennas, there are four mode gains for each set. For example, the rightmost CDF is for the strongest mode for the $J = 0$, or no interference, case. The next rightmost solid curve is the CDF of the second-strongest mode gain for the $J = 0$ case. The table shows the means values of the gains. The largest $4-J$ gains of the channel with J interfering streams are greater than the gains, $\widetilde{\lambda}^{(k)}, k = 2, \ldots, 4-J+1$, of the channel with $J-1$ interfering streams for all meaningful values of J $(J = 1, 2, 3)$. Therefore, our assumption results in a weaker channel than what we actually get.

With the additional interference stream directly degrading

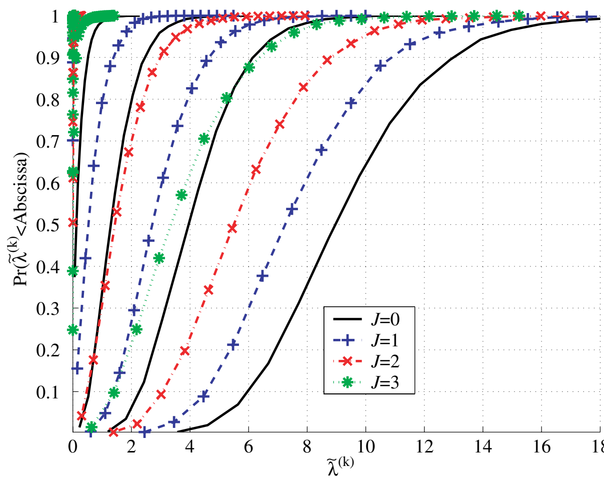


Fig. 4. CDFs of the channel mode gains, $\widetilde{\lambda}^{(k)}$, of a 4×4 whitened channel degraded by different number of interference streams.

TABLE I
MEANS OF THE CHANNEL MODE GAINS OF A 4×4 MIMO CHANNEL DEGRADED BY J INTERFERING STREAMS.

	$J = 0$	$J = 1$	$J = 2$	$J = 3$
$\widetilde{\lambda}^{(1)}$	9.7732	8.0567	6.2165	4.0279
$\widetilde{\lambda}^{(2)}$	4.4431	3.1664	1.8357	*0.0737*
$\widetilde{\lambda}^{(3)}$	1.5803	0.8056	*0.0183*	*0.0082*
$\widetilde{\lambda}^{(4)}$	0.2496	*0.0033*	*0.0014*	*0.0009*

the kth mode, the change in the channel mode gain would be

$$\widetilde{\lambda}_{12}^{(1)} \rightarrow \frac{\widetilde{\lambda}_{12}^{(1)}}{1 + r_{32}}, \tag{7}$$

where r_{32} is the noise-normalized power of the stream at node 2. In this case, transmit power would be reallocated to the modes of l_{12}. The contribution of the kth mode to the capacity would decrease, while the capacity due to the other modes would increase. Hence, the reduction in the capacity of l_{12}, or ΔC_{12}, would be bounded by

$$\Delta C_{12} \leq \log_2 \left[\frac{\mu_{12} \widetilde{\lambda}_{12}^{(k)}}{\mu'_{12} \frac{\widetilde{\lambda}_{12}^{(k)}}{1+r_{32}}} \right] < \log_2 (1 + r_{32}) \, . \tag{8}$$

Here, μ_{12} and μ'_{12} are the top *water levels* found by (3) before and after the extra interference stream is excited, respectively, and $\mu_{12} < \mu'_{12}$.

With the additional stream, the increase in the capacity of the link between nodes 3 and 4 is

$$\begin{aligned} \Delta C_{34} &= \sum_{k=1}^{L+1} \log_2 \left[\mu_{34}^{(L+1)} \widetilde{\lambda}_{34}^{(k)} \right] - \sum_{k=1}^{L} \log_2 \left[\mu_{34}^{(L)} \widetilde{\lambda}_{34}^{(k)} \right] \\ &= \log_2 \left\{ \frac{\left[\mu_{34}^{(L+1)} \right]^{(L+1)}}{\left[\mu_{34}^{(L)} \right]^{L}} \widetilde{\lambda}_{34}^{L+1} \right\}, \end{aligned} \tag{9}$$

where $\mu_{34}^{(L)}$ and $\mu_{34}^{(L+1)}$ are the top levels with L and $L+1$ modes, respectively.

To assure gain in total throughput we need $\Delta C_{34} > \Delta C_{12}$. Requiring the upper bound given on the right side of (8) to be less than the term on the second line of (9), we get the following condition that assures a positive change in the total network throughput:

$$\alpha_{34}^{(L+1)} g_i < \left[\frac{\mu_{34}^{(L+1)}}{\mu_{34}^{(L)}}\right]^L \left(1 + \alpha_{34}^{(L+1)} \widetilde{\lambda}_{34}^{(L+1)}\right) - 1. \quad (10)$$

In this relation, r_{32} has been replaced by $\alpha_{34}^{(L+1)} g_i$, where $\alpha_{34}^{(L+1)}$ is the power allocated to the channel mode by node 3, and g_i is the corresponding channel gain along the interference path from node 3 to 2. Note that $\alpha_{34}^{(L+1)}\widetilde{\lambda}_{34}^{(L+1)}$ is the SNR of the additional stream at node 4, and $\alpha_{34}^{(L+1)} g_i$ is the INR caused by this stream at node 2.

The largest value g_i can take is largest mode gain, $\lambda_{32}^{\max}$, of the the channel between nodes 3 and 2. Assuming M and N are the numbers of antenna elements at nodes 2 and 3, respectively, an upper bound on $\lambda_{32}^{\max}$ is given in [12] as

$$\lambda_{32}^{\max} < \frac{(\sqrt{M} + \sqrt{N})^2}{R^n}. \quad (11)$$

Replacing g_i in (10) with the upper bound given in (11) and solving for the interference path length, we get

$$R^n > \frac{\alpha_{34}^{(L+1)} (\sqrt{M} + \sqrt{N})^2}{\frac{\left[\mu_{34}^{(L+1)}\right]^{L+1}}{\left[\mu_{34}^{(L)}\right]^{L}} \widetilde{\lambda}_{34}^{(L+1)} - 1}. \quad (12)$$

A more relaxed, yet simpler constraint is obtained by omitting the "1"s corresponding to noise power in (10). Then, (10) reduces to

$$g_i < \left[\frac{\mu_{34}^{(L+1)}}{\mu_{34}^{(L)}}\right]^L \widetilde{\lambda}_{34}^{(L+1)}, \quad (13)$$

and (12) reduces to

$$R^n > \left[\frac{\mu_{34}^{(L)}}{\mu_{34}^{(L+1)}}\right]^L \left(\sqrt{M} + \sqrt{N}\right)^2 \frac{1}{\widetilde{\lambda}_{34}^{(L+1)}}. \quad (14)$$

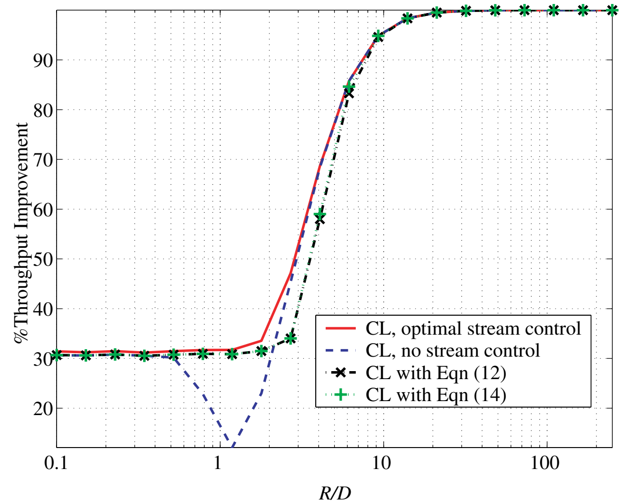
With large P_T, the power ratio approaches

$$\frac{\mu_{34}^{(L)}}{\mu_{34}^{(L+1)}} \to \frac{L+1}{L}, \quad (15)$$

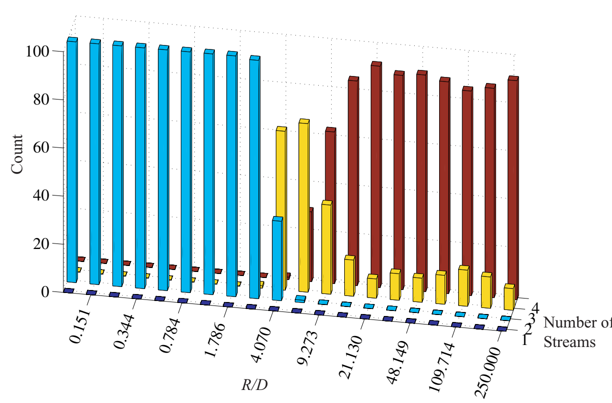
which further simplifies the relation.

B. Results

Figure 5(a) shows the throughput improvement over TDMA achieved by limiting the number of streams according to the criteria in (12) and (14). With 4 antenna elements at each node, each link is assigned 2 streams initially. Each link checks to see if the condition in (12) is true to get one more stream. Figure 5(b) shows the histograms of the number of streams of one of the links at different values of R/D. The algorithm allows no stream increase for $R/D < 3$, and the throughput drop due to excessive streams is eliminated.



(a) Throughput improvement with respect to TDMA.



(b) Histograms of the number of streams of one the two links for different R/D values with stream control algorithm of (12).

Fig. 5. Performance of the distributed stream control algorithm.

To test the stream control algorithm with multiple interfering links, we consider the symmetric 3-link network of Figure 6. Each node has 3 antenna elements. Each link initially is assigned one stream, and can increase its number of streams if the distance to the closest interfered node satisfies the condition in (12).

Figure 7 shows the throughput improvement relations as R/D is varied. We observe that stream control is necessary for all values of $R/D < 3$, since at least one of the interfering transmitters is in the critical distance range. Next, we see that the combination of stream control and spatial filtering at the transmitter gives about 40% higher throughput than either one by itself. Finally, we observe that the stream control algorithm using either (12) or (14) gives nearly optimal performance.

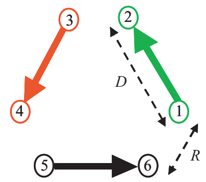


Fig. 6. 3-link network.

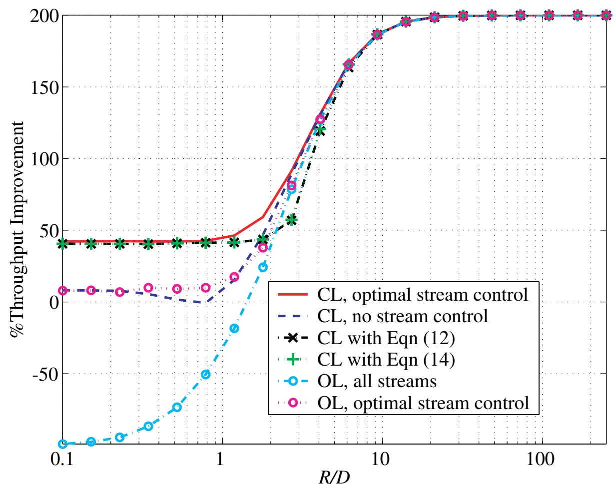


Fig. 7. Throughput improvement of the 3-link network, 3 antennas.

Figure 8 shows the throughput improvements for the 3-link network with 4 antenna elements at each node. One of the links is initially assigned 2 streams, and the other two are assigned one stream. The throughputs with stream control according to (12) and (14) are close to that with optimal stream control. For small R/D, there is a small difference, since the initial choice of which link gets two streams is not optimal.

VI. Conclusion

The distributed link adaptation methods of [4], [5] cannot find the best number of streams to optimize network throughput, since network throughput is not the cost function being optimized. We have introduced a constraint to limit the number of streams transmitted by each transmitting node in a distributed way. We have shown through simulation for two example topologies that the stream control algorithm yields throughputs very close to those obtained with optimal stream control. The extra throughput achieved when stream control is combined with adaptive spatial filtering at the transmitter suggests that closed-loop schemes may be desirable in applications with slowly varying CSI.

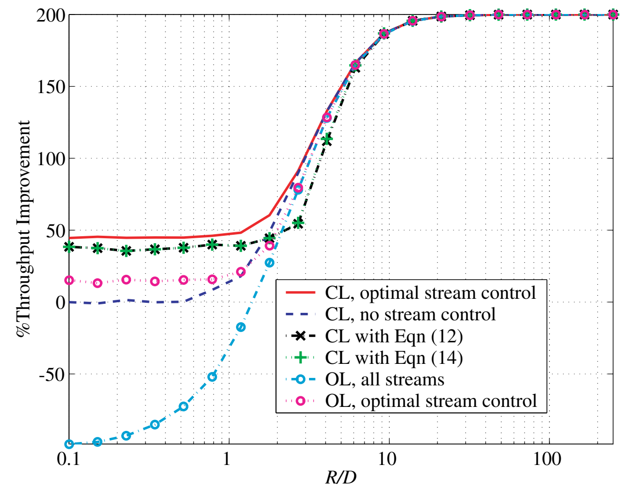


Fig. 8. Throughput improvement of the 3-link network, 4 antennas.

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