A Stochastic Approach in Modeling Cooperative Line Networks

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***Abstract*—We consider a quasi-stationary Markov chain as a model for a decode and forward wireless multi-hop cooperative transmission system that forms successive Opportunistic Large Arrays (OLAs). This paper treats a linear network topology, where the nodes form a one-dimensional horizontal grid with equal spacing. In this OLA approach, all nodes are intended to decode and relay. We derive the transition probability matrix of the Markov chain based on the hypoexponential distribution of the received power at a given time instant assuming that all the nodes have equal transmit power and the channel has Rayleigh fading and path loss with an arbitrary exponent. The Perron-Frobenius eigenvalue and the corresponding eigenvector of the sub-stochastic matrix indicates the signal-to-noise ratio (SNR) margin that enables a given hop distance.**

I. Introduction

Wireless multi-hop communications, where radios forward the packets of other radios, has a wide variety of applications, not only in the cellular and sensor networking regimes, but in technologies like wireless computer networking and mobile computing. One promising, very fast, and low-overhead wireless transmission technique is the Opportunistic Large Array (OLA) [1], in which all radios that decode a message relay the message together very shortly after reception, without coordination with other relays. Synchronization can be achieved based on a packet preamble that all cooperators receive [2] or from GPS. When paired with a transmission threshold, OLA broadcasting is an energy-efficient candidate for large dense wireless sensor networks [1].

In this paper, we model a special case of the decode and forward (DF) OLA network, where the nodes are uniformly spaced along a line. This topology can be considered a precursor to a strip shaped network for the finite density case. The wireless channel is modeled with path loss and flat Rayleigh fading. All the nodes that can decode the source packet correctly, relay the packet concurrently in orthogonal channels, thereby providing transmit diversity. Then, all the nodes that can decode that OLA transmission will relay in the next hop, and this process proceeds until it fails. Specifically, we assume that the conditional probability that the kth node in cluster decodes, given that the previous cluster had at least one node transmitting, is the same for each cluster. This allows us to apply the well-established theory of quasi-stationary discrete time Markov chains with an absorbing state [6]. The absorbing state represents when the transmissions stop propagating. Once we have the quasi-stationary distribution, we can determine network performance, such as packet delivery ratio and latency over a given distance as a function of system parameters such as transmit power, inter-node distance, and path loss exponent.

The authors in [1], [3], and [4] studied large dense networks, using the continuum assumption. Under this assumption, the number of nodes goes to infinity while the power per unit area is kept fixed. These papers derived conditions under which broadcasting over an infinite disk or strip is guaranteed. In contrast, in this paper, we obtain closed-form theoretical results without the continuum assumption, by deploying a simple one-dimensional network where the nodes are uniformly spaced on a grid. By applying the quasi-stationary Markov chain analysis, we show that there is no condition guaranteeing infinite propagation of OLAs, however, there is only a probability of successfully delivering a packet over a given distance. In [5], the authors studied cluster to cluster transmission over a linear network, where the cluster size is fixed over the entire transmission. However, in this paper, we have shown that the fixed cluster scenario appears to be a special case of our more general approach.

The rest of the paper is organized as follows. In the next section, we define the network parameters and propose a Markov chain model in Section III. In Section IV, we derive the transition probability matrix and we propose an iterative algorithm for optimizing the system parameters in Section V. The results and system performance is given in Section VI. The paper then concludes with certain recommendations in Section VII.

II. System Description

Consider an infinite line of nodes where adjacent nodes are a distance d apart from one another, as shown in Figure 1. We assume that the nodes transmit synchronously in OLAs or levels, and that a *hop* occurs when nodes in one level transmit a message and at least one node in the following cluster is able to decode the message for the first time. Correct decoding is achieved when a node's received SNR at the output of the diversity-combiner, from the previous level only, is greater than or equal to a modulation-dependent threshold, τ. Exactly one time slot later, all the nodes that just decoded the message

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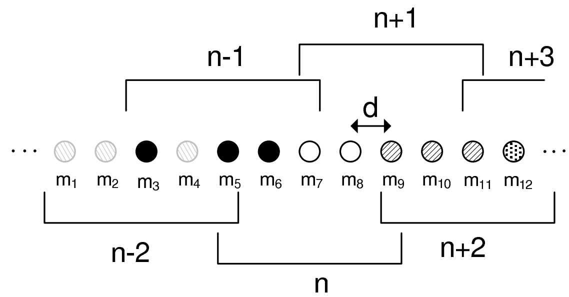

Fig. 1. The overlapping windows with $M = 5$ and $h_d = 2$

relay the message. These nodes are said to *decode and forward* (DF). Once a node has relayed a message, it will not relay that message again. Let $p_n(m)$ be the *membership* probability that the mth node transmits in the nth level, given that at least one node transmitted in the $(n-1)th$ level. Also let M be at least the width of the region of support of $p_n(m)$. In other words, there exists some M_0 such that $p_n(m) \geq 0$ for $M_0 \leq m \leq M_0 + M - 1$ and $p_n(m) = 0$ otherwise. As we will show later, the quasi-stationary property implies that there exists a hop distance, h_d, such that $p_{n-1}(m - h_d) = p_n(m)$. The main distinction between this paper and [5] is that in this paper $M > h_d$ is considered, while in [5] only the restricted case of $M = h_d$ is considered. Hence h_d can be considered as a shift to the window of size M. A sample outcome of the transmissions is shown in Figure 1 where the window size, M, is 5 and the hop distance or the shift in window, h_d, is 2. The nodes m_1, m_2, and m_4 are able to decode the message and become part of level $n-2$. These nodes will relay the message in the next time slot and only the nodes in level $n-1$ may decode that message. Since m_4 has already participated in level $n-2$, so it cannot be part of any other level including $n-1$. Thus the candidate nodes are m_3, m_5, m_6, and m_7, out of which m_3, m_5, and m_6 become DF nodes in level $n-1$ and this process continues.

We assume that all the nodes transmit with the same transmit power P_t. A node receives superimposed copies of the message signal from the nodes that decoded the message correctly in the previous level, over orthogonal fading channels using equal gain combining (EGC). Let us define $\mathbb{N}_n = \{1, 2, ..., k_n\}$, where k_n is the cardinality of the set $\mathbb{N}_n$, to be the set of indices of those nodes that decoded the signal perfectly at the time instant (or hop) n. For example, from Figure 1, $\mathbb{N}_n = \{3, 4\}$ and $\mathbb{N}_{n+1} = \{3, 4, 5\}$. The received power at the jth node at the next time instant $n+1$ is given by

$$Pr_j(n+1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{N}_n} \frac{\mu_{mj}}{|h_d - m + j|^\beta}, \quad (1)$$

where the summation is over the DF nodes in the previous level. μ_{mj} is the flat fading Rayleigh channel gain from node m in the previous level to node j in the current level. The elements of μ are independently and identically distributed (i.i.d) and are drawn from an exponential distribution with the parameter σ_μ^2=1; β is the path loss exponent with a usual range of 2-4. Consequently, the received SNR at the jth node is given as $\gamma_j = Pr_j/\sigma_j^2$, where σ^2 is the variance of the noise in the receiver. We assume perfect timing and frequency recovery at each receiver, and we also assume that there is sufficient transmit synchronization between the nodes of a level, such that all the nodes in a level transmit to the next level at the same time [2]. In other words, the transmissions only occur at discrete instants of time $n, n+1, ...$ such that the hop number and the time instants can be defined by just one index n. By the overlapping nature of the windows, we have the following proposition and corollary.

Proposition 1: Given M and h_d, a node at a position x can become part of several levels n, such that $\forall x > M - h_d$

$$\left\lceil \frac{x - M}{h_d} \right\rceil + 1 \leq n \leq \left\lfloor \frac{x - 1}{h_d} \right\rfloor + 1. \quad (2)$$

Corollary: $\forall x \leq M - h_d$, we have $n = 1, ..., \left\lceil \frac{x}{h_d} \right\rceil$.

One goal of this study is to find the hop distance as a function of the values of system parameters such as relay transmit power and inter-node distance. However, because of the discrete nature of the hop distance, solving the problem is this manner is quite tedious. Hence in this paper we follow the inverse approach, i.e., for a given hop distance, we will find the optimal values of the window size, M, and the system parameters that generate this hop distance.

III. Modeling by Markov Chain

At a certain time n, a node from the nth level will take part in the next transmission, if it has decoded the data perfectly at the current time, or it will not take part, if it did not decode correctly or it has already decoded the data in one of the previous levels. The decisions of all the nodes in the nth level can be represented as $X(n) = [\mathbb{I}_1(n), \mathbb{I}_2(n), ..., \mathbb{I}_M(n)]$, where $\mathbb{I}_j(n)$ is the tertiary indicator random variable for the jth node at the nth time instant and is 0 if node j does not decode, 1 if node j decodes and 2 if node j has decoded at some earlier time. Thus each node is represented by either 0, 1 or 2 depending upon the successful decoding of the received data. For example, from Figure 1, we have $\mathbb{I}_1(n) = \mathbb{I}_2(n) = 2$, $\mathbb{I}_3(n) = \mathbb{I}_4(n) = 1$ and $\mathbb{I}_5(n) = 0$. We observe that the outcomes of $X(n)$ are ternary M-tuples, each outcome constituting a state, and there are 3^M number of states, which are enumerated in decimal form $\{0, 1, ..., 3^M - 1\}$. Let i_n be the outcome at time n. For example, $i_n = \{22110\}$ in ternary, and $i_n = 228$ in decimal in Figure 1. Then we may write

$$\begin{aligned} \mathbb{P}\{X(n) = i_n | X(n-1) = i_{n-1}, ..., X(1) = i_1\} = \\ \mathbb{P}\{X(n) = i_n | X(n-1) = i_{n-1}\}, \end{aligned} \quad (3)$$

where $\mathbb{P}$ indicates the probability measure. Equation (3) implies that $X(n)$ is a discrete-time finite-state Markov Process. Assuming the statistics of the channel are same for all the hops in the network, the Markov chain can be regarded as a homogeneous one.

It can be further noticed that at any point in time, there is a probability that the Markov chain can go into an absorbing state, thus terminating the transmission. That can be a state when all the nodes at a particular hop cannot decode the message perfectly and thus Markov chain will be in the 0 state (decimal). It can be further noticed, that any possible combination of 0 and 2 will also make the state an absorbing state. Since we are enumerating the states using ternary words, the total number of states appears to be 3^M. But since all the transitions are not possible because of the overlapping nature of the window, we will have the possible number of states that can be reached during transitions is $\hat{N} = 3^{M-h_d} \times 2^{h_d}$, including 2^{M-h_d} number of absorbing states.

Hence we consider the Markov chain, X, on a state space $A \cup S$, where A is the set of absorbing states, where

$$\lim_{n\to\infty} \mathbb{P}\left\{X(n) \in A\right\} \nearrow 1 \quad \text{a.s.} \tag{4}$$

On the other hand, the states in S (where cardinality of S is $|S| = \hat{N} - 2^{M-h_d}$) make an irreducible state space, i.e., there is always a non-zero probability to go from any transient state to another transient state. We will define two matrices to describe the Markov Chain. The first, $\tilde{\mathbf{P}}$, is the full transition probability matrix for all the states in the set $A \cup S$. Each row in $\tilde{\mathbf{P}}$ sums to one. The second matrix, $\mathbf{P}$, is the submatrix of $\tilde{\mathbf{P}}$ that is formed by striking each column and row that involves transitions to and from the absorbing states in A. Therefore, $\mathbf{P}$ is the matrix corresponding to the states in S. It can be noticed that the transition probability matrix $\mathbf{P}$ on the state space S is not right stochastic, i.e., the row entries of $\mathbf{P}$ do not sum to 1 because of the *killing probabilities* given as

$$\kappa_i = 1 - \sum_{j\in S} \mathbf{P}_{ij}, \qquad i \in S. \tag{5}$$

Since $\mathbf{P}$ is a square irreducible nonnegative matrix, then by the Perron-Frobenius theorem [8], there exists a unique maximum eigenvalue, ρ, such that the eigenvector associated with ρ is unique and has strictly positive entries. For proof, please refer to [8]. Overall our assumptions for $\mathbf{P}$ imply that

$$0 < \rho < 1. \tag{6}$$

From the theory of Markov chains [8], we know that a distribution $\mathbf{u} = (u_i, i \in S)$ is called ρ-invariant distribution if $\mathbf{u}$ is the left eigenvector of the transition matrix $\mathbf{P}$ corresponding to the eigenvalue ρ, i.e.

$$\mathbf{uP} = \rho\mathbf{u}. \tag{7}$$

We are now interested in the limiting behavior of this Markov chain as time proceeds. Since $\forall n, \mathbb{P}\left\{X(n) \in A\right\} > 0$, eventual killing is certain. But we are interested in finding the distribution of the transient states, before the killing occurs. The so-called limiting distribution is called the quasi-stationary distribution of the Markov chain, which is independent of the initial conditions of the process. From [6] and [7], this unique distribution is given by the ρ-invariant distribution for one step transition probability matrix of the Markov chain on S. We can find the quasi-stationary distribution by getting the *maximum* eigenvector, $\hat{\mathbf{u}}$ of $\mathbf{P}$, then defining $\mathbf{u} = \hat{\mathbf{u}} / \sum_{i=1}^{\hat{N}} \hat{\mathbf{u}}_i$ as a normalized version of $\hat{\mathbf{u}}$ that sums to one.

Thus we can define the unconditional probability of being in state j at time n as

$$\mathbb{P}\left\{X(n) = j\right\} = \rho^n u_j, \quad j \in S, \quad n \geq 0. \tag{8}$$

We also let $T = \inf\left\{n \geq 0 : X(n) \in A\right\}$ denote the end of the survival time, i.e., the time at which killing occurs. It follows then,

$$\mathbb{P}\left\{T > n + m | T > n\right\} = \rho^m, \tag{9}$$

while the quasi-stationary distribution of the Markov chain is given as

$$\lim_{n\to\infty} \mathbb{P}\left\{X(n) = j | T > n\right\} = u_j, \quad j \in S. \tag{10}$$

We also note that the membership probability can be expressed as

$$p_n(m) = \sum_{j\in\theta} u_j, \tag{11}$$

where $\theta = \left\{X(n) \in S : \mathbb{I}_m(n) = 1\right\}$.

IV. Formulation of the Transition Probability Matrix

In this section, we will find the state transition matrix $\mathbf{P}$ for our model, the eigenvector of which will give us the quasi-stationary distribution. Let i and j denote a pair of states of the system such that $i, j \in S$, where each i and j are the decimal equivalents of the tertiary words formed by the set of indicator random variables. Now for each node m, the probability of being able to decode at time n is given as

$$\mathbb{P}\left\{\text{node } m \text{ of level } n \text{ will decode}\right\} = \mathbb{P}\left\{\gamma_m(n) > \tau\right\}. \tag{12}$$

where

$$\mathbb{P}\left\{\gamma_m(n) > \tau\right\} = \int_\tau^\infty p_{\gamma_m}(y)dy. \tag{13}$$

$p_{\gamma_m}(y)$ is the probability density function (PDF) of the received SNR at the mth node. We note that a node can have three possible states, where the initial state of a node is always 0. A node can make the transitions shown in Figure 2. Hence each individual node is a state machine, although $\mathbb{I}_m(n)$ is not a Markov chain itself; the probabilities of transition for a single node are defined only at certain times. P_{01} from Figure 2, i.e., the conditional probability of success of the mth node in the nth level, is given as

$$P_{01} = \mathbb{P}\left\{\gamma_m(n) > \tau | \mathbb{I}_m(k) = 0, X(n-1) \in S\right\}. \tag{14}$$

for $k = n-1, n-2, ...$ Hence the probability of perfect decoding is based on the PDF of the received power which is the hypoexponential distribution given as

$$p_Y(y) = \sum_{k=1}^{K} C_k \lambda_k \exp\left(-\lambda_k y\right), \tag{15}$$

where

$$C_k = \prod_{\zeta \neq k} \frac{\lambda_\zeta}{\lambda_\zeta - \lambda_k}. \tag{16}$$

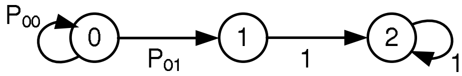

Fig. 2. State transition diagram of a node

Let us define a set which consists of all those nodes that decoded the data perfectly in the previous hop as $\mathbb{N}_{n-1} = \{m_i : \mathbb{I}_{m_i}(n-1) = 1\} \;\; \forall i = 1, 2, ...M$, then P_{01} from (14) is given as

$$P_{01} = \sum_{k \in N_{n-1}} C_k \exp\left(-\lambda_k^{(m)} \tau\right), \tag{17}$$

where $\lambda_k^{(m)}$ is given as

$$\lambda_k^{(m)} = \frac{d^\beta \left|h_d - k + m\right|^\beta \sigma^2}{P_t}. \tag{18}$$

Let a superscript on the indicator functions show the value of the indicator given the ith state. For example, if $i = \{22110\}$, then $\mathbb{I}_5^{(i)}(n) = 0$. Therefore, the one-step transition probability going from the state i in level $n-1$ to state j in level n is always 0 when either of the following conditions is true:
Condition I: $\mathbb{I}_k^{(j)}(n) \in \{0, 1\}$ and $\mathbb{I}_{M-h_d+k}^{(i)}(n-1) \in \{1, 2\}$,
Condition II: $\mathbb{I}_k^{(j)}(n) = 2$ and $\mathbb{I}_{M-h_d+k}^{(i)}(n-1) = 0$.

Thus the one step transition probability for going from state i to state j given the above conditions do not hold is given as

$$\begin{aligned} P_{ij} = \prod_{k \in \mathbb{N}_{n-1}^{(i)}} \left(\sum_{m \in \mathbb{N}_{n-1}^{(i)}} C_m \exp\left(-\lambda_m^{(k)} \tau\right) \right) \bullet \\ \prod_{k \in \overline{\mathbb{N}}_n^{(j)}} \left(1 - \sum_{m \in \mathbb{N}_{n-1}^{(i)}} C_m \exp\left(-\lambda_m^{(k)} \tau\right) \right) \end{aligned} \tag{19}$$

where $\mathbb{N}_n^{(j)}$ and $\overline{\mathbb{N}}_n^{(j)}$ are the indices of those nodes which are 1 and 0, respectively, in state j at level n. Thus it can be seen that the transition probability matrix will contain a large number of zeros. The smaller the hop distance, the larger are the number of zeros in the matrix. Thus the resulting matrix is highly sparse which helps in evaluating the Perron-Frobenius eigenvalue pretty quickly.

V. ITERATIVE APPROACH

In the previous section, we have shown how to compute the quasi-stationary distribution and the membership probabilities for a given specification of system parameters, such as transmit power, path loss exponent, inter-node distance, hop distance, and for the one artificial constraint, the window width. Therefore, an infinite variety of possible solutions exist, depending on the choice of these parameters. In this section, we eliminate the artificial constraint and show how the design space dimension can be further reduced through parameter normalization and by optimizing the shape of the membership probability function.

M is an *artificial* constraint because there is no real physical need for it, however, it strongly impacts the size of the state space and therefore the computational complexity of finding the quasi-stationary distribution. Therefore, we would like for M to be as small as possible without significantly impacting the system performance results. Since the transmissions from nodes at the trailing edge of a large window will have only a small contribution to the formation of the next OLA, because of disparate path loss (especially in a line-shaped network), and therefore, their contribution can be neglected. This suggests that an energy efficient solution will be a unimodal membership probability function with a narrow region of support, and therefore a small M can support it. We note that the number of nodes that relay in each hop determines the diversity order in this finite density scenario, so the most narrow membership function (a Kronecker delta) is not desirable. A final consideration is that for the broadcast application, ideally, we want every node to decode the message, and so, under our assumption that every node that decodes for the first time also relays, we have that for a hop distance of h_d, we want at least h_d nodes to relay.

Based on all of these considerations, we decided to choose the solution that yields a membership probability function that most closely resembles a square pulse of unit height that is h_d nodes wide, and takes the value of zero everywhere else on a window that is M nodes wide. We find M by increasing it until the one-hop success probability (i.e., the Perron-Frobenius eigenvalue) ceases to change significantly.

To further decrease the design space dimension, we observe that the transition matrix in (19) depends on the product $\lambda_m^{(k)} \tau$, from which we can extract the normalized parameter

$$\Upsilon = \frac{\gamma_0}{\tau} = \frac{P_t}{d^\beta \sigma^2} \frac{1}{\tau}, \tag{20}$$

which can be interpreted as the SNR margin from a single transmitting node a distance d away. However, Υ is not the only independent parameter, because β and h_d also separately impact the value of $\lambda_m^{(k)} \tau$, in (18) through the factor $\left|h_d - k + m\right|^\beta$.

We now formally describe our optimization procedure. We define our ideal membership probability function as

$$\hat{q}(k) = u(k-a) - u(k - (a + h_d - 1)) \quad k \geq 1, \tag{21}$$

where u is the unit step function and $a = \left\lfloor \frac{M - h_d}{2} \right\rfloor + 1$. We can express the membership probabilities for a given level in vector form as, $\mathbf{q} = \{p_{m_1}, p_{m_2}, ..., p_{m_M}\}$, where the values of $p_{m_k}(n)$ can be found using either (11) or as

$$\begin{gathered} p_{m_k}(n) = \sum_{j=1}^{|S|} \mathbb{P}\left\{\mathbb{I}_{m_k} = 1 | X(n) = j\right\} \mathbb{P}\left\{X(n) = j\right\} \\ \forall k = \{1, 2, ..., M\} \text{ and } j \in S. \end{gathered} \tag{22}$$

Then the problem of finding the best Υ can be formulated as

$$\min_{\Upsilon > 0} \;\; \Xi = \frac{1}{M} \left\| \mathbf{q} - \hat{\mathbf{q}} \right\|^2, \tag{23}$$

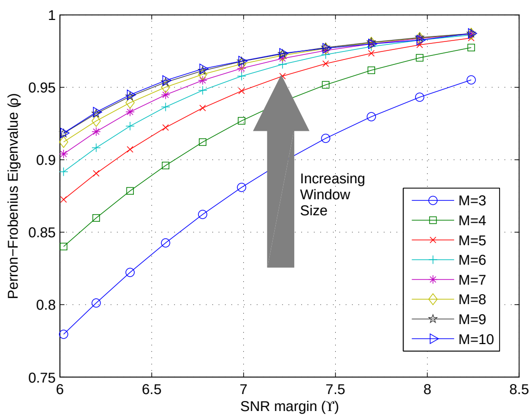

Fig. 3. Behavior of ρ as M increase for $h_d = 2$ and $\beta = 2$

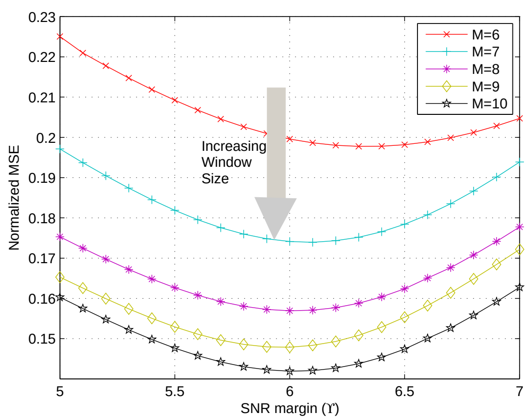

Fig. 4. Error curves for different window sizes for $h_d = 2$ and $\beta = 2$

The iterative algorithm in this case is given as follows.

Algorithm 1:

1) Given h_d, initialize the algorithm with a window size of $M = 2h_d$.
2) Compute the Perron-Frobenius eigenvalue, $\rho(M)$, over a range of SNR margin.
3) Increment the window size by one, and compute $\rho(M+1)$ using Step 2.
4) If $|\rho(M+1) - \rho(M)| < \epsilon$, for $\epsilon > 0$, M is the desired window size and the convergence is achieved. Otherwise go to step 3.

By using the iterative technique, we are able to find the optimal M over a range of SNR margin. To choose the SNR Margin that gives a close approximation to (21), minimize (23) over the SNR margin range to get the best value of SNR margin where we achieve the minimization. This value of Υ is the one that ensures the given h_d with maximum probability.

VI. Results and System Performance

In this section, we will show the convergence of iterative algorithm and show some results for system performance. Figure 3 depicts the trend of eigenvalues as we increase the SNR margin for different window sizes and a hop distance of 2. The behavior is quite obvious that increasing SNR margin increases the probability of survival of the transmissions. It can be further noticed that for a given value of SNR margin, the curves start to converge as we increase the window size, thereby indicating that after a specific window size, even if we increase M, there is no change in the transmissions outcome which agrees with the iterative algorithm from Section V.

Figure 4 shows the error surfaces for the overlapping window case, generated by (23) for a hop distance of 2 and different window sizes. It can be seen that the error surface is convex that contains a minimum for a particular value of SNR margin, Υ. It can be further noticed, that as we increase the window size the difference between the errors becomes smaller in the same vicinity of Υ. Thus, for a window size of 10 and a hop distance of 2, we can select the SNR margin of around 6dB to give us desired membership probability function.

Figure 5 shows the numerical simulation result for conditional membership probabilities of the nodes to different levels, where the values Υ and M are taken from Figures 3 and 4. Based on the previous state (assuming an initial distribution of nodes at the first hop) and Rayleigh fading channel gains, we use the received power at each node to set the indicator functions as either 0,1, or 2 depending upon the threshold criterion. These indicator functions will form the current state and the process continues. We finally obtain the distribution of the chain by simulating over 20,000 trials. From Figure 5 it can be seen that the distance between the peaks of any two membership functions is always 2. Thus a window size of 10 seems reasonable to get a hop distance of 2 with an SNR margin of approximately 6dB. The inset figure in the right top corner shows the analytical membership function obtained from (22) by using the quasi-stationary distribution.

From the deployment perspective of the network, it is sometimes desirable to optimize the values of certain parameters like transmit power of relays or distance between them. This optimization is done to obtain a certain quality of service (QoS). For example, we are interested in finding the probability of delivering the message over a certain distance without having entered the absorbing state, and we desire this probability to be at least η, where $\eta \sim 1$ ideally. We can use (9) to get a nice upper bound on the value of m (the number of hops) one can go with a given η, i.e. $\rho^m \geq \eta$, which gives

$$m \leq \frac{\ln \eta}{\ln \rho}. \tag{24}$$

Thus if the destination is far off, we require more hops, which will require a larger value of ρ. Figure 6 shows the relationship between required SNR margin to reach the destination node at a particular normalized distance for different values of hop distance. The normalized distance, which is the true distance divided by d, is defined as the product of h_d and the number of hops, (made to reach the destination). We observe that the performance of all the cooperative cases exceeds that of non-cooperative case for a particular value of SNR margin.It can be further noticed that the transmissions with cooperative case

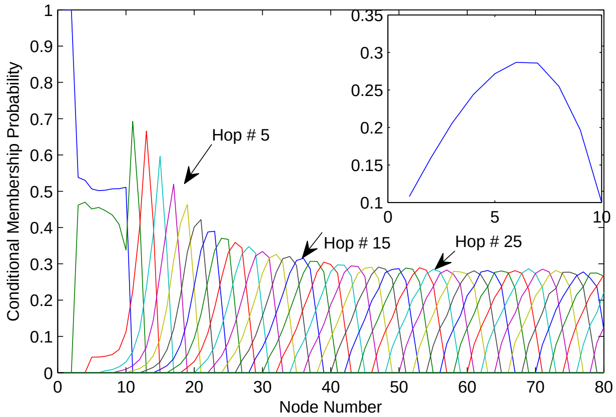

Fig. 5. Conditional membership probabilities of the nodes for $h_d = 2$ for a window size of 10 and $\Upsilon = 6dB$

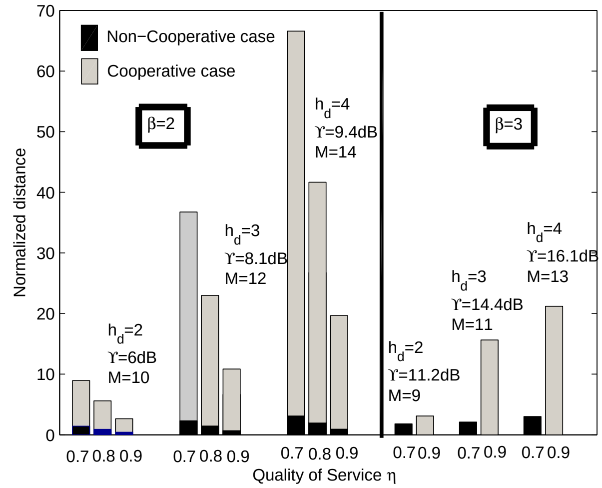

Fig. 6. Normalized distance for various cooperative vs. non-cooperative cases

can reach a particular point in two ways, i.e., keeping both the hop distance and SNR margin small or having a higher hop distance with a higher SNR margin, where the latter has lower latency, i.e., fewer hops and higher QoS, η. The results are also plotted for a higher path loss exponent, i.e., $\beta = 3$. Thus we observe that if we increase the path loss exponent and also the SNR margin, we get results similar to that of small path loss exponent with small SNR margin. The non-cooperative results show that only a small distance can be reached with a small success probability when we use the same SNR margin as for high path loss exponent.

From the broadcast perspective, another important parameter is to find the fraction of nodes in the network that have decoded. If we assume that the Markov chain is in the quasi-stationary state, and has not entered the absorbing state over a linear network of interest, then the fraction of decoded nodes in the network is the same as the fraction of the nodes in any one hop. From Figure 5, we can see that we do not exactly get a rectangular membership function, which implies that not all the nodes in the network may have decoded the data. Let N_d be a random variable that denotes the number of DF nodes such that n_{d_j} are the realizations of this variable where $j = 1, 2, ... |S|$. Hence the average number of the nodes that have decoded the data is given as

$$\mathbb{E}(N_d) = \sum_{j=1}^{|S|} n_{d_j} u_j. \tag{25}$$

Hence for the cases that are described in Figure 6, the results are summarized in Table I. It can be seen that as we increase the hop distance (and SNR margin consequently), we get more nodes that are able to decode in a given hop.

TABLE I
FRACTION OF DF NODES FOR VARIOUS HOP DISTANCES

Hop distance, h_d	2	3	4
% of nodes decoded, $\beta = 2$	92.30	94.67	97.02
% of nodes decoded, $\beta = 3$	93.54	95.98	98.21

VII. CONCLUSION AND RECOMMENDATIONS

In this paper, we have shown that a one-dimensional multi-hop network can be modeled as a quasi-stationary Markov chain in discrete time and we derived the sub-stochastic matrix of this chain. The Perron-Frobenius eigenvalue and the corresponding eigenvector of this matrix helps in determining different parameters for achieving better performance in delivering the message to a destination. As an extension to this work, it is recommended to obtain a framework where the nodes are aligned on a two dimensional grid, which will mimic a strip shaped OLA network.

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