Linear and Non-linear Receiver Processing in MIMO Ad-hoc Networks

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Abstract **– A collision model is proposed for ns2 simulations of a multi-hop ad-hoc network with multi-antenna nodes. We analyze the network throughput performance of linear minimum mean squared error (LMMSE) and successive interference cancellation (SIC) receiver-processing techniques for the flat fading channel. We compare two MAC protocols, one that uses single-input-multiple-output (SIMO) links and the other that uses point-to-point multiple-input-multiple-output (MIMO) links. We also consider two power policies: constant power per stream and constant power per node.**

Key words: Cross-layer, ad-hoc network, CSMA/CA, MIMO, interference cancellation

1. INTRODUCTION

This paper analyses the throughput of ad-hoc networks for two types of multiple-input-multiple-output (MIMO) architectures. In the first, both the transmitter and receiver have multiple antenna elements. In the second, the receivers have multiple antennas, but the transmitters only use one antenna. In this later case, the receiver can receive several users' transmissions at once.

The high degree of flexibility in MIMO architectures makes networks with MIMO physical (PHY) layers excellent candidates for medium access control (MAC)-PHY cross-layer design. The MAC is designed to take the special features of the PHY layer and the states of the channel into account in admission decisions. Cross-layer approaches have been shown to increase the throughputs of networks, compared to conventional approaches [1, 2]. Most of the research in cross-layer design presents theoretical results with the average analysis for centralized network architectures [1, 3-5]. Very little is known about ad-hoc networks especially because the theory becomes intractable for multi-hop networks. To the best knowledge of the authors there are very few existing works that analyze the performance of a multi-hop ad-hoc network with anything but a "path-loss only" type PHY layer processing [6]. The authors in [1] have also highlighted the need of a good collision model.

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Furthermore, there are no works the authors are aware of that provide a good collision model for the MIMO PHY layer. For example, in [2], the collision model is deterministic.

The performance of MIMO depends on the type of receiver signal processing algorithm. We consider two popular algorithms, linear minimum mean squared error (LMMSE) and successive interference cancellation (SIC) (a nonlinear technique) [7]. They represent a tradeoff in complexity versus performance [8], with SIC providing better performance because it gives higher diversity gains to the weakest signals. SIC was considered in cross-layer design in [9], however [9] did not consider MIMO.

In this paper, we analyze the impact of LMMSE and SIC receiver-processing techniques on the throughput of multi-hop ad hoc networks with either single-input-multiple-output (SIMO) or MIMO flat fading links. In doing so, we make the following contributions i) collision models for multi-packet reception using MMSE and SIC, and ii) a throughput comparison of two receiver processing techniques on a multi-hop SIMO/MIMO ad-hoc network with two different MAC protocols.

The first MAC protocol, which we call the simple MAC (S-MAC) protocol, allows a user to transmit from at most one antenna. The receiver is capable of handling M simultaneous streams, where M is the number of receiver antennas. The second one is called the collision sense multiple access/collision avoidance allowing k streams (CSMA/CA(k)) MAC protocol, first proposed in [2]. For this paper, the protocol requires the same user to transmit all the $k = M$ streams.

We also consider two power policies [18]. One attempts to force the two protocols to expend equal energy by making the total power within the sensing range constant. This is achieved by maintaining constant power per stream (CPPS). The other power policy constrains each node to transmit the same power, even when only one stream is transmitted. This policy is called constant power per node (CPPN). CPPN results in reduced energy per stream for CSMA/CA(k), compared to CPPS. For a fair comparison of the two protocols we use range-preserving strategy [22] such that SNR per stream is the same at the boundary of decoding and sensing ranges; this results in CSMA/CA(k) having smaller ranges due to lower power per stream.

In Section II, we describe the proposed collision model for LMMSE and SIC receivers. Section III presents the results of network simulations and discussion. The paper is concluded in Section IV.

2. Collision models

A landmark SISO collision or multi-packet reception model was proposed [10]. This model discussed the capture probability, P_{cap}, which is the probability that a packet will be correctly decoded if its signal to interference and noise ratio (SINR) is greater than a preset threshold, γ_{th}:

$$P_{cap} = \Pr\{SINR \geq \gamma_{th}\}. \tag{1}$$

P_{cap} depends upon the coding and the modulation. However, [10] considered only path-loss-dependent received signal strength. Later, a detailed study of capture probability, in presence of multipath, shadowing and the near-far effect was done in [11]. To our best knowledge, almost all the work till now uses the collision model proposed in [11] for PHY layer abstraction to be used with MAC layer [3,4,12]. We will extend this model to LMMSE and SIC receivers.

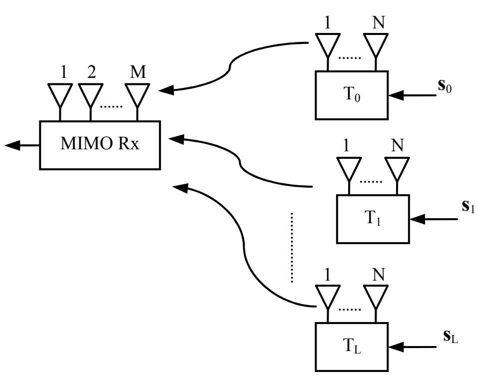

Figure 1: Interference scenario in ad-hoc wireless system with V-BLAST transmitters

We consider a MIMO system with M and N antennas at the receiver and transmitter respectively and a total of $L+1$ transmitters. Fig. 1 shows the receiver under consideration in an ad-hoc wireless network. In Fig. 1 s_i is the BPSK symbol with period T_s. Though all the links in the figure are MIMO-capable we analyze two special scenarios. In the first, the desired transmitter does V-BLAST transmission, i.e. the symbol sequence into the transmitter is converted to N parallel streams before transmission [7], with $N = M$ and $L = 0$. In the second, each transmitting node excites only a single omni-directional antenna from the antenna array such that $N = 1$ and $L = M - 1$; the antenna is selected deterministically. In both these cases the receiver is fully loaded, which means that the total number of incident streams is equal to the number of receive antennas. For ease of model representation we exploit the fact that even for V-BLAST transmission, all the streams other than the one being demodulated act as interferers. Hence the following system model can be used for both the scenarios

$$\mathbf{x} = \sum_{i=0}^{L} \alpha_i \mathbf{v}_i \mathbf{s}_i + \mathbf{n}, \tag{2}$$

where $\mathbf{x}$ is a $M \times 1$ received signal vector, α_i is the received signal amplitude representing path loss, and $\mathbf{v}_i$ is a vector of independent unit variance complex rayleigh random variables representing the faded channel of the i^{th} stream (first scenario) or i^{th} user (second scenario), and $\mathbf{n}$ is the receiver noise. The desired user is indexed 0.

The post processing SINR of the optimum LMMSE combiner [14] is given by

$$\gamma = \alpha_0^2 \mathbf{v}_0^H \mathbf{\Phi}^{-1} \mathbf{v}_0, \tag{3}$$

where $\mathbf{\Phi}$ is interference-plus-noise covariance matrix. Assuming noise and interference are uncorrelated and flat fading such that the gain remains constant over the whole packet we have

$$\mathbf{\Phi} = \sigma^2 \mathbf{I} + \sum_{i=1}^{L} \alpha_i^2 \mathbf{v}_i \mathbf{v}_i^H, \tag{4}$$

where $\mathbf{I}$ is the identity matrix and we assume that all branches have same noise power.

To compute the probability of packet drop we need the cumulative distribution function (CDF) of SINR as a function of fading statistics and path-loss. For this, we apply the approximate semi-analytical solution of the CDF [16], which is accurate when the total number of incident streams, $(L+1) \times N$, is not greater than M; such a receiver is called a non-overloaded receiver.

From the definition of P_{cap} in Eq. 1 we need to define the SINR threshold, γ_{th}, which is determined by the outage probability of the conditional probability of bit error, P_e. We assume the interference is Gaussian. A simple analysis shows that this assumption provides an upper bound on P_e. Hence, $P_D = 1 - P_{cap}$ estimated by this method gives a conservative figure. In this paper we consider outage to be the case when $P_e \geq 10^{-5}$ for BPSK modulation. For BPSK modulation, P_e is given by

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \tag{5}$$

where $\gamma = E_b / N_0$ is the SNR per bit. γ_{th} corresponds to a non-faded channel and $P_e = 10^{-5}$, which we can get by inverting (5):

$$\gamma_{th} = \tfrac{1}{2}\left(Q^{-1}\left(10^{-5}\right)\right)^2 = 9.12 \qquad (6)$$

To compute P_D we use the post-processor SINR probability distribution function, $p(\gamma)$ [16], as follows

$$P_D = \int_0^{\gamma_{th}} p(\gamma) d\gamma . \qquad (7)$$

A closed form expression for P_D where different streams have different path losses is difficult to derive. Hence, we generate a table of P_D for k users located on a 300x300m grid with a resolution of 10m, where $0 \le k \le M$. The reason we choose a maximum of M streams is because we assume the MAC prevents overloading.

LMMSE and SIC receivers are compared against the ideal receiver that always perfectly decodes all the streams/packets when it is not overloaded. However, when overloaded the ideal receiver rejects all the streams/packets. We note that [9] assumed the ideal receiver model.

2.1. Simple MAC (S-MAC) protocol

S-MAC requires that the desired user is single stream and should be within the decoding range. S-MAC allows a maximum of $M-1$ interferers to be anywhere within the sensing range, because the MIMO receiver can suppress them. We assume centralized control in this paper, so S-MAC never admits more than M streams within the sensing range of a receiver. However, a distributed version might sometimes erroneously allow receiver overloading. Fig. 2(a) shows the functionality of the distributed S-MAC protocol with $M = 4$. The upright triangles represent receivers; the inverted triangles represent transmitters, with filled and unfilled triangles indicating the desired and interfering nodes, respectively. The packet will be dropped with $P_D = 1$ if all the four interfering links are active, however, if any three of these links are active then the packet will be dropped with the P_D obtained from the table.

In case of the SIC receiver, the users are sorted based on their average power and cancelled in order of descending average power. We assume perfect channel estimation and cancellation. This implies that the stream demodulated first will not get any diversity gain, the second stream will have a diversity order of 2, third will have order 3 and so on.

2.2. CSMA/CA(k) MAC protocol

The CSMA/CA(k) protocol requires a node to sense the channel before transmitting, and V-BLAST transmission with all M streams occurs only if the channel is free; otherwise, the transmitter does a back-off and attempts V-BLAST transmission after a random back-off time [2]. Fig. 2(b) shows the functionality of CSMA/CA(k). This protocol is conservative in the sense that it does not allow any transmissions in the sensing range; only the desired user within the decoding range is allowed to transmit. All M streams must be demodulated without error for the packet to be correctly received. Also it needs to be noted that average power of all the M streams will be same since they are coming from the same transmitter. When the MMSE receiver is analyzed, the capture probability, P_{cap}^{MMSE-k} , is given by $P_{cap}^{MMSE-k} = \left(P_{cap}\right)^M$, where P_{cap} is capture probability of one stream, and it is obtained from the table described earlier. Here we have assumed that all the streams are independent of each other (V-BLAST). In case of a SIC receiver, the capture probability, P_{cap}^{SIC-k} , is given by

$$P_{cap}^{SIC-k} = \prod_{i=1}^{M} P_{cap}^{i} \qquad (8)$$

where P_{cap}^{i} is the capture probability of the i^{th} stream given that other $i-1$ streams are correctly demodulated.

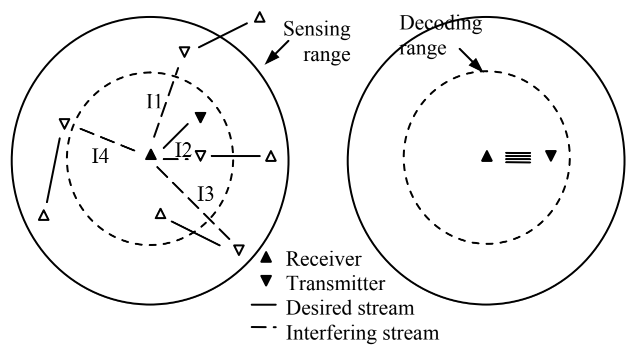

Figure 2: Illustration of the two MAC protocols

Performance of the receivers in the network is analyzed using flow throughput. Here a flow is defined as the collection of all the links between desired transmitter and receiver. The flow throughput is defined as the throughput of the bottleneck link. We assume that each stream is transmitted at a rate of 1 Mbps, and a packet is 8ms long. Therefore, under S-MAC each packet contains 1000 Bytes, and under CSMA/CA(k), each packet contains M×1000 Bytes. $M = 4$ is used for the results in this paper. We analyze network throughput as a function of network load. Network load is defined as number of active transmitter-receiver pairs.

3. Simulation and Results

The received signal power is computed according to a two-slope partitioned model [15] where links with distances less than d_b have a free space path loss exponent, $n_1 = 2$. d_b is the longest distance between the transmitter and receiver for which first Fresnel Zone does not touch the flat ground. For additional distance beyond d_b, the path loss exponent is $n_2 = 3.5$. For these simulations, we assume that the transmitter and receiver heights are 1.5 m and the center frequency is 2.4 GHz; therefore, d_b=200m.

In our CPPS simulations, the transmit power is 0.4mW per stream, hence the total power in the sensing range is 1.6mW for both the protocols. Under CPPN, each node transmits 0.4mW; therefore under S-MAC, each stream is transmitted with 0.4mW and under CSMA/CA(*k*), each stream is transmitted with 0.1mW. The noise spectral density is assumed to be -103.98dBm.

Under CPPN, the nodes have a decoding range of 101m and a sensing range of 202m for CSMA/CA(*k*). This corresponds to $P_D = 0.04$ at 101m and $P_D = 0.14$ at 202m. The other combinations of the MAC protocol and power schemes have the decoding and sensing ranges equal to 200m and 300m respectively. There are 100 nodes in the network. P_D is chosen based on the distance of the transmitters from the receiver in the decoding range and is mapped to the closest distance in the grid.

Fig. 3 shows throughput versus load for the CPPS power scheme. We observe that both the MAC protocols have almost the same performance as for the ideal receiver. Also, SIC performs significantly better than LMMSE (around 35%). S-MAC and CSMA/CA(*k*) perform similarly because the total data transmission power within the sensing range for the two protocols is the same. SIC performs about as well as the ideal receiver because we have set the decoding ranges for SIC and LMMSE to be the same, even though SIC performs much better than LMMSE, especially with perfect channel estimation and cancellation. This means that most SIC links will have $P_e << 10^{-5}$.

Fig. 4 shows throughput versus load for the CPPN power scheme. We observe that CSMA/CA(*k*) performs better than the S-MAC for both SIC and LMMSE. As in the CPPS case, SIC performs as well as ideal receiver. Also, SIC throughput is better than LMMSE by about 35% for CSMA/CA(*k*) and by about 47% for S-MAC.

We observe that S-MAC under the two power schemes give the same throughput because the notion of stream and node is same for this protocol.

It is interesting to observe that SIC with CSMA/CA(*k*) performs so much better than SIC with S-MAC (by about 100%). This seems to contradict the physical layer knowledge that SIC benefits from the stream power disparity [17] that naturally results from S-MAC. We had also observed that a stream control strategy similar to S-MAC outperformed CSMA/CA(*k*) for a toy network, using a throughput definition based on Shannon capacity [18]. We attribute the contrary results in this paper to the higher spatial reuse in a large multi-hop network that results from the smaller decoding and sensing ranges of CSMA/CA(*k*) under CPPN, because shortening range is known to increase throughput in ad hoc networks [21].

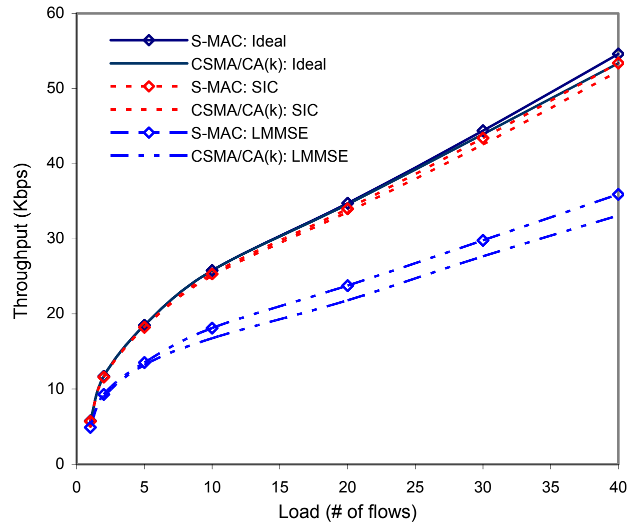

Figure 3: Throughput of an ad-hoc wireless network employing ideal, SIC, and LMMSE receivers with CPPS power scheme

We anticipate that under CPPS we might see more of a gap between SIC-S-MAC and SIC-CSMA/CA(*k*)` if we shorten the decoding range, so that P_D has more of an impact on throughput. Another important consideration not investigated in this paper that may give SIC-S-MAC a stronger advantage over SIC-CSMA/CA(*k*) is that S-MAC can be combined with selection diversity at the transmitter for a small cost in signaling overhead [19], or with space-time block coding transmit diversity [20] for an additional complexity cost in the receiver signal processing.

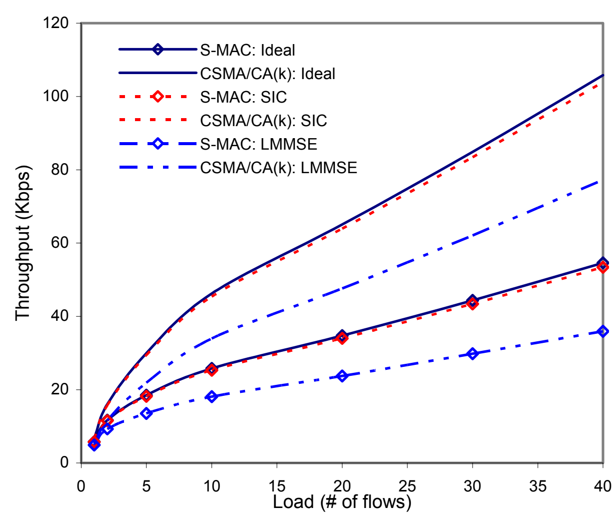

Figure 4: Throughput of an ad-hoc wireless network employing ideal, SIC, and LMMSE receivers with CPPN power scheme

The CPPN results in Fig. 4 show an interesting cost-performance trade-off, in that the less complex (and hence less expensive) Linear MMSE receiver with CSMA/CA(*k*) can perform better than the more complex non-linear SIC with S-MAC when employed in a large multi-hop ad hoc network.

S-MAC can be generalized to allow more than one stream per user [2,18]. The advantage of using such a scheme is that some links in the network can allocate more resources if they need a higher data rate. The disadvantage is that a small overhead is required to inform the transmitter how many streams it can transmit. We anticipate that the throughput in this case would lie in between the S-MAC (single stream case) and CSMA/CA(*k*) (all streams case).

4. SUMMARY AND CONCLUSION

We have proposed a collision model for the study of a multi-hop MIMO ad-hoc wireless network. Network throughputs are compared for LMMSE, SIC, and ideal receivers, for CSMA/CA(*k*) and S-MAC protocols, and for CPPS and CPPN power schemes. We conclude that SIC and CSMA/CA(k) is a high-throughput combination under a CPPN constraint, because of the spatial reuse in large multi-hop networks. We show that power scheme has significant impact on the network performance. With CPPS the two MAC protocols have very similar behavior but the receiver processing has an impact. With CPPN, both MAC protocol and receiver processing play important roles.

Although SIC and CSMA/CA(*k*) is a powerful combination it will have a higher number of hops on average to reach the destination because of shorter decoding range, which could negatively impact time-sensitive applications such as video streaming. Also, this work considers a flat fading channel, and therefore does not capture the diversity advantage of wideband channels under frequency selective fading. We intend to address these issues in future work.

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