Multiple Repeater Placement for Assisting Long-Range LOS MIMO Links

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***Abstract* — A wireless MIMO link is considered in a LOS configuration with multiple SISO wireless repeaters assisting. A theoretical analysis is presented in which repeater positions are considered and sufficient conditions proposed and evaluated for maximizing the MIMO multiplexing gain. The assisting repeaters would be useful when space constraints, for example, on a building top, would prohibit the wide element spacings needed by a MIMO terminal to reach full capacity over a long range LOS link. Sufficient conditions include, among other things, mutual orthogonality of steering vectors from one MIMO terminal pointing toward the opposite terminal and toward each of the repeaters. Once $n-1$ such repeaters have been properly placed, where n represents the smaller dimension of the channel matrix, the matrix becomes full rank and the multiplexing gain is maximized.**

***Index Terms* — MIMO, wireless repeater, capacity enhancement**

I. INTRODUCTION

The use of repeaters has been proposed as a means of creating a virtual multiple-input multiple-output (MIMO) array and improving multiplexing and/or diversity through cooperative communications [1-10]. The impact of the position of such repeaters has been studied to some extent, but usually related to diversity gain [7-10]. We present here a theoretical analysis of the impact of positioning of multiple repeaters on achievable multiplexing gain in line of sight (LOS) environments by considering the steering vectors pointing toward the various repeaters. The analysis shows that full multiplexing may be achieved by ensuring mutually orthogonal steering vectors from the perspective of both arrays pointing toward the opposite array and toward each repeater. This analysis may potentially aid in network deployment and relaying strategies, configuring MIMO-enabled point-to-point microwave links, and potentially enabling MIMO for LOS cellular channels. The results may also be useful in understanding the impact of scattering environments on available MIMO capacity.

The use of repeaters in LOS environments is motivated by the fact that achieving maximum capacity in LOS channels requires relatively large spacing between the elements of the transmit (TX) and receive (RX) arrays [11-13]. For example, for a 4x4 MIMO link operating at 2.4GHz at a range of 900m, the required antenna spacing is approximately 5.3m, implying the need for a distance of approximately 15.9m between the two outer elements of the array. Considering practical restrictions, for example, on roofs of buildings, the required array size may not be possible. Using repeaters to assist the link allows for much smaller TX/RX arrays, with spacings of $\lambda/2$ at a minimum (or 6.25cm at 2.4GHz).

We denote the concept of improving multiplexing gain using repeaters as "repeater-assisted capacity enhancement (RACE)". The authors previously demonstrated the utility of the RACE concept in enabling multiplexing for a 2x2 point-to-point link [14]. This paper offers a more complete analysis of repeater positioning for a general $n_R x n_T$ link. A conceptual system diagram of the RACE concept applied to a 4x4 MIMO system with 3 single-input single-output (SISO) wireless repeaters is shown in Figure 1. In the figure, triangles represent MIMO antenna elements, stars represent repeaters, and dashed lines represent channel coupling. These lines have been drawn to illustrate the LOS channel response and a single repeater path response. The other two repeaters also contribute to the channel response, but these channel couplings have not been illustrated.

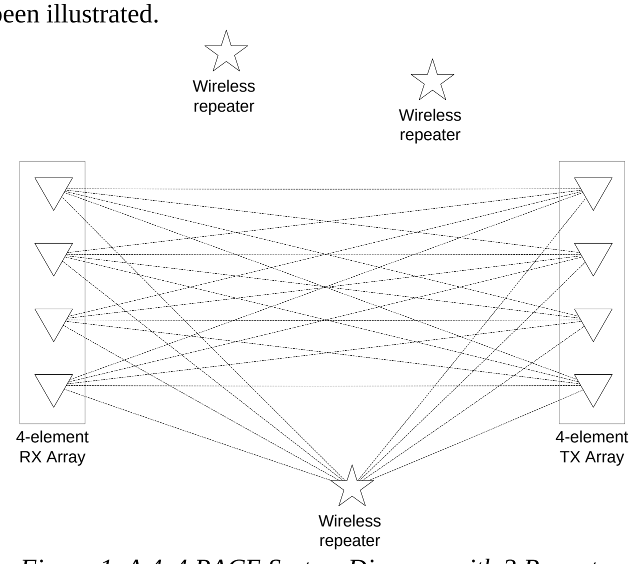

Figure 1. A 4x4 RACE System Diagram with 3 Repeaters

The repeaters in this paper are assumed to be SISO with two directional antennas, one pointed at the TX array and the other at the RX array. These antennas are likely separated in space, possibly with obstructions or absorbent material between them to mitigate TX/RX feedback. This feedback mitigation enables the repeaters to operate in a full-duplex, amplify-and-forward (FDAF) mode while maintaining stability. This FDAF configuration is assumed for all repeaters in this analysis.

In Section II, we present five sufficient conditions for achieving full MIMO multiplexing with wireless repeaters;

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Section III describes the channel model; and Section IV presents a proof of the sufficiency of the conditions in Section II. In Section V, we offer simulation results from a 4x4 MIMO system to illustrate the RACE concept; we explore tradeoffs associated with suboptimally-placed repeaters in Section VI; and discuss conclusions in Section VII.

II. Sufficient Conditions

We will show in Section IV that the following are sufficient conditions for achieving maximum multiplexing gain in an $n_R x n_T$ MIMO link in a LOS environment using $n-1$ SISO full-duplex amplify-and-forward repeaters. Here, we define $n = min\{n_R, n_T\}$.

1. Each of the n signals (one direct path and $n-1$ repeated signals) have equal power as seen by the RX array.
2. The n TX steering vectors, pointing in the direction of the center of the receive (RX) array and in the direction of the $n-1$ repeaters, are mutually orthogonal.
3. The n RX steering vectors, pointing in the direction of the center of the transmit (TX) array and in the direction of the $n-1$ repeaters, are mutually orthogonal.
4. The TX and RX arrays must be in the far-field of one another.
5. The $n-1$ repeaters must be in the far-field of both TX and RX arrays.

By a simple extension of the results in [15], we note that if $n_R \leq n_T$, multiplexing gain is considered to be maximized if $HH^H = mI_n$ for some positive, real-valued m. Likewise, if $n_R \geq n_T$, the multiplexing gain is maximized if $H^H H = mI_n$.

III. Channel Model

Consider an $n_R x n_T$ MIMO link in a LOS configuration where the RX node has n_R antennas and the TX node has n_T antennas. Applying Condition #4 from Section II, we assume that each node is in the far field of the other array (the range is large relative to the array size), so we may approximate the LOS channel response of the direct path without multipath as an outer product of two steering vectors:

$$H_0 \approx A_0 e^{j\xi_0} \vec{v}_{R,0} \vec{v}_{T,0}^T, \tag{1}$$

where $\vec{v}_{R,0}$ is the RX steering vector in the direction of the center of the TX array, $\vec{v}_{T,0}$ is the TX steering vector in the direction of the center of the RX array, A_0 is a positive, real-valued variable representing the path loss of the direct path, and ξ_0 is a phase term to account for fractional wavelength distances. This phase term is necessary to construct actual channel gains because the steering vector accounts for direction only and is blind to range, but the phase term disappears in the analysis, so we do not compute its value. The steering vectors are given by [16]

$$\vec{v}_{R,0} = \begin{pmatrix} e^{-j\vec{k}_{R,0}^T \vec{p}_{R,1}} \\ \vdots \\ e^{-j\vec{k}_{R,0}^T \vec{p}_{R,n_R}} \end{pmatrix}, \vec{v}_{T,0} = \begin{pmatrix} e^{-j\vec{k}_{T,0}^T \vec{p}_{T,1}} \\ \vdots \\ e^{-j\vec{k}_{T,0}^T \vec{p}_{T,n_T}} \end{pmatrix}, \tag{2}$$

where $\vec{k}_{R,0}$ is the wavenumber vector pointing from the center of the RX array to the center of the TX array, $\vec{k}_{T,0}$ is the wavenumber vector pointing from the center of the TX array to the center of the RX array, $\vec{p}_{R,r}$ is the vector from the center of the RX array to the r$^{\text{th}}$ RX antenna, $\vec{p}_{T,t}$ is the vector from the center of the TX array to the t$^{\text{th}}$ TX antenna, and $(\ \ )^T$ denotes the transpose operator. From [16], we note that the norm of any wave number vector $\vec{k}$ is given by $\|\vec{k}\| = \frac{2\pi}{\lambda}$ where λ is the wavelength of the carrier.

Assuming the $n-1$ repeaters are in the far-field of both TX and RX arrays (Condition #5 from Section II), we may similarly approximate the channel response of the path through the q$^{\text{th}}$ repeater as:

$$H_q \approx A_q e^{j\xi_q} \vec{v}_{R,q} \vec{v}_{T,q}^T, \tag{3}$$

where $\vec{v}_{R,q}$ is the RX steering vector in the direction of the q$^{\text{th}}$ repeater, $\vec{v}_{T,q}$ is the TX steering vector in the direction of the q$^{\text{th}}$ repeater, A_q is a positive, real-valued variable representing the path loss of the path through the q$^{\text{th}}$ repeater (including loss from two paths and the gain of the repeater), and ξ_q is a phase term to account for fractional wavelength distances.

The composite LOS channel response (H) may then be approximated as the sum of the channel responses of the various paths

$$H = \sum_{q=0}^{n-1} H_q \approx \sum_{q=0}^{n-1} A_q e^{j\xi_q} \vec{v}_{R,q} \vec{v}_{T,q}^T. \tag{4}$$

IV. Proof

Having applied the 4$^{\text{th}}$ and 5$^{\text{th}}$ Conditions from Section II to construct an approximate channel model, we now apply the first 3 Conditions, which we rewrite as follows based on the parameter definitions above:

1. $A_i = A_j$ for all $i, j = 1,2, \dots, n$
2. $\vec{v}_{R,i}^H \vec{v}_{R,j} = n_R \delta_{ij}$
3. $\vec{v}_{T,i}^H \vec{v}_{T,j} = n_T \delta_{ij}$

where δ_{ij} is the Kronecker delta. Assuming far-field placement of all elements, we now show that $HH^H = mI_n$ for the case $n_R \leq n_T$. The case $n_R \geq n_T$ follows a very similar analysis, which will not be presented here because of its redundancy.

$$HH^H = \left(\left(\sum_{q=0}^{n-1} A_q e^{j\xi_q} \vec{v}_{R,q} \vec{v}_{T,q}^T \right) \left(\sum_{p=0}^{n-1} A_p e^{-j\xi_p} \vec{v}_{T,p}^* \vec{v}_{R,p}^H \right) \right) \tag{5}$$

$$= \sum_{q=0}^{n-1} n_T A_q^2 \vec{v}_{R,q} \vec{v}_{R,q}^H \tag{6}$$

$$= \begin{pmatrix} \frac{\vec{v}_{R,0}}{\sqrt{n_R}} & \frac{\vec{v}_{R,1}}{\sqrt{n_R}} & \cdots & \frac{\vec{v}_{R,n-1}}{\sqrt{n_R}} \end{pmatrix} \tag{7}$$

$$\begin{pmatrix} n_R n_T A_0^2 & 0 & \cdots & 0 \\ 0 & n_R n_T A_1^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & n_R n_T A_{n-1}^2 \end{pmatrix} \begin{pmatrix} \frac{\vec{v}_{R,0}^H}{\sqrt{n_R}} \\ \frac{\vec{v}_{R,1}^H}{\sqrt{n_R}} \\ \vdots \\ \frac{\vec{v}_{R,n-1}^H}{\sqrt{n_R}} \end{pmatrix}$$

$$= n_R n_T A_0^2 \begin{pmatrix} \frac{\vec{v}_{R,0}}{\sqrt{n_R}} & \frac{\vec{v}_{R,1}}{\sqrt{n_R}} & \cdots & \frac{\vec{v}_{R,n-1}}{\sqrt{n_R}} \end{pmatrix} \begin{pmatrix} \frac{\vec{v}_{R,0}^H}{\sqrt{n_R}} \\ \frac{\vec{v}_{R,1}^H}{\sqrt{n_R}} \\ \vdots \\ \frac{\vec{v}_{R,n-1}^H}{\sqrt{n_R}} \end{pmatrix} \quad (8)$$

$$= n_R n_T A_0^2 I_{n_R}, \quad (9)$$

where the last step is accomplished by noting that $\begin{pmatrix} \frac{\vec{v}_{R,0}}{\sqrt{n_R}} & \frac{\vec{v}_{R,1}}{\sqrt{n_R}} & \cdots & \frac{\vec{v}_{R,n-1}}{\sqrt{n_R}} \end{pmatrix}$ is a unitary matrix (from Condition #2).

Having demonstrated that $HH^H = mI_{n_R}$ where $m = n_R n_T A_0^2$ for the $n_R \leq n_T$ case and noting, without proof, that for the $n_R \geq n_T$ case, $H^H H = mI_{n_T}$ where $m = n_R n_T A_0^2$, we conclude that the conditions stated in Section II are sufficient to ensure full multiplexing gain for an $n_R x n_T$ MIMO system in a LOS environment using $n-1$ SISO wireless repeaters.

V. A 4X4 EXAMPLE

Having demonstrated certain conditions as sufficient for achieving full MIMO multiplexing using $n-1$ wireless repeaters, we present an example of achieving full multiplexing (approximately four times the baseline capacity, that is, the capacity without repeater assistance) with a 4x4 MIMO system assisted by 3 strategically placed SISO wireless repeaters. For the simulations, we use a more exact channel model than described in Section III and an amplify-and-forward repeater model with results for both noiseless and noisy repeaters incorporating the effects of noise amplification and coloring. More details on these models and other simulation results may be found in [14]. For locations that meet the far-field conditions described in Section II, the channel model of Section III is sufficient and simulation results based on that model are almost identical to those presented here.

For the results presented here, we assume a range of 900m, a center frequency of 2.4GHz, and an inter-element spacing of 0.75m (for a total TX/RX array length of 2.25m). Figure 2a shows the capacity of the system with a single noiseless repeater as a function of the position of that repeater. The colorbar to the right of the figure identifies the values of the various colors. For the capacity plots, these values have units of bits per second per Hertz (bps/Hz). The null-space metric is unitless and is useful as a relative metric to identify good repeater positions. We also assume the TX, RX, and repeaters are in the x-y plane and define positions in two dimensions. The MIMO link capacity is given by [17]

$$C = log_2\left(det\left(I_{n_R} + \frac{\rho}{n_T}\widetilde{H}\widetilde{H}^H\right)\right), \quad (10)$$

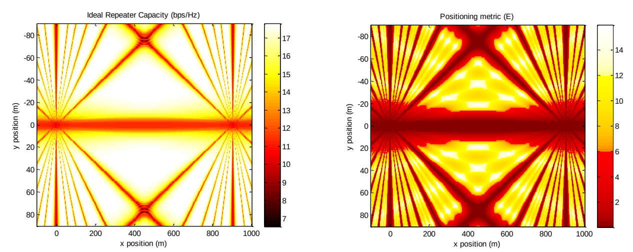
where ρ is the RX SNR and $\widetilde{H} = \frac{4\pi R}{\lambda}H$ is the normalized channel matrix.

Figure 2b shows the results of a positioning metric (E):

$$E = \prod_{i=1}^{Q}\prod_{j=0}^{i-1}\left(n_R - \left|\langle \vec{v}_{R,i}, \vec{v}_{R,j}\rangle\right|\right)\left(n_T - \left|\langle \vec{v}_{T,i}, \vec{v}_{T,j}\rangle\right|\right), \quad (11)$$

where Q is the number of wireless repeaters. Notice that the metric is maximized when the TX and RX steering vectors are mutually orthogonal, which satisfies Conditions 2 and 3 of Section II.

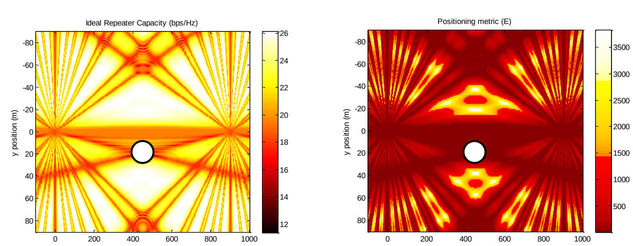
This metric gives additional insight and offers a practical methodology for ideal placement of the repeaters. The method involves converting the MIMO arrays into conventional beamformers and steering the main beams toward the opposite node. An optimal repeater location may be identified by measuring the power received from both of these beamformers and selecting a location where the power is minimized. This is equivalent to placing it to ensure orthogonality of steering vectors. When placing successive repeaters, power should be measured when the beams are steered toward the opposite nodes and when they are steered toward incumbent repeaters.



a) Capacity (C) b) Positioning metric (E)

Figure 2. Capacity and positioning metric as a function of the 1st repeater's position for a 4x4 system

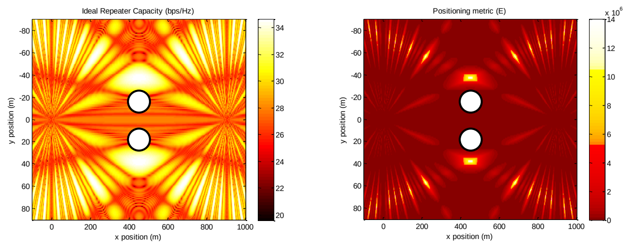
Notice the existence of nine areas of optimal placement in Figure 2b that are blurred into one large high-capacity area in Figure 2a. By placing the first repeater at one of these nine locations (450m,19m), we can plot the capacity and positioning metric as a function of a 2nd repeater's position. Figure 3 shows the results with a white circle representing the first repeater's location.



a) Capacity (C) b) Positioning metric (E)

Figure 3. Capacity and positioning metric as a function of a 2nd repeater's position for a 4x4 system

Placing a second repeater at (450m,-19m), we now consider the capacity and positioning metric as a function of the 3rd repeater's position, shown in Figure 4. Again, white circles represent the positions of the first two repeaters. We observe that the options for repeater placement diminish with each successive placement.



a) Capacity (C) b) Positioning metric (E)

Figure 4. Capacity and positioning metric as a function of the 3rd repeater's position for a 4x4 system

Figure 5 shows a plot of the $x = 450m$ cross-section of 1) the ideal capacity of Figure 4a ("Ideal Repeater") and 2) the capacity using a noisy repeater accounting for repeater-induced noise amplification/coloring ("Noisy Repeater"). Also shown in the figure are 3) the optimal capacity using 3 repeaters ("Optimal Repeater"), 4) the optimal capacity that could be achieved without repeaters when the TX/RX arrays employ proper spacing ("Optimal 4x4"), 5) the baseline capacity, or the capacity obtained by the LOS configuration without repeater assistance ("Baseline"), and 6) the worst case capacity when the channel matrix is a matrix of ones ("Worst Case"). The "Optimal Repeater" curve is higher than the "Optimal 4x4" curve simply because of the increase in SNR due to the presence of the repeaters. The baseline SNR is 20dB, but with three repeaters, the actual SNR is closer to 26dB.

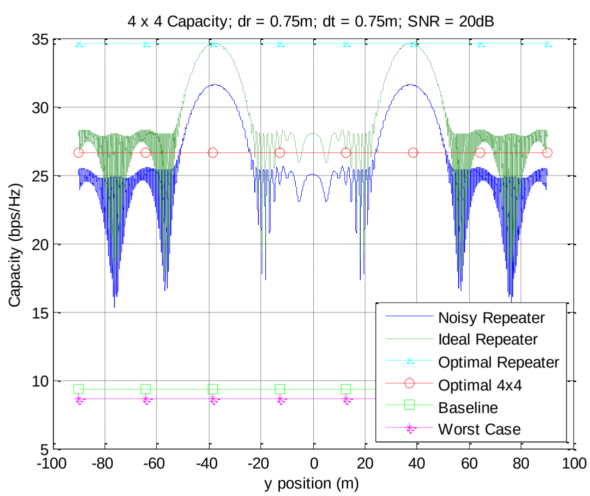


Figure 5. Capacity cross-section (x=450m) as a function of 3rd repeater position for a 4x4 system

Placing the third repeater at (450m,38m) will yield an optimal capacity (34.6 bps/Hz) for the ideal repeater model and for a realistic model, about 31.6 bps/Hz. This 9% degradation in capacity is the result of noise amplification and coloring introduced by the three repeaters. Table 1 shows the capacities associated with the various curves when three repeaters are placed at (450m,19m), (450m,-19m), and (450m,38m).

Table 1. Link Capacities for various 4x4 assumptions

Noisy Repeater	31.6 bps/Hz
Ideal Repeater	34.6 bps/Hz
Optimal Repeater	34.6 bps/Hz
Optimal 4x4	26.6 bps/Hz
Baseline	9.3 bps/Hz
Worst Case	8.6 bps/Hz

VI. SUBOPTIMAL REPEATER PLACEMENT

Noting the existence of local maxima and minima of the position metric within the high-capacity regions of Figure 2b, consider the results of C and E when repeaters are placed in the local minima of the position metric. Figure 6 shows results when the first repeater is placed at (450m,27.5m). Again, the white circles represent the position of this first repeater.

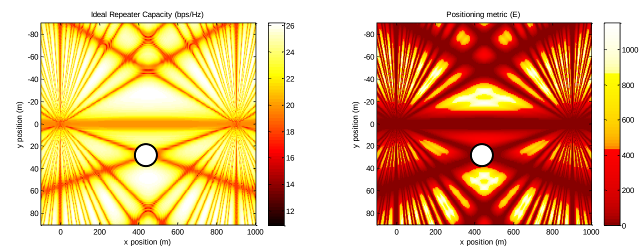


Figure 6. C and E as a function of the 2nd repeater's position with a suboptimally-placed initial repeater

Comparing these results with those shown in Figure 3, we note that the available capacity is still quite high for a second repeater's placement, but the pattern does change. The optimal placements are compressed downward since we have moved the repeater farther down the plot. Now consider the results of C and E as a function of a third repeater's position when we place a second repeater at (450m,-27.5m) as shown in Figure 7.

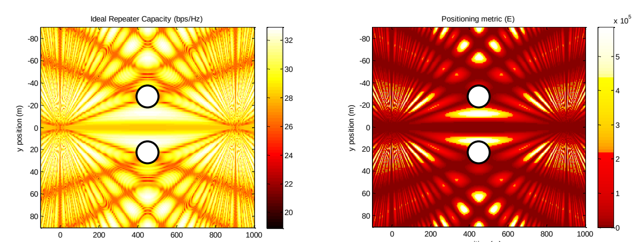


Figure 7. C and E as a function of the 3rd repeater's position with two suboptimally-placed initial repeaters

Comparing these results with Figure 4, note that the suboptimal placement of two repeaters has reduced the available capacity of the link from a maximum of approximately 34.7bps/Hz to 32.9bps/Hz, a 5% degradation. However, the robustness in available positions for the final repeater are actually improved by these initial suboptimal placements. For the optimally placed repeaters (Figure 4), approximately 15.0% of the positions simulated exceed a capacity of 31.2bps/Hz (or 90% of the 34.7bps/Hz maximum). For the suboptimally placed repeaters (Figure 7), 40.7% of the positions exceed a capacity of 29.6bps/Hz (90% of the

32.9bps/Hz maximum) and 15.5% of the positions exceed a capacity of 31.2bps/Hz (90% of the 34.7bps/Hz maximum). In terms of percentage of maximum, there is a significant improvement in repeater position robustness at the expense of a small degradation in capacity. Relative to an absolute capacity, there is a small improvement in position robustness by placing initial repeaters in suboptimal locations.

Figure 8 shows the CCDF curves associated with both optimal and suboptimal initial repeater placements over the entire simulated area. Noting that the shape of the curves will be determined by the arbitrary cutoff of our range of simulated positions, we are not concerned with the values of the probabilities, but rather the comparison between them. If the link can be considered successful with capacities at or below 31bps/Hz, for example, we will likely enjoy better robustness in the placement of our repeaters using suboptimal placements than we would by using optimal placements. The amount of robustness obviously depends on the specific implementation, but the example given here illustrates an interesting possibility in designing higher-order RACE systems for MIMO links.

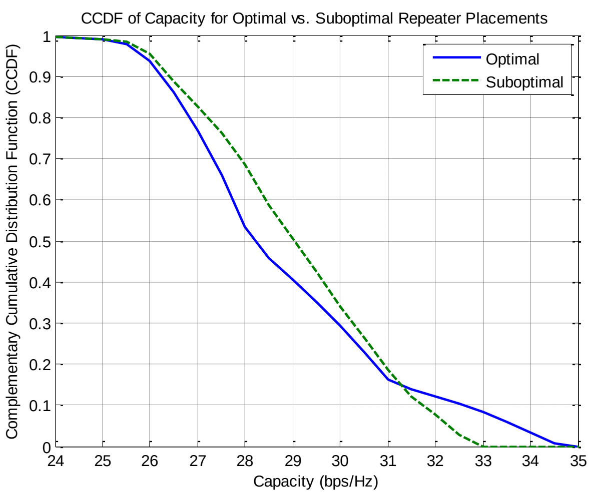


Figure 8. Ideal Capacity CCDFs over simulated positions for optimally- and suboptimally-placed repeaters.

VII. CONCLUSION

We have demonstrated the potential for achieving full MIMO multiplexing gain in a LOS environment using $n-1$ SISO wireless repeaters. In order to achieve this under these assumptions, the opposite array and repeaters must be in the far field of both TX and RX arrays, repeater gains must be calibrated to ensure equal power from all signal paths at the RX, and TX/RX steering vectors toward the opposite array and toward each repeater must be mutually orthogonal. Minor deviations from these conditions will yield very good capacity with small variations in the relative capacity of the spatial subchannels, but the analysis presented here provides the conditions for optimal capacity.

This analysis may be useful in considering cooperative communications relaying strategies and configuring MIMO-enabled links in LOS environments. Considering the orthogonality constraints on steering vectors, it becomes clear that a distribution of repeaters that is widely dispersed in angle relative to both MIMO arrays is desirable. By considering the repeaters to be scatterers, it is interesting to note the relationship between MIMO array size and the required spread in arrival angle of the multipath components. When the array size is small, the required spread is quite large, but as the array size grows, introducing more grating lobes in an equivalent beamforming array, the requirement for orthogonal steering vectors may be met by a much smaller angular spread of scatterers. This is similar to the results of certain studies relating angular spread to MIMO capacity in NLOS environments [17-19].

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