Repeater-Assisted Capacity Enhancement (RACE) for MIMO Links in a Line-of-Sight Environment

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***Abstract* — A single on-frequency, full duplex wireless repeater is considered for enhancing the capacity of a long-distance 2x2 multiple-input multiple-output (MIMO) wireless link with a dominant line-of-sight (LOS) component. Such links might be used for high-speed communication between buildings or towers. For practical reasons, the aperture sizes of the transmit (TX) and receive (RX) arrays may be limited, which tends to make the rank of the long-distance MIMO channel matrix close to unity. The paper shows that the addition of just one repeater can bring the channel to full rank, approximately doubling the capacity of the link. Typical values of repeater isolation are assumed. For small TX and RX array element spacings, there is considerable robustness in optimal repeater location. Even with some multipath, most of the capacity improvement is retained.**

***Index Terms* — LOS, MIMO, wireless communications, capacity, repeater.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technology has been revolutionary in its ability to increase capacity and/or improve the robustness of a wireless communication link. To improve the capacity of the link, MIMO technology usually relies on statistically uncorrelated channel coupling in order to effectively retrieve the multiplexed transmitted data. Channels that experience high correlation between channel gain coefficients are usually thought to have lower capacity limits. Line of sight (LOS) channels have often been included in this category because their channel gains are highly inter-dependent and they often experience degraded capacities. However, "correlation" cannot properly be applied to these channels since they are increasingly deterministic as the Rician K-factor increases, with channel gains based almost solely on the physical configuration of the link. Although low capacities are common in LOS, a substantial body of research concludes that certain configurations can achieve the maximum capacity [1-7] by ensuring the channel matrix is full rank. One result is the derivation of an optimal inter-element antenna spacing [3-5] for a given link's range and frequency. When the MIMO arrays have this optimal spacing, the channel is orthogonalized and the maximum MIMO capacity is achieved. This spacing, however, may be quite large for some applications as the range between transmitter and receiver grows.

We propose the use of wireless repeaters operating as "active reflectors" to improve the richness of the multipath environment, thereby reducing the effective Rician K-factor without blocking the LOS component, and orthogonalizing the channel matrix. This configuration may serve to improve the MIMO capacity for configurations with suboptimal inter-element antenna spacings. We call this concept "repeater-assisted capacity enhancement (RACE)". This paper presents an analysis of a 2x2 MIMO channel's capacity using a single repeater.

Repeaters are typically used in cellular, WiFi, and other wireless applications to extend the range of coverage or to illuminate areas that would otherwise have weak signal reception due to blockage or other fading problems [8-13]. In such configurations, the repeater may 1) mix the signal it receives to another channel or band before relaying it; 2) buffer the signal in time and use a second time slot to relay the signal (half-duplex repeater); or 3) relay the signal on the same frequency at the same time it receives it (full-duplex repeater). This third type of repeater is sometimes called an "on-frequency repeater" and will be considered for this analysis. An important parameter of repeaters is isolation, which is defined as the attenuation of the feedback path in the repeater's amplifier. The first two repeater types may be used to ensure sufficient TX/RX isolation such that the repeater gain necessary for effective operation won't cause the repeater to become unstable. While these types could be considered, the use of extra time and/or spectrum would reduce the effective capacity of the system. With on-frequency repeaters, other means must be used to ensure sufficient isolation. Directional antennas (one for relay input, one for relay output), circulators, and obstructions may be used for this purpose. Some studies have proposed using a repeater that injects a low-power signal into the relayed signal, which can be used to estimate the feedback channel. This estimation can then be used to back off the amplifier gain or attempt to filter out the feedback path to ensure stability [10-11]. Other methods have also been proposed to enhance the isolation by filtering the feedback channel using gain dithering and MEMS reconfigurable parasitics [12-13].

The type of repeater we assume has also been called a "full-duplex amplify-and-forward (AF) relay" in the context of cooperative diversity. Although the earliest information theory research on cooperative diversity was based on full-duplex relaying [14-15], almost all the more recent work assumes half-duplex relays [16-17]. In particular [18-19] address a problem similar to ours: that of using AF relays to assist a rank-deficient MIMO channel, but they also assume half-duplex operation. Half-duplex has been assumed necessary because sufficient isolation for full-duplex is considered too difficult to achieve in typical ad-hoc and sensor networks environments [20].

In this paper, however, we restrict our attention to free-space channels or Rician channels with a high K-factor, such as might be encountered in building-top or tower-mounted long-distance MIMO microwave links. For such applications, use of directional antennas on the repeater (or relay) is

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reasonable and sufficient measured isolations are available [8-9]. We consider a point-to-point LOS MIMO link with a dedicated on-frequency repeater designed solely for enhancing the channel capacity. We determine the optimal placement of the repeater and the sensitivity of the capacity to deviations from that optimal location, under the constraint that the TX and/or RX antenna spacings are significantly smaller than the optimal spacing without the repeater. In this context, low sensitivity to deviations from the optimal placement of the repeater is an advantage, since service providers may have limited choices for positioning the repeater.

II. CHANNEL MODEL

The wireless configuration we propose to analyze is shown in Figure 1. In the figure, the triangles represent antennas, the black squares are the centers of the MIMO arrays, and the star represents the repeater. The inter-element antenna spacings are given by "d_R" and "d_T", the range by "R", and the angles the array normals make with the line connecting the centers of the arrays are given by ϕ_R and ϕ_T. The distances between RX/TX antennas and the repeater are given by d_{Rm} and d_{Tn} where $m \in \{1,2\}$, and $n \in \{1,2\}$. Though not shown in the figure, the distance between the m[th] RX antenna and the n[th] TX antenna is given by d_{mn}. In the present analysis, we restrict our model to two spatial dimensions.

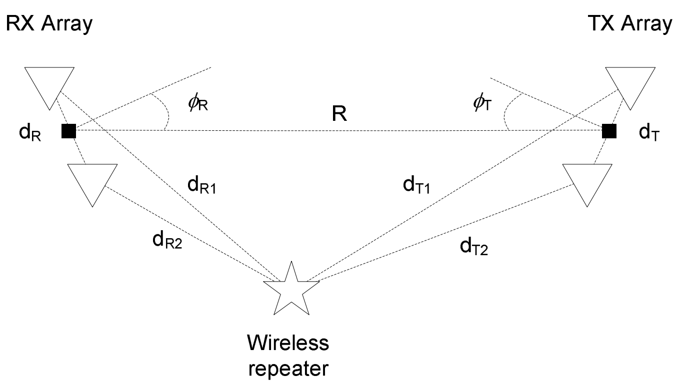

Figure 1. MIMO wireless repeater configuration

Friis Path Loss

With a single repeater assisting the link, the free-space channel matrix (H_{LOS}) may be modeled as the summation of two channel responses:

$$H_{LOS} = H_0 + e^{j\phi_1} H_1. \tag{1}$$

Here, H_0 is the direct path response, H_1 is the repeated path response, and ϕ_1 is a random phase associated with the repeated signal. We demonstrate in Section V that the value of ϕ_1 has little impact on the capacity and may be set to zero. We model the channel responses with the Friis transmission equation. The (m,n)[th] element of H_0 is given by

$$h_{0,mn} = \frac{e^{-jkd_{mn}}}{2kd_{mn}}, \tag{2}$$

depending only on the distance between the m[th] RX element and the n[th] TX element (d_{mn}) and the wave number ($k = \frac{2\pi}{\lambda}$). Similarly, the (m,n)[th] element of H_1 is given by

$$h_{1,mn} = \frac{e^{-jkd_{Rm}}}{2kd_{Rm}} \sqrt{G_1} \frac{e^{-jkd_{Tn}}}{2kd_{Tn}}, \tag{3}$$

where d_{Tn} and d_{Rm} are the distances between the repeater and TX/RX elements, and G_1 is the repeater's power gain.

Multipath Fading Model

Although we are primarily concerned with the effect of the repeater in a pure LOS environment, we also wish to analyze the effect of multipath fading to determine how our analysis degrades with increasing multipath power. To account for NLOS fading, we introduce a Rician K-factor defined as $K = \frac{E\{\|H_{LOS}\|_F^2\}}{E\{\|H_{NLOS}\|_F^2\}}$ or the ratio of the power in the LOS signal to the power in the NLOS multipath reflections arriving at the receiver. The operator $\|H\|_F$ represents the Frobenius norm of H. We model the NLOS channel response (H_{NLOS}) as a complex Gaussian random variable with zero mean and unit variance. Thus, the channel matrix is modeled as

$$H = \sqrt{\frac{K}{K+1}} H_{LOS} + \frac{\|H_{LOS}\|_F}{\sqrt{n_R n_T}} \sqrt{\frac{1}{K+1}} H_{NLOS}. \tag{4}$$

III. REPEATER MODEL

For this analysis, we assume a repeater with sufficient isolation and gain to overcome the path loss from any location while maintaining stability. We assume for now that the repeater must amplify the signal such that the signal power through the repeated path is equal to the signal power through the direct path as seen by the receiver. We will explore the reason for this assumption in Sections IV and V, but here, we compute the required gain (G_1) based on the equal power assumption.

Repeater Gain

If the range is much greater than the size of either array ($R \gg d_R$ and $R \gg d_T$), then the direct path distances are all approximately equal to R ($d_{mn} \approx R \; \forall \, m, n$) [4]. If we also assume that the repeater is sufficiently far away from both arrays ($d_{Rm} \gg d_R$ and $d_{Tn} \gg d_T$), then we can assume $d_{R1} \approx d_{R2}$ and $d_{T1} \approx d_{T2}$. Let $\overline{H}_0 = \frac{H_0}{A_0}$ and $\overline{H}_1 = \frac{H_1}{A_1}$ where $A_0 = \frac{1}{2kR}$ and $A_1 = \frac{1}{2kd_{R1}} \sqrt{G_1} \frac{1}{2kd_{T1}}$. With these definitions, the above assumptions, and using (2) and (3), we note that the absolute value of each of the elements of $\overline{H}_0$ and $\overline{H}_1$ is approximately equal to one. In order to ensure the direct and repeated paths have equal power, we ensure that the elements of H_0 and H_1 have equal magnitude by setting $A_0 = A_1$ and solving for G_1:

$$G_1 = \left(\frac{2kd_{R1} d_{T1}}{R} \right)^2. \tag{5}$$

In Section V, we will analyze this gain for the proposed scenario and determine whether our isolation assumption is valid by comparing with experimental isolation values. In the future, it would be prudent to incorporate a more realistic model for isolation, but the present analysis demonstrates the feasibility of the RACE concept.

Repeater Noise

The repeater will add some amount of receiver noise to the signal as it relays it to the MIMO receiver. This will introduce noise coloring and amplification [18], which we explore here. Let $H_1 = H_{1R}\sqrt{G_1}H_{1T}$, where H_{1R} is the repeater-to-RX column vector response and H_{1T} is the TX-to-repeater row vector response. Then the received signal may be written as $y = H_0x + H_{1R}\sqrt{G_1}(H_{1T}x + n_1) + n_0 = Hx + n_c$ where n_0 is the noise introduced by the MIMO receiver, n_1 is the noise introduced by the repeater, and $n_c = H_{1R}\sqrt{G_1}n_1 + n_0$ is the colored noise seen by the MIMO receiver. In order to use this signal model for our analysis, we follow the pattern in [18] and whiten the noise as follows.

The autocorrelation matrix of the noise power at the RX is given by $R_{n_c} = H_{1R}G_1H_{1R}^H P_{n,1} + P_{n,0}I_2$ where $P_{n,0} = k_BT_0B$ is the noise power introduced by the MIMO receiver and $P_{n,1} = k_BT_1B$ is the noise power introduced by the repeater. Here, k_B is Boltzmanns' constant, T_0 and T_1 are the system noise temperatures of the RX and repeater respectively, and B is the signal bandwidth, which we assume to be 20MHz. The noise figure of each system is assumed to be 3dB.

$\tilde{R}_{n_c} = \frac{R_{n_c}}{P_{n,0}} = H_{1R}G_1H_{1R}^H\frac{P_{n,1}}{P_{n,0}} + I_2$ is then decomposed as $\tilde{R}_{n_c} = U\Lambda U^H$ where U contains the eigenvectors of $\tilde{R}_{n_c}$ and Λ is a diagonal matrix of the eigenvalues. We then construct a whitening filter $W = \Lambda^{-1/2}U^H$. The resultant noise power after whitening is equal to $P_{n,0}$. The channel matrix after whitening is given by $\tilde{H} = WH$. An ideal (noiseless) repeater is modeled by using H instead of $\tilde{H}$. Some results from the ideal model will be shown in Section V for comparison.

IV. Repeater Parameter Analysis

Two metrics will be considered in analyzing the impact of the repeater as a function of position. The first metric is Shannon's capacity limit [21] given for MIMO systems [22]:

$$C = log_2\left(det\left(I_{n_R} + \frac{P_T}{n_TP_{n,0}}\tilde{H}\tilde{H}^H\right)\right) \tag{6}$$

where P_T is the transmit power and $P_{n,0}$ is the noise power introduced by the receiver. The transmit power is fixed to ensure a predetermined average SNR of ρ for the configuration without the repeater by $P_T = \rho P_{n,0}P_L$. Here, $P_L = (2kR)^2$ represents the path loss for the direct path (TX to RX) modeled by the Friis transmission equation.

The second metric is derived from the capacity by assuming a sufficiently large SNR [23] and is given by

$$D = det(\bar{H}\bar{H}^H) \leq 4 \tag{7}$$

Recall from Section III that the magnitude of each element of $\bar{H}$ is equal to one. Because of this, the determinant metric (D) is bounded by $D \leq 4$ [23] by the result of Hadamard's maximum determinant problem. It can also be shown that D is proportional to the square of the product of the singular values of H, so when any one singular value is close to zero, the metric (7) is close to zero. This would indicate at least one degenerate sub-channel (i.e. less than full multiplexing gain capacity). Therefore, when the capacity improves from a boost in SNR or the use of more antennas on one side or the other, the determinant should remain largely unaffected assuming the channel rank is limited by the environment to less than full rank. This may be a useful metric in terms of maximizing the multiplexing gain.

Optimal Inter-Element Spacing

Assuming TX and RX have the same inter-element spacing, the optimal spacing for a 2x2 MIMO system is given by [3-5]

$$d_{opt} = \sqrt{\frac{\lambda R}{2cos\phi_T cos\phi_R}} \tag{8}$$

This optimal spacing ensures that the MIMO capacity will be maximized for a LOS environment. This is equivalent to making the normalized channel matrix $\bar{H}$ subject to $\bar{H}\bar{H}^H = n_TI_{n_R}$ [23]. As an example, at 2.4GHz and a range of $900m$, $d_{opt} = 7.5m$. The RACE concept will be useful when the array element spacings are constrained to be much smaller than d_{opt}.

Free Space Positioning Metric

The following analysis aids in understanding the optimal position of a repeater serving a LOS MIMO link. This leads directly to a positioning metric that may be used to design and deploy such a system.

Let the direct path channel response H_0 (1) be a rank 1 matrix with singular value decomposition (SVD)

$$H_0 = \begin{pmatrix} U_1 & U_2 \end{pmatrix}\begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}\begin{pmatrix} V_1^H \\ V_2^H \end{pmatrix} \approx \sigma_1U_1V_1^H \tag{9}$$

where σ_1 represents the non-zero singular value, σ_2 is the zero or near-zero singular value, $span(U_1)$ is the left signal subspace, $span(U_2)$ is the left null space, $span(V_1)$ is the right signal subspace, and $span(V_2)$ is the right null space. This is a valid model for a long-range LOS channel with no multipath and small antenna spacing.

In this decomposition, let $a_0 \in span(U_1) = \mu U_1$ represent the RX steering vector in the direction of the TX array and $b_0 \in span(V_1) = \nu V_1$ the TX steering vector in the direction of the RX array. Here, μ and ν are complex scalars of magnitude such that $\|a_0\|^2 = \|b_0\|^2 = 2$.

If the TX array is in the far-field of the RX array and vice versa, which is satisfied in the long-range LOS MIMO scenarios we are targeting, we can approximate the channel response of the direct path as $H_0 \approx A_0a_0b_0^H$.

We previously defined $H_1 = H_{1R}\sqrt{G_1}H_{1T}$ and note that we may write this as an outer product of two vectors representing scaled versions of the TX/RX steering vectors in the direction of the repeater $H_1 = A_1a_1b_1^H$. A_1 was defined previously as $A_1 = \frac{1}{2kd_{R1}}\sqrt{G_1}\frac{1}{2kd_{T1}}$. Note that $\|a_1\|^2 = \|b_1\|^2 = 2$.

If a_1 is an RX steering vector such that a conventional, untapered beamformer steering its main beam toward the center of the TX array will place a null in the direction of a_1, then we may write $U_1^Ha_1 = a_0^Ha_1 = 0$.

Similarly, if b_1 is a TX steering vector such that a beamformer steering its main beam toward the RX array will place a null in the direction of b_1, then we may write $V_1^H b_1 = b_0^H b_1 = 0$.

Theorem 1: A repeater placed in the overlapping TX/RX nulls of two hypothetical conventional, untapered beamformers, assuming a repeater gain such that the repeated and direct paths have equal power, maximizes the MIMO multiplexing gain.

Proof: Mathematically, we need to show that $HH^H = mI$ for some non-zero real-valued m, which is equivalent to maximizing the multiplexing gain [23]. H may be written as $H = H_0 + H_1 = A_0 a_0 b_0^H + A_1 a_1 b_1^H$. We may write the null placement constraint as $a_0^H a_1 = b_0^H b_1 = 0$ and the equal power constraint as $A_0 = A_1$. Therefore,

$$HH^H = (A_0 a_0 b_0^H + A_1 a_1 b_1^H)(A_0 a_0 b_0^H + A_1 a_1 b_1^H)^H$$
$$= A_0^2 a_0 b_0^H b_0 a_0^H + A_0 A_1 a_0 b_0^H b_1 a_1^H + A_0 A_1 a_1 b_1^H b_0 a_0^H + A_1^2 a_1 b_1^H b_1 a_1^H$$
$$= A_0^2 \|b_0\|^2 a_0 a_0^H + A_1^2 \|b_1\|^2 a_1 a_1^H$$
$$= 2A_0^2 a_0 a_0^H + 2A_1^2 a_1 a_1^H$$
$$= \begin{pmatrix} a_0 & a_1 \end{pmatrix} \begin{pmatrix} 2A_0^2 & 0 \\ 0 & 2A_1^2 \end{pmatrix} \begin{pmatrix} a_0^H \\ a_1^H \end{pmatrix}$$
$$= \begin{pmatrix} \frac{a_0}{\sqrt{2}} & \frac{a_1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4A_0^2 & 0 \\ 0 & 4A_1^2 \end{pmatrix} \begin{pmatrix} \frac{a_0}{\sqrt{2}} & \frac{a_1}{\sqrt{2}} \end{pmatrix}^H$$
$$= 4A_0^2 \begin{pmatrix} \frac{a_0}{\sqrt{2}} & \frac{a_1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{a_0}{\sqrt{2}} & \frac{a_1}{\sqrt{2}} \end{pmatrix}^H$$
$$= 4A_0^2 I \text{ by noting that } \begin{pmatrix} \frac{a_0}{\sqrt{2}} & \frac{a_1}{\sqrt{2}} \end{pmatrix} \text{ has unit norm.}$$

Based on this analysis, we determine that in order to maximize the MIMO multiplexing gain, the repeater should be located in the nulls of two imaginary beamformers based on the MIMO TX/RX array locations when their beams are steered toward one another. This offers a metric for determining optimal repeater locations and also suggests a method for designing such a system. Supposing we have flexibility in the placement of our repeater, we can fix the TX and RX antennas, use them as beamformers, and find the repeater position where the power received from either one of these beamformers is minimal when the power coupled between the beamformers is maximized by beam steering. If we don't have such flexibility, we may fix the repeater's position and then move the MIMO antennas while adaptively beamforming toward one another until the power at the repeater is minimized.

We therefore propose the following repeater positioning metric, which, when maximized, enables the system to achieve the maximum MIMO multiplexing gain:

$$y(p_1) = (4 - |y_R(p_1)|^2)(4 - |y_T(p_1)|^2). \quad (10)$$

Here, p_1 is the proposed repeater position and y_R and y_T are the RX and TX beamformer responses in the direction of a proposed repeater position.

Based on the assumptions stated above, the metric will fail to correlate with system capacity as the antenna spacing approaches the optimal (8). However, the repeater becomes less useful as the spacing increases since the baseline capacity gets larger and potential improvement is reduced, so we typically want to consider much smaller than optimal spacings. For the simulations we performed, the metric was very accurate for spacings as large as half the optimal spacing.

This analysis may be extended to achieving a full-rank channel matrix for an NxN MIMO system. This requires introducing (N-1) repeaters subject to some orthogonality constraints, but that analysis will not be presented here.

V. Simulation Results

For our simulations, we use a carrier frequency of 2.4 GHz, so $\lambda = 0.125m$. Let $\phi_R = \phi_T = 0$, so that the array normals lie on the x-axis. Also let $R = 900m\ (2953ft)$ and $d_R = d_T = 0.75m\ (2.46ft) = 6\lambda$. We have chosen a range that might reasonably model a building-to-building wireless connection in a downtown urban environment. The antenna spacing is chosen to be significantly less than the optimal, which for this range (8), is $d_{opt} = 7.5m\ (24.6ft) = 60\lambda$. Although various values of antenna spacing were simulated, we restrict most of the results to the $0.75m$ case for the sake of brevity. The SNR is set to 20dB. These parameters will be kept constant unless otherwise noted. Figure 2 shows the capacity of the resultant LOS channel as a function of the repeater's (x,y) position. For each repeater position, the LOS channel matrix (1) is computed, which is then used to compute the capacity (6). This capacity is then plotted for all (x,y) repeater position pairs. The figure is a 2-dimensional image that utilizes a color-bar to show values by shading. The color-bar ranges from 0 to 15.3bps/Hz. For comparison, the capacity associated with H_0 (the configuration without the repeater) is approximately 7.67bps/Hz, marked in the figure. Notice that the high-capacity regions achieve approximately double the capacity of the baseline. The figure also shows two dashed lines representing a null angle for TX/RX beamformers when they steer their beams in the direction of the opposite array. Because the antenna spacing is larger than the wavelength, there are multiple nulls for each beamformer, one of which is shown in the figure.

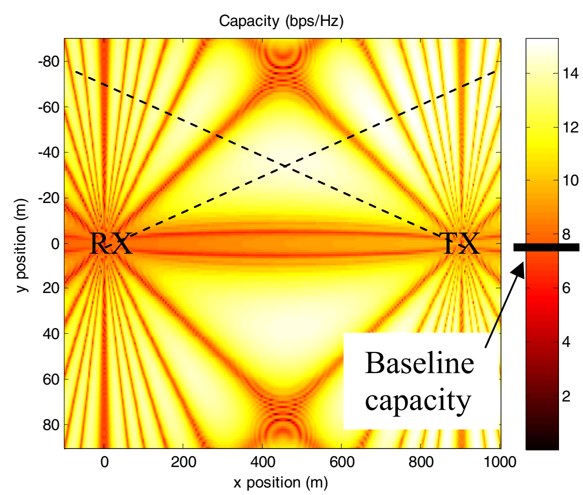

Figure 2. Capacity as a function of repeater position for d = 0.75m

A clear pattern emerges with high capacity regions at irregular intervals corresponding to the nulls of the hypothetical TX/RX beamformers. Notice the point where two of the nulls (represented by dashed lines) overlap corresponds

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to a high capacity region. Notice the slightly higher capacity for positions closer to the TX (on the right). As the repeater moves closer to the RX node, the SNR at the repeater's receiver decreases and the noise amplification increases, reducing the system capacity. Along the mid-point between nodes (x = 450), we find the noise is amplified by a factor of approximately 1.25, so if we choose carefully, this effect should have minimal impact on the capacity. Notice also that the array normals lie on the x-axis leading to symmetry in the figure along the x-axis. When the MIMO arrays are rotated, this symmetry does not typically hold.

To ensure the RX sees equal power from the direct and repeated paths, the repeater (including its antennas) needs to have a gain of as much as 87dB depending on its position. If the repeater antennas are 60^o sector antennas, for example, with gains of 14dBi, the amplifier gain (ignoring line losses) would need to be 87 – 28 = 59 dB. To avoid oscillation, the amplifier gain should be at least 15 dB less than the isolation between the two repeater antennas [8-9]. This implies that the isolation should be at least 59 + 15 = 74 dB. Fortunately, measured isolations with sector antennas usually exceed this [8-9]; for example, 8dBi gain repeater antennas at 2.15 GHz produced > 74 dB of isolation for horizontal separations of 3 meters or more or with vertical separations (on a pole) of 5 meters [9].

In Figure 3, a cross section of the 2-D capacity is plotted by looking at the line x = 450m, halfway between the two nodes. This curve is compared to an ideal, noiseless repeater model, an optimal, noiseless repeater case where $\overline{H}\overline{H}^H = 4I$ (the power is doubled by using a repeater), the optimal 2x2 case where $\overline{H}\overline{H}^H = 2I$, a baseline case where we consider $H = H_0$ (free space LOS without the repeater), and the worst case where H is a matrix of 1's. Notice the last two lines almost overlap one another because of the small antenna spacing.

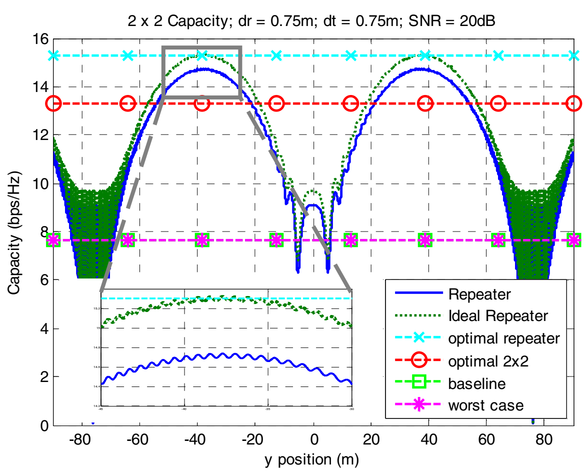

Figure 3. Capacity cross-section for d = 0.75m (realistic and ideal repeater models)

The capacity is maximized at approximately (450,+/-37.5). Notice how the capacity varies slightly in the zoomed portion of the figure. The period of this variation appears to be on the order of 0.75m, which is also the antenna spacing for the TX and RX arrays. As we vary the random phase of the repeated signal (ϕ_1 in (1)), the variation shifts, but the general shape remains the same. For areas of high capacity, the relative phasing due to positioning has very little impact on the capacity. However, in lower capacity areas, such as near +/-75m, the variation is much more severe. Since we are only interested in placing the repeater in a position that will yield high capacity, we can safely ignore the effect of the relative phasing due to position.

Positioning With Multipath

Consider the effect of multipath on the system capacity as a function of repeater position. We assume $K = 10dB$ (4) and compute both the 1% outage capacity and the average capacity. A plot is not shown here because of space limitations.

The outage capacity largely retains the same shape as the pure LOS case indicating the same positioning concepts will still hold for a Rician fading channel. The average capacity suffers only a slight degradation compared to the pure LOS case, but the lower capacity areas experience larger variations with an overall increase in average capacity in those areas. We conclude that the optimal positions do not change significantly for typical Rician channels, but the presence of multipath adds "noise" to the result and tends to flatten the 2-D capacity surface. Note that as K approaches zero, the repeater becomes useless since the channel becomes full rank. Near the optimal position, average and outage capacities are approximately 14.35 and 12.7 bps/Hz respectively. It seems reasonable then that a systems engineer could adequately design the MIMO configuration based solely on the free-space model without regard to multipath.

Repeater Power Analysis

Let $d_R = d_T = 1.5m$ (larger distance) and set the repeater location to (450,19.1), one of the higher capacity positions for that spacing. The spacing was increased to highlight the difference between the "baseline" and "worst case" curves in Figure 4, which shows the MIMO capacity as a function of the repeated-to-direct path power ratio where H is normalized by $\sqrt{A_0^2 + A_1^2}$. This is similar to the normalization done in (7) and is intended to allow for a comparison of MIMO multiplexing gain while holding the RX SNR constant.

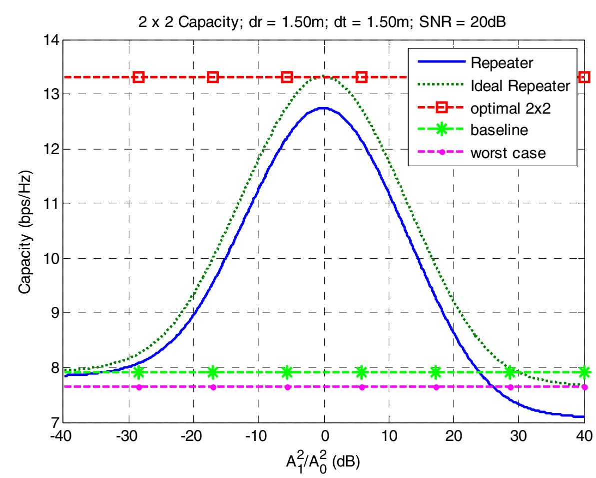

Figure 4. Capacity as a function of repeated-to-direct path power ratio for d=1.5m

As noted in Theorem 1, the optimal capacity is achieved when the receiver sees equal power from both sources. If we consider the repeater signal as a multipath reflection, this would be equivalent to reducing the K-factor to a value of one. As the ideal repeater's power decreases from this optimal point, we approach the baseline capacity. As the repeater power becomes the dominant signal, we approach the worst case capacity with the ideal model. A power offset of 10dB yields 88% of the maximum capacity for this configuration.

Delay Spread Considerations

Because the optimal repeater positions usually require a longer repeated signal path than the direct signal path, it is important to consider the delay spread induced by the use of the repeater at various locations and ensure that a typical system can function with such a delay spread. For the repeater positions previously simulated, the delay spread varies from 0 to approximately 796ns.

The utility of the RACE concept will depend on the how much delay spread the standard can accommodate. For example, the following table shows some delay spread tolerances for various bandwidths and cyclic prefix lengths of a WiMax system based on the 802.16 standard [24] assuming an OFDM symbol length of 256 samples.

Table 1. Delay spread tolerances for various bandwidths and cyclic prefix lengths

Cyclic Prefix Length	3.5 MHz	5 MHz	20 MHz
1/16 (16 bits)	4μs	2.8μs	694ns
1/8 (32 bits)	8μs	5.6μs	1.4μs
1/4 (64 bits)	12μs	11.1μs	2.8μs

For the configuration considered, the delay spread introduced by the repeater should not present a problem for most of the available settings in a typical WiMax system. As the range increases, antenna spacing decreases, or repeater placement is limited, delay spread could become an issue and should be considered as a design parameter.

VI. CONCLUSION

It has been observed in this analysis that LOS MIMO multiplexing gain may be improved by the use of a single wireless on-frequency repeater, if the repeater is positioned properly. For smaller antenna spacings, the areas of optimal position are larger, offering robustness in placement, but also farther away from the MIMO arrays, requiring more repeater gain and isolation for the same range. The RACE concept may be useful in long-range LOS links such as building-top or tower-mounted microwave links. Cellular backhaul and high-speed wireless bridges are two potential candidates.

We note that the repeater's power is a concern and must be considered in a system deployment. In a static configuration, the repeater gain should be carefully calibrated to ensure the direct and repeated signals have nearly equal power at the receiver. In a mobile configuration or in other scenarios where the channel's impact on power coupling varies significantly, this limitation could potentially be overcome by 1) feedback from the receiver to enable the repeater to adapt its gain or 2) multiple repeaters with sufficient gain to overwhelm the TX/RX LOS signal. The first option requires a smarter repeater than we have discussed while the second option might be considered wasteful of resources.

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