Synchronization for Cascaded Distributed MIMO Communications

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***Abstract*—In this paper, we propose a general synchronization scheme and a common model for both frequency and time offsets in the cascaded distributed MIMO communications. By cascaded, we mean that cluster A transmits to cluster B, and cluster B transmits to cluster C, and so on, yet there is never any coordination between nodes within a cluster; for example, there is no signal redistributed within a cluster. The proposed scheme includes two parts: distributed pre-synchronization of the MIMO transmitters and the weighted combination at a receiver of estimates associated with different relays. The analysis shows that, with the power-weighted combination, although both the absolute variance and covariance of pre-synch errors keep increasing from hop to hop, the expectation of sample variance will converge to a relatively small value after several hops. The convergence property of the expected sample variance makes the multi-hop transmission possible. Both simulation and experimental results verify our analysis.**

I. INTRODUCTION

A distributed multiple-input-multiple-output (MIMO) link comprises a transmit cluster and a receive cluster such that each cluster is composed of multiple radios that are not connected by wire. Distributed MIMO is a flexible way to obtain high spectral efficiency through spatial multiplexing, or high reliability through cooperative diversity [1, 2]. In a transmit cluster, or virtual transmit array, unlike a real transmit array, each antenna transmits a signal with a distinct time offset (TO) and a distinct carrier frequency offset (CFO) because of noise and typical inaccuracies of oscillators. If these offsets are small enough, a receiver will not distinguish them from multipath delays and Doppler shifts, and decoding will proceed normally. In cascaded (or consecutive) distributed MIMO links, by which we mean that one cluster transmits to the next cluster, and then that cluster immediately transmits to the next, and so on [3, 4]. We assume that each node in a cluster synchronizes based on all the preambles received from the previous cluster. In other words, there is no cluster leader selected to redistribute a synchronization preamble within the cluster, as in [5].

In this paper, we propose a general scheme for both timing and frequency synchronization for cascaded distributed MIMO communications. We assume orthogonal channels for the transmission of preambles for synchronization. Our scheme includes two parts: pre-synchronization at transmitters and the combination at a receiver of estimates associated with different relays. Analysis simulations and experimental results show the convergence of the sample variance of the synchronization parameter as a function of hop count.

This paper is organized as follows. In Section II, we introduce the related work. Section III gives the generic system model. A general synchronization scheme and a common model for both timing error and CFO are given in Section IV. The proposed scheme is analyzed in Section V. Simulation results for CFO synchronization and experimental results for timing synchronization are presented in Section VI. The paper is concluded in Section VII.

II. RELATED WORK

The existing frequency synchronization schemes for cooperative communications can be classified into three types: post-synchronization, self-cancellation, and pre-synchronization. In the post-synchronization schemes [6-8], all the CFOs are estimated and compensated at the receiver, so the complexity is very high. An effective ICIs self-cancellation scheme is proposed by [9] for OFDM systems based on the symmetric conjugate mapping, but its bandwidth efficiency is low, and it may not be applicable when the number of relays is larger than two. A pre-synchronization method is proposed by [10-11], in which the CFOs between source and relays are pre-compensated at relays. However, this scheme is just designed for the two-relay, two-hop scenario, in which the source is required to reach the destination, so it is not suitable for multi-hop scenarios. Another pre-synchronization scheme, frequency advance, is proposed in [12] for uplink in the mobile communication systems. In this scheme, the CFOs between users and the base station are estimated during the downlink communication; and these CFOs are then pre-compensated for the uplink transmissions. But this scheme is not suitable for cascaded transmissions, because, except the first hop, there is no a "base station" that can distribute synchronization signal in other hops. In contrast, our scheme is based on all the preambles received from the previous cluster, so multihop is supported.

As to timing synchronization, [13] shows that jitters as large as 10% of the bit duration do not have much effect on the BER performance of the system. Reference [14] proposed to use a globally synchronized clock by network time protocol or a GPS for cooperative transmission. However, this scheme does not suitable to the cascaded distributed MIMO communications because of the lack of central control.

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III. GENERIC SYSTEM MODEL

A. Network Structure

We consider a cascaded multi-hop transmission topology (Fig. 1), in which one source transmits a message, and the message is relayed along a sequence of clusters of nodes. We assume the number of effective relays (R) is the same for each cluster. All the channel gains between clusters are independent Rayleigh fading (with parameter σ_h). For convenience, we assume the average power gain of the channel $2\ \sigma_h^2=1$. In Fig.1, $h_{kr}^{(j)}$ represents the channel gain between the k-th receiver in the current cluster (j-th cluster) and the r-th transmitter in the previous cluster. We assume the receive SNR for the first hop is SNR_1, and the average receive SNR from each transmitter in the previous cluster is SNR_r (this implies that nodes are co-located within each cluster and that clusters are equi-spaced). When R is large, it's reasonable to have $SNR_1 >> SNR_r$.

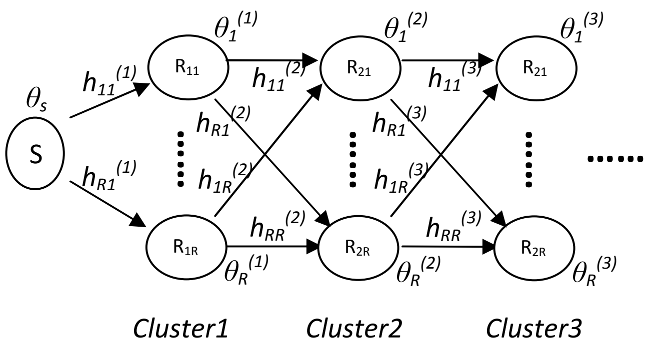

Fig. 1. Multi-hop Distributed MIMO Transmission

B. Preamble-Based Estimation and Orthogonal Assumption

In this paper, we apply preamble-based estimation for both TO and CFO. We assume that within a cluster each relay transmits a preamble in a distinct orthogonal (i.e. diversity) channel. In Fig. 1, $\theta_k^{(j)}$ represents the transmit parameter (transmit time $T_k^{(j)}$ or carrier frequency $f_k^{(j)}$) of the k-th node in the j-th cluster. The determination of $\theta_k^{(j)}$ is based on the observations that the node makes in each diversity channel, of each of the respective preambles transmitted from the previous cluster. In other words, by correlating to a specific preamble, a receiver is able to estimate the synchronization parameter associated with the signal that is transmitted by exactly one transmitter in the previous cluster.

C. Definition of two types of Error and three types of Variance

We set the transmit parameter of the source to be $\theta_s = (T_s$ or $f_s)$. We ignore all deterministic components of the time synchronize parameter, which for our assumed topology means that every time offset can be interpreted as the error in the estimation of the original source transmit time. The equivalent assumption for frequency is no Doppler shifts. We define $e_k^{(j)} = \theta_k^{(j)} - \theta_s$ as the absolute error of the k-th node in the j-th cluster, and define $w_{kr}^{(j)}$ as the estimation error between the k-th receiver and the r-th transmitter in the j-th hop caused by the noise in the kr-th diversity channel. Because $e_k^{(j)}$ is zero mean (see Section IV), we can define: (1) $\sigma_j^2 = E[(e_k^{(j)})^2]$ as the absolute variance for the j-th cluster; (2) $\gamma_j = E[(e_k^{(j)} e_l^{(j)})]$ as the covariance of errors within the j-th cluster; (3) $\sigma_{Sj}^2 = E\left[\frac{1}{R-1}\sum_{r=1}^{R}\left(e_r^{(j)} - \overline{e}^{(j)}\right)^2\right]$ as the expectation of the sample variance, where $\overline{e}^{(j)} = \frac{1}{R}\sum_{r=1}^{R} e_r^{(j)}$ is the sample mean.

In this paper, we assume all the errors (TO or CFO) for every link are within the acquisition range.

IV. GENERAL SYNCHRONIZATION SCHEME

The general synchronization scheme includes estimation of error from each transmitter, combination of estimates and pre-synchronization based on the combination result.

A. Begin with the First hop

In the first hop, the source transmits to all the nodes in the first cluster. Each node (e.g. k-th) estimates the packet arriving time or CFO (ε_k, normalized by the sampling frequency) based on the preamble [15 - 18], and perform pre-synchronization based on the estimate. For CFO, each node adjusts its transmit carrier frequency through multiplying the n-th complex sample by $e^{-j2\pi n\varepsilon k}$. For TO, each node decides its transmission time by adding a fixed period of time to the estimated reception time of the packet [18]. In this paper, we assume the nodes in one cluster are co-located, so the differences of the propagation times are negligible.

For the nodes in the first cluster, the estimation error is only caused by the noise, so $e_k^1 = w_{k1}^1$. For reasonable SNR, the variance $\sigma_1^2 = c/\mathrm{SNR}_1$ [15], where c is a constant related to the structure of the preamble. Actually, as we will see in Section V, σ_1^2 is just an initial value, which does not influence the convergence value of σ_{Sj}^2.

B. Continue to the Second hop

Each node in the second cluster will have one estimate for each transmit nodes in the first cluster. Therefore, at the k-th node in the second cluster, the estimate error relative to the source from the r-th transmitter should be

$$\hat{e}_{kr}^{(2)} = e_r^{(1)} + w_{kr}^{(2)} \qquad (1)$$

$w_{kr}^{(2)}$ is the "local" estimate error, which depends on the channel gain $h_{kr}^{(j)}$ and noise in the kr-th diversity channel. For reasonable receive SNR, this error can be expressed as [15] $w_{kr}^{(2)} \approx \sqrt{c/SNR_r}\, x_{kr}^{(2)} \Big/ \left|h_{kr}^{(2)}\right|$, where $x_{kr}^{(2)}$ is a standard normal distributed random variable. The variance of $w_{kr}^{(2)}$ is c/SNR_{kr}, where $SNR_{kr} = SNR_r |h_{kr}^{(2)}|^2$. At each receiver, R estimates as (1) are got. Because of the linearity of (1), to minimize the variance of $e_k^{(2)}$, we can apply the best linear unbiased estimator (BLUE) to combination the R estimates as [19]

$$e_k^{(2)} = \sum_{r=1}^{R} \alpha_{kr}^{(2)} \hat{e}_{kr}^{(2)} = \sum_{r=1}^{R} \alpha_{kr}^{(2)} e_r^{(1)} + \sum_{r=1}^{R} \alpha_{kr}^{(2)} w_{kr}^{(2)} \quad , \qquad (2)$$

where $\alpha_{kr}{}^{(2)}$ are the components of $\underline{\alpha}_k{}^{(2)} = \frac{C_k{}^{-1}\underline{1}}{\underline{1}^T C_k{}^{-1}\underline{1}}$, and the final variance is $\mathrm{var}(e_k{}^{(2)}) = \frac{1}{\underline{1}^T C_k{}^{-1}\underline{1}}$. C_k is the covariance matrix of vector $[\hat{e}_{k1}{}^{(2)}\ \hat{e}_{k2}{}^{(2)} \ldots \hat{e}_{kR}{}^{(2)}]^T$ and $\underline{1}=[1,1, \ldots, 1]^T$ is a vector of R ones. Because $e_r{}^{(1)}$ (r = 1, 2, …, R) are uncorrelated and zero mean, and $w_{kr}^{(2)}$ is independent of $e_r^{(1)}$ and also zero mean, from (2) we can have $E[e_k^{(2)}] = 0$ and

$$\begin{aligned} C_k{}^{(2)} &= diag(\mathrm{var}(\hat{e}_{k1}{}^{(2)}), \mathrm{var}(\hat{e}_{k2}{}^{(2)}), ..., \mathrm{var}(\hat{e}_{kR}{}^{(2)})) \\ &= c \times diag(\frac{1}{SNR_1} + \frac{1}{SNR_{k1}}, \frac{1}{SNR_1} + \frac{1}{SNR_{k2}}, ..., \frac{1}{SNR_1} + \frac{1}{SNR_{kR}}) \end{aligned} \quad .(3)$$

Because $SNR_1 >> SNR_r$, we can have an approximation

$$\underline{\alpha}_k{}^{(2)} \approx \left[\left|h_{k1}{}^{(2)}\right|^2, \left|h_{k2}{}^{(2)}\right|^2, ..., \left|h_{kR}{}^{(2)}\right|^2 \right] \Big/ \sum_{r=1}^{R} \left|h_{kr}{}^{(2)}\right|^2 \quad (4)$$

With (4), the results of BLUE (2) are used to adjust the transmit parameters of each node in the second cluster. Since (4) depends on channel gains, it seems that channel estimation is necessary. However, we can show for both TO and CFO that estimation of a single parameter after an equal-gain combiner automatically produces the coefficients of (4) (because of the orthogonality of the preambles) with the need for channel estimation.

C. For other hops

Similar to the second hop, in the j-th cluster (j > 2), the estimation errors at the k-th node can be expressed as $\hat{e}_{kr}{}^{(j)} = e_r{}^{(j-1)} + w_{kr}{}^{(j)}$, where $e_r{}^{(j-1)}$ is the error of previous cluster and $w_{kr}{}^{(j)} \approx \sqrt{c/SNR_r}\, x_{kr}{}^{(j)} \Big/ \left|h_{kr}{}^{(j)}\right|$ is the local error. Applying the same kind of weighted combination, we can have the relationship between the error of current cluster (j) and that of the previous cluster (j-1) as

$$e_k{}^{(j)} = \sum_{r=1}^{R} a_{kr}{}^{(j)} e_r{}^{(j-1)} + \sum_{r=1}^{R} a_{kr}{}^{(j)} w_{kr}{}^{(j)} \quad (5)$$

in which the combination coefficients are still $a_{kr}{}^{(j)} = \left|h_{kr}{}^{(j)}\right|^2 \Big/ \sum_{r=1}^{R} \left|h_{kr}{}^{(j)}\right|^2$. Because $e_r{}^{(j-1)}$ and $w_{kr}{}^{(j)}$ are both zero mean and uncorrelated, so $E[e_k^{(j)}] = 0$.

Need to point out that, because $e_r^{(j)}$ (r = 1, 2, …, R) are correlated for j > 2 (see Section V.B), the covariance matrix $C_k^{(j)}$ is not a diagonal matrix, so $a_{kr}{}^{(j)} = \left|h_{kr}{}^{(j)}\right|^2 \Big/ \sum_{r=1}^{R} \left|h_{kr}{}^{(j)}\right|^2$ will not produce a BLUE. However, we think this weighted combination based on power of each branch is reasonable, simple, and also give good results (see Section IV), so we still use it at each relay in every cluster.

V. Analysis of the Synchronization Scheme

In this section, we derive the iterative expressions for the absolute variance σ_j^2, covariance γ_j and sample variance σ_{Sj}^2. Based on these iterative forms, we get the linear property of σ_j^2 and γ_j, and the convergence value of σ_{Sj}^2.

A. Absolute Variance σ_j^2

Substituting from (5), because $E[e_k^{(j)}] = 0$, the variance of the estimation error at the k-th relay in the j-th cluster (j > 1) is

$$\begin{aligned} E\left[\left(e_k{}^{(j)}\right)^2\right] &= E\left[\left(\sum_{r=1}^{R} a_{kr}{}^{(j)} e_r{}^{(j-1)}\right)^2\right] + E\left[\left(\sum_{r=1}^{R} a_{kr}{}^{(j)} w_{kr}{}^{(j)}\right)^2\right] \\ &\quad + 2E\left[\left(\sum_{p=1}^{R} a_{kp}{}^{(j)} e_p{}^{(j-1)}\right)\left(\sum_{q=1}^{R} a_{kq}{}^{(j)} w_{kq}{}^{(j)}\right)\right] \end{aligned} \quad (6)$$

For the first term of (6), because $a_{kr}{}^{(j)}$ and $e_r{}^{(j-1)}$ are uncorrelated, we have

$$E\left[\left(\sum_{r=1}^{R} a_{kr}{}^{(j)} e_r{}^{(j-1)}\right)^2\right] = \frac{2}{R+1} E\left[\left(e_r{}^{(j-1)}\right)^2\right] + \frac{R-1}{R+1} E\left[e_p{}^{(j-1)} e_q{}^{(j-1)}\right] \quad (7)$$

The derivation for (7) is given in the Appendix I.

As shown in Appendix II, the second term of (6) is

$$E\left[\left(\sum_{r=1}^{R} a_{kr}{}^{(j)} w_{kr}{}^{(j)}\right)^2\right] = \frac{c}{SNR_r} \frac{1}{R-1} = \frac{\sigma_w{}^2}{R-1}. \quad (8)$$

Finally, because $w_{kq}{}^{(j)}$ (q = 1,2, …, R) are i.i.d. zero mean random variables, it's easy to see that the third term in (6) is zero.

Combining these results, we have

$$E\left[\left(e_k{}^{(j)}\right)^2\right] = \frac{2}{R+1} E\left[\left(e_r{}^{(j-1)}\right)^2\right] + \frac{R-1}{R+1} E\left[e_p{}^{(j-1)} e_q{}^{(j-1)}\right] + \frac{\sigma_w{}^2}{R-1}$$

or
$$\sigma_j{}^2 = \frac{2}{R+1}\sigma_{j-1}{}^2 + \frac{R-1}{R+1}\gamma_{j-1} + \frac{\sigma_w{}^2}{R-1} \quad (10)$$

B. Covariance γ_j

According to (5), because $E[e_k^{(j)}] = 0$, the covariance between the estimation errors at relays in the j-th cluster (j > 1) is

$$\begin{aligned} &E\left[e_k{}^{(j)} e_l{}^{(j)}\right] \\ &= E\left[\sum_{p=1}^{R} a_{kp}{}^{(j)} e_p{}^{(j-1)} \cdot \sum_{q=1}^{R} a_{lq}{}^{(j)} e_q{}^{(j-1)}\right] + E\left[\sum_{r=1}^{R} a_{kp}{}^{(j)} w_{kp}{}^{(j)} \cdot \sum_{r=1}^{R} a_{lq}{}^{(j)} w_{lq}{}^{(j)}\right] \\ &\quad + 2E\left[\sum_{p=1}^{R} a_{kp}{}^{(j)} e_p{}^{(j-1)} \cdot \sum_{r=1}^{R} a_{lq}{}^{(j)} w_{lq}{}^{(j)}\right] \end{aligned} \quad (11)$$

Because $w_{kq}^{(j)}$ and $w_{lq}^{(j)}$ ($p, q = 1,2, \ldots, R$) are i.i.d. zero mean random variables, the second and third term of (11) equal 0. Then we have

$$\begin{aligned}E\left[e_k^{(j)}e_l^{(j)}\right] &= E\left[\sum_{p=1}^{R}a_{kp}^{(j)}e_p^{(j-1)}\cdot\sum_{q=1}^{R}a_{lq}^{(j)}e_q^{(j-1)}\right]\\ &= \sum_{p=1}^{R}\sum_{q=1}^{R}E\left[a_{kp}^{(j)}a_{lq}^{(j)}\right]E\left[e_p^{(j-1)}e_q^{(j-1)}\right]\\ &= R\cdot E\left[a_{kp}^{(j)}a_{lq}^{(j)}\right]E\left[\left(e_p^{(j-1)}\right)^2\right]\\ &\quad +R(R-1)\cdot E\left[a_{kp}^{(j)}a_{lq}^{(j)}\right]E\left[e_p^{(j-1)}e_q^{(j-1)}\right]\end{aligned} \tag{12}$$

Because $a_{kp}^{\ j}$ and $a_{kp}^{\ j}$ are uncorrelated and $E\left[a_{kr}^{(j)}\right]=1/R$ [20], we have $E\left[a_{kp}^{(j)}a_{lq}^{(j)}\right] = E\left[a_{kp}^{(j)}\right]E\left[a_{lq}^{(j)}\right]=1/R^2$. So, finally we have $E\left[e_k^{(j)}e_l^{(j)}\right]=\frac{1}{R}E\left[\left(e_p^{(j-1)}\right)^2\right]+\frac{R-1}{R}E\left[e_p^{(j-1)}e_q^{(j-1)}\right]$,

or $$\gamma_j=\frac{\sigma_{j-1}^{\ 2}}{R}+\frac{R-1}{R}\gamma_{j-1}\ . \tag{13}$$

C. Convergence of σ_{Sj}^2

According to [21], we have $\sigma_{Sj}^{\ 2}=\sigma_j^{\ 2}-\gamma_j$. Bring this relationship into (12), we get the iterative form of the sample variance as

$$\sigma_{Sj}^{\ 2}=\frac{2}{R+1}\sigma_{S(j-1)}^{\ 2}+\frac{\sigma_w^{\ 2}}{R-1}+\left(\gamma_{j-1}-\gamma_j\right). \tag{14}$$

From (13), we have

$$\gamma_j-\gamma_{j-1}=\frac{1}{R}\left(\sigma_{j-1}^{\ 2}-\gamma_{j-1}\right)=\frac{1}{R}\sigma_{S(j-1)}^{\ 2}, \tag{15}$$

Bring (15) into (14), we get

$$\sigma_{Sj}^{\ 2}=\left(\frac{2}{R+1}-\frac{1}{R}\right)\sigma_{S(j-1)}^{\ 2}+\frac{\sigma_w^{\ 2}}{R-1}. \tag{16}$$

Then we can get the convergence value of σ_{Sj}^2 as

$$\sigma_S^{\ 2}=\frac{R(R+1)}{(R-1)(R^2+1)}\sigma_w^{\ 2} \tag{17}$$

Note that this value is only related to the local error for one branch （σ_w^2） and the number of relays (R). For large R, the approximation of (19) is

$$\sigma_S^{\ 2}\approx\frac{1}{R}\frac{c}{SNR_r}\ , \tag{18}$$

in which the factor R in the denominator can be seen as the array gain we get by doing the weighted combination.

D. Linear Divergence of σ_j^2 and γ_j

Bring (17) into (10) and (13) we get

$$\begin{aligned}\sigma_j^{\ 2}-\sigma_{j-1}^{\ 2} &= \frac{1}{R-1}\sigma_w^{\ 2}-\frac{R-1}{R+1}\sigma_S^{\ 2}\\ &= \frac{(R+1)}{(R-1)(R^2+1)}\sigma_w^{\ 2}\end{aligned}\ , \tag{19}$$

and

$$\begin{aligned}\gamma_j-\gamma_{j-1} &= \frac{1}{R}\left(\sigma_{j-1}^{\ 2}-\gamma_{j-1}\right)=\frac{1}{R}\sigma_{S(j-1)}^{\ 2}\\ &= \frac{(R+1)}{(R-1)(R^2+1)}\sigma_w^{\ 2}\end{aligned}\ , \tag{20}$$

for $j > j_B$. (19) and (20) tell that the slopes of σ_j^2 and γ_j converge to the same value, which is only related to the local error （σ_w^2） and the number of relays (R). In another word, σ_j^2 and γ_j are linearly divergent for $j > j_B$.

VI. Simulations and Experiments

A. Simulation for CFO synchronization

MATLAB simulations for CFO synchronization are used to check the theoretical results about the model (5) in Section IV. In the simulation, we assume a the time-division scheme between relays for the transmission of preambles. In one time slot, each receiver applies the method based on repeated training blocks [16] to estimate the CFO from the corresponding transmitter. The training block we use is a simple extension of the long training sequence defined in [18]. The constant $c=1/(4\pi^2(M-1)^2L^3)$, where M (=5) is the number of training blocks and L (=64) is the length of one training block. We set SNR_I = 25dB and SNR_r = 10dB. Fig. 2 shows the divergence of σ_j^2 and γ_j, the convergence of σ_{Sj}^2, and the effect of R. From Fig. 2 we see that σ_{Sj}^2 get convergent after the third hop. When we increase R from 4 to 8, σ_{Sj}^2 decreases by more than half according to (17), and the slopes of σ_j^2 and γ_j decrease dramatically according to (19) and (20).

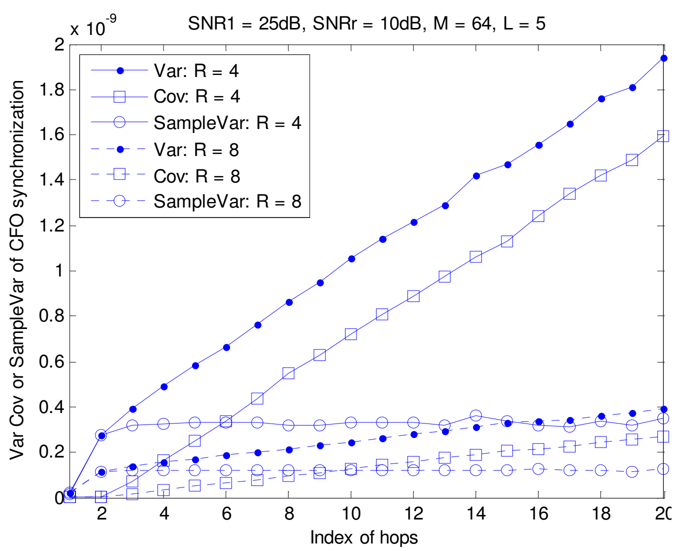

Fig. 2. Simulation for CFO synchronization

B. *Experiments for Transmit Time Synchronization*

For the time synchronization, software-defined radios (SDRs) are used to evaluate the convergence property. Each wireless node in this experiment is composed of a RF-daughterboard (RFX-2400), an Universal Software Radio Peripheral (USRP1) board, a personal computer (PC), and the GNU radio software. Binary frequency shift keying (BFSK) with non-coherent envelope detection was used for the experiments. Orthogonal preamble was achieved by choosing orthogonal center frequencies.

The reception time of each orthogonal preamble is estimated by finding a mean of the output of the preamble correlator [18]. The transmission time is scheduled by combining reception times with combining coefficients (4) and adding a fixed amount of time to avoid a random processing time of SDR systems.

For the sake of a large number of multi-hop cooperative transmissions (CTs), we designed the "ping-pong" experiment that two groups of cooperative nodes transmit the source message back and forth up to 10 CTs. The experiment had conducted on the fifth floor of Centergy Building in Georgia Institute of Technology and was repeated 500 times to get 500 trials of the sample variance. The experimental setup is shown in Fig. 3.

Fig. 4 shows the empirical rms transmit time spread (RTTS) of cooperative nodes. It is noted that RTTS is equal to the square root of the sample variance σ_{Sj}^2. Each curve represents an empirical cumulative density function (CDF) of RTTS of each CT. In this result, we can observe that RTTS tends to be convergent after the third hop. This provides strong evidence that our analysis of convergence property is correct.

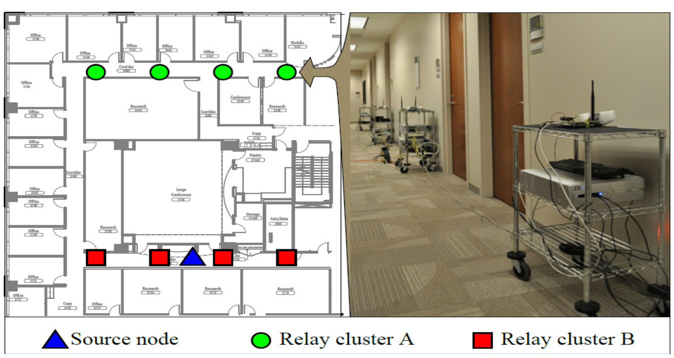

Fig. 3. The experimental setup for "ping-pong" experiment

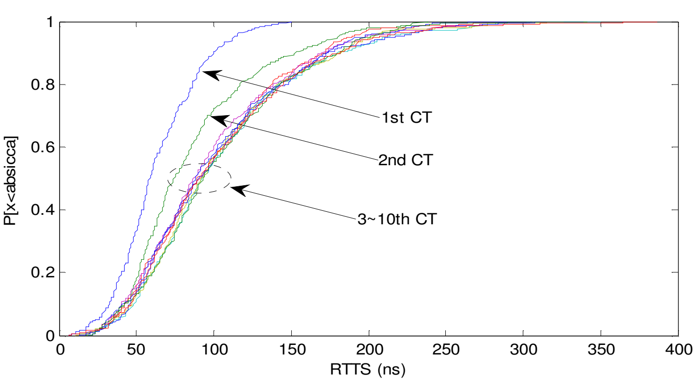

Fig. 4. Measured RTTS of "ping-pong" experiment

VII. CONCLUSIONS

In this paper, we propose a general synchronization scheme and a common model for both timing errors and CFOs in the cascaded distributed MIMO communications. Our scheme includes pre-synchronization at relays and weighted combination of estimates at receivers. The pre-synchronization is based on the result of the weighted combination. Our analysis shows that, using the proposed simple scheme, both of the absolute variance and covariance of the transmit parameters will keep increasing from hop to hop, but the expectation of sample variance will get convergence after several hops. This convergence property makes the multi-hop cooperative transmission feasible. Both simulation and experimental results prove our conclusions.

APPENDIX I

$$E\left[\left(\sum_{r=1}^{R} a_{kr}^{(j)} e_r^{(j-1)}\right)^2\right]$$
$$= E\left[\sum_{r=1}^{R}\left(a_{kr}^{(j)} e_r^{(j-1)}\right)^2 + \sum_{p=1}^{R}\sum_{q=1,q\neq p}^{R}\left(a_{kp}^{(j)} a_{kq}^{(j)} e_p^{(j-1)} e_q^{(j-1)}\right)\right]$$
$$= \sum_{r=1}^{R}\left(E\left[\left(a_{kr}^{(j)}\right)^2\right] E\left[\left(e_r^{(j-1)}\right)^2\right]\right)$$
$$+ \sum_{p=1}^{R}\sum_{q=1,q\neq p}^{R}\left(E\left[a_{kp}^{(j)} a_{kq}^{(j)}\right] E\left[e_p^{(j-1)} e_q^{(j-1)}\right]\right)$$

Because $\left(\sum_{r=1}^{R} a_{kr}^{(j)}\right)^2 = \sum_{r=1}^{R}\left(a_{kr}^{(j)}\right)^2 + \sum_{p=1}^{R}\sum_{q=1,p\neq q}^{R}\left(a_{kp}^{(j)} a_{kq}^{(j)}\right) = 1$, if define $P = E\left(\sum_{r=1}^{R}\left(a_{kr}^{j}\right)^2\right) = \sum_{r=1}^{R} E\left[\left(a_{kr}^{j}\right)^2\right]$, we have

$$E\left(\sum_{p=1}^{R}\sum_{q=1,p\neq q}^{R}\left(a_{kp}^{j} a_{kq}^{j}\right)\right) = \sum_{p=1}^{R}\sum_{q=1,p\neq q}^{R} E\left[a_{kp}^{j} a_{kq}^{j}\right] = 1 - P .$$

Assume $X = \left|h_{kr}^{j}\right|^2, Y = \sum_{r=1}^{R}\left|h_{kr}^{j}\right|^2$, according to [20], we can have

$$E\left[\left(a_{kr}^{j}\right)^2\right] = \frac{E\left[\left(\left|h_{kr}^{j}\right|^2\right)^2\right]}{E\left[\left(\sum_{r=1}^{R}\left|h_{kr}^{j}\right|^2\right)^2\right]} = \frac{E\left[X^2\right]}{E\left[Y^2\right]} .$$

Based on the following properties of related distribution

$$\left|h_{kr}^{(j)}\right| \sim Rayleigh(\sigma_h) \Rightarrow$$
$$\begin{cases} X \sim \exp(\lambda) \Rightarrow \begin{cases} E[X] = 1/\lambda \\ Var(X) = 1/\lambda^2 \end{cases} \Rightarrow E\left[X^2\right] = 2/\lambda^2 \\ Y \sim Gamma(1/\lambda, R) \Rightarrow \begin{cases} E[Y] = R/\lambda \\ Var(X) = R/\lambda^2 \end{cases} \Rightarrow E\left[Y^2\right] = (R+1)R/\lambda^2 \end{cases}$$

in which $\lambda = 1/2\sigma_h^2$, we have

$$P = \sum_{r=1}^{R} E\left[\left(a_{kr}{}^{j}\right)^2\right] = R\frac{E\left[X^2\right]}{E\left[Y^2\right]} = R\frac{2/\lambda^2}{(R+1)R/\lambda^2} = \frac{2}{R+1} .$$

Then we get

$$\begin{aligned} E\left[\left(\sum_{r=1}^{R} a_{kr}{}^{j} e_r{}^{j-1}\right)^2\right] &= PE\left[\left(e_r{}^{j-1}\right)^2\right] + (1-P)E\left[e_p{}^{j-1} e_q{}^{j-1}\right] \\ &= \frac{2}{R+1}E\left[\left(e_r{}^{j-1}\right)^2\right] + \frac{R-1}{R+1}E\left[e_p{}^{j-1} e_q{}^{j-1}\right] \end{aligned} .$$

APPENDIX II

$$\begin{aligned} \left(\sum_{r=1}^{R} a_{kr}{}^{(j)} w_{kr}{}^{(j)}\right)^2 &= \sum_{r=1}^{R}\left(a_{kr}{}^{(j)} w_{kr}{}^{(j)}\right)^2 + \sum_{p=1}^{R}\sum_{q=1,p\neq q}^{R} a_{kp}{}^{(j)} w_{kp}{}^{(j)} a_{kq}{}^{(j)} w_{kq}{}^{(j)} \\ &= \frac{c}{SNR_r}\left(\sum_{r=1}^{R}\frac{\left|h_{kr}{}^{(j)}\right|^2\left(x_{kr}{}^{(j)}\right)^2}{\left(\sum_{r=1}^{R}\left|h_{kr}{}^{(j)}\right|^2\right)^2} + \sum_{p=1}^{R}\sum_{q=1,p\neq q}^{R}\frac{\left|h_{kp}{}^{(j)}\right|}{\sum_{p=1}^{R}\left|h_{kp}{}^{(j)}\right|^2}\frac{\left|h_{kq}{}^{(j)}\right|}{\sum_{q=1}^{R}\left|h_{kq}{}^{(j)}\right|^2} x_{kp}{}^{(j)} x_{kq}{}^{(j)}\right) \end{aligned} . \quad (21)$$

Because $x_{kp}{}^{(j)}$ and $x_{kq}{}^{(j)}$ are i.i.d. random variables with unit variance, the expected value of (21) can be expressed as

$$\begin{aligned} &E\left[\left(\sum_{r=1}^{R} a_{kr}{}^{(j)} w_{kr}{}^{(j)}\right)^2\right] \\ &= \frac{c}{SNR_r}\sum_{r=1}^{R} E\frac{\left|h_{kr}{}^{(j)}\right|^2\left(x_{kr}{}^{(j)}\right)^2}{\left(\sum_{r=1}^{R}\left|h_{kr}{}^{(j)}\right|^2\right)^2} = \frac{c}{SNR_r}\sum_{r=1}^{R} E\frac{\left|h_{kr}{}^{(j)}\right|^2}{\left(\sum_{r=1}^{R}\left|h_{kr}{}^{(j)}\right|^2\right)^2} \\ &= E\sum_{r=1}^{R}\frac{\left|h_{kr}{}^{(j)}\right|^2}{\left(\sum_{r=1}^{R}\left|h_{kr}{}^{(j)}\right|^2\right)^2} = E\left[\frac{1}{Y}\right] \end{aligned} , \quad (22)$$

where Y is a random variable defined in Appendix I. Because $Y \sim Gamma(1/\lambda, R)$, its -1 moment can be calculated as

$$u_{-1} = \lambda\frac{\Gamma(R-1)}{\Gamma(R)} = \frac{\lambda}{R-1}$$, in which $\lambda=1/2\sigma_h{}^2=1$.

Finally, if we define $\sigma_w{}^2 = c/SNR_r$, (22) becomes

$$E\left[\left(\sum_{r=1}^{R} a_{kr}{}^{(j)} w_{kr}{}^{(j)}\right)^2\right] = \frac{c}{SNR_r}\frac{1}{R-1} = \frac{\sigma_w{}^2}{R-1} .$$

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