Maximum Multi-hop Range Using Cooperative Transmission With a Fixed Number of Nodes

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***Abstract*—We explore range expansion along an appointed direction using a fixed number of mobile nodes that do distributed multiple-input-multiple-output (MIMO) in the diversity configuration, also known as cooperative transmission (CT). The emphasis is on optimal groupings and locations of groups to maximize the multi-hop reach along a line; such information may inform the motion of robots that are deployed as far away as possible from a fixed base station yet still maintain connectivity. Also the impact of message-sharing within a cluster before relaying is considered. The simulation results show that for different fixed number of nodes, Rayleigh fading and arbitrary path loss exponent, there are preferred deployments depending on path loss exponent and sharing strategy.**

I. INTRODUCTION

In this paper, we explore how to best arrange a fixed number of relays along a line, to cover the longest distance, when the network is capable of cooperative transmission (CT). The cost function is end-to-end outage probability, when there is only one transmission or CT per hop (i.e., no retransmissions are considered). The original motivation for this study was a robotics problem, where the robots need to stay in contact with a fixed station, but can move themselves out as far as possible in some specified direction. We also note that a linear network arrangement might be a low-cost way for a team of robots to periodically monitor a large circular area (fixed base station in the center) by sweeping the line of robots around.

This robot deployment problem was studied in [1], for the non-CT case. In this paper we consider only the optimal (final) locations of the robots; we do not address the robot motion control to achieve those locations. To our knowledge, no one has addressed the robot or relay placement issue in the CT case, when the total number of nodes is fixed and when the hop distances and cluster sizes can be un-equal.

While there have been many studies of the SNR advantage of CT [2], [3], including using the SNR advantage for range extension [4], most of these studies consider only two hops (source to relay cluster to destination); they do not address the question of whether re-arranging the same fixed number of nodes into a 3+ hop topology would extend range further. Other studies address coverage area when CT is used [5]-[10], however, the two-dimensional nature of the coverage area problem makes its solution not applicable to the line network. We note that other authors have considered how to arrange nodes into equi-spaced, same-sized clusters along an infinite line network with CT [11], [12]; While this yields some insights into what cluster sizes yield the lowest outage probability per unit distance as a function of path loss exponent, the quasi-stationary method of analysis in those papers makes their results not directly applicable to the fixed small number of nodes (e.g., six nodes), which might be more likely in a robotic deployment. As we will later show in this paper, the optimal clusters are not equi-spaced and not always same-sized.

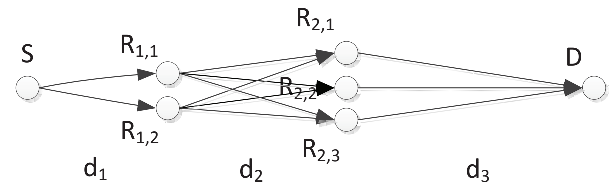

Fig. 1. 7 nodes Wireless Network Model

II. SYSTEM MODEL

Consider a wireless network with N single-antenna nodes deployed along a straight line. An $N = 7$ 3-hop example is shown in Fig. 1. The first node, the source, is always isolated, but the other $N - 1$ nodes can be in clusters. The first hop is the link between the source and Cluster 1. The second hop is the link between Cluster 1 and Cluster 2, and so forth. In Fig. 1, Cluster 1 has two nodes, Cluster 2 has three nodes, and the destination is isolated. We refer to this as a $1-2-3-1$ topology. We also consider the case when the destination is part of a cluster.

We assume all transmitters have exactly one antenna emitting the same power P_t, and that the transmissions within a cluster are in orthogonal channels [2]. A receiving node is assumed to be able to do maximal ratio combining (MRC) of the copies in each orthogonal channel. We note that this type of CT is practical and was demonstrated using software defined radios in [13].

We model the channels with path loss, as a function of path loss exponent n, and with independent Rayleigh fading. We assume that nodes within a cluster are co-located and we let d_i denote the distance of hop i. However we assume the nodes within a cluster are sufficiently separated (e.g., by at least a quarter wavelength in a rich scattering environment) to be able to assume uncorrelated fading of all the channels. We assume block fading (i.e., the fading outcome is the same for all symbols in the packet).

We consider two relay strategies: autonomous decode and forward (ADF) and “message sharing” (MS). The ADF strategy is the standard “decode and forward” strategy, wherein each node in the cluster relays the packet if it is able to correctly decode the packet. The MS strategy assumes that all nodes in the cluster will transmit the correct packet if at least one node in the cluster is able to correctly decode the packet. MS implies that the correctly decoded packet is shared with the nodes in the cluster that were not able to correctly decode originally. We expect the MS strategy to give better performance because more nodes in the cluster will be transmitting and therefore giving more diversity and array gains, however, MS will incur more delay. In either strategy, we assume that nodes in a cluster receive packets only from the previous cluster. In other words, in DF, for example, if at least one node in a cluster is able to relay in a certain time interval after receiving the packet for the first time, then no other node in that cluster will transmit in a later time interval.

We assume the jth receiver in the ith cluster can decode if its received signal-to-noise ratio (SNR), after MRC combining, exceeds a specified threshold τ. Our intent is for the threshold to represent the case when the packet error rate is 0.1. Therefore τ would generally depend on many things, such as the type of modulation and demodulation, the type of error correction coding, and the packet length. However, in this study, we simply specify the value of τ. If the destination is part of a cluster, then we assume the destination receives the packet correctly if any node within the destination cluster is able to correctly decode the packet, which is effectively selection combining.

We compute the total length of the network as the sum of the hop distances. We perform numerical optimization to find what combination of (1) the number of clusters, (2) the numbers of nodes in the different clusters, and (3) the various hop distances, yields the longest network, such that the destination is able to decode the packet correctly. Of course, in order for the destination to receive the packet, at least one node in each intermediate cluster must correctly decode the packet.

For Rayleigh fading, the PDF of the combined SNR, γ, is given by

$$p(\gamma) = \frac{1}{(M-1)!(E_b/N_0)^M}\gamma^{M-1}e^{\frac{-\gamma}{E_b/N_0}}, \qquad (1)$$

where $\frac{E_b}{N_0}$ is the bit energy to noise ratio and M is the number of combining signals at any received antenna.Therefore,

$$\frac{E_b}{N_0} = \frac{P_t T}{N_0 K} = \frac{P_0 T}{N_0 K}(\frac{d_0}{d})^n = \gamma_0(\frac{d_0}{d})^n, \qquad (2)$$

where d is the hop distance, $\gamma_0 = \frac{P_0 T}{N_0 K}$, T is the symbol period, and the number of bits per symbol is K. P_0 is the average power received at a reference distance d_0, and n is the path loss exponent. Then the probability of success for a receiver is:

$$P(\gamma \geq \tau) = \int_{\tau}^{+\infty} p(\gamma)d\gamma. \qquad (3)$$

Let D define the total distance from the source to the last node along the direction of the network. Then,

$$D = \sum_{i=1}^{k} d_i, \qquad (4)$$

A. Non-cooperative Model

In Fig. 2, we consider the special case with only one node in every hop, which we term the non-cooperative network. The received SNR in the ith hop is γ_i.

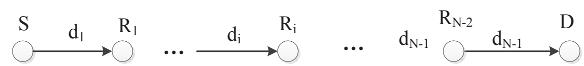

Fig. 2. N Nodes Non-cooperative model

The maximum value of D is achieved by having:

$$P(\gamma_1 \geq \tau) = P(\gamma_2 \geq \tau) = \cdots = P(\gamma_{N-1} \geq \tau). \qquad (5)$$

Based on (3), we can get

$$P_{out} = 1 - \prod_{i=1}^{N-1} P(\gamma_i \geq \tau) = 1 - P(\gamma_1 \geq \tau)^{N-1}. \qquad (6)$$

Therefore,

$$d_i = \sqrt[n]{\frac{ln^{0.9} d_0^n \gamma_0}{\tau(1-N)}}. \qquad (7)$$

The maximum distance from the source to the destination by non-cooperative transmission is

$$D_{max} = (N-1)d_i. \qquad (8)$$

B. Cooperative Model

In the cooperative model, we consider an idealized multihop network model as shown in Fig.1. The ith relay cluster includes L_i relay nodes. We assume that the signal transmitted by a certain node can be heard only by the nodes in its neighboring relay cluster and neglect the signal from source to destination, for we are trying to get the maximum value of distance.

We consider the 1-2-3-1 topology in Fig. 1 as an example to show how we find the maximum distance from source to the destination.

Let the jth receiver in the ith hop be denoted as $R_{i,j}$. Let the event that $R_{i,j}$ decodes the packet correctly be denoted $D_{i,j}$, and the event that it fails to decode be denoted $\overline{D}_{i,j}$. For the 1-2-3-1 network, the optimization problem is to maximize $\sum_{i=1}^{3} d_i$, subject to the constraint $P(D_{3,1}) = 0.1$. $R_{3,1}$ cannot decode unless at least one relay in the previous hop decoded, therefore, $P(D_{3,1}|\overline{D}_{2,1} \cap \overline{D}_{2,2} \cap \overline{D}_{2,3}) = 0$.

To ease calculation, we express $\overline{\overline{D}_{2,1} \cap \overline{D}_{2,2} \cap \overline{D}_{2,3}}$ as the union of single transmitter events, such as $D_{2,1} \cap \overline{D}_{2,2} \cap \overline{D}_{2,3}$, two transmitter events, such as $\overline{D}_{2,1} \cap D_{2,2} \cap D_{2,3}$, and the 3-transmitter event $D_{2,1} \cap D_{2,2} \cap D_{2,3}$. With this partition of $\overline{\overline{D}_{2,1} \cap \overline{D}_{2,2} \cap \overline{D}_{2,3}}$, we can write

$$\begin{aligned} &P(D_{3,1}) = P(D_{3,1} | \overline{\overline{D}_{2,1} \cap \overline{D}_{2,2} \cap \overline{D}_{2,3}}) \\ &= P(D_{3,1} | D_{2,1} \cap \overline{D}_{2,2} \cap \overline{D}_{2,3}) P(D_{2,1} \cap \overline{D}_{2,2} \cap \overline{D}_{2,3}) \\ &+ \dots \end{aligned} \quad (9)$$

C. Optimal Placements

If we pick 7 nodes as an example, they can deployed in different ways, such as $1-1-1-1-1-1-1$, $1-2-3-1$, $1-6$, $1-3-1-1-1$, and so on, so how to get the optimal deployment topology is a key part of our work. We use the following algorithm to find out the optimal topology to achieve the maximum distance expansion.

```
Input: d_0, n, γ_0 and SNR threshold τ;
Output: d_i and D;
for everytopology do
    for i = 1 : N − 1 do
        Calculate P(γ_i ≥ τ);
        if P(γ_i ≥ τ) ≥ 0.9 then
            Calculate d_i and D;
        end
    end
end
Compare the results for all the topologies; Get out the
optimal D_max.
```

Algorithm 1: Optimal Placement

III. Simulation Results

We considered networks of 4, 5 and 6 nodes, with the parameters τ=5, 10, 15, 20, $\gamma_0 = 10^3$, 10^4 and 10^5, and n from 2 to 5. For a given number of nodes, we found that only n causes the best topology to change, so we display the results for only $\tau = 10$, $\gamma_0 = 10^3$.

We first consider the 4-nodes case. In Table I, we can see that for any value of n, the $1-2-1$ topology has the best result among cooperative topologies as well as the SISO topology. When $n = 2$, the distance expansion result by cooperative models are all much longer than non-cooperative model, but as the path loss exponent, n increases, the CT advantage over SISO diminishes. We observe $1-2-1$ is always better than $1-1-2$, because MRC is well known to be better than selection.

For the 5 node network, there is a non-cooperative topology and 7 cooperative topologies. But $1-2-1-1$ and $1-1-2-1$ have the same distance, so we consider only one. Fig. 3 shows the simulation results for the 5 nodes network with n from 2 to 5, reference SNR $\gamma_0 = 30dB(10^3)$, and decoding threshold 10 dB (10). We observe that for all values of n except $n = 5$, CT yields the longest range. The gap between the best CT range and SISO range is the largest for lower n. This is consistent with the observation that the best way to "move power" across a multi hop network at the high path loss is with SISO links [14]. However, even at $n = 5$, the best CT range is about equal to the non-CT range and may be desirable from the view of energy required to move mobile agents into place. We also observe that topologies $1-2-2$ and $1-3-1$ give almost equal performance no matter how n changes and are both close to SISO at $n = 5$. Therefore, we can say that if every cluster has more than one node, the network would have a very good range expansion result. We also simulated for $\tau = 5$, 15 and 20, to capture impact of different types of modulation and coding, however we don't show the results because of space limitations, and because although the ranges decrease for higher τ, we observed the same trends as for the $\tau = 10$ case. We also simulated $\tau = 5$ and a higher reference SNR of $\gamma_0 = 10^4$. Again, although the ranges are generally larger with the higher power (or lower noise), the trends observed are the same as above.

TABLE I
4 Nodes Distance Expansion for Different Topologies $\tau = 10$, $\gamma_0 = 10^3$

n	Style	d_1	d_2	d_3	total D
2	SISO	1.8740	1.8740	1.8740	5.6220
	1-2-1	4.7230	7.1770	0	11.9000
	1-1-2	2.0870	5.2980	0	7.3850
	1-3	7.8980	0	0	7.8980
3	SISO	1.5200	1.5200	1.5200	4.5600
	1-2-1	2.9720	3.5740	0	6.5460
	1-1-2	1.7640	2.9270	0	4.6910
	1-3	3.9660	0	0	3.9660
4	SISO	1.3690	1.3690	1.3690	4.1070
	1-2-1	2.2880	2.5750	0	4.8630
	1-1-2	1.5590	2.2110	0	3.7700
	1-3	2.8100	0	0	2.8100
5	SISO	1.2856	1.2856	1.2856	3.8568
	1-2-1	1.9470	2.1230	0	4.0700
	1-1-2	1.4370	1.8760	0	3.3130
	1-3	2.2850	0	0	2.2850

Between $1-1-1-2$ and $1-2-1-1(1-1-2-1)$, we still can see the same situation as 4 nodes: the MRC result is better than the selection result.

Fig. 4 shows the different deployment topologies for 6 nodes. Like 5 nodes, $1-2-1-1-1$, $1-1-2-1-1$ and $1-1-1-2-1$ give the same performance, so we just choose one in Fig. 4 as an example. We can see that there are more topologies for 6 nodes than for 5 nodes. During the CT topologies, $1-2-2-1$ and $1-3-2$ always perform well as n changes.

Comparing 4, 5 and 6 nodes, we can find when n is very high, we would like to choose the SISO deployment, however among the CT, we would like the topology which would have at least two nodes per intermediate cluster.

IV. Simple Model with Message Sharing

In this section, we assume the message can be shared among the nodes in the same cluster; this means there would be at least one node in the same cluster that could decode

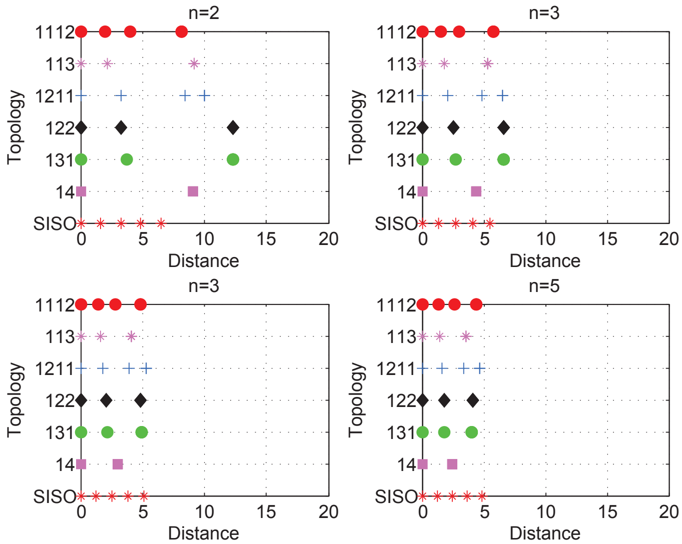

Fig. 3. 5 Nodes Deployment Simulation $\tau = 10, \gamma_0 = 10^3$

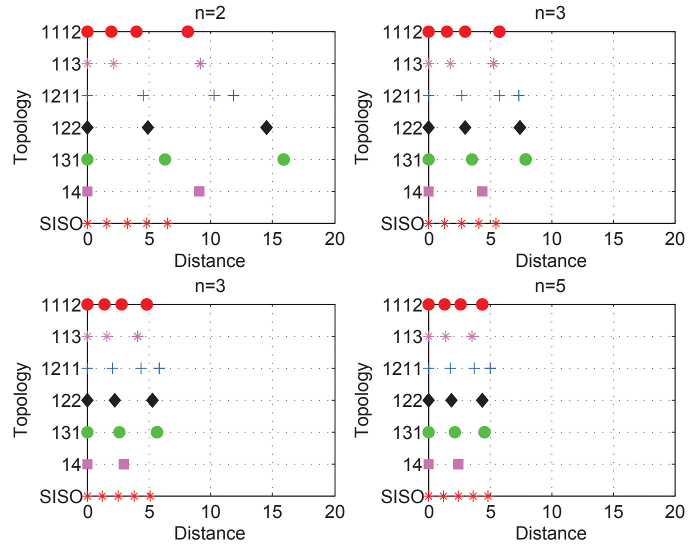

Fig. 5. 5 Nodes Message Sharing Deployment Simulation $\tau = 10, \gamma_0 = 10^3$

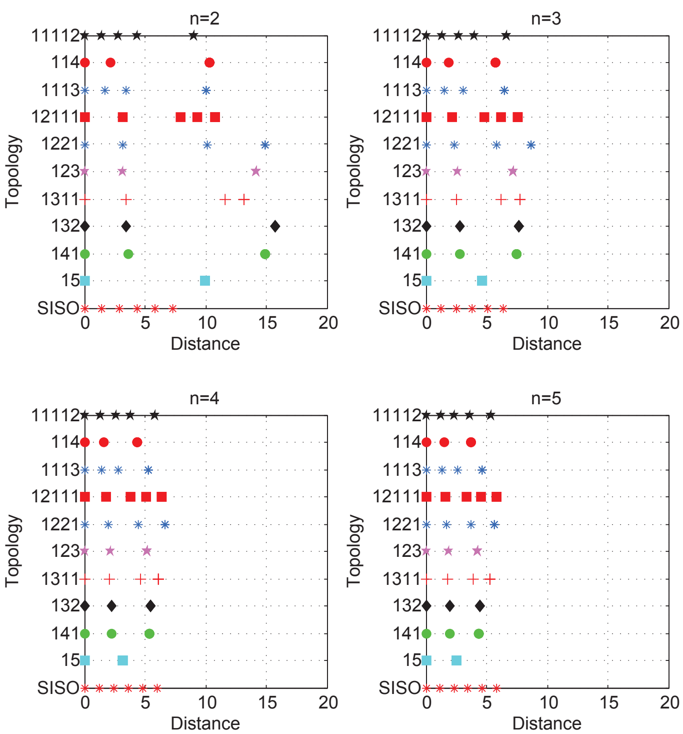

Fig. 4. 6 Nodes Deployment Simulation $\tau = 10, \gamma_0 = 10^3$

the message from the previous hop. The message sharing assumption simplifies the analysis significantly. Given that at least one node in a cluster decodes, we can assume all nodes in the cluster will retransmit the message. In this way, the model simplifies and the calculation time is less than before.

With message sharing, the probability of one receiver in the ith hop failing to achieve the SNR threshold, given at least one node in the previous cluster decoded, is

$$P_i(\gamma < \tau) = \int_0^\tau p_i(\gamma) d\gamma, \quad (10)$$

where $p_i(\gamma)$ is just (1) with M equal to the total number of nodes in the previous cluster, and (2) with $d = d_i$.

So we can get the probability that the message will be decoded by the last hop as

$$P(success) = \prod_{i=1}^{\alpha} 1 - P_i(\gamma < \tau)^{L_i} \quad (11)$$

where α is the total hops in the network, and L_i is the number of nodes in the ith hop.

Under the constraint $P(success) \geq 0.9$, we can easily get the distance expansion result by optimizing (11) over D_{max}. Now in order to compare with the above result, we still choose 5 and 6 nodes as an example, but by the sharing assumption, analyzing more nodes would be easier than before.

We performed simulations using the same parameters as in the non-sharing case. A sample result is shown in Fig. 5 and Fig. 6. We can compare the differences between sharing message and no sharing with the same nodes number.

For 5 nodes, we can see that the SISO, $1-4$, $1-1-3$ and $1-1-1-2$ give the same range expansion as for the no-sharing case because there is no message sharing during these topologies. Similarly, for 6 nodes, $1-5$, $1-1-1-1-2$, $1-1-1-3$ and $1-1-4$ are the non-sharing topologies. There is message sharing for the other topologies, and the message sharing result is better than no message sharing.

We observe that CT always yields a longer distance, regardless of the n values we consider. We define φ_A as the increase rate of the distance expansion after message sharing by the A topology. The equation

$$\varphi_A = \frac{D_{sharing} - D_{non-sharing}}{D_{non-sharing}} \times 100\% \quad (12)$$

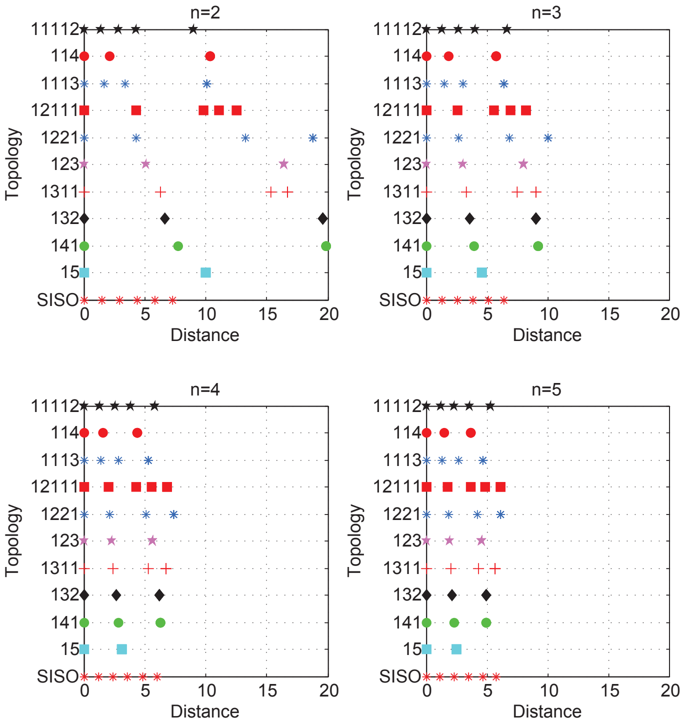

Fig. 6. 6 Nodes Message Sharing Deployments Simulation $\tau = 10, \gamma_0 = 10^3$

We choose the 5 and 6 node cases for discussing. Comparing the $1-3-1$, $1-2-2$ and $1-2-1-1$ topologies, we can see that message sharing increases the distance by 6%-31% or more. For 6 nodes, all the topologies which could have message sharing would increase 5%-33% of expansion distance than non-sharing.

Table II shows that for the same SNR threshold and with the increase of path loss exponent, the distance increase rate would decrease. We also can see that for the same path loss exponent n, the same topology, but different SNR threshold τ which is 5, 10, 15 and 20, we get all most the same increase rate. So it means the SNR threshold doesn't influence the distance expansion. The results for 6 nodes are similar and not shown.

TABLE II
INCREASE RATE OF 5 NODES BY MESSAGE SHARING

Topology	τ	n=2	n=3	n=4	n=5
131	5	30.36%	20.76%	15.38%	11.77%
	10	29.49%	20.53%	14.84%	14.33%
	15	30.20%	21.93%	15.55%	13.97%
	20	31.12%	21.42%	15.39%	13.74%
122	5	17.82%	12.41%	9.25%	7.35%
	10	17.82%	12.41%	9.26%	7.34%
	15	17.82%	12.40%	9.25%	7.34%
	20	17.82%	12.61%	9.23%	7.66%
1211	5	19.15%	11.20%	8.12%	7.96%
	10	18.70%	12.77%	9.62%	8.09%
	15	19.63%	12.28%	9.38%	6.36%
	20	20.00%	11.73%	8.67%	7.80%

V. CONCLUSION AND FUTURE WORK

The objective of this paper was to maximize the multi-hop reach using cooperative transmission with a fixed number of nodes, 4, 5, and 6 nodes were considered under the outage probability constraint.

Using the MRC method, the cooperative transmission method has a much better result than the non-cooperative approach, and message sharing widens the gap further, such that the CT longest distance network is always longer than the non-CT method.

The overall conclusion is that a fixed number of nodes should be arranged for cooperative transmission to extend range. And message sharing method is effective and the model is more simple and useful than in the non-sharing case.

We will continue our work on the range expansion problem based on the energy and time constraints. Then we will focus on the condition that there are obstacles on the deployment direction.

REFERENCES

[1] M. Arguedas, C. Perez, M. Carvalho, K. Hoback and W. Kraus, *A Directed Expansion Topology Control Algorithm for Mobile Ad-Hoc Networks*, in *proc. IEEE MILCOM*, San Jose, CA, USA, OCT. 2010, pp. 1293-1298.

[2] J.N. Laneman, D.N.C Tse, and G. W. Wornell, *Cooperative Diversity in Wireless Networks: Efficient Protools and Outage Behavior*, in *proc. IEEE trans. Infromation Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[3] A. Sendonaris, E. Erkip and B. Aazhang, *User Cooperation Diversity Part I: System Description*, in *proc. IEEE trans. Commun.*, vol. 51, no.11, pp. 1927-1938, Nov. 2003.

[4] H. Jung, Y.J. Chang, and M.A. Ingram, *Experimental Range Extension of Concurrent Cooperative Transmission in Indoor Environments at 2.4GHz*, in *proc. IEEE MILCOM*, San Jose, CA, Nov. 2010, pp. 148-153.

[5] A.K. Sadek, Z.Han, and K.J. Ray Liu, *Distributed relay-assignment protocols for coverage expansion in cooperative wireless networks*, *IEEE Trans. Mobile Comput.*, vol. 9, no. 4, pp. 2531- 2550, APR. 2010.

[6] A.K. Sadek, Z. Han, and K. J. Ray Liu, *Multi-node cooperative resource allocation to improve coverage area in wireless networks*, in *proc. IEEE Global Telecommunication Conf. (Globecom)*, St. Louis, MO, USA, Nov. 2005, pp. 3058-3062.

[7] I.D. Garcia, K. Sakaguchi, and K. Araki, *Cell planning for cooperative transmission*, in *proc. IEEE WCNC*, MAR. 2008, pp. 1769-1774.

[8] Zhu Han, K. J. Ray Liu, *Lifetime Maximization via Cooperative Nodes and Relay Deployment in Wireless Networks*, *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 305-317, FEB. 2007.

[9] T. Himsoon, W. Pam Siriwongpairat, Z. Han, K. J. Ray Liu, *Lifetime Maximization by Cooperative Sensor and Relay Deployment in Wireless Sensor Networks*, in *proc. IEEE WCNC*, Las Vegas, NV, USA, APR. 2006, pp. 439-444.

[10] J.W. Jung, M.A. Ingram, *Lifetime Optimization of Multi-hop Wireless Sensor Networks by Regulating the Frequency of Use of Cooperative Transmission*, in *proc. IEEE MILCOM*, Baltimore, MD, NOV. 2011, pp. 878-883.

[11] S.A. Hassan, M.A. Ingram, *A Quasi-Stationary Markov Chain Model of a Cooperative Multi-Hop Linear Network*, in *IEEE Trans. Wireless Commum.*, vol. 10, no. 7, pp. 2306-2315, Jul. 2011.

[12] B. Sirkeci-Mergen, A. Scaglione, *A continuum Approach to Dense Wireless Networks with Cooperation*, in *proc. IEEE INFOCOM*, Miama, FL, USA, MAR. 2005, pp. 2755-2763.

[13] Y.J. Chang, Haejoon Jung, and M.A. Ingram, *Demonstration of a New Degree of Freedom in Wireless Routing: Concurrent Cooperative Transmission*, in *HotEMNETS 2010 Workshop on Hot Topics in Embedded Networked Sensors*, Killarney, Ireland, June 28-29 2010.

[14] Ayfer Ozgur, Ramesh Johari, David N.C. Tse, Olivier Leveque, *Information-Theoretic Operating Regimes of Large Wireless Networks*, in *proc. IEEE Trans. Inf. Theory*, vol. 56, no.1, pp 427-437, JAN, 2010.