EE 6082 HW2

1 4.1

(a) $d_{min} = 3 \quad (= d\{(00100), (10010)\})$

(b) max. weight for detection = $d_{min} - 1 = 2$

(c) max. weight for correction = $\lfloor (d_{min}-1)/2 \rfloor = 1$

(d) not Linear : no all-zeros CW or $(00100)+(10010) \notin \mathcal{C}$

(e) Use hamming bound; $n - \log_2 M = 3 \neq \log_2 \underbrace{\left\{ \sum_{j=0}^{1} \binom{n}{j} \right\}}_{V_2(5,1)} = 6$

$\therefore$ not perfect.

2 4.2

(a) $r = (00000) \rightarrow \hat{c} = (00100)$
(b) $r = (00111) \Rightarrow \hat{c} = (00100)$ or (11111)
(c) $r = (01101) \Rightarrow \hat{c} = (01001)$
(d) $r = (10110) \Rightarrow \hat{c} = (10010)$
(e) $r = (01010) \Rightarrow \hat{c} = (10010)$ or (01001)

3 4.3

(a) $V_2(n,1) = \sum_{j=0}^{1} \binom{n}{j} = 1 + n$

(b) $V_q(n,1) = \sum_{j=0}^{1} (q-1)^j \binom{n}{j} = 1 + n(q-1)$

(c) $V_q(n,n-1) = \sum_{j=0}^{n} (q-1)^j \binom{n}{j} - (q-1)^n = q^n - (q-1)^n$

3

4 4.6

For repetition code, $\mathcal{C} = \{(111,\dots,1)\,(000,\dots,0)\}$

$d_{min} = n$ & $t = \lfloor (n-1)/2 \rfloor = (n-1)/2$ for n odd;

also, $k = 1$; so $n - k = n - 1$, and, (true for n odd)

$\log_2 V_2(n, (n-1)/2) = \log_2 \left\{ \sum_{j=0}^{(n-1)/2} \binom{n}{j} \right\} = \log_2 (2^{n-1}) = n-1$

hence, Hamming bound is satisfied with equality; code is thus perfect.

5 4.8

(a) $n = 7$, $k = 3$, $d_{min} = 3$ (columns 1, 6, 7)

(b) $n = 7$, $k = 3$ $d_{min} = 4$.

6 4.11

One example is $H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} = [I_{n-k} \,\vdots\, P]$

with a corresponding

$$G = [-P^T \,\vdots\, I_k] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$