7 4.12

The standard array decoding table will have $2^4 = 16$ columns and since Hamming codes are perfect & single-error-correcting, 8 rows. The row headers will consist of all weight 1 error patterns & no others.

8 4.13

For the H in problem 11, with $\underline{r} = (r_0, r_1, r_2, \ldots, r_6)$

$\underline{S}$	error location
100	r_0
010	r_1
001	r_2
110	r_3
011	r_4
101	r_5
111	r_6

9 4.14

$$A(x) = 1/8\left[(1+x)^7 + 7(1-x)(1-x^2)^3\right]$$

$$= 1 + 7x^3 + 7x^4 + x^7$$

10 4.15

From the MacWilliams identity

$$B(x) = 2^{-4}(1+x)^7\left\{A(x)\Big|_{x\to\left(\frac{1-x}{1+x}\right)}\right\} = 1 + 7x^4.$$

EXTRA PROBLEMS

PROBLEM 1

a) *False* — 9 is not prime.
b) *False* — $\underline{v}$ is not a linear combination of any of the rows of $\boldsymbol{G}$.
c) *False* — The columns of $\boldsymbol{H}$ can be *any* ordering of all the nonzero m-tuples.
d) *False* — e.g. in GF(2) 1^{-1} under $(+) = 1^{-1}$ under $(*) = 1$.

PROBLEM 2

a) $\boldsymbol{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$, $\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

b) $\underline{c} = \{(00000), (11010), (11101), (00111)\}$

c)

Standard Array				Syndrome $\underline{S}$
00000	**11101**	**11010**	**00111**	000
00001	11100	11011	00110	111
00010	11111	**11000**	00101	**110**
00100	11001	11110	**00011**	**001**
01000	**10101**	10010	01111	010
10000	01101	01010	**10111**	**100**
01001	10100	**10011**	01110	101
10001	01100	**01011**	10110	**011**

d) Since $d_{min} = 3$, code can correct 1 error and detect 2.

PROBLEM 3

a) Yes, any linear combination of the codewords is a codeword.

b) $|C| = 4 \therefore k = \log_2 4 = 2$.

c) One basis is $\{(0101), (1010)\}$.

d) $d_{min} = 2$, therefore code can detect 1 error but cannot correct any.

e) One $\boldsymbol{G} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ and $\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ also. Thus $\underline{C^{\perp} = C}$ so that $\dim(C^{\perp}) = \dim(C) = 2$.
Self dual code !!

f) A basis for $C^{\perp}$ is $\{(1111), (1010)\}$. another is same basis as in c)