ECE6605
Information Theory

HW #4

1) Problem 2, Chapt 5.

2) Problem 4, Chapter 5

3) Problem 6, Chapter 5

4) Problem 9, Chapter 5

Hint: Let X be a binary random variable, X {A,B} with p(A) = p, p(B) = 1-p. For any $\varepsilon > 0$, construct a distribution (i.e. find p) such that the optimum codeword length for a code where X's are taken one at a time is greater than H(X) + 1-ε. You will need to use the fact that H(X) for a binary random variable as a function of p looks like

5) Design a code using the Shannon-Fano-Elias coding scheme for a source with alphabet A = {a,b,c} where p(a) = 1/9, p(b) = 1/3, p(c) = 5/9. Compute the average codeword length and compare it to the entropy. (You will probably have to write a short C or Matlab program to get the binary expansion of 1/9,1/3 and 5/9. Recall for example the binary expansion of 3/8 = 0.011000000 . Notice that according to the algorithm you only need a finite precision estimate of the fraction).

6) Let a source emit IID random variables X from an alphabet A={1,2,3,4,5,6,7} with respective probabilities 1/3, 1/3, 1/9, 1/9, 1/27, 1/27, 1/27. A company provides two different services. Service 1 accepts source outputs, encodes them one at a time into binary digits and transmits the digits at $2.00 per digit. Service 2 encodes source outputs one at a time into ternary digits at $3.50 per digit. You must select the service and design a code to minimize the expected cost.

a) Would you select Service 1 or 2?
b) Specify the code you would use.
c) What is the expected cost?
d) If the binary cost were changed at what cost would you change your mind?