ECE 6605
Information Theory

HW #5

1) Lempel Ziv problem This problem concerns the Lempel-Ziv (LZ) data compression algorithm. Answer parts a) and b) below. *The sources and codes in parts a) and b) are unrelated.*

a) **Encoding.** Given that the LZ encoder starts with the following table, construct the table and indicate the transmitted codewords for the source output $\underline{X}$. You may assume that the source outputs either α or β. (You can number the codewords 1,2,3...)

X	C(X)
α	1
β	2

Source output: $\underline{X}$= α α α α β β α β β β α α α β α α α α

b) **Decoder.** In this part you will be the decoder. Given the following sequence of codewords, decode it, and construct the table as you go along. You will start with the same table the encoder starts with and assume the source outputs two letters c and d.

X	C(X)
c	1
d	2

Codeword sequence to be decoded: 1 2 1 3 4 5 1 7 3 (decode this message and indicate whether this is a valid **encoding**. If it is not, give the correct codeword for the source output).

2) Arithmetic coding problem

Consider a binary source that outputs a's and b's independently with probabilities

P(a) = 0.3 and P(b) = 0.7

Using arithmetic coding, find the codeword for the following source output and compare it with the source entropy:

a b a b b b a a

3) Problem 1, Chapter 8 . You may assume $X \rightarrow Y \rightarrow g(Y)$ is a Markov chain

4) Problem 2, Chapter 8
Hint: Be very careful in part a). You have to compute $p(X=a_1 \mid Y=000)$, $p(X=a_1 \mid Y=001)$ ... You need to use Baye's rule here!!!

5) Problem 5, Chapter 8. Modulo arithmetic is what you might expect. For instance in this problem, mod 11 arithmetic means: $11 \pmod{11} = 0$, $12 \pmod{11} = 1$ and so on.

6) Problem 7, Chapter 8.

7) The two-input EXCLUSIVE-OR (XOR) function can be viewed as a communications channel. Let X and Z be the inputs and Y = X xor Z be the output. Assume that X {0,1} is the input to the channel, Y is the output of the channel, and Z {0,1} is some noise introduced in the channel where p(Z=1) = p .

a) Find the probability transition matrix P.

b) Draw a diagram of the channel with inputs, outputs, and label any connections between them with the crossover probabilities.

c) What is the more common name of this channel and what is its capacity?