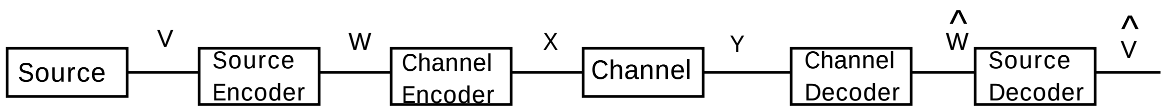
ECE 6605
Information Theory

HW #6

1) Prove the Fano inequality given in class

2) (optional-source channel coding theorem not discussed in calss) We are given the following communications system:



The source: Emits IID binary digits V_i {0,1} where $p(V_i = 0) = .1$
The channel: A binary symmteric channel with crossover probability $\varepsilon = .2$

a) Consider a source code that encodes source outputs one at a time into bits. Design the optimum source code and find R_S , the average codeword length in bits per source output . What is the maximum channel code rate R_C at which perfect reproduction of the source is possible?

b) Now consider a source code that encodes 2 consecutive source outputs into bits. Design the optimum source code and find R_S , the average codeword length in bits per source output . What is the maximum channel code rate R_C at which perfect reproduction of the source is possible?

c) Give an interpretation of why the rate R_C in b) is less than that in a).

3) For the following channel determine capacity and the maximizing input distribution.

$$P = \begin{bmatrix} 1-\alpha-\beta & \beta & \alpha \\ \alpha & \beta & 1-\alpha-\beta \end{bmatrix}$$

4) In a joint pair of random variables XY the “specific mutual information” $I(x_i, y_j) = \log[p(x_i, y_j)/ p(x_i) p(y_j)]$ is a random variable. Our definition of mutual information (I(X;Y)is just the average “specific mutual information” or its mean. In this problem we consider its variance.

a) Prove that $\mathrm{var}[I(x_i, y_j)] = 0$ if and only if there is a constant α such that for all (x_i, y_j) with $p(x_i, y_j) \neq 0$,

$$p(x_i, y_j) = \alpha\, p(x_i)\, p(y_j)$$

a) Express I(X;Y) in terms of α and interpret in special case of $\alpha=1$.

b) For the channel below find the input distribution such that $I(X;Y) \neq 0$ and $\mathrm{var}[I(x_i, y_j)] = 0$. Calculate I(X;Y)