Math 1553 Reading Day Spring 2023

This is a preview of the published version of the quiz

Started: Mar 18 at 12:08pm

Quiz Instructions

Question 1

If \( \{u, v, w\} \) is a set of linearly dependent vectors, then \( w \) must be a linear combination of \( u \) and \( v \).

- True
- False

Question 2

Find the value of \( k \) that makes the following vectors linearly dependent:

\[
\begin{pmatrix}
-3 \\
0 \\
3
\end{pmatrix}, \quad
\begin{pmatrix}
3 \\
-3 \\
k
\end{pmatrix}, \quad
\begin{pmatrix}
3 \\
-1 \\
-1
\end{pmatrix}
\]

Question 3

If \( \{u, v\} \) is a basis for a subspace \( W \), then \( \{u - v, u + v\} \) is also a basis for \( W \).
Question 4

Which of the following are subspaces of $\mathbb{R}^4$?

(1) The set $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 : 2x - y - z = 0 \right\}$.

(2) The set of solutions to the equation $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

○ (1) is a subspace but (2) is not a subspace

○ neither is a subspace

○ both are subspaces

○ (2) is a subspace but (1) is not a subspace

Question 5

Let $W$ be the set of vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ in $\mathbb{R}^3$ with $abc = 0$. Then $W$ is closed under addition, meaning that if $v$ and $w$ are in $W$, then $v + w$ is in $W$.

○ True

○ False
### Question 6

Match the transformations given below with their corresponding $2 \times 2$ matrix.

A. \[
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\]

B. \[
\begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}
\]

C. \[
\begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}
\]

D. \[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

E. \[
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\]

- Counter-clockwise rotation by 90 degrees: [Choose] ✓
- Reflection about the line $y=x$: [Choose] ✓
- Clockwise rotation by 90 degrees: [Choose] ✓
- Reflection across the x-axis: [Choose] ✓
- Reflection across the y-axis: [Choose] ✓

### Question 7

Find the value of $k$ so that the matrix transformation for the following matrix is not onto.
Question 8  1 pts

Find the nonzero value of $k$ that makes the following matrix not invertible.

$$
\begin{pmatrix}
1 & -1 & 0 \\
-1 & 1 & 5 \\
\end{pmatrix}
$$

Enter an integer as your answer. Note that 0 is not the correct answer, since the question asks for a nonzero value of $k$.

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Question 9  1 pts

Match the following definitions with the corresponding term describing a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$.

Each definition should be used exactly once.

A. For each $y$ in $\mathbb{R}^n$ there is at most one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.
B. For each $y$ in $\mathbb{R}^n$ there is at least one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.
C. For each $y$ in $\mathbb{R}^n$ there is exactly one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.
D. For each $x$ in $\mathbb{R}^m$ there is exactly one $y$ in $\mathbb{R}^n$ so that $T(x) = y$.

T is a transformation  [ Choose ]

T is one-to-one
Question 10
1 pts

Suppose $A$ is a $4 \times 6$ matrix. Then the dimension of the null space of $A$ is at most 2.

○ True
○ False

Question 11
1 pts

Complete the entries of the matrix $A$ so that $\text{Col}(A) = \text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ and $\text{Nul}(A) = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$.

$A = \begin{pmatrix} r & 1 \\ s & 2 \end{pmatrix}$, where $r =$ and $s =$

Question 12
1 pts

Suppose $T : \mathbb{R}^7 \to \mathbb{R}^9$ is a linear transformation with standard matrix $A$, and suppose that the range of $T$ has a basis consisting of 3 vectors. What is the
dimension of the null space of $A$?

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**Question 13**  
1 pts

Define a transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ by $T(x, y, z) = (0, x - y, y - x, z)$. Which one of the following statements is true?

- $T$ is one-to-one and onto.
- $T$ is onto but not one-to-one.
- $T$ is one-to-one but not onto.
- $T$ is neither one-to-one nor onto.

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**Question 14**  
1 pts

Suppose that $A$ is a $7 \times 5$ matrix, and the null space of $A$ is a line. Say that $T$ is the matrix transformation $T(v) = Av$. Which of the following statements must be true about the range of $T$?

- It is a 6-dimensional subspace of $\mathbb{R}^7$
- It is a 4-dimensional subspace of $\mathbb{R}^5$
- It is a 6-dimensional subspace of $\mathbb{R}^5$
- It is a 4-dimensional subspace of $\mathbb{R}^7$

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**Question 15**  
1 pts
Say that $S : \mathbb{R}^2 \to \mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^4$ are linear transformations. Which of the following must be true about $T \circ S$?

- It is not one-to-one
- It is one-to-one
- It is onto
- The composition is not defined
- It is not onto

**Question 16**

Suppose that $A$ is an invertible $n \times n$ matrix. Then $A + A$ must be invertible.

- True
- False

**Question 17**

Suppose $A$ is a $3 \times 3$ matrix and the equation $Ax = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ has exactly one solution.

Then $A$ must be invertible.

- True
- False
Question 18  
1 pts

Suppose that $A$ and $B$ are $n \times n$ matrices and $AB$ is not invertible. Which one of the following statements must be true?

- None of these
- $A$ is not invertible
- At least one of the matrices $A$ or $B$ is not invertible
- $B$ is not invertible

Question 19  
1 pts

Suppose $A$ and $B$ are $3 \times 3$ matrices, with $\det(A) = 3$ and $\det(B) = -6$. Find $\det(2A^{-1}B)$.

Question 20  
1 pts

Let $A$ be the $3 \times 3$ matrix satisfying $Ae_1 = e_3$, $Ae_2 = e_2$, and $Ae_3 = 2e_1$ (recall that we use $e_1$, $e_2$, and $e_3$ to denote the standard basis vectors for $\mathbb{R}^3$). Find $\det(A)$.

Question 21  
1 pts
Suppose \( A \) is a square matrix and \( \lambda = -1 \) is an eigenvalue of \( A \).

Which one of the following statements must be true?

- For some nonzero \( x \), the vectors \( Ax \) and \( x \) are linearly dependent.
- \( \text{Nul}(A + I) = \{0\} \)
- The equation \((Ax = x)\) has only the trivial solution.
- \( A \) is invertible.
- The columns of \( A + I \) are linearly independent.

**Question 22**

Suppose \( A \) is a 4 \( \times \) 4 matrix with characteristic polynomial \(- (1 - \lambda)^2 (5 - \lambda) \lambda\).

What is the rank of \( A \)?

**Question 23**

Let \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) be the transformation that reflects across the line \( x_2 = 2x_1 \).

Find the value of \( k \) so that \( A \begin{pmatrix} 2 \\ k \end{pmatrix} = \begin{pmatrix} 2 \\ k \end{pmatrix} \).

**Question 24**
Find the value of $k$ such that the matrix $\begin{pmatrix} 1 & k \\ 1 & 3 \end{pmatrix}$ has one real eigenvalue of algebraic multiplicity 2. *Enter an integer value below.*

**Question 25**

1 pts

Suppose that $A$ is a $5 \times 5$ matrix with characteristic polynomial $(1 - \lambda)^3 (2 - \lambda)(3 - \lambda)$ and also that $A$ is diagonalizable. What is the dimension of the 1-eigenspace of $A$?

**Question 26**

1 pts

Find the value of $t$ such that 3 is an eigenvalue of $\begin{pmatrix} 1 & t & 3 \\ 1 & 1 & 1 \\ 3 & 0 & 3 \end{pmatrix}$. *Enter an integer answer below.*

**Question 27**

1 pts
Say that $A$ is a $2 \times 2$ matrix with characteristic polynomial $(1 - \lambda)(2 - \lambda)$. What is the characteristic polynomial of $A^2$?

- $(1 - \lambda)(2 - \lambda)$
- $(1 - \lambda)^2(2 - \lambda)^2$
- $(1 - \lambda^2)(2 - \lambda^2)$
- $(1 - \lambda^2)(4 - \lambda^2)$
- $(1 - \lambda)(4 - \lambda)$

**Question 28**

1 pts

Suppose that a vector $x$ is an eigenvector of $A$ with eigenvalue 3 and that $x$ is also an eigenvector of $B$ with eigenvalue 4. Which of the following is true about the matrix $2A - B$ and $x$:

- $x$ is an eigenvector of $2A - B$ with eigenvalue 1
- None of these
- $x$ is an eigenvector of $2A - B$ with eigenvalue 4
- $x$ is an eigenvector of $2A - B$ with eigenvalue 2
- $x$ is an eigenvector of $2A - B$ with eigenvalue 3

**Question 29**

1 pts

Suppose that $A$ is a $4 \times 4$ matrix with eigenvalues 0, 1, and 2, where the eigenvalue 1 has algebraic multiplicity two.

Which of the following must be true?

1. $A$ is not diagonalizable
(2) \( A \) is not invertible

- Both (1) and (2) must be true
- Neither statement is necessarily true
- (2) must be true but (1) might not be true
- (1) must be true but (2) might not be true

**Question 30**

1 pts

Suppose \( A \) is a \( 5 \times 5 \) matrix whose entries are real numbers. Then \( A \) must have at least one real eigenvalue.

- True
- False

**Question 31**

1 pts

Suppose \( A \) is a positive stochastic matrix and \( A \left( \begin{array}{c} 3/5 \\ 2/5 \end{array} \right) = \left( \begin{array}{c} 3/5 \\ 2/5 \end{array} \right) \). Let \( \mathbf{v} = \left( \begin{array}{c} 5 \\ 95 \end{array} \right) \).

As \( n \) gets very large, \( A^n \mathbf{v} \) approaches the vector \( \left( \begin{array}{c} r \\ s \end{array} \right) \), where:

\[ r = \quad \text{and} \quad s = \quad \]

**Question 32**

1 pts
Suppose that $A$ is a $4 \times 4$ matrix of rank 2. Which one of the following statements must be true?

- none of these
- $A$ is diagonalizable
- $A$ cannot have four distinct eigenvalues
- $A$ is not diagonalizable
- $A$ must have four distinct eigenvalues

**Question 33**

1 pts

Suppose $A$ is a $2 \times 2$ matrix whose entries are real numbers, and suppose $A$ has eigenvalue $1 + i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1 + i \end{pmatrix}$.

Which of the following must be true?

- $A$ must have eigenvalue $1 - i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1 + i \end{pmatrix}$
- None of these
- $A$ must have eigenvalue $1 - i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1 - i \end{pmatrix}$
- $A$ must have eigenvalue $1 + i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1 - i \end{pmatrix}$

**Question 34**

1 pts

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates the plane clockwise by 45 degrees, and let $A$ be the standard matrix for $T$.

Which one of the following statements is true?
Question 35

Suppose \( u \) and \( v \) are orthogonal unit vectors (to say that a vector is a unit vector means that it has length 1). Find the dot product

\[
(3u - 8v) \cdot 4u.
\]

Question 36

Find the value of \( k \) that makes the following pair of vectors orthogonal.

\[
\begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} k \\ 1 \\ -6 \end{pmatrix}
\]

Your answer should be an integer.

Question 37

If \( W \) is a subspace of \( \mathbb{R}^{100} \) and \( v \) is a vector in \( W^\perp \) then the orthogonal projection of \( v \) to \( W \) must be the 0 vector.
Question 38

Suppose \( W \) is a subspace of \( \mathbb{R}^n \). If \( x \) is a vector and \( x_W \) is the orthogonal projection of \( x \) onto \( W \), then \( x \cdot x_W \) must be 0.

- True
- False

Question 39

Suppose that \( A \) is a \( 3 \times 3 \) invertible matrix. What is the dot product between the second row of \( A \) and third column of \( A^{-1} \) equal to?

- 1
- 2
- 0
- -2
- -1
- Not Enough Information is Given

Question 40

Find the orthogonal projection of \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) onto \( \text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\} \).
The orthogonal projection is \( \begin{pmatrix} a \\ b \end{pmatrix} \), where: \( a = \) \( \) and \( b = \).

Enter integers or fractions as your entries.

**Question 41**

1 pts

Compute the orthogonal projection of the vector \( \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix} \) to the plane spanned by the vectors \( \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \). What is the first coordinate of the projection? Your answer should be an integer.

**Question 42**

1 pts

Suppose \( B \) is the standard matrix for the transformation \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) of orthogonal projection onto the subspace \( W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + y + 2z = 0 \right\} \).

What is the dimension of the 1-eigenspace of \( B \)?

**Question 43**

1 pts
Let \( W \) be the subspace of \( \mathbb{R}^4 \) given by all vectors \( \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \) such that \( x - y + z + w = 0 \). Find dimension of the orthogonal complement \( W^\perp \).

**Question 44**  
1 pts

If \( b \) is in the column space of the matrix \( A \) then every solution to \( Ax = b \) is a least squares solution.

- True
- False

**Question 45**  
1 pts

If \( A \) is an \( m \times n \) matrix, \( b \) is in \( \mathbb{R}^m \), and \( \hat{x} \) is a least squares solution to \( Ax = b \), then \( \hat{x} \) is the point in \( \text{Col}(A) \) that is closest to \( b \).

- True
- False

**Question 46**  
1 pts

Find the least squares solution \( \hat{x} \) to the linear system
\[
\begin{pmatrix}
6 \\
-2 \\
-2
\end{pmatrix}
\begin{pmatrix}
x \\
14 \\
-2 \\
0
\end{pmatrix}.
\]

If your answer is an integer, enter an integer.

If your answer is not an integer, enter a fraction.

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**Question 47**

Find the best fit line \(y = \quad x + \quad\) for the data points \((-7, -22), (0, -2),\) and \((7, 6)\) using the method of least squares. Your answers should both be integers.

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**Question 48**

Let \(A = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} -3 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}^{-1}.\)

Find \(r\) and \(s\) so that \(A^3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}.\)

\(r = \quad\)

\(s = \quad\)

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**Question 49**

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If $A$ is a diagonalizable $6 \times 6$ matrix, then $A$ has 6 distinct eigenvalues.

- True
- False

**Question 50**

Find the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$ and write them in increasing order.

The smaller eigenvalue is $\lambda_1 = \phantom{0}$.

The larger eigenvalue is $\lambda_2 = \phantom{0}$.