Computing with Dynamical Systems in the Post-CMOS Era

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In the pursuit for building hardware accelerators to compute optimization problems researchers realize that the challenges in achieving this objective lie not only in implementing the hardware but also in the formulating the computing fundamentals of such designs. Neural network algorithms are considered most suited for this task, as there is usually a direct description of distributed computing entities, called "neurons", and their interactions which can be mapped to both electronic and non-electronic hardware. In this regard, coupled oscillator systems have been studied where individual oscillators correspond to neurons and the information is encoded in either phase or frequency. But as is the case with neural networks, the computational power of the network depends on complexity of interactions among oscillators, and it is a challenge to implement oscillator networks with complex simultaneous interactions among multiple oscillators. Sinusoidal oscillators with assumption of weak linear phase coupling, akin to Kumamoto models, have been studied in theory but implementing such oscillators with weak couplings and encoding information in phase or frequency have been a challenge. Examples of using novel devices for making neural network hardware include memristor based neuromorphic synapses [1] and spin-torque oscillator (STO) based systems [2]. In our work, we use relaxation oscillators coupled using passive elements - capacitances or resistances - without the assumption of weak linear phase couplings. Our theoretical models are derived from circuit implementations, instead of the other way round, which means there are only engineering challenges in implementing the hardware, and no modeling discrepancies. We have explored two kinds of implementations - (a) simple pairwise coupling scheme with information encoded as frequency for pattern matching and associative computing, and (b) complex global coupling with information encoded in phase for the NP-hard graph coloring problem. We
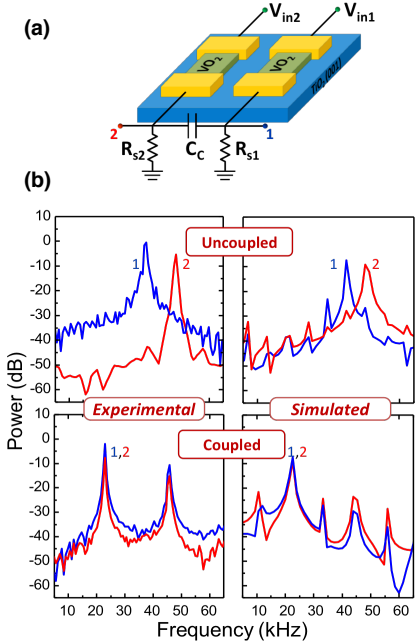

Figure 2: (a) Schematic of two capacitively coupled VO₂ oscillators tuned to different oscillating frequencies with $R_{s1}=38k\Omega$ (=6Rc) and $R_{s2}=47k\Omega$ (=7.5Rc). (b) Experimental and simulated power spectrum (power expressed as mean square amplitude in dB) of both oscillators before and after coupling. The individual oscillator frequencies lock to a new frequency when coupled capacitively (Cc = 680 pF).
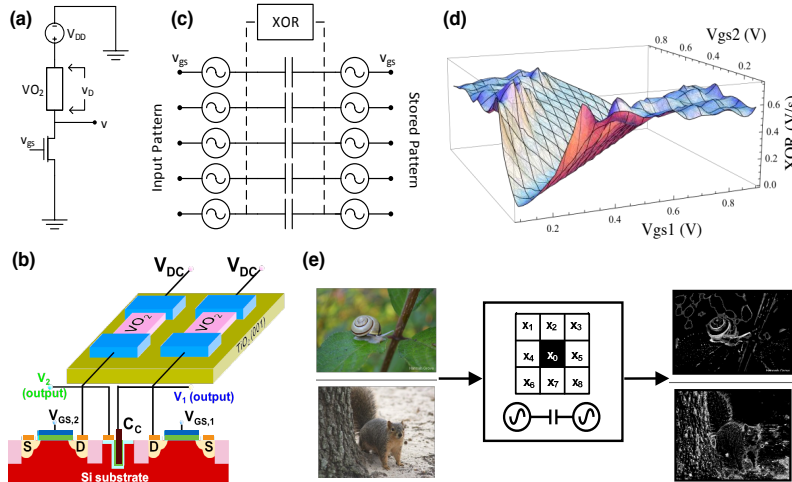
have been demonstrated in theory, using simulations and experimental implementations using VO₂ devices, the working of such coupled relaxation oscillator networks.

The oscillators we use are of the relaxation type, made using a VO₂ device in series with a resistance (Figure 1a) [3]. The VO₂ device is a two terminal *metal-insulator-transition* device which changes state from metallic to insulating based on the voltage applied across it. When the voltage across it increases above a threshold v_h, the device switches to a low resistance metallic state, and when the voltage across it reduces below a

Figure 1 : (a) VO2 oscillator with a series transistor (b) Schematic of the capacitively coupled HVFET oscillators ($L_{VO2}=4\mu m$; $W_{VO2}=40\mu m$, $C_C=2.2nF$) (c) System of pattern matching using pairwise coupled VO2 oscillators (d) Surface plot of averaged XOR output for different values of Vgs1 and Vgs2 (e) Saliency detection in images using coupled HVFET oscillators in sensor frontends

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threshold v_l it changes to a high resistance insulating state. The threshold voltages v_h and v_l are not equal and there is a hysteresis in the operation, i.e. when the voltage across the device is in between v_h and v_l the device retains the last state it was in. Because of this hysteresis, and also due to an internal capacitance associated with the device, when such a device is connected in series with a resistance of appropriate value, the circuit shows relaxation oscillations due to continuous charging and discharging of the internal capacitance of the device.

When two VO_2 relaxation oscillators with frequencies close to each other are coupled using a capacitor as shown in figure Fa, they synchronize to a common frequency (figure Fb) but the locking is out-of-phase [4]. Information can be encoded as frequency of such oscillators by using a transistor in place of the series resistance (Figures 2a and 2b) called hybrid VO_2-MOSEFT (HVFET) oscillators and the steady state phase
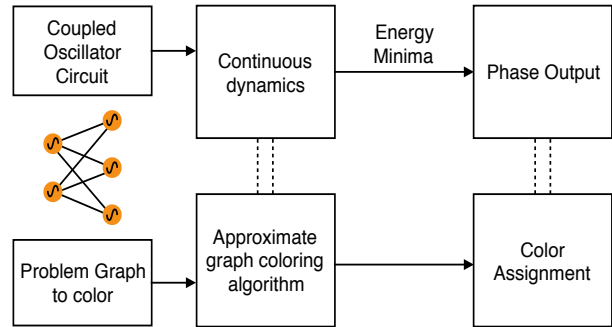

Figure 3 : Overview of the proposed system for graph coloring. First step is a coupled relaxation oscillator circuit connected using capacitors in the same graph structure as the problem graph. The time evolution of the dynamical system created by the coupled circuit correspond to an aproximate graph coloring algorithm because of which the order of the steady state phases of the oscillators correspond to the color assignment given by the graph coloring algorithm.

difference can be calculated using a threshold and XOR operation (Figure 2c) [5]. When the frequencies are close but not equal, the steady state phase differences reduce as the difference in frequencies increase. As such, if two analog quantities are encoded as the frequency of the relaxation oscillators, such coupling gives an easy way to calculate a measure of difference. We observe from simulations that this measure resembles a squared distance norm (Figure 2d) which is 0 for equal frequencies and saturates to $0.5V_{DD}$ for larger frequency differences. Using pairwise coupling of arrays of such oscillators, we can calculate a simple l_1 norm distance of input and stored vector patterns. From the viewpoint of power and computational complexity, this scheme reduces the time and power by parallelizing the operation of calculating the l_1 norm distance between vector patterns which can be a substantial reduction in an overall table lookup kind of pattern matching [6,7]. Also, such difference measure can be used for saliency detection. In comparison with 11 nm CMOS technology node, the oscillators are expected to provide a power reduction of ~20X [5].

The full computational power of such coupled dynamical systems can be harnessed using complex global interactions among oscillators instead of just pairwise interactions. Using a global coupling scheme which can be implemented in hardware, we demonstrate, using theory and experimental implementations that such relaxation oscillators could be used to approximate the solutions of the NP-hard *minimum graph coloring* problem. For a given graph with some nodes and edges, the *minimum graph coloring* problem attempts to assign a single color to each node such that no two nodes with the same color are connected by an edge, and while doing that tries to minimize the number of colors used. For solving such a minimum graph coloring problem, the oscillators are connected in the same graph structure as the problem graph whose coloring needs to be computed (Figures 3), where each oscillator corresponds to a node and each edge to a capacitive coupling with equal capacitances. In the steady state, the oscillators synchronize and get phase locked in such a way that the steady state phases can be used to find a correct approximate solution to the graph coloring problem with approximately minimum colors.

Complex dynamical systems can thus pave the way to next generation computing systems and harness the true potential of post-CMOS devices by matching the "physics" of these devices to the "mathematics" of the computing.

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