Spring 06

~~EE~~E4803 HW #5 Sol'ns

1) $f_c \approx \frac{1}{2\pi R_{big}} \frac{R_{small}(19.2\, I_{CON})}{C} = \frac{100}{20K} \frac{19.2}{100pF} I_{CON}$

$\approx \boxed{1.52 \times 10^8 I_{CON}}$

2) a) $s^2$ terms dominate, so $\boxed{\text{gain} = 1}$

b) $|H(j\omega)|^2 = \left| \frac{-\omega^2}{-\omega^2 + \frac{\omega_c}{Q} j\omega + \omega_c^2} \right|^2$

$= \left( \frac{-\omega^2}{-\omega^2 + \frac{\omega_c}{Q} j\omega + \omega_c^2} \right) \left( \frac{-\omega^2}{-\omega^2 - \frac{\omega_c}{Q} j\omega + \omega_c^2} \right)$

$= \boxed{\frac{\omega^4}{(\omega^2 - \omega_c^2)^2 + \frac{\omega_c^2 \omega^2}{Q^2}}}$

c) $\frac{\omega^4}{(\omega^2 - \omega_c^2)^2 + \frac{\omega_c^2 \omega^2}{Q^2}} = \frac{1}{2}$

$(\omega^2 - \omega_c^2)^2 + \frac{\omega_c^2}{Q^2}\omega^2 = 2\omega^4$

$\omega^4 - 2\omega_c^2 \omega^2 + \omega_c^4 + \frac{\omega_c^2 \omega^2}{Q^2} = 2\omega^4$

$-\omega^4 + \left(\frac{1}{Q^2} - 2\right)\omega_c^2 \omega^2 + \omega_c^4 = 0$

$\omega^4 + \left(2 - \frac{1}{Q^2}\right)\omega_c^2 \omega^2 - \omega_c^4 = 0$

$\omega^2 = \frac{\omega_c^2\left(\frac{1}{Q^2} - 2\right) + \sqrt{\left(2 - \frac{1}{Q^2}\right)^2 \omega_c^4 + 4\omega_c^4}}{2}$ (choosing + branch)

$\omega^2 = \frac{\omega_c^2\left(\frac{1}{Q^2} - 2\right) + \frac{\omega_c^2}{Q^2}\sqrt{(2Q^2 - 1)^2 + 4Q^4}}{2}$

$$= \cancel{\omega_c^2\left[\frac{\frac{1}{Q}-2+\sqrt{\frac{1}{Q}\left(\frac{1}{Q}-4\right)}}{2}\right]}$$

$$\omega = \sqrt{\frac{\omega_c^2\left(\frac{1}{Q^2}-2\right)+\frac{\omega_c^2}{Q^2}\sqrt{8Q^4-4Q^2+1}}{2}}$$

$$= \boxed{\frac{\omega_c}{Q}\sqrt{\frac{(2-Q^2)+\sqrt{8Q^4-4Q^2+1}}{2}}}$$

(see next page also)

d) Ask: ~~Where~~ when does $\dfrac{(\omega^2-\omega_c^2)^2+\frac{\omega_c^2\omega^2}{Q^2}}{\omega^4}$ have a valley?

$$\frac{\omega^4-2\omega_c^2\omega^2+\omega_c^4+\frac{\omega_c^2\omega^2}{Q^2}}{\omega^4}$$

$$= 1+\left(\frac{\omega_c^2}{Q^2}-2\omega_c^2\right)\omega^{-2}+\omega_c^4\omega^{-4}$$

Take deriv w.r.t $\omega$

$$-2\omega_c^2\left(\frac{1}{Q^2}-2\right)\omega^{-3}-4\omega_c^4\omega^{-5}$$

$$2\omega_c^2\left(2-\frac{1}{Q^2}\right)\omega^2 = 4\omega_c^4$$

exists for
$2Q^2-1>0$
$Q^2>\frac{1}{2}$
$Q>\frac{1}{\sqrt{2}}$

$$\omega = \sqrt{\frac{4\omega_c^4}{2\omega_c^2\left(2-\frac{1}{Q}\right)}}$$

$$\omega = \boxed{\omega_c\sqrt{\frac{2Q^2}{(2Q^2-1)}}} = \omega_c\sqrt{\frac{1}{1-\frac{1}{2Q^2}}}$$

Regarding (c):

One student wrote it like this, which I now think looks clearer:

$$w^2 = w_c^2\left[\left(\frac{1}{2Q^2}-1\right)+\sqrt{\left(1-\frac{1}{2Q^2}\right)^2+1}\right]$$

$$w_{1/2} = w_c\sqrt{\left(\frac{1}{2Q^2}-1\right)+\sqrt{\left(1-\frac{1}{2Q^2}\right)^2+1}}$$