Spring 2009, Aaron Lanterman

ECE 6279: Spatial Array Processing Homework 6

Due: Friday 3/6/09 at the *start* of class - homeworks turned in later in the hour may be penalized at my discretion

Late due date (30 point penalty): Monday 3/9/09

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, your solutions should be your own work and should be written up by yourself; feel free to discuss things, but don't be looking at someone else's paper when you are writing your solution. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

Looking at solutions to homeworks and quizzes of previous offerings of ECE6279 is expressly forbidden.

A 30 point penalty will be assessed on late homeworks (even homeworks turned in later the day it is due); I will distribute solutions shortly after class on Monday, so I will not accept solutions after that. If you cannot make a class, please make arrangements to get your homework to me ahead of time.

1 Required Problems

1. We derived the weights for the MVDR beamformer as the solution to the optimization problem

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad \mathbf{e}^H \mathbf{w} = 1$$

   Suppose we assume there are N_s signals in white noise; denote the resulting subspace eigendecompositions as

$$\begin{array}{ll} \text{Signal + Noise Eigenvectors:} & \mathbf{V_{S+N}} = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_{N_s}] \\ \text{Noise Eigenvectors:} & \mathbf{V_N} = [\mathbf{v}_{N_S+1} \quad \ldots \quad \mathbf{v}_M] \end{array}$$

   Suppose we want to find the weights as given by the constrained optimization problem above, *but we also wanted to constrain the weights to lie within the signal+noise subspace.* To solve this augmented problem, we need to restate it in the form of our standard quadratic minimization problem with linear constraints:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad \mathbf{C}\mathbf{w} = \mathbf{c}$$

   (a) What should $\mathbf{C}$ and $\mathbf{c}$ be in this case? Assume that $\mathbf{e}$ lies in the signal+noise subspace. Hints: 1) $\mathbf{C}$ should be expressed in terms of the vector $\mathbf{e}$, the matrix $\mathbf{V_{S+N}}$ and/or the matrix $\mathbf{V}_N$. 2) If you have two orthogonal subspaces that span a complete space, way to make sure something lies in one subspace is to make sure it *doesn't lie in the other subspace.* 3) You will need to specify $\mathbf{C}$ and $\mathbf{c}$ as "stacks" of some kind in order to incorporate all the constraints.

(b) Compute are the resulting optimal weights $\mathbf{w}_\diamond$. Use various orthogonality properties to simplify your answer as much as possible.

(c) What common beamforming technique discussed in lecture does your answer in part (b) correspond to? Notice the above analysis provides another way to justify of this technique.

2. In this problem and the next problem, we will further explore the $m = 9$-element array from Homework #4; hence, you will be able to reuse a lot of your code from that homework. (For both this problem and the next problem, be sure to turn in a listing of your code. Obviously, you don't need to turn in a separate listing for every parameter variation given in each problem; just one will suffice.)

   In this problem, you will create some example graphs comparing the steered responses of the "power" of the conventional, MVDR, EV, and MUSIC algorithms as a function of θ. Plot the results of all four algorithms on a single graph. (To get the conventional result on a similar scale, you will need to divide by M^2.) For each scenario given below, present two graphs, one using the "ideal" covariance and one using an "empirical" covariance from a limited number of snapshots. Use the same number of snapshots for all the scenarios. Choose enough snapshots that you get decent results, but don't choose so many that the ideal and empricial results look the same. You will need to experiment to find a good number of snapshots.

   Consider two sources with power 2 and a noise power of 10. Set the true ϕ^0 angles for both targets to 60°, and plot your steered response as a function of θ for a fixed look angle $\phi = 60$. You will need to find good example θ^0 angles to use in your demonstrations via experimentation. For the EV and MUSIC algorithms, assume $N_s = 2$ (the correct number of targets); we'll explore the effect of assuming the wrong number of targets in the next problem.

   - Scenario 1: Present an example in which the sources can be resolved by the MVDR beamformer but not by the conventional beamformer.
   - Scenario 2: Now, by slowly separating the sources and looking at the results, present an example in which the sources are resolved by both the conventional and MVDR beamformer.
   - Scenario 3: Now, by starting with the θ^0 parameters you used in Scenario 1 and slowly moving the sources closer to each other, present an example in which the sources are not resolved by either the conventional or MVDR methods.

   Above, when I say "slowly move," I intend you to manually experiment with different values; there's no need to write any sort of complicated code to automatically find interesting values. The point of this problem is the intuition you will gain by playing around with different parameters.

3. Here we'll keep playing with that same 9-element array.

   We'll now dispense with the business of using the "ideal" covariance matrix. However, to make sure there's not too much variability from solution to solution, let's use

100 snapshots to form our covariance estimate. (There will still be some variability, so you may want to run the experiment a few times to watch overall trends and make sure you didn't get just one "lucky" case. You need only turn in one set of plots, though.)

Let's have two sources, one at $\phi_1^0 = 45°$ and $\theta_1^0 = 30°$, and another $\phi_2^0 = 45°$ and $\theta_2^0 = 60°$. Let's let each source have power 1/4 (so you'll want to multiply your steering vectors by 1/2 when making the "seen by array" variables), and let's make the noise have power 2. Notice we're really pushing our algorithms hard now; the signals are pretty weak compared to the noise.

To best show details, please plot everything on a decibel scale (i.e., it's time to pull out MATLAB's `log10` function).

(a) Plot the power of the steered response of the eigenvalue method beamformer and the MUSIC beamformer as a function of θ, for $0 \leq \theta < 360°$, for a fixed $\phi = 45°$. Do this assuming zero signals, one signal, two signals, and finally three signals.

To see the effects I want you to see, it will really help to put all the eigenvector method cases on one single plot, and all the MUSIC cases on another plot.

(Note: In this particular experiment, I wasn't able to see any big differences between the EV and MUSIC techniques. If you manage to find some, let me know!)

(b) What method does EV technique correspond to for the $N_s = 0$ case?

(c) Does the MUSIC technique give you anything useful for the $N_s = 0$ case?

(d) For each technique, how big does N_s need to be before you can clearly see both targets?

(e) What difference do you see between the $N_s = 2$ and $N_s = 3$ cases?

(f) Finally, plot the steered response of the Pisarenko harmonic decomposition beamformer as a function of θ, for $0 \leq \theta < 360°$, for a fixed $\phi = 45°$. Comment on the usefulness of the results.