Spring 2009, Aaron Lanterman

ECE 6279: Spatial Array Processing
Homework 7

Due: Friday 3/13/09 at the *start* of class - homeworks turned in later in the hour may be penalized at my discretion

Late due date (30 point penalty): Monday 3/16/09

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, your solutions should be your own work and should be written up by yourself; feel free to discuss things, but don't be looking at someone else's paper when you are writing your solution. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

Looking at solutions to homeworks and quizzes of previous offerings of ECE6279 is expressly forbidden.

A 30 point penalty will be assessed on late homeworks (even homeworks turned in later the day it is due); I will distribute solutions shortly after class on Monday, so I will not accept solutions after that. If you cannot make a class, please make arrangements to get your homework to me ahead of time.

Note the late turnin is over spring break; if you need to use it contact me to make arrangements for turning it in.

1 Required Problems

1. On the website, you will find a file called `hw7_data09.mat`. This consists of a MATLAB array called `real_data` that has 128 snapshots from a linear array of 32 equally spaced elements (with half-wavelength spacing) along the x-axis, centered around the origin. The scene contains four sources, all with azimuthal angles $\theta = 0$; you may assume that the number of sources is known. The data was simulated using the usual independent, identically distributed electronic receiver noise.

   (a) Estimate the incident angles ϕ using root MUSIC.

   (b) Estimate the incident angles ϕ using total least squares ESPRIT.

   (Note: It wouldn't hurt to go ahead and estimate the angles using one of the techniques you've already implemented, such as regular MUSIC, the Eigenvalue method, MVDR, or whatever. This isn't required, but it might provide a "sanity check" on your answers.)

   As usual, please include listings of your programs.

2. In class we derived an expression for the optimal weights derived from constrained optimization algorithms that employ uncertainty constraints on the signal model:

$$\mathbf{w}_\diamond = -\lambda_1 \left(\mathbf{R} - \frac{\lambda_1^2}{\lambda_2} \mathbf{I} \right)^{-1} \mathbf{e} \tag{1}$$

However, we did not fully explore the difficulties of explicitly finding the Lagrange multipliers associated with this problem.

(a) Show that $\mathbf{w}_\diamond^H \mathbf{R} \mathbf{w}_\diamond = -\lambda_1$, thereby indicating that this multiplier is negative. (Hint: it's easier to use the gradients of the Lagrangian and the constraints than it is to use equation (1).)

(b) Assuming that the spatial correlation matrix has the simple form $\mathbf{R} = \sigma^2 \mathbf{I}$, use part (a) and the equation (1) to show that

$$M\sigma^2 = \left(\sigma^2 - \frac{\lambda_1^2}{\lambda_2} \right)^2 / (-\lambda_1).$$

(c) Use the constraints and the gradients of the Lagrangian to show that under the same conditions as in part (b), the other simultaneous equation for the Lagrange multipliers is

$$\lambda_1 = -\sigma^2 \epsilon^2 \left(\frac{\lambda_2^2}{\lambda_1^2} \right).$$