


ECE4893A/CS4803MPG:
MULTICORE AND GPU PROGRAMMING FOR VIDEO GAMES




Arbitrary Rotations
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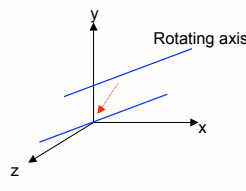


Rotation along an arbitrary axis

1. Transform the axis to cross origin (0,0,0)
2. Transform the axis around X onto the XZ plane
3. Transform the axis around Y onto the Z axis
4. Perform rotation (angle= θ)
5. Inverse transform of 3.
6. Inverse transform of 2.
7. Inverse transform of 1.



Step 1: Translate to (0, 0, 0)




Rotating axis

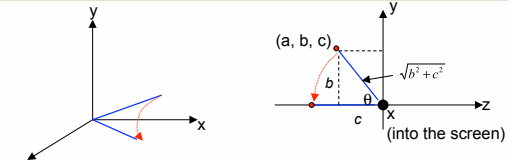
$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -T_x & -T_y & -T_z & 1 \end{bmatrix}$$

$$T_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

- For easier viewing, I use RHS.



Step 2: Rotate around X, project to XZ



(a, b, c)


(into the screen)

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

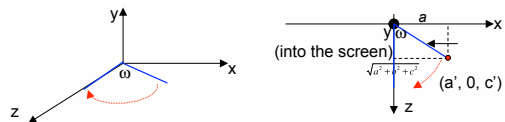
$$T_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos \theta = \frac{c}{\sqrt{b^2 + c^2}}$ and $\sin \theta = \frac{b}{\sqrt{b^2 + c^2}}$ $\cos \theta = \frac{c}{\sqrt{b^2 + c^2}}$ and $\sin \theta = \frac{b}{\sqrt{b^2 + c^2}}$

- Let's pick up a random point (a, b, c) on the axis to figure out $\cos \theta$



Step 3: Transform around Y onto the Z axis



$$T_3 = \begin{bmatrix} \cos \omega & 0 & -\sin \omega & 0 \\ 0 & 1 & 0 & 0 \\ \sin \omega & 0 & \cos \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^{-1} = \begin{bmatrix} \cos \omega & 0 & \sin \omega & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \omega & 0 & \cos \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos \omega = \frac{\sqrt{b^2 + c^2}}{\sqrt{a^2 + b^2 + c^2}} \text{ and } \sin \omega = \frac{a}{\sqrt{a^2 + b^2 + c^2}} \quad \cos \omega = \frac{\sqrt{b^2 + c^2}}{\sqrt{a^2 + b^2 + c^2}} \text{ and } \sin \omega = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

Step 4: Perform Rotation (angle=θ) on Z

$$T_4 = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combine all matrices with steps 5, 6, 7

$$[x', y', z', 1] = [x, y, z, 1] \cdot T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_3^{-1} \cdot T_2^{-1} \cdot T_1^{-1}$$