On the state liveness of some classes of guidepath-based transport systems and its computational complexity ⋆

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Abstract

Guidepath-based transport systems is a pertinent abstraction for the traffic that is generated in many contemporary applications, ranging from industrial material handling and robotics, to computer game animations and the qubit transport systems employed in quantum computing. A particular problem that must be effectively addressed for the systematic operation of these systems, is the preservation of their "liveness", i.e., the preservation of the ability of the system agents to complete their current assignments and engage successfully to similar assignments in the future operation of the system. This paper provides a systematic and comprehensive characterization of the notion of "liveness" for the entire spectrum of the aforementioned transport systems, and it further investigates the implications of this characterization for the deployment of maximally permissive liveness-enforcing supervision for the underlying traffic. It is shown that the computational complexity of the sought supervisors is contingent upon certain structural and operational attributes of the considered transport systems, that define, thus, a useful taxonomy for these environments. The paper proposes effective and efficient liveness-enforcing supervisors for each member of this taxonomy. Furthermore, the concluding part of the paper indicates how the obtained results can be integrated in a broader control framework for the considered transport systems that will also address time-related performance considerations for these environments, like the maximization of their throughput.

Key words: Guidepath-based Transport Systems; Liveness; Supervisory Control; Discrete Event Systems

1 Introduction

Guidepath-based transport systems and the problem of their liveness enforcing supervision: This paper concerns the traffic that is generated by a set of agents circulating on a connected graph which is known as the "(supporting) guidepath network". The "mission" trips of these agents on the guidepath network are specified by edge sequences that must be visited by the agents in the indicated order. Furthermore, during their trips to these destinations, the agents must observe certain regulations that are dictated by safety considerations, and essentially stipulate that two agents cannot cohabitate on the same edge of the guidepath network at any point in time during their trips. This stipulation is enforced by a traffic coordinator, and it turns the agent traveling towards their various destinations into a *sequential resource allocation process* (Reveliotis (2017)), with the negotiated resources being the edges – also known as "zones" – of the guidepath network. A complete characterization of the structure and the traffic dynamics of the considered transport systems is provided in Section 2.

From an application standpoint, the traffic problems outlined in the previous paragraph arise naturally in the real-time operations of various automated unit-load material handling (MH) systems, like the AGV, the overhead monorail and the complex crane and gantry systems that are used in many production and distribution facilities (Heragu (2008), Weiss (1996)). They also arise in the physical medium that implements the various elementary operations taking place in the context of quantum computing (Daugherty et al. (2019), Daugherty (2017)). In addition, similar guidepath-based traffic models have drawn recently the attention of the robotics community (e.g., Standley & Korf (2011), Sajid et al. (2012), Yu & LaValle (2016), Ma et al. (2016)), while, in the past, they have been studied even by the broader Computer Science community in the context of some classical games like the, so called, "15-puzzle" where 15 uniquely numbered "pebbles" located on a 4×4 grid have to be re-arranged in the row-major order by "peb-

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ble sliding" through the single unoccupied vertex of the grid (Wilson (1974), Kornhauser et al. (1984)).

A primary concern for the zone allocation function that takes place in the various instantiations of the aforementioned transport systems, is the establishment of a high throughput for their operations, through the facilitation of expedient traveling of the running agents to their various destinations. This objective is attained through (i) a pertinent coordination of the agent traversal of the various contested edges, and (ii) the effective utilization of the routing flexibility that is defined by the topology of the underlying guidepath network (Daugherty et al. (2019), Reveliotis (2019)). But an additional important concern for the traffic coordinator and the corresponding resource allocation process, is to preserve the *"liveness"* of the generated traffic, i.e., the ability of all the system agents to complete their current assignments and engage successfully to similar assignments in the future operation of the system. In the considered transport systems, this ability can be compromised by a potential formation of deadlocks and livelocks among the traveling agents. Hence, the system controller must restrict the system traffic so that no such formation ever takes place in it. Furthermore, this restriction must be of a minimal nature, so that it does not compromise unnecessarily the time-based performance of the system.

A brief, critical review of the literature on the notion of "liveness" of the considered transport systems and the corresponding supervisory control problem: In fact, the issues of liveness and liveness-enforcing supervision for various instantiations of the considered transport systems have already drawn the attention of the control-systems community. In particular, traffic coordination of AGV systems for deadlock avoidance has been a "pet" application for researchers that work on supervisory control (SC) of Discrete Event Systems (DES); the works of Krogh & Holloway (1991), Brandin (1996), Wonham (2006), Girault et al. (2016) provide some indicative examples of this activity. Furthermore, the same problem of deadlock avoidance in AGV systems has a prominent position in the research activity of a particular group of researchers within the DES community that works on the broader problem of liveness-enforcing supervision of complex resource allocation systems (RAS) (Reveliotis (2017)); some indicative examples coming from this line of work are those presented in Reveliotis (2000), Wu & Zhou (2001), Fanti (2002), Roszkowska & Reveliotis (2008), Reveliotis & Roszkowska (2010).

But those past works and their results have appeared in the corresponding literature in a fragmented and scattered manner. More specifically, the first set of the aforementioned works have used the problem of deadlock avoidance in guidepath-based transport systems primarily as an "application example" that demonstrates the efficacy and the application potential of some more general DES SC theory developed in those works. On the other hand, the works that focus more explicitly on the particular problems of liveness assessment and enforcement for the considered transport systems, tend to target specific configurations of these systems, customizing their results to the particular features of these configurations, and seeking to provide practical solutions that are synthesized around these features rather than a complete formal theory. Furthermore, while it is generally acknowledged that the resulting SC problems are "hard", there has been only limited effort to formally characterize the complexity of these problems and the factors that shape this complexity.

The intended contribution and the basic structure of this work: This work seeks to address the theoretical gaps that were described in the previous paragraph by (i) providing a systematic investigation of the aforementioned problems of assessing and enforcing liveness in the considered transport systems, in the least restrictive manner, and (ii) characterizing the computational complexity of these problems. In some more specific terms, the developments that are presented in this manuscript support the following triple role: (a) First, they define a unifying framework for the investigation of the targeted liveness-related problems across the various instantiations of the considered transport systems, and use this framework as an instrument for the further organization of the corresponding results that already exist in the literature. (b) In addition, they complement those past results with new results regarding the considered traffic-liveness problems and their computational complexity that pertain to guidepath-based transport systems not addressed by the past literature. (c) Finally, they also identify a number of additional open problems that should get the attention of the corresponding research community.

As it will be revealed in the subsequent developments, the presented results are strongly contingent upon certain structural and operational characteristics of the underlying transport system. Among these characteristics, some of the most prominent ones are: (i) the ability of an agent to freely reverse its motion on any given edge of the guidepath network; (ii) the availability of a "depot" location where the agents retire upon the completion of their mission trips; and (iii) the degree of prespecificity of the routes to be followed by the traveling agents as they try to reach their target nodes. These three attributes define the three "dimensions" of a taxonomy that will be instrumental for the organization and exposition of the presented material. We elaborate further on these three attributes and the induced taxonomy in Section 2, where we provide a more systematic description of the guidepath-based transport systems considered in this work.

Finally, in view of the above positioning of the paper content and its intended contribution, the rest of it is organized as follows: Section 2 introduces the guidepath-based transport systems considered in this work in terms of their structural and operational elements, formalizes the taxonomy of these systems along the lines that were discussed in the previous paragraph, and introduces the basic notion of traffic liveness. Sections 3 and 4 provide the main results of the paper on liveness assessment and preservation, organizing them along the primary axes of the aforementioned taxonomy. Section 5 concludes the paper, and suggests some directions for future work. In addition, the main results of the paper are summarized in a structured manner in an appendix, for the readers' convenience. On the other hand, due to the page limitations that are imposed to this publication, some of the proofs of the supported results are only sketched in the manuscript, while the complete versions of these proofs are provided in an electronic supplement that is accessible through the author's personal website. Finally, we also notice, for completeness, that a preliminary, much more concise version of this work, has appeared in Reveliotis (2018).

2 The considered guidepath-based transport systems, a useful taxonomy, and the fundamental notion of traffic liveness

The basic abstracting ingredients of the guidepath-based transport systems considered in this work: The basic structure of the guidepath-based transport systems considered in this work is formally represented by a tuple $\langle G, \mathcal{A} \rangle$, where:

(1) $G = (V, E)$ is an undirected, connected graph [1] that represents the supporting "guidepath network". More specifically, each edge $e \in E$ of graph G represents a "zone" of the underlying guidepath network that can be traversed by a traveling agent in either direction, but it cannot be occupied by more than one agent at any time. Hence, the guidepath edges $e \in E$ are "reusable resources" of the considered traffic system, in the spirit of Reveliotis (2017), and their allocation to the system agents is dynamically controlled by a traffic coordinator.

(2) $\mathcal{A} = \{a_1, \ldots, a_n\}$ is the set of agents that circulate in this system. Agents execute "mission" trips that are externally specified and are elaborated in a later part of this section. Furthermore, these "mission" trips are continually updated as new "service requirements" are dynamically posed to the underlying transport system.

[1] We remind the reader that a graph G is *undirected* if each of its edges does not have a sense of direction associated with it, and that an undirected graph G is *connected*, if for every pair of vertices v_1, v_2, there is a "path" of edges, π, that connects these two vertices.

Example: As a concretizing example of the above abstraction, consider the familiar Automated Guided Vehicle (AGV) systems that are used in various production and distribution facilities (Heragu (2008)). In this case, the traveling agents $a \in \mathcal{A}$ are the system AGVs, which are used to transport materials among the various locations of the facility. At any point in time, each vehicle might be assigned a sequence of such transport tasks that must be executed in the specified order by visiting the corresponding pickup and delivery locations. The transport-task sequences associated with each vehicle are also dynamically updated as new transport requirements arise in the underlying system.

The guidepath network for these AGV systems is defined either physically (e.g., through some colorful duct tape that is deployed on the shop-floor and must be traced by the vehicle scanners), or virtually (e.g., through some radio signals that must be traced and processed by the vehicle sensors). The exact specification of the system guidepath network, in any of the aforementioned manners, intends to confine the AGV traffic in particular corridors and, in this way, separate it from the remaining activity that takes place in the surrounding environment, due to safety and other efficiency considerations. Finally, in an effort to avoid collisions among the traveling vehicles, the various corridors of the guidepath network are split into "zones" that must be occupied by at most one AGV at any point in time; these zones define the edges $e \in E$ of the abstracted guidepath network G.

A classification of the considered transport systems: Next, we introduce some additional attributes that qualify further the operation of the considered transport systems, and induce a taxonomy for these systems that will help us structure the investigation of the control problems that are considered in this work.

I. "Open" vs. "closed" guidepath-based transport systems: In many practical instantiations of the considered transport systems – including most of the AGV systems that were described in the above example – the guidepath network avails of a "depot" location where the system agents can retire upon the completion of their running missions, and possibly receive some maintenance service, recharge their batteries, etc. For the representational needs of this work, we shall model this "depot" location by augmenting the zone-modeling edge set E with a set of $|\mathcal{A}|$ self-loop edges, one for each agent $a \in \mathcal{A}$, all connected to the same vertex v_h. Vertex v_h will be called the *"home" vertex*. Also, the self-loop edge corresponding to agent a will be denoted by $e_h(a)$, will be used exclusively by agent a, and it will referred to as the *"home" edge* of agent a. [2]

[2] We want to emphasize that the modeling of the "depot" location through the vertex v_h and the edges $e_h(a)$, $a \in \mathcal{A}$, as described above, intends to capture the fact that the agents

In the following, guidepath-based transport systems that possess the "depot" location – or, equivalently, the "home" structure – that was described in the previous paragraph, will be characterized as *"open"*; the remaining ones will be said to be *"closed"*. Furthermore, the presence of a "home" structure introduces a "regenerative" element in the dynamics of the underlying traffic, which will further function as a decomposing mechanism in the study of the corresponding "liveness"-related problems that are considered in this work. Hence, the above classification of the considered transport systems into "open" and "closed" will play a significant role in the developments of Sections 3 and 4.

II. "Reversible" vs. "irreversible" guidepath-based transport systems: In some of the considered transport systems, agents $a \in \mathcal{A}$ can reverse the direction of their motion in their currently allocated edge e, while in the remaining ones such motion reversal is not possible. For instance, in many of the typical AGV systems that have been deployed in various industrial settings, vehicles are effectively moving only in one direction of their longitudinal axis. And even in those cases where the system vehicles have a substantial capability of "backing up", such an operation might be rendered cumbersome and unsafe due to the spatial constraints that are imposed by the vehicle loads, their sensing capabilities, etc. On the other hand, there are also robotic applications where reversibility of the agent motion is practically feasible. Furthermore, such motion reversibility is naturally supported in the guidepath-based transport systems that abstract the qubit traffic in quantum computing (Daugherty et al. (2019), Daugherty (2017)), and the various games that have been studied in the CS context (Wilson (1974), Kornhauser et al. (1984)).

In the following, we shall refer to the guidepath-based transport systems that support reversibility of the agent motion in their current zone as *"reversible"*, and to the remaining ones as *"irreversible"*. For the purposes of the "liveness"-related studies that are pursued in this work, "irreversibility" is important because it introduces a potential for the development of deadlocks and livelocks. The simplest example of such a deadlock is that caused by a number of traveling agents that converge to a single junction from all possible directions. On the other hand, "reversible" guidepath-based transport systems can cope with such developing deadlocks by "backtracing" their moves that led to those formations. Hence, the above classification of the considered transport systems into "reversible" and "irreversible" will also have a prominent role in the subsequent developments.

III. "Statically" vs. "dynamically"-routed guidepath-based transport systems: A natural, formal characterization of the "mission" trip for any traveling agent $a \in \mathcal{A}$ is through an edge sequence $\Sigma_a = \langle e_1, e_2, \ldots, e_k \rangle$; $e_i \in E$, $\forall i = 1, \ldots, k$, that constitutes the set of zones that must be visited by agent a in the corresponding order. Furthermore, in open transport systems, it is implicitly assumed that the last edge to be visited by agent a in its current mission trip, is the corresponding "home" edge $e_h(a)$. And as already discussed, agent "mission" trips can be extended dynamically, as new service requirements arise in the underlying system.

When traveling from edge e_i to edge e_{i+1}, agent a will follow a walk [3] between these two edges that is consistent with the operational assumptions of the underlying transport system. In many cases, the exact determination of the aforementioned walks will take place in real-time, and it will be contingent upon the prevailing traffic conditions. Guidepath-based transport systems where the agent "mission" trips are fully defined and maintained in this way, are characterized as *"dynamically routed"*. Furthermore, it can be argued that dynamic routing is the most natural routing scheme for the considered transport systems. But in order to provide a comprehensive treatment of the notion of "liveness" and its complexity in the considered transport systems, we shall also consider an alternative routing scheme where the agent routes are completely predetermined by an external entity. In this case, the "mission" trip of each traveling agent $a \in \mathcal{A}$ is an externally specified walk W_a that defines completely the sequence of the edges $e \in E$ that this agent must traverse till the completion of its trip; such a routing scheme will be characterized as *"static"* in the following. Furthermore, in the following, we shall restrict the study of "static" routing schemes into the class of "open" transport systems only, and we shall further assume that each of the aforementioned walks $W_a,\ a \in \mathcal{A}$, implicitly terminates at the corresponding "home" edge $e_h(a)$. [4]

The notion of "traffic liveness" in the consid-

retiring in this location do not interfere with the traffic that takes place in the main guidepath network that is defined by the edge set E of the graph G that was introduced at the beginning of this section. The presented model is just one of a number of models that can capture this effect, and for the purposes of the subsequent analysis, all these models would be equivalent.

[3] We remind the reader that a walk in an undirected graph G is a sequence $\langle v_0, e_1, v_1, \ldots, v_{i-1}, e_i, v_i, \ldots, v_{k-1}, e_k, v_k \rangle$ where, for all $i = 1, \ldots, k$, edge e_i is incident upon the vertices v_{i-1} and v_i.

[4] The notion of "static routing" is not easily defined in closed guidepath-based transport systems, since agents will still have to move around after the completion of their designated "mission" trips in order to permit the completion of the "mission" trips of the other agents. Furthermore, static routing might over-constrain the traffic dynamics in these environments, to the point that it might not be possible to generate feasible routing plans.

ered transport systems and the role of the introduced taxonomy: In the operational regimes for the giuidepath-based transport systems that were outlined in the previous parts of this section, *traffic liveness* can be naturally defined as *the preservation of the ability of each traveling agent to complete its current "mission" trip, and engage successfully in similar "mission" trips in the future operation of the system.*

Furthermore, as explained in the earlier parts of this section, loss of liveness can result from an irreversible motion of the system agents within their allocated edges, and the analysis of the corresponding traffic dynamics can be affected by additional attributes of the underlying transport system like the "open" or the "closed" structure of its guidepath network, and the dynamic or static nature of the routing scheme that is supported by it. In the rest of this paper, we provide a formal characterization of the "liveness" concept as it materializes in the various classes of the taxonomy of the considered guidepath-based transport systems that is induced by the aforementioned attributes, and we also investigate the corresponding supervisory control (SC) problems of liveness assessment and enforcement. We organize these developments into two major sections, with the first section focusing on dynamically routed guidepath-based transport systems, and the second one dealing with their statically routed counterparts.

3 Liveness characterizations and enforcement for dynamically routed guidepath-based transport systems

3.1 Preamble

An abstracting finite state automaton: We begin the developments of this section, by formalizing further the basic motion dynamics of dynamically routed guidepath-based transport systems by means of a finite state automaton (FSA) $\Phi = \langle S, Q, f, s_0 \rangle$ (Cassandras & Lafortune (2008)). A formal definition of the state s of this automaton that serves the needs of the subsequent analysis, is as follows:

Definition 1 *At any point in time, the state s of the automaton Φ that will represent the untimed dynamics of the* dynamically routed *guidepath-based transport systems introduced in Section 2, is defined by the following two elements:*

(1) The placement of the system agents $a \in \mathcal{A}$ on the edges of the guidepath network G.
(2) The direction of motion of each agent $a \in \mathcal{A}$ in its allocated edge.

In the following, the distribution of the agents $a \in \mathcal{A}$ over the edges of the guidepath network G, in any given state s, will be formally represented by the function $\epsilon(\cdot; s) : \mathcal{A} \rightarrow \hat{E}$, where $\hat{E} = E$, in the case of closed systems, and $\hat{E} = E \cup \{e_h(a) : a \in \mathcal{A}\}$, in the case of open systems. Also, the direction of motion of an agent $a \in \mathcal{A}$ with $\epsilon(a; s) = \{i, j\} \in E$, can be formally expressed by one of the two ordered pairs (i, j) and (j, i) (i.e., by assigning a sense of direction to the underlying undirected edge $\{i, j\}$).

The event set Q that advances the state s of the considered automaton Φ, contains all those events q that advance a single agent $a \in \mathcal{A}$ from its current edge $\epsilon(a; s)$ to a free neighboring edge e', under the further condition that this advancement is also compatible with the direction of motion of the corresponding agent a on its current edge $\epsilon(a; s)$. Furthermore, in the case of reversible systems, an event $q \in Q$ might simply reverse the direction of motion of an agent $a \in \mathcal{A}$ in its current edge $\epsilon(a; s)$. Finally, all events $q \in Q$ are supposed to be controllable by the supervisory controller that coordinates the traffic of the considered transport system.

The state transition function $f : S \times Q \rightarrow S$ of the automaton Φ provides a formal representation of the transitional dynamics that are implied by the above definitions of state s and the event set Q. Furthermore, following Cassandras & Lafortune (2008), we assume f to be a partial function that is defined only for those (s, q) pairs for which the corresponding state transition is feasible under the operational assumptions that define the complete dynamics of the underlying guidepath-based transport system. We also extend f, in the standard manner, to the set $S \times Q^*$, where Q^* denotes the Kleene closure of Q (i.e., Q^* consists of all the finite sequences of elements of Q, including the empty sequence ε). And we use the notation $R(s)$ to denote the states s' of Φ that are reachable from a given state s through the dynamics that are defined by the extended function f; i.e., $\forall s' \in S,\ s' \in R(s) \iff \exists \sigma \in Q^* : s' = f(s, \sigma)$.

Finally, in the tuple that defines the considered FSA Φ, s_0 denotes a generic initial state; this state will be given different interpretations in different parts of the subsequent developments.

A formal characterization of liveness for dynamically routed guidepath-based transport systems: The operational definition of liveness that was provided in the closing part of the previous section, when combined with the arbitrary structure of the "mission" trips that can be assigned to the system agents, motivate the following characterization of "liveness" for the traffic dynamics that are modeled by automaton Φ:

Definition 2 *Consider the automaton Φ abstracting the unitmed dynamics of a dynamically routed guidepath-based transport system, and let $s_0 \in S$ be an arbitrary initial state for this automaton. Then, the traffic that is* *represented by automaton Φ is* live*, if and only if*

$$\forall s \in R(s_0),\ \forall a \in \mathcal{A},\ \forall e \in E,\ \exists s' \in R(s) : \epsilon(a; s') = e$$

State liveness and maximally permissive liveness-enforcing supervision for the considered transport systems: While Definition 2 is motivated naturally from the operational dynamics of the considered transport systems, it is not straightforwardly testable on any given instantiation of these systems. A first step to develop a more straightforward test is provided by the following proposition.

Proposition 3 *Consider the FSA Φ abstracting the dynamics of a dynamically routed guidepath-based transport system, and let $s_0 \in S$ be an arbitrary initial state for this automaton. Then, the resulting traffic that is represented by automaton Φ is* live *if and only if, for every state $s \in R(s_0)$, the state transition diagram (STD) of the corresponding subspace $R(s)$ contains a strongly connected component $\Psi(s)$ that satisfies the following condition:*

$$\forall (a, e) \in \mathcal{A} \times E,\ \exists s' \in \Psi(s) : \epsilon(a; s') = e$$

A complete proof for this proposition can be found in the electronic supplement for this paper. A brief exposition of the basic logic of this proof is as follows: The sufficiency of the condition of Proposition 3 for the liveness of the traffic of the underlying transport system follows immediately from the content of this condition, the controllability of the considered dynamics, and Definition 2. On the other hand, in order to prove the necessity part of Proposition 3, we consider the directed graph $\mathcal{G}$ that is defined by the maximal strongly connected components of the STD of the considered FSA Φ and their connectivity. By its definition, the digraph $\mathcal{G}$ is acyclic. Furthermore, if the condition of Proposition 3 does not hold, then, at any node n of $\mathcal{G}$ (or, equivalently, maximal strongly component Ψ of the STD of Φ), we shall be able to identify a pair $(a, e) \in \mathcal{A} \times E$ such that there is no state s in Ψ with $\epsilon(a; s) = e$. Then, the only way that we can possibly satisfy a request for placing the considered agent a on the corresponding edge e, is by moving to some state s' belonging to some node n_1 of the subgraph of digraph $\mathcal{G}$ that emanates from node n. We can repeat the above argument at the reached node n', generally with a different pair (a', e'), reaching a new node n'' in the subgraph of $\mathcal{G}$ that emanates from node n'. But since digraph $\mathcal{G}$ is finite and acyclic, any such sequence of advancements through it will be finite, and therefore, eventually we shall reach a state $\tilde{s}$ and a pair $(\hat{a}, \hat{e})$ that will be unattainable from state $\tilde{s}$.

The result of Proposition 3 motivates naturally the following definition:

Definition 4 *A state $s \in S$ of the FSA Φ modeling a dynamically routed guidepath-based transport system $\langle G, \mathcal{A} \rangle$ will be characterized as* live *if and only if it satisfies the condition of Proposition 3. The set of live states of FSA Φ will be denoted by S_l.*

Furthermore, for any initial state $s_0 \in S$, the maximally permissive liveness-enforcing supervisor (LES) *for this transport system is the supervisor that admits any state $s \in R(s_0)$ if and only if $s \in S_l$.*

On the other hand, assessing state liveness for any given traffic state s through the characterization of Proposition 3 requires a global view of the corresponding subspace $R(s)$, and therefore, this test will not be easily tractable for most practical instantiations of the considered transport systems. Hence, in the remaining parts of this section, we investigate possible restatements of the liveness condition of Proposition 3 that take into consideration additional structural and operational attributes of the underlying transport system, and, in this way, they might end up being more easily testable than the original condition of this proposition.

A necessary condition for the traffic liveness of irreversible, dynamically routed guidepath-based transport systems: We close this subsection by stating a structural condition that must be satisfied by the guidepath graph G in order to be able to preserve traffic liveness in any *irreversible*, dynamically routed guidepath-based transport system. This condition is formally stated as follows:

Condition 1 *The guidepath graph G has a minimal vertex degree of 2.*[5]

The necessity of this condition for the traffic liveness of any irreversible, dynamically routed guidepath-based transport system results from the fact that, under the irreversibility assumption, any agent accessing a vertex v of G of degree 1 would get deadlocked at this location.

3.2 An alternative characterization of state liveness for open, dynamically routed guidepath-based transport systems

Deriving an alternative characterization of state liveness for open, dynamically routed guidepath-based transport systems: In the case of open, dynamically routed guidepath-based transport systems, we can also define the notion of the *"home" state s_h* in the semantics of the underlying FSA Φ, as follows:

[5] We remind the reader that the degree of a vertex v of an undirected graph G is the number of edges that are incident to vertex v.

Definition 5 *The* "home" state s_h *of an open, dynamically routed giuidepath-based transport system* $\langle G, \mathcal{A} \rangle$ *is the state where* $\forall a \in \mathcal{A},\ \epsilon(a; s_h) = e_h(a)$.

The "home" state s_h plays a central role in the characterization of state liveness in open, dynamically routed giuidepath-based transport systems. We proceed to establish this result, starting with the following proposition:

Proposition 6 *The "home" state s_h of a reversible, open, dynamically routed guidepath-based transport system is live. Also, the "home" state s_h of an irreversible, open, dynamically routed guidepath-based transport system that satisfies Condition 1 is live.*

A formal proof for the results of Proposition 6 can be provided by showing that, under the stated conditions, any single agent $a \in \mathcal{A}$ can be taken to any edge $e \in E$ of the underlying guidepath graph G and returned successfully to its "home" edge $e_h(a)$. Then, state s_h satisfies the liveness condition of Proposition 3. Filling in the details of this argument is quite straightforward, and it is left to the reader.

Proposition 6 subsequently leads to the following theorem that provides an alternative, very practical characterization of liveness in open, dynamically routed guidepath-based transport systems

Theorem 7 *A state $s \in S$ of an open, dynamically routed, reversible guidepath-based transport system is live if and only if it is co-reachable to the home state s_h (i.e., $s_h \in R(s)$). Also, a state $s \in S$ of an open, dynamically routed, irreversible guidepath-based transport system is live if and only if it is co-reachable to the home state s_h and the underlying guidepath graph G satisfies Condition 1.*

A statement and proof of the result of Theorem 7 for the case of open, dynamically routed and irreversible guidepath-based transport systems appeared recently in Reveliotis & Masopust (2019*b*). We also provide a complete proof for Theorem 7 in the electronic supplement for this paper. The necessity of the co-reachability condition of Theorem 7 for the liveness of the considered state s results from (i) the fact that, according to Definition 2, any agent a with $\epsilon(a; s) \neq e_h(a)$ must be able to reach edge $e_h(a)$, and (ii) the further realization that any such advancement of agent a to edge $e_h(a)$ can take place without relocating any other agent a' with $\epsilon(a'; s) = e_h(a')$. The sufficiency of the co-reachability condition of Theorem 7 for the liveness of the considered state s results from the liveness of state s_h that was established in Proposition 6.

Implications of Theorem 7 for the liveness and the liveness-enforcing supervision of *reversible*, open, dynamically routed guidepath-based transport systems. The next corollary is a further important implication of Theorem 7.

Corollary 8 *In an open, dynamically routed, reversible guidepath-based transport system, every state $s \in S$ is live.*

To the best of our knowledge, a first explicit statement and proof of the result of Corollary 8 appeared only recently in Daugherty et al. (2017, 2019), and the proof is reproduced in the electronic supplement of this paper. The proof relies on the following two facts: (i) In open, dynamically routed, reversible guidepath-based transport systems, it is always possible to reach state s_h from any state s by routing agents a with $\epsilon(a; s) \neq e_h(a)$ to their corresponding edges $e_h(a)$ one at a time, giving priority to those agents that are closer to vertex v_h. (ii) Also, in the considered class of transport systems, we can always reach any valid traffic state s from state s_h, by routing agents $a \in \mathcal{A}$ to their corresponding destinations one at a time, starting with those agents that are heading to the furthest destinations.

From a more practical standpoint, Corollary 8 further implies that in open, dynamically routed, reversible guidepath-based transport systems, preservation of the traffic liveness is immediately guaranteed by the system structure and dynamics, and, therefore, there is no need for any externally imposed LES.

Implications of Theorem 7 for the liveness and the liveness-enforcing supervision of *irreversible*, open, dynamically routed guidepath-based transport systems. As already remarked in the earlier parts of this paper, in this class of guidepath-based transport systems, traffic liveness can be compromised by the formation of deadlocks among the traveling agents. We formally define this notion of "deadlock" as follows:

Definition 9 *In the dynamics of the FSA Φ that models a dynamically routed, irreversible guidepath-based transport system, a set $\mathcal{A}^D \subseteq \mathcal{A}$ of the system agents is in deadlock if every possible advancement of each agent $a \in \mathcal{A}^D$ in the underlying guidepath graph G is blocked by the presence of another agent $a' \in \mathcal{A}^D$.*

Figure 1(a) depicts such a deadlock formation. Furthermore, irreversible, open, dynamically routed guidepath-based transport systems will also possess an additional set of non-live traffic states that will not contain any deadlocks, but deadlocks will be unavoidable from these states. Figure 1(b) depicts such a deadlock-free but non-live traffic state.

Detection of deadlock in any given traffic state s is an easy task. A simple algorithm for this task will start with the given traffic state s, and it will iteratively scan

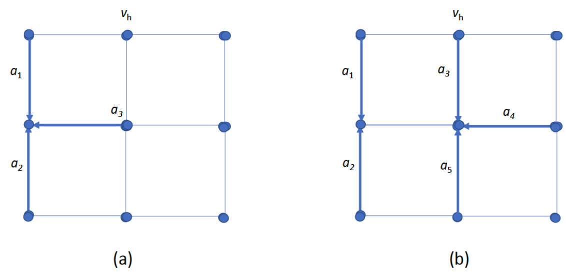


Fig. 1. The left part of the figure depicts a traffic state of an irreversible, open, dynamically routed guidepath-based transport system that contains a deadlock. In the adopted representation, the edge $\epsilon(a; s)$ of a traveling agent $a \in \mathcal{A}$ in the depicted traffic state s, and the direction of its motion in this edge, are jointly represented by a directed arc that is labeled by agent a. The right part of the figure depicts a state from the same class of systems that contains no deadlocks but it is not live, since from the depicted state, the formation of a deadlock is unavoidable. We also notice, for completeness, that in both parts of this figure we have depicted the "home" vertex v_h, but we have omitted the "home" edges $e_h(a),\ a \in \mathcal{A}$.

the considered traffic state in order to detect an agent $a \in \mathcal{A}$ that can advance to a neighboring edge under the applying zone-allocation protocol. Every time that such an agent is detected, it will be removed from the system, and its edge will be released for possible usage by the remaining agents. If all agents $a \in \mathcal{A}$ are removed through the aforementioned iterations, then the considered state s is deadlock-free. Otherwise, the set of agents that were not removed by the algorithm, define one or more deadlocks in s.

On the other hand, detecting a deadlock-free non-live state might not be an easy task. In fact, to the best of our knowledge, the computational complexity of the decision problem of assessing the state liveness of any given traffic state s of an irreversible, open, dynamically routed guidepath-based transport system remains an open problem. [6]

"Ordered" states and Banker's-type algorithms for open, dynamically routed, irreversible guidepath-based transport systems. The current lack of a polynomial algorithm for assessing traffic-state liveness for open, dynamically routed, irreversible guide-path-based transport systems, has been addressed by the corresponding research community through the adaptation to this problem of the notion of "ordered state" and Banker's algorithm (Dijkstra (1965), Reveliotis (2000)). These results enable computationally efficient liveness-enforcing supervision for the considered transport systems, at the expense of non-maximal permissiveness.

In the considered transport systems, an "ordered" traffic state is formally defined as follows:

Definition 10 *A state s of an irreversible, open, dynamically routed guidepath-based transport system satisfying Condition 1 is* "ordered" *if there exists an ordering $[\cdot] : \{1, \ldots, |\mathcal{A}|\} \to \mathcal{A}$, of the agent set $\mathcal{A}$, such that each agent $a_{[i]},\ i = 1, \ldots, |\mathcal{A}|$, can advance to its "home" edge $e_h(a_{[i]})$ from its current edge $\epsilon(a_{[i]}; s)$ while agents $a_{[j]},\ j = i+1, \ldots, |\mathcal{A}|$, maintain the original edges $\epsilon(a_{[j]}; s)$ that they held in state s.*

For further reference, we shall denote the set of ordered states by S_o. Establishing that any given state $s \in S$ is ordered, can be performed through the construction of an ordering for the traveling agents in that state that satisfies the conditions of Definition 10. The search for such an ordering of the agent set $\mathcal{A}$ can be performed in a "greedy" manner (i.e., without the need for any backtracking) since the placement of any agent $a \in \mathcal{A}$ at the "home" edge $e_h(a)$ increases the set of free edges that can be used by the remaining agents a' for reaching their "home" edges $e_h(a')$. The algorithmic details for organizing such a search scheme can be found in Reveliotis (2000), and as already noticed, this algorithm can be perceived as a (nontrivial) adaptation of Dijkstra's Banker's algorithm to the considered problem context.

Furthermore, in Lawley et al. (1998) it is shown how the set of ordered states that is admitted by any efficient realization of Banker's algorithm, can be effectively expanded into the complement state set $S \setminus S_o$ through *controlled partial search* that will guarantee the return to state set S_o within a bounded number of steps.

Finally, it is easy to see that the "home" state s_h, and also any state s that has only a single traveling agent on some edge $e \in E$, are ordered. This remark further implies that it is, indeed, possible to attain traffic liveness while operating within the set of ordered states, S_o. More specifically, a traffic controller that will start the underlying traffic system from its natural initial state s_h, and will use the algorithmic tools provided in Reveliotis (2000) and Lawley et al. (1998) in order to resolve the admissibility of any tentative transition to a new traffic state, will be able to maintain the operation of the underlying guidepath-based transport system in a subset S' of its state space S with the following properties: (a) $S_o \subseteq S' \subseteq S_l$; (b) S' is efficiently recognizable; and (c) the resulting supervision will ensure live operation for the generated traffic.

[6] Some recent partial results on this problem can be found in Reveliotis & Masopust (2019*b*, 2020, 2019*a*). These works have identified important special structure for the considered traffic states s that enables liveness assessment of polynomial complexity with respect to the size of the underlying transport system, and for the remaining cases, they also provide a liveness assessment algorithm with an empirical computational complexity that is expected to be very benign.

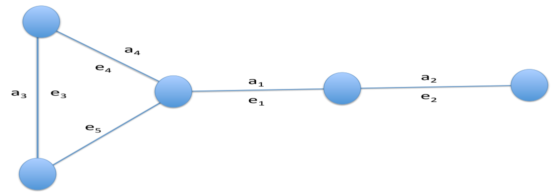


Fig. 2. A counter-example establishing that, for closed, dynamically routed, reversible guidepath-based transport systems, the condition $|\mathcal{A}| < |E|$ is not adequate for ensuring the ability of an agent $a \in \mathcal{A}$ to advance from its current edge e to any target edge e'.

3.3 *An alternative characterization of state liveness for closed, dynamically routed, reversible guidepath-based transport systems*

A structural condition necessary for the liveness of this class of transport systems: We start our discussion for this class of guidepath-based transport systems by noticing that, for any meaningful realization of these systems, we must have $|\mathcal{A}| < |E|$ since, otherwise, no agent motion is possible. This inequality implies that there is always a free edge in the guidepath network; to facilitate the subsequent discussion, we shall refer to such a free edge as a *"hole"*. Then, we have the following lemma:

Lemma 11 *Consider a traffic state s of a closed, reversible, dynamically routed guidepath-based transport system with $|\mathcal{A}| < |E|$, and an edge $e \in E$. Then, there is a state $s' \in R(s)$ in which edge e is a "hole".*

The gist of the argument that establishes the result of Lemma 11, is the observation that anyone of the nearest "holes" to edge e in state s can be "transferred" to edge e by advancing each agent on the path leading from edge e to the considered "hole" by one edge; the detailed formal proof is provided in the electronic supplement.

But while the condition $|\mathcal{A}| < |E|$ guarantees the effective move of a "hole" to any edge of graph G, it is not sufficient to ensure that any agent $a \in \mathcal{A}$ can move from its current location to a target destination. A counter-example establishing the truth of this statement is presented in Figure 2. In the depicted situation, agent a_1 wants to move to edge e_2, and it also holds that $|\mathcal{A}| = 4 < |E| = 5$. But it is easy to check that the required transfer of agent a_1 is not feasible.

The problem in the example of Figure 2 arises from the presence of the path e_1e_2. This path is characterized by the fact that all of its edges do not belong on any cycle[7] of the corresponding graph G, and in the following discussion, we shall characterize the maximal paths of graph G that possess this property as *"singular"*. Also, we shall denote the set of singular paths in graph G by $\mathcal{P}_S$, and for any path $p \in \mathcal{P}_S$, $|p|$ will denote the "length" of p as defined by the number of its edges.

Then, our main result for the considered class of guidepath-based transport systems can be stated as follows.

Theorem 12 *In the class of closed, reversible, dynamically routed guide-path-based transport systems, a sufficient condition guaranteeing that any agent $a \in \mathcal{A}$ can move from its current edge e to any other edge $e' \in E$ of the guidepath network G is that $|\mathcal{A}| \leq |E| - 1 - \max_{p \in \mathcal{P}_S}\{|p|\}$.*

The proof of Theorem 12 is by construction of an event sequence that will effect the requested agent transfer; the details of this construction are rather technical, and they are provided in the electronic supplement of this paper. Furthermore, in the case where $\mathcal{P}_S = \emptyset$, the resulting condition of Theorem 12 is also necessary for ensuring the ability of any agent $a \in \mathcal{A}$ to move from its current edge e to any other edge $e' \in E$ of the guidepath network G. On the other hand, when $\mathcal{P}_S \neq \emptyset$, the condition of Theorem 12 is only sufficient; characteristically, the reader can check that in the example of Figure 2, the circulating agents can reach any edge of the depicted guidepath graph as long as $|\mathcal{A}| \leq |E| - \max_{p \in \mathcal{P}_S}\{|p|\}$. Finally, the perusal of the proof of Theorem 12 further implies that the condition $|\mathcal{A}| \leq |E| - 1 - \max_{p \in \mathcal{P}_S}\{|p|\}$ is also necessary as long as there exists a maximal-length singular path p that connects two cyclical components, G_i and G_j, of graph G.

The implications of Theorem 12 for the liveness-enforcing supervision of the corresponding class of transport systems: The condition of Theorem 12 is a structural condition for the underlying guidepath-based transport system that can be validated off-line. Furthermore, once this condition is established, the construction of the event sequence that is sought in the proof of Theorem 12 also provides the necessary mechanism for transferring any agent $a \in \mathcal{A}$ from its current edge $\epsilon(a; s)$ to a target edge e'. Furthermore, this mechanism involves only the identification of (shortest) paths for the necessary transfers of agent a and of the "holes" that facilitate the agent motion during the various legs of its trip, and therefore, it is also computationally efficient.

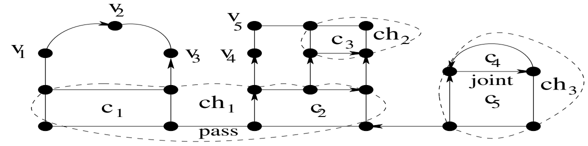
We close this subsection by noticing that the result of Theorem 12 resembles, in its basic structure, the result

[7] Following standard terminology of graph theory, in this work we define a cycle in an undirected graph as a simple path with coinciding starting and ending nodes. Furthermore, a path is simple if it does not revisit any of its vertices (except possibly the first and the last ones, in the case of a cycle).

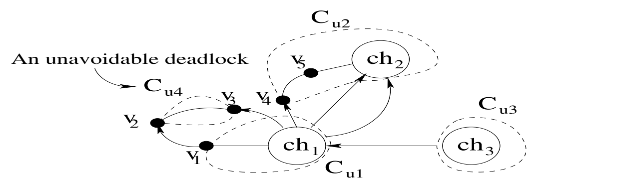
of Corollary 8 for the open, dynamically routed and reversible subclass of the considered transport systems: in both cases, once some structural condition for the underlying guidepath-based transport system has been established, the liveness of the generated traffic is immediately guaranteed.

3.4 An alternative characterization of state liveness for closed, dynamically routed, irreversible guidepath-based transport systems

As in the case of open and irreversible, dynamically routed guidepath-based transport systems, closed and irreversible, dynamically routed guidepath-based transport systems need more active real-time supervision for ensuring the liveness of the underlying traffic. A set of results concerning the structural characterization of state liveness in this class of transport systems was originally developed in Roszkowska & Reveliotis (2008). In this section, we overview the main points of these past developments, and we also discuss their implications for the liveness-enforcing supervision of the corresponding class of transport systems.



(a) A PDG G(s) and the "chain" structure that is recognized in it.



(b) The condensation C(G(s)) of the above PDG G(s), and its u−connected components

Fig. 3. The content of this figure is adapted from Roszkowska & Reveliotis (2008), and it exemplifies the PDG-related concepts and definitions that are provided in Section 3.4. We also notice, for completeness, that the concept of a "u-connected component" of the condensation graph $\mathcal{C}(s)$ is defined by a chain of this graph together with all the paths of free edges that are incident upon this chain; but this concept is not explicitly necessary for the statement of the key result of Section 3.4.

The PDG-based representation of the traffic state and the induced notion of a "chained" traffic state: Central in the developments of Roszkowska & Reveliotis (2008) is a convenient representation of the traffic state s through a *partially directed graph (PDG)* $G'(s)$ that is defined as follows: [8]

Definition 13 *Given a state s of the considered class of guidepath-based transport systems, the corresponding PDG $G'(s)$ is induced from state s and the guidepath graph G, by substituting the edge $\epsilon(a; s)$ of G, for each agent $a \in \mathcal{A}$, with a* directed *edge that indicates the orientation / direction of motion of agent a on this edge.*

Next, we introduce a series of concepts that are defined on the PDG $G'(s)$ and are instrumental for communicating the main results of this section; the reader is referred to Figure 3 for a more concrete demonstration of the most involved of these concepts and their accompanying definitions.

A *(simple) path* p in PDG $G'(s)$ is defined as any (simple) path p in the original graph G where, however, all the directed edges introduced in the definition of $G'(s)$ have the same sense of direction. Furthermore, a *cycle* c in $G'(s)$ is a simple path with coinciding initial and terminal nodes. A *joint* between two cycles c and c' is a simple path that is a sub-path for both c and c'. On the other hand, a *pass* between two cycles c and c' is a path of the PDG $G'(s)$ with its first node lying on c, its last node lying on c', all of its edges being undirected, and with none of its edges belonging on any cycle of $G'(s)$. Finally, the next set of concepts are at the core of the sought characterization of liveness for the considered class of guidepath-based transport systems:

Definition 14 *A* chain *in PDG $G'(s)$ is the subgraph defined by a sequence $ch = \langle c_1, p_2, c_2, p_3, \ldots, p_n,\ c_n \rangle$, $n \geq 1$, such that (i) c_i, $i = 1, \ldots, n$, are cycles, (ii) p_i, $i = 2, \ldots, n$, are simple paths, and (iii) each path p_i is a joint or a pass between cycles c_{i-1} and c_i. Two edges $e, e' \in E$ are* chain-connected *– or, simply,* chained *– if there exists a chain that contains, both, e and e'. Furthermore, PDG $G'(s)$ and the corresponding state s are said to be* "chained" *if every two edges $e, e' \in E$ are chained.*

Chain connectivity defines a relationship in the edge set E that is symmetric and transitive, and the subgraphs of $G'(s)$ that are induced by the maximal chains of E are the *chained components* of $G'(s)$. Also, the PDG $\mathcal{C}(s)$ that is obtained from $G'(s)$ by replacing each of its chained components by a simple node, is called the *condensation* of $G'(s)$. Obviously, chained PDGs $G'(s)$ have condensations that correspond to a single node. Finally, Roszkowska & Reveliotis (2008) also provides an efficient algorithm for obtaining the condensation $\mathcal{C}(s)$ for any given PDG $G'(s)$.

A structural characterization of state liveness for the considered transport systems: In view of the above definition of chain connectivity and of chained states, and considering also the notion of the singular

[8] In fact, the notion of PDG $G'(s)$ that is introduced in Definition 13 was already used in the representation of the two states that are depicted in Figure 1.

paths $p \in \mathcal{P}_S$ of the guidepath graph G that was introduced in Section 3.3, we can state the main result of Roszkowska & Reveliotis (2008) that is of interest to this work, as follows:

Theorem 15 *In a closed, dynamically routed and irreversible guidepath-based transport system with $|\mathcal{A}| \leq |E| - \sum_{p \in \mathcal{P}_S} |p| - 2$, a given state s is live if and only if the corresponding set $R(s)$ contains a chained state s'.*

In Roszkowska & Reveliotis (2008) it is also argued that the condition $|\mathcal{A}| \leq |E| - \sum_{p \in \mathcal{P}_S} |p| - 1$ is necessary for being able to establish traffic liveness for closed, dynamically routed and irreversible guidepath-based transport systems, and furthermore, the case of $|\mathcal{A}| = |E| - \sum_{p \in \mathcal{P}_S} |p| - 1$ can give rise to certain configurations with unavoidable livelocks. Hence, the condition $|\mathcal{A}| \leq |E| - \sum_{p \in \mathcal{P}_S} |p| - 2$ that is eventually used in the statement of Theorem 15, can be perceived as *practically* necessary for being able to establish live traffic in the considered class of transport systems.

The computational complexity of the state-liveness characterization of Theorem 15 and its implications for the liveness-enforcing supervision of the considered class of transport systems: Currently we do not avail of an efficient test to check the reachability of a chained state s' from any given state s that might arise in the considered class of transport systems. In fact, to the best of our knowledge, the characterization of the computational complexity of this particular decision problem remains an open problem. In view of this limitation, Roszkowska & Reveliotis (2008) proposes to establish the liveness of the considered class of transport systems by confining their operation in states that are either chained or *semi-chained*; the latter are obtained from chained states by transferring a single agent between two cycles c and c' over a pass p that connects these cycles. The developments of Roszkowska & Reveliotis (2008) for the proof of Theorem 15 guarantee that the aforementioned restriction will maintain the liveness of the underlying traffic. Of course, the resulting LES is not maximally permissive anymore, but such a restriction is similar, in spirit, to the restriction that is imposed by the concept of the "ordered" state in the case of open, dynamically routed, irreversible guidepath-based transport systems. [9]

[9] Also, the techniques that have been developed in Reveliotis & Masopust (2019*b*, 2020, 2019*a*) for assessing state liveness in open, dynamically routed, irreversible guidepath-based transport system might be adaptable to the corresponding problem of state-liveness assessment in their closed counterparts; this is another issue currently open to further investigation.

4 Liveness characterizations and enforcement for statically routed, open guidepath-based transport systems

Characterizing liveness in statically routed, open guidepath-based transport systems: In this section, we provide a brief coverage of the notion of "traffic liveness" and its enforcement in the context of statically routed, open guidepath-based transport systems. We start by reminding the reader that, in these systems, a "mission" trip for any traveling agent $a \in \mathcal{A}$ is defined as a walk W_a on the guidepath graph G that starts from the current position of agent a in G and ends at its "home" edge $e_h(a)$. In particular, a complete "mission" trip for any agent $a \in \mathcal{A}$ is a walk that originates and ends at the corresponding "home" edge $e_h(a)$. In this operational regime, *liveness implies the preservation of the ability of each agent $a \in \mathcal{A}$ to execute successfully each walk W_a that is assigned to it, (starting from its "home" edge $e_h(a)$ and ending up at the same edge).*

Furthermore, as in the case of open, dynamically routed guidepath-based transport systems, we shall define the "home" traffic state s_h of a statically routed guidepath-based transport system as the state where all agents $a \in \mathcal{A}$ are in their "home" edges $e_h(a)$.

Then, we can easily see that when the system is started at state s_h, any set of walks $\{W_a, a \in \mathcal{A}\}$ that are compatible with the topology of the underlying guidepath network G and the motion dynamics of the system agents, will be executable by having the agents performing their corresponding walks one at a time. Hence, as in the case of open, dynamically routed guidepath-based transport systems, *preservation of traffic liveness is tantamount to preservation of reachability of the "home" state s_h in the underlying traffic dynamics (c.f. Theorem 7 in Section 3.2); traffic states that satisfy this reachability property, (once again) will be characterized as "live".*

However, in the case of statically routed, open guidepath-based transport systems, the reachability of the "home" state s_h that characterizes any live traffic state s, must be attained under the more restricted dynamics that result from the confinement of each agent $a \in \mathcal{A}$ on a specific walk W_a of the underlying guidepath graph G. As we shall see in the rest of this section, this confinement has some very strong implications for the supervisory control problems of assessing and enforcing traffic liveness in this particular class of transport systems.

The computational complexity of assessing traffic-state liveness in statically routed transport systems: For statically routed guidepath-based transport systems, the following result was recently established in Reveliotis & Masopust (2019*b*).

Theorem 16 *The problem of assessing the liveness of any given traffic state s of a statically routed, open* *guidepath-based transport system is NP-complete in the strong sense.*

The above result was established in Reveliotis & Masopust (2019*b*) through a polynomial reduction (Garey & Johnson (1979)) from the decision problem of "assessing the state safety in a linear, single-unit resource allocation system (L-SU-RAS)", which has been shown to be NP-complete in the strong sense in Reveliotis & Roszkowska (2010). In order to maintain the completeness of this work, we replicate the corresponding results of Reveliotis & Masopust (2019*b*) in the electronic supplement.

The reader should also notice that the above statement of Theorem 16 does not differentiate between reversible and irreversible guidepath-based transport systems, and therefore, the result of Theorem 16 applies to both cases. This finding further implies that the super-polynomial complexity of state liveness in the statically routed, open guidepath-based transport systems that are considered by Theorem 16, is the result of the complete pre-specification of the agent routes that is enforced by this class of transport systems, and not an implication of the irreversibility of the agent motion, which was found to be the primary source of complexity in the case of dynamically routed, open guidepath-based transport systems. Finally, the detailed development of the result of Theorem 16 that is provided in the electronic supplement also highlights the fact that, in the context of statically routed guidepath-based transport systems, the notion of "state" that must be employed in any formal reasoning regarding the liveness characterization and assessment in these environments, must also contain the walks W_a on the guidepath graph G that represent the remaining "mission" trips for each agent $a \in \mathcal{A}$, since these walks determine the advancing path of each agent a towards its final destination $e_h(a)$.

Efficient LES for statically routed guidepath-based transport systems: The developments on the complexity of state liveness for statically routed guidepath-based transport systems that are described in the previous part of this section, also reveal the strong affinity that exists between the qualitative dynamics of the statically routed guidepath-based transport systems that are considered in this work, and the corresponding dynamics of the L-SU-RAS class of Reveliotis (2017). In fact, the supervisory control problems of traffic(-state) liveness assessment and enforcement in statically routed transport systems can be effectively addressed through a straightforward adaptation of the corresponding results in the current L-SU-RAS theory. In particular, in view of the result of Theorem 16, the notion of "ordered" (traffic) state and the related Banker's algorithm can be applied to the liveness enforcement of statically routed guidepath-based transport systems in exactly the same way that these two concepts have been applied to the preservation of liveness in L-SU-RAS. The reader is referred to Reveliotis (2017) for all the relevant details.

5 Conclusion

This paper has provided a comprehensive treatment of the notion of "liveness" and its enforcement for the traffic generated by a set of agents that circulate over the edges of a supporting guidepath network. It was shown that, both, the operational and the computational complexity of this concept depends, in a strong manner, on certain structural and operational attributes of the underlying transport system. The paper characterized clearly these dependencies, and it also provided practical efficient solutions to the problem of liveness enforcement for the cases that the corresponding maximally permissive supervisor might not be computationally tractable.

The presented results can also define a starting base for a systematic resolution of the complementary problem of performance-oriented control of the considered transport systems. In the non-stationary settings that frequently characterize the operation of these environments, the system performance can be optimized by means of a pertinently defined Model Predictive Control (MPC) scheme (Kouvaritakis & Cannon (2015)), where the notion of "liveness" plays a role similar to that of the notion of "stability" in more classical control applications of the MPC framework. An implementation of this idea for the case of dynamically routed, open, reversible guidepath-based transport systems can be found in Daugherty et al. (2019), while a first attempt to extend the MPC framework of Daugherty et al. (2019) to dynamically routed, open but irreversible guidepath-based transport systems can be found in Reveliotis (2019).

Furthermore, the presented framework can be expanded to include additional features, constraints and requirements regarding the behavior of the underlying transport systems, like the synchronized advancement of the traveling agents that is considered in Yu & Rus (2015), or the collaborating behavior that is investigated in Ma et al. (2016). When moving in this direction, it is also possible to consider the sequential satisfaction of a series of "formation" requirements that stipulate the placement of the system agents on specific edges of the underlying guidepath network, possibly with specific orientations, as well; in fact, the investigation of such "formation"-related problems for the particular class of closed, dynamically routed and reversible guidepath-based transport systems, was the content of the works of Wilson (1974), Kornhauser et al. (1984), that were among the very first to formulate and study reachability problems in the traffic of guidepath-based transport systems of the type that are considered in this work.

Concluding this paper, and on the basis of all the above, we can say that the class of guidepath-based transport systems considered in it is very rich in terms of, both, application potential but also open problems and research challenges. By taking a comprehensive and systematic view of these systems and their behavioral dynamics, the paper has tried to further define and articulate all this potential.

A Appendix: A summary of the main results of Sections 3 and 4

All the following statements presume that the underlying guidepath graph G is undirected and connected. Furthermore, for dynamically routed traffic systems, the employed notion of traffic liveness is that provided in Definition 2. On the other hand, for statically routed traffic systems, the employed notion of traffic liveness is that introduced in the opening part of Section 4.

Open, dynamically routed, reversible: State liveness is equivalent to co-reachability of the "home" state s_h. Every traffic state $s \in S$ is live. No need for an externally imposed LES.

Open, dynamically routed, irreversible: A necessary structural condition for traffic liveness is that the guidepath graph G has a minimal vertex degree of 2. Then, state liveness is equivalent to co-reachability of the "home" state s_h. The characterization of the (worst-case) computational complexity of this co-reachability problem for any given state $s \in S$ is an open problem, but there is a recently developed set of results that can resolve this co-reachability problem with polynomial worst-case computational complexity for certain classes of state s, and with low empirical computational complexity for the remaining cases. Also, the notion of "ordered" state can function as a substitute of state liveness in order to obtain a correct but non-maximally permissive LES of polynomial complexity with respect to the size of the underlying traffic system.

Open, statically routed, reversible: State liveness is equivalent to co-reachability of the "home" state s_h. The corresponding decision problem is NP-complete in the strong sense. The notion of "ordered" state can function as a substitute of state liveness in order to obtain a correct but non-maximally permissive LES of polynomial complexity with respect to the size of the underlying traffic system.

Open, statically routed, irreversible: A necessary structural condition for traffic liveness is that the guidepath graph G has a minimal vertex degree of 2. Then, the rest of the results are similar to those stated above for the class of open, statically routed, reversible guidepath-based transport systems.

Closed, dynamically routed, reversible: A sufficient structural condition for traffic liveness is that $|\mathcal{A}| \leq |E| - 1 - \max_{p \in \mathcal{P}_S}\{|p|\}$, where $\mathcal{P}_S$ is the set of the singular paths of the guidepath graph G and $|p|$ denotes the number edges in such a path. This condition is also necessary if there are no singular paths in guidepath graph G, or there exists a maximal-length singular path p connecting two cyclical components, G_i and G_j, of graph G. When the aforestated condition is satisfied, every state $s \in S$ is live. Hence, there is no need for an externally imposed LES.

Closed, dynamically routed, irreversible: For transport systems from this class, $|\mathcal{A}| \leq |E| - \sum_{p \in \mathcal{P}_S} |p| - 2$ is a practically required condition for being able to establish traffic liveness. Under this condition, a state $s \in S$ is live if and only if its reachability space $R(s)$ contains a "chained" state. Testing whether a given state $s \in S$ is chained, is a task of polynomial worst-case complexity with respect to the size of the underlying transport system. But the worst-case computational complexity of the decision problem: "$\exists s' \in R(s) : s'$ is chained", that is defined by any traffic state $s \in S$, is an open problem. A correct, computationally efficient but non-maximally permissive LES for this class of systems can be obtained by admitting only "chained" and "semi-chained" states.

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