10.2 ~ More examples ~

~ Example 1 ~

According to a polling organization, 23% of adults in a large region consider themselves to be liberal. A survey asked respondents to disclose their political philosophy: Conservative, Liberal, Moderate. Treat the results of the survey as a random sample of adults in this region. Do the survey results suggest the proportion is higher than that reported by the polling organization? Use an $\alpha = 0.01$ level of significance.

Click the icon to view the survey results.

Identify the null and alternative hypotheses for this test. Let p represent the population proportion of adults in this region who consider themselves to be liberal.

A. $H_0: p < 0.23$ $H_1: p = 0.23$

B. $H_0: p = 0.23$ $H_1: p \neq 0.23$

C. $H_0: p = 0.23$ $H_1: p > 0.23$

D. $H_0: p \neq 0.23$ $H_1: p = 0.23$

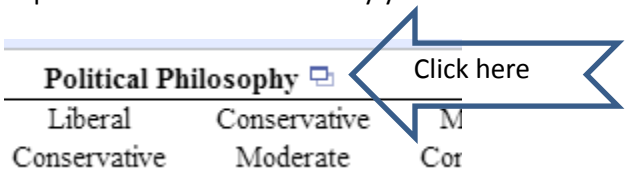
E. $H_0: p = 0.23$ $H_1: p < 0.23$

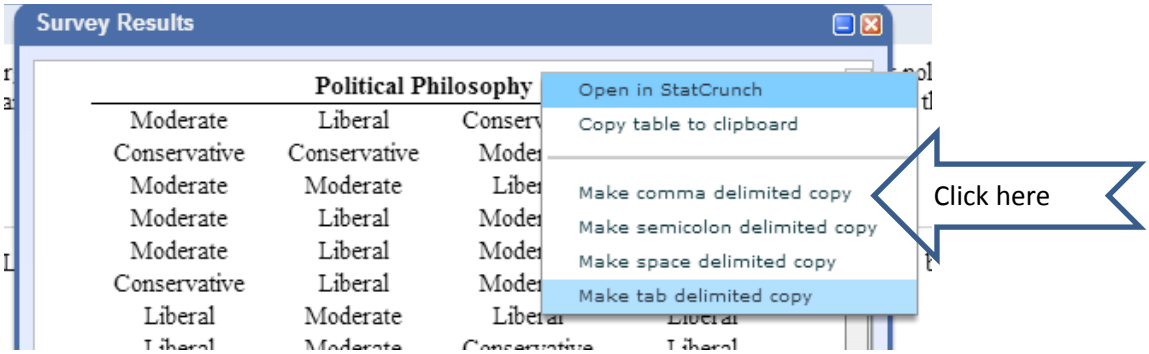
F. $H_0: p > 0.23$ $H_1: p = 0.23$

When you view the survey results it lists 200 (yes, 200!) results of the survey as to which philosophy each person chose.

There is no way you want to sit there and count by hand how many there are of each one, so copy it into Excel by doing this:

**please note – there's no way you will have to do this on the final since you won't have Excel!!





Then open Excel and click "paste" and you will have this column full of your data.

	A
1	Moderate
2	Conservative
3	Moderate
4	Moderate
5	Moderate
6	Conservative
7	Liberal
8	Liberal
9	Liberal
10	Liberal
11	Liberal
12	Moderate
13	Liberal
14	Moderate
15	Conservative
16	Conservative
17	Conservative
18	Liberal
19	Conservative
20	Moderate
21	Moderate
22	Conservative
23	Liberal
24	Conservative
25	Moderate
26	Conservative
27	Liberal
28	Liberal
29	Liberal
30	Moderate
31	Liberal
32	Liberal
33	Conservative
34	Liberal
35	Moderate
36	Conservative

and so on .. . .

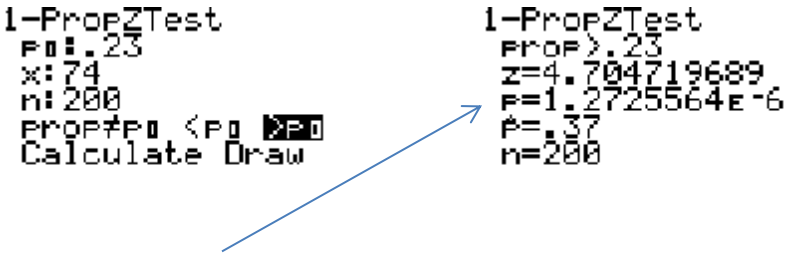
1	Conservative
2	Conservative
3	Conservative
4	Conservative
5	Conservative
6	Conservative
7	Conservative
8	Conservative
9	Conservative
10	Conservative
11	Conservative
12	Conservative
13	Conservative
14	Conservative
15	Conservative
16	Conservative
17	Conservative
18	Conservative
19	Conservative
20	Conservative
21	Conservative
22	Conservative
23	Conservative
24	Conservative
25	Conservative
26	Conservative
27	Conservative
28	Conservative
29	Conservative

Near the top choose “sort” and choose “sort A to Z”.

Now you easily can see that there are 200 entries, and you can also see how many of each you have of Liberal, Conservative, and Moderate because of the nice “numbering” Excel gives you down the left side.

In this chart we have 50 people who answered Conservative, 74 who answered Liberal, and 76 who answered Moderate.

Since we surveyed 200 people, n = 200. x is 74 because 74 people answered “liberal”.



The p-value is 1.2725564E-6 which means we move the decimal size places to the left

So the p-value is 0.0000012725564.

If the p-value is less than α, we reject the null hypothesis.

0.00000127 is definitely less than our α, which was 0.01, so we reject the null.

By rejecting the null, we are saying there is sufficient evidence that the proportion of liberals is higher than 23% (which the organization reported).

~ Example 2 ~

In February 2008, the Gallup organization surveyed 1,040 adults between 30 and 64 years of age and found that 538 were worried that they will outlive their money after they retire. Does the sample evidence suggest that a majority of 30- to 64-year-olds in the United States are worried they will outlive their money? Use the $\alpha = 0.01$ level of significance.

a) Perform the test using the classical approach.

b) Perform the test using the P-value approach.

The question says*: "does the evidence suggest that a MAJORITY are worried?"*

The definition of a majority means more than 50%, so in this case, $p_0 = 0.50$.

So the null hypothesis will be H_0: $p = 0.50$ and the alternative hypothesis is H_1: $p > 0.50$ because we want to know if it's a majority, meaning greater than, 50%.

Testing Hypotheses Regarding a Population Proportion, *p*

Use Steps 1–5 provided that

- The sample is obtained by simple random sampling or the data result from a randomized experiment.
- $np_0(1 - p_0) \geq 10$.
- The sampled values are independent of each other.

Step 1 Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: p = p_0$	$H_0: p = p_0$	$H_0: p = p_0$
$H_1: p \neq p_0$	$H_1: p < p_0$	$H_1: p > p_0$

Note: p_0 is the assumed value of the population proportion.

Step 2 Select a level of significance α, depending on the seriousness of making a Type I error.

Classical Approach

Step 3 Compute the **test statistic**

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Use your calculator to find the test statistic.

P-Value Approach

By Hand Step 3 Compute the **test statistic**

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Use your calculator (invNorm) to find the critical value.

	Two-Tailed	Left-Tailed	Right-Tailed
Critical value	$-z_{\frac{\alpha}{2}}$ and $z_{\frac{\alpha}{2}}$	$-z_\alpha$	z_α

Step 4 Compare the critical value with the test statistic.

Two-Tailed	Left-Tailed	Right-Tailed
If $z_0 < -z_{\frac{\alpha}{2}}$ or $z_0 > z_{\frac{\alpha}{2}}$, reject the null hypothesis.	If $z_0 < -z_\alpha$, reject the null hypothesis.	If $z_0 > z_\alpha$, reject the null hypothesis.

Use your calculator to find the P-value.

Two-Tailed	Left-Tailed	Right-Tailed
The sum of the area in the tails is the *P*-value	The area left of z_0 is the *P*-value	The area right of z_0 is the *P*-value
P-value $= 2P(z > \lvert z_0 \rvert)$	*P*-value $= P(z < z_0)$	*P*-value $= P(z > z_0)$

Technology Step 3 Use a statistical spreadsheet or calculator with statistical capabilities to obtain the *P*-value. The directions for obtaining the *P*-value using the TI-83/84 Plus graphing calculator, MINITAB, Excel, and StatCrunch are in the Technology Step-by-Step on page 448.

Step 4 If *P*-value $< \alpha$, reject the null hypothesis.

Step 5 State the conclusion.

a) Classical approach:

This is a right-tailed test (because we are testing a = null vs. > alternative)

So if the test statistic is greater than the critical value, we reject the null hypothesis.

Test statistic, from calculator, is 1.116 (rounded to three places)

```
1-PropZTest
 p0:.5
 x:538
 n:1040
 prop≠p0 <p0 >p0
 Calculate Draw
```

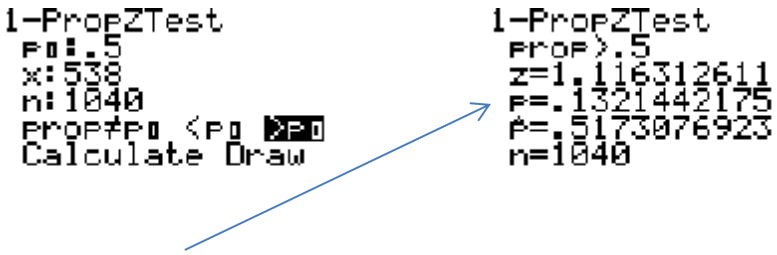
```
1-PropZTest
 prop>.5
 z=1.116312611
 p=.1321442175
 p̂=.5173076923
 n=1040
```

α is given and is 0.01. So $z_{0.01} = 2.33$ (we use invNorm(0.99) because this is a right tailed test, we want the area to the right to be 0.01, but the calculator only "understands" are to the LEFT. To get the area to the left, we have to do 1 – 0.01 = 0.99)

```
invNorm(0.99)
      2.326347877
```

Test statistic 1.116 < critical value 2.33 so **we do not reject the null hypothesis.**

b) P-value approach



P = 0.132 (rounded to three places)

If the P-value < alpha, we reject the null hypothesis.

0.132 > 0.01 so **we do not reject the null hypothesis.**