Helping Students Learn How to Approach Problem-Solving Situations

Dr. Brooke Skelton

Dr. Marjorie Lewkowicz

Perimeter College, Georgia State University

Knowledge Transfer

Journal of Experimental Psychology:
Learning, Memory, and Cognition
1989, Vol. 15, No. 1, 153–166

0278-7393/89/$00.75

Interdomain Transfer Between Isomorphic Topics in Algebra and Physics

Miriam Bassok
University of Pittsburgh

Keith J. Holyoak
University of California, Los Angeles

Knowledge Transfer

Bothered by Abstraction: The Effect of Expertise on Knowledge

Transfer and Subsequent Novice Performance

Pamela J. Hinds

Michael Patterson

Management Science & Engineering

Jeffrey Pfeffer

Graduate School of Business

Stanford University

Stanford, CA 94305-4024

This paper was published in 2001 in the Journal of Applied Psychology, Vol. 86, pp. 1232-1243.

I Am STEM
Science Technology Engineering Math

National Science Foundation
Georgia State University | Perimeter College

Mindset and *Belongingness* In Underrepresented Populations

I Am STEM

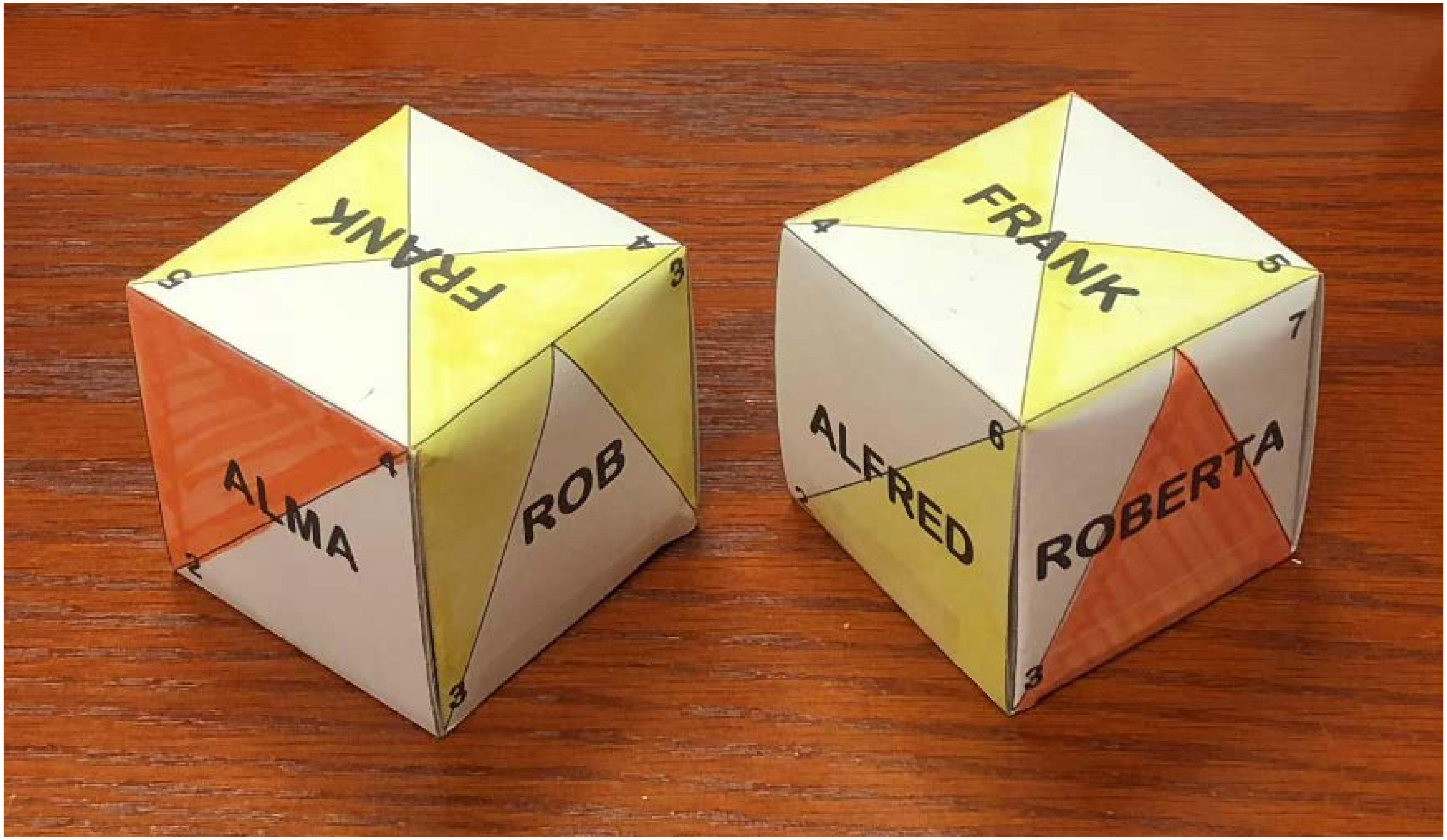
Science Technology Engineering Math

Field Trips and Presentations

Enrichment in Gateway Courses

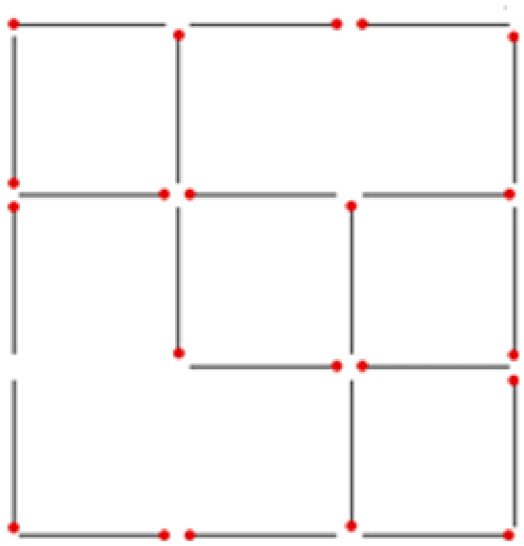
Critical Thinking:
Hands-On Problem Solving

Critical Thinking:
Logical Problem Solving

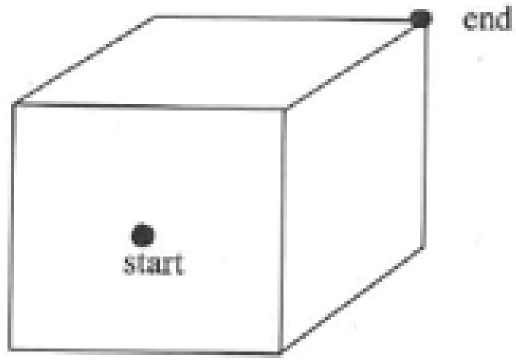


Critical Thinking: Visualization in Problem Solving

How many squares do you see?



What is the shortest path for an ant crawling on the surface of a unit cube from the starting point to the ending point shown? What is the distance the ant has to crawl?



Algebra looks different in physics…

Solve for the indicated variable:

1. $8 = 6 + 2x$, *for x*
2. $54 = 5\times6 + 3x\times2^2$, *for x*
3. $12 = 6\times\frac{18x}{3^2}$, *for x*
4. $4^2 = 8^2 + 2\times4(x - 5)$, *for x*

Solve for the indicated variable:

1. $v = v_0 + at$, *for t*
2. $s = v_0t + \frac{1}{2}at^2$, *for a*
3. $F = k\frac{q_1q_2}{r^2}$, *for q_2*
4. $v^2 = v_0^2 + 2a(x - x_0)$, *for x*

Side-by-Side Comparisons

Algebra

Solve the system of linear equations using the substitution method:

(1) $x - 7y = 9$

(2) $2y - x = 1$

Take equation (1) and isolate "x"

$$x - 7y = 9$$
$$x = 9 + 7y$$

Now, take equation (2) and substitute $9+7y$ for "x"

$$2y - x = 1$$
$$2y - (9+7y) = 1$$
$$2y - 9 - 7y = 1$$
$$-5y - 9 = 1$$
$$-5y = 10$$
$$y = -2$$

Since

$$x = 9+7y$$
$$x = 9+7(-2)$$
$$x = 9-14$$
$$x = -5$$

$(-5, -2)$

Physics

Solve the system of linear equations using the substitution method:

(1) $F_T - m_1 g = m_1 a$

(2) $m_2 g - F_T = m_2 a$

Take equation (1) and isolate "F_T"

$$F_T - m_1 g = m_1 a$$
$$F_T = m_1 a + m_1 g$$

Now, take equation (2) and substitute $m_1 a + m_1 g$ for "F_T"

$$m_2 g - F_T = m_2 a$$
$$m_2 g - (m_1 a + m_1 g) = m_2 a$$
$$m_2 g - m_1 a - m_1 g = m_2 a$$
$$m_2 g - m_1 g = m_2 a + m_1 a$$
$$\frac{g(m_2 - m_1)}{(m_2+m_1)} = \frac{a(m_2+m_1)}{(m_2+m_1)}$$

$$\rightarrow a = g\left(\frac{m_2 - m_1}{m_2 + m_1}\right)$$

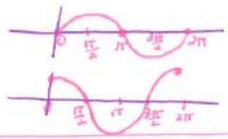
Algebra Misconceptions

Determine whether the statements are true or false. If false, explain why and correct the error. If true, discuss why the statement is true.

1. The solution to $x^2 = 25$ is $x = 5$.
2. $\frac{a+b}{a+c} = 1 + \frac{b}{c}$
3. $(x - 5)^2 = x^2 - 25$
4. $\sqrt{x^2 + y^2} = x + y$
5. $\frac{a^2+a}{a+b} = \frac{a+1}{b}$
6. $\frac{5y-6x}{2x} = 5y - 3$
7. $-4^2 = 16$
8. $\log(x + y) = \log x + \log y$

"Things you just need to know"

graph of $y = \sin\theta$
$y = \cos\theta$



Identities: $\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

$\sec x = \frac{1}{\cos x}$ $\quad \csc x = \frac{1}{\sin x}$

$\cot x = \frac{1}{\tan x}$

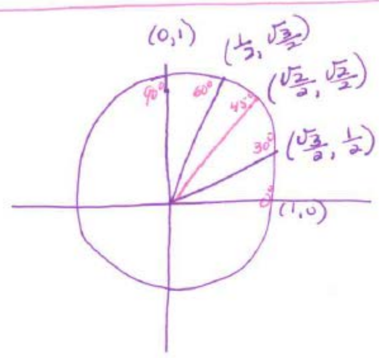
$\sin^2 x + \cos^2 x = 1$

$\sin 30^\circ = \frac{1}{2}$
$\cos 30^\circ = \frac{\sqrt{3}}{2}$

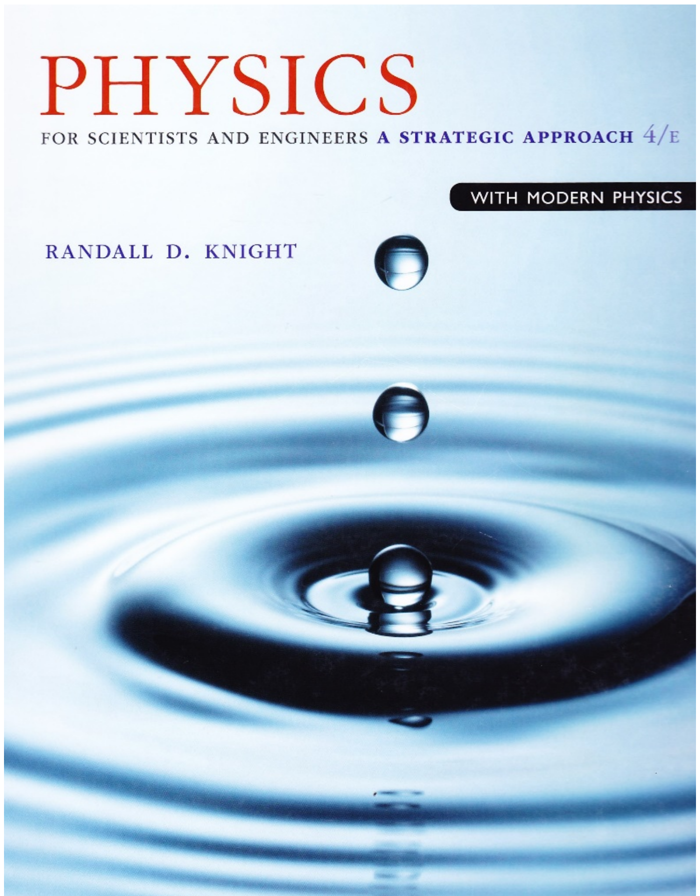
$\sin 45^\circ = \frac{\sqrt{2}}{2}$
$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$
$\cos 60^\circ = \frac{1}{2}$

$\sin 90^\circ = 1$
$\cos 90^\circ = 0$



Problem Solving Instructions



GENERAL PROBLEM-SOLVING STRATEGY

MODEL It's impossible to treat every detail of a situation. Simplify the situation with a model that captures the essential features. For example, the object in a mechanics problem is often represented as a particle.

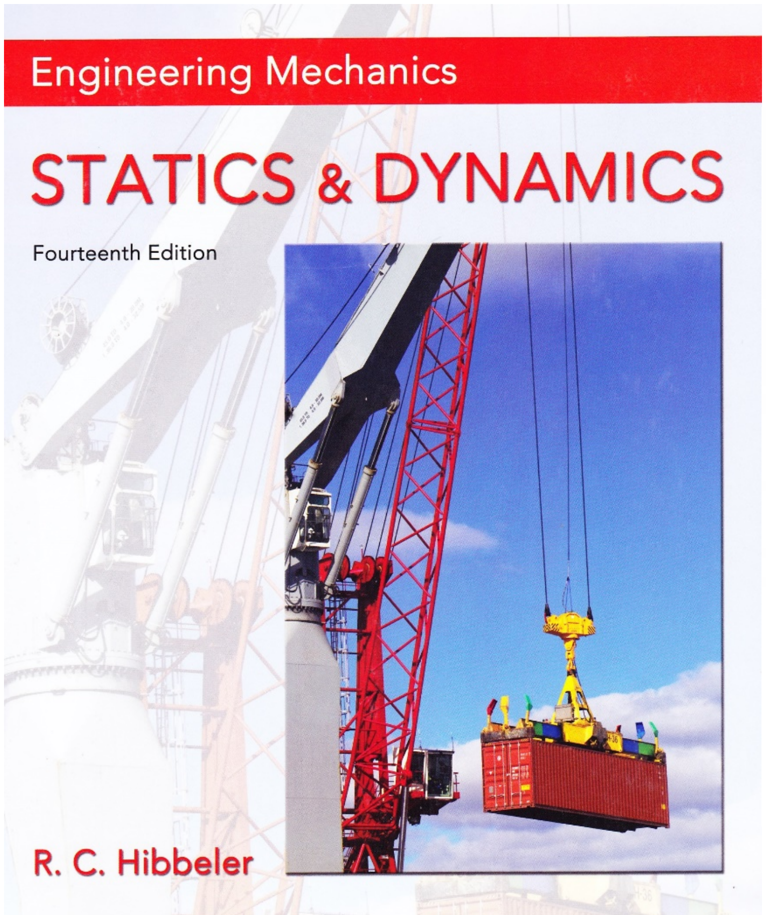
VISUALIZE This is where expert problem solvers put most of their effort.

- Draw a *pictorial representation.* This helps you visualize important aspects of the physics and assess the information you are given. It starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these representations; they need not be done in any particular order.

SOLVE Only after modeling and visualizing are complete is it time to develop a *mathematical representation* with specific equations that must be solved. All symbols used here should have been defined in the pictorial representation.

ASSESS Is your result believable? Does it have proper units? Does it make sense?

Problem Solving Instructions

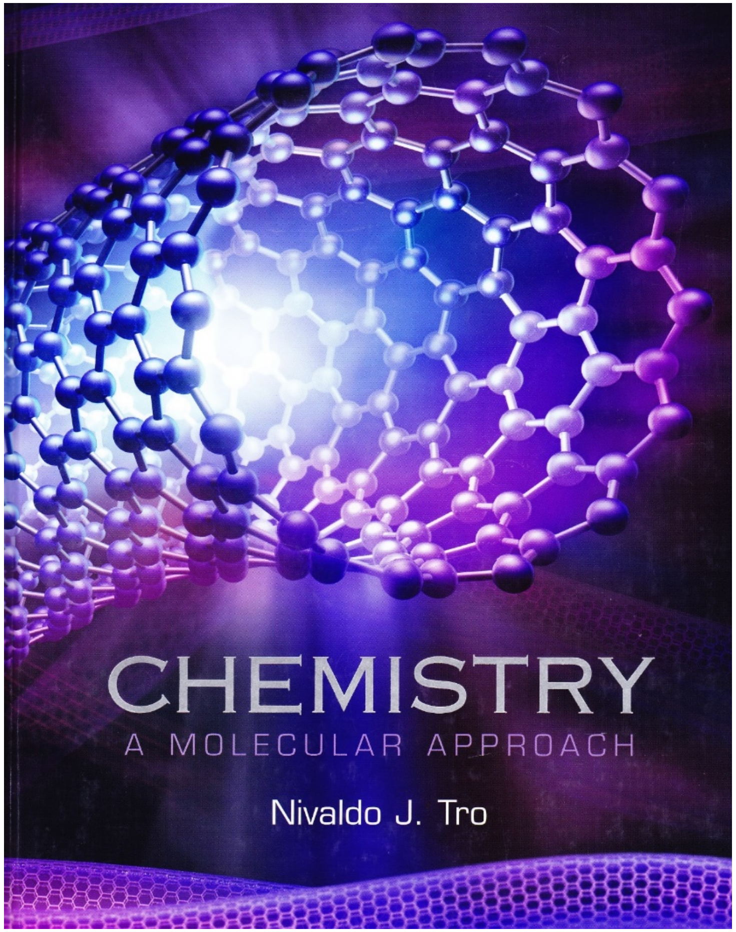


1.6 General Procedure for Analysis

Attending a lecture, reading this book, and studying the example problems helps, but **the most effective way of learning the principles of engineering mechanics is to *solve problems*.** To be successful at this, it is important to always present the work in a *logical* and *orderly manner*, as suggested by the following sequence of steps:

- Read the problem carefully and try to correlate the actual physical situation with the theory studied.
- Tabulate the problem data and *draw to a large scale* any necessary diagrams.
- Apply the relevant principles, generally in mathematical form. When writing any equations, be sure they are dimensionally homogeneous.
- Solve the necessary equations, and report the answer with no more than three significant figures.
- Study the answer with technical judgment and common sense to determine whether or not it seems reasonable.

Problem Solving Instructions



General Problem-Solving Strategy

In this book, we use a standard problem-solving procedure that can be adapted to many of the problems encountered in general chemistry and beyond. Solving any problem essentially requires you to assess the information given in the problem and devise a way to get to the information asked for. In other words, you must

- Identify the starting point (the ***given*** information).
- Identify the end point (what you must ***find***).
- Devise a way to get from the starting point to the end point using what is given as well as what you already know or can look up. (We call this guide the ***conceptual plan.***)

In graphic form, we can represent this progression as

Given ⟶ Conceptual Plan ⟶ Find

One of the main difficulties beginning students have when trying to solve problems in general chemistry is simply not knowing where to start. While no problem-solving procedure is applicable to all problems, the following four-step procedure can be helpful in working through many of the numerical problems you will encounter in this book.

1. **Sort.** Begin by sorting the information in the problem. *Given* information is the basic data provided by the problem—often one or more numbers with their associated units. *Find* indicates what information you will need for your answer.
2. **Strategize.** This is usually the hardest part of solving a problem. In this process, you must develop a *conceptual plan*—a series of steps that will get you from the given information to the information you are trying to find. You have already seen conceptual plans for simple unit conversion problems. Each arrow in a conceptual plan represents a computational step. On the left side of the arrow is the quantity you had before the step; on the right side of the arrow is the quantity you will have after the step; and below the arrow is the information you need to get from one to the other—the relationship between the quantities.

Most problems can be solved in more than one way. The solutions we derive in this book will tend to be the most straightforward but certainly not the only way to solve the problem.

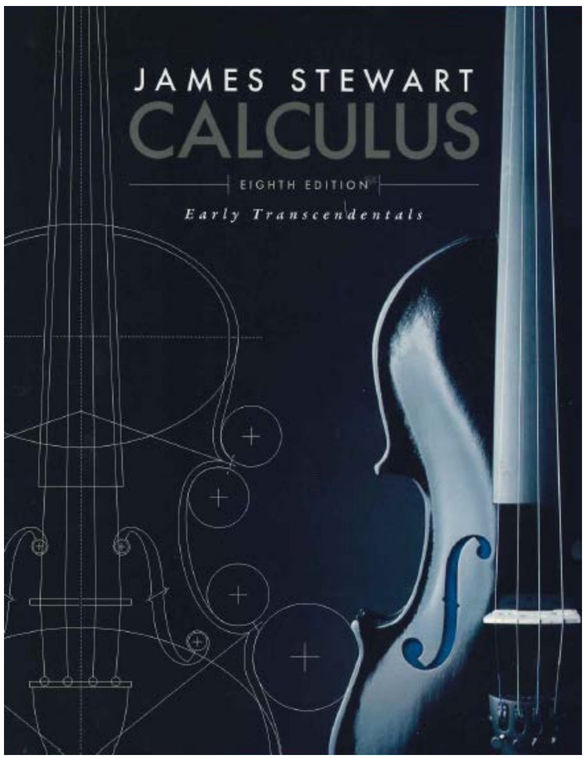
Often such relationships will take the form of conversion factors or equations. These may be given in the problem, in which case you will have written them down under "Given" in step 1. Usually, however, you will need other information—which may include physical constants, formulas, or conversion factors—to help get you from what you are given to what you must find. You must recall this information from what you have learned or look it up in the chapter or in tables within the book.

In some cases, you may get stuck at the strategize step. If you cannot figure out how to get from the given information to the information you are asked to find, you might try working backwards. For example, you may want to look at the units of the quantity you are trying to find and try to find conversion factors to get to the units of the given quantity. You may even try a combination of strategies; work forward, backward, or some of both. If you persist, you will develop a strategy to solve the problem.

3. **Solve.** This is the easiest part of solving a problem. Once you set up the problem properly and devise a conceptual plan, you simply follow the plan to solve the problem. Carry out any mathematical operations (paying attention to the rules for significant figures in calculations) and cancel units as needed.
4. **Check.** This is the step most often overlooked by beginning students. Experienced problem solvers always go one step further and ask, does this answer make physical sense? Are the units correct? Is the number of significant figures correct? When solving multistep problems, errors easily creep into the solution. You can catch most of these errors by simply checking the answer. For example, suppose you are calculating the number of atoms in a gold coin and end up with an answer of 1.1×10^{-6} atoms. Could the gold coin really be composed of one-millionth of one atom?

Below we apply this problem-solving procedure to unit conversion problems. The procedure is summarized in the left column and two examples of applying the procedure are shown in the middle and right columns. This three-column format will be used in selected examples throughout the text. It allows you to see how a particular procedure can be applied

Problem Solving Instructions



Principles of Problem Solving

There are no hard and fast rules that will ensure success in solving problems. However, it is possible to outline some general steps in the problem-solving process and to give some principles that may be useful in the solution of certain problems. These steps and principles are just common sense made explicit. They have been adapted from George Polya's book *How To Solve It.*

1 UNDERSTAND THE PROBLEM

The first step is to read the problem and make sure that you understand it clearly. Ask yourself the following questions:

What is the unknown?

What are the given quantities?

What are the given conditions?

For many problems it is useful to

draw a diagram

and identify the given and required quantities on the diagram.

Usually it is necessary to

introduce suitable notation

In choosing symbols for the unknown quantities we often use letters such as a, b, c, m, n, x, and y, but in some cases it helps to use initials as suggestive symbols; for instance, V for volume or t for time.

2 THINK OF A PLAN

Find a connection between the given information and the unknown that will enable you to calculate the unknown. It often helps to ask yourself explicitly: "How can I relate the given to the unknown?" If you don't see a connection immediately, the following ideas may be helpful in devising a plan.

Try to Recognize Something Familiar Relate the given situation to previous knowledge. Look at the unknown and try to recall a more familiar problem that has a similar unknown.

Try to Recognize Patterns Some problems are solved by recognizing that some kind of pattern is occurring. The pattern could be geometric, or numerical, or algebraic. If you can see regularity or repetition in a problem, you might be able to guess what the continuing pattern is and then prove it.

Use Analogy Try to think of an analogous problem, that is, a similar problem, a related problem, but one that is easier than the original problem. If you can solve the similar, simpler problem, then it might give you the clues you need to solve the original, more difficult problem. For instance, if a problem involves very large numbers, you could first try a similar problem with smaller numbers. Or if the problem involves three-dimensional geometry, you could look for a similar problem in two-dimensional geometry. Or if the problem you start with is a general one, you could first try a special case.

Introduce Something Extra It may sometimes be necessary to introduce something new, an auxiliary aid, to help make the connection between the given and the unknown. For instance, in a problem where a diagram is useful the auxiliary aid could be a new line drawn in a diagram. In a more algebraic problem it could be a new unknown that is related to the original unknown.

Take Cases We may sometimes have to split a problem into several cases and give a different argument for each of the cases. For instance, we often have to use this strategy in dealing with absolute value.

Work Backward Sometimes it is useful to imagine that your problem is solved and work backward, step by step, until you arrive at the given data. Then you may be able to reverse your steps and thereby construct a solution to the original problem. This procedure is commonly used in solving equations. For instance, in solving the equation $3x - 5 = 7$, we suppose that x is a number that satisfies $3x - 5 = 7$ and work backward. We add 5 to each side of the equation and then divide each side by 3 to get $x = 4$. Since each of these steps can be reversed, we have solved the problem.

Establish Subgoals In a complex problem it is often useful to set subgoals (in which the desired situation is only partially fulfilled). If we can first reach these subgoals, then we may be able to build on them to reach our final goal.

Indirect Reasoning Sometimes it is appropriate to attack a problem indirectly. In using proof by contradiction to prove that P implies Q, we assume that P is true and Q is false and try to see why this can't happen. Somehow we have to use this information and arrive at a contradiction to what we absolutely know is true.

Mathematical Induction In proving statements that involve a positive integer n, it is frequently helpful to use the following principle.

> **Principle of Mathematical Induction** Let S_n be a statement about the positive integer n. Suppose that
>
> **1.** S_1 is true.
>
> **2.** S_{k+1} is true whenever S_k is true.
>
> Then S_n is true for all positive integers n.

This is reasonable because, since S_1 is true, it follows from condition 2 (with $k = 1$) that S_2 is true. Then, using condition 2 with $k = 2$, we see that S_3 is true. Again using condition 2, this time with $k = 3$, we have that S_4 is true. This procedure can be followed indefinitely.

3 CARRY OUT THE PLAN

In Step 2 a plan was devised. In carrying out that plan we have to check each stage of the plan and write the details that prove that each stage is correct.

4 LOOK BACK

Having completed our solution, it is wise to look back over it, partly to see if we have made errors in the solution and partly to see if we can think of an easier way to solve the problem. Another reason for looking back is that it will familiarize us with the method of solution and this may be useful for solving a future problem. Descartes said, "Every problem that I solved became a rule which served afterwards to solve other problems."

These principles of problem solving are illustrated in the following examples. Before you look at the solutions, try to solve these problems yourself, referring to these Principles of Problem Solving if you get stuck. You may find it useful to refer to this section from time to time as you solve the exercises in the remaining chapters of this book.

EXAMPLE 1 Express the hypotenuse h of a right triangle with area 25 m^2 as a function of its perimeter P.

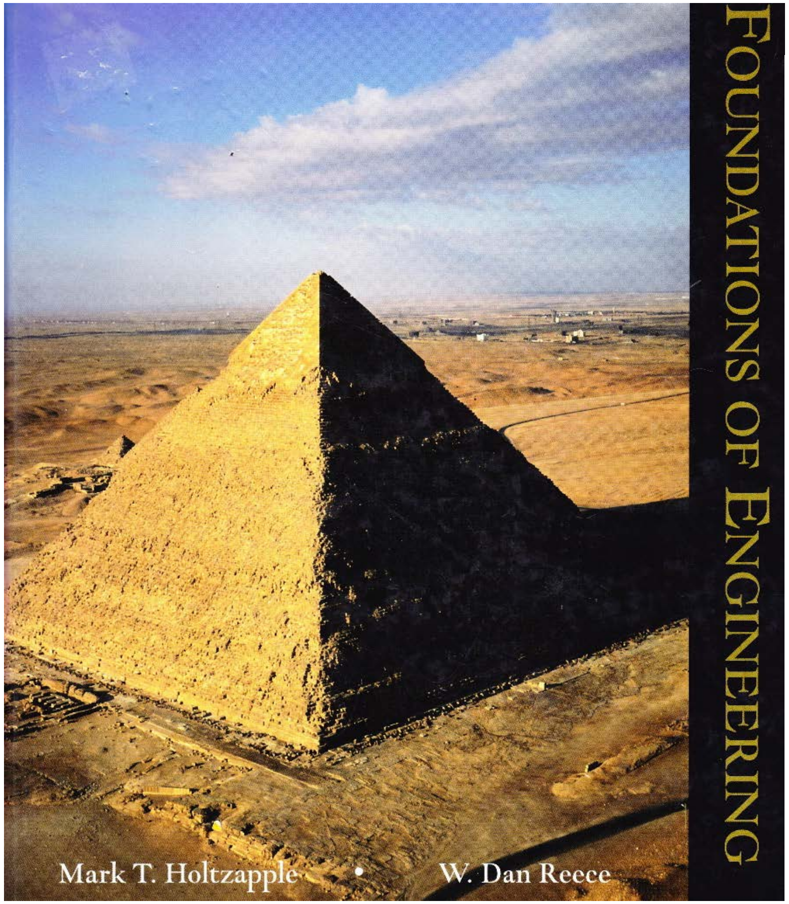
PS Understand the problem

SOLUTION Let's first sort out the information by identifying the unknown quantity and the data:

Unknown: hypotenuse h

Given quantities: perimeter P, area 25 m^2

Problem Solving Instructions



3.4 TECHNIQUES FOR ERROR-FREE PROBLEM SOLVING

Although we can never be certain our answer is correct, we can increase the probability of calculating a correct answer using the following procedure.

Given:

1. Always draw a picture of the physical situation.
2. State any assumptions.
3. Indicate all given properties on the diagram **with their units**.

Find:

4. Label unknown quantities with a question mark.

Relationships:

5. From the text, write the *main equation* that contains the desired quantity. (If necessary, you might have to derive the appropriate equation.)
6. Algebraically manipulate the equation to isolate the desired quantity.
7. Write *subordinate equations* for the unknown quantities in the main equation. Indent to indicate that the equation is subordinate. You may need to go through several levels of subordinate equations before all the quantities in the main equation are known.

Solution:

8. After all algebraic manipulations and substitutions are made, insert numerical values **with their units**.
9. Ensure that units cancel appropriately. Check one last time for a sign error.
10. Compute the answer.
11. Clearly mark the final answer. **Indicate units.**
12. Check that the final answer makes physical sense!
13. Ensure that all questions have been answered.

A General Problem-Solving Method

1. Understand the problem.
   a. Read the problem carefully.
   b. What are you looking for?
   c. What technique are you going to use?

2. Draw pictures or diagrams.

3. Identify knowns and unknowns.
   a. Assign symbols or variables for all quantities involved – known AND unknown.
   b. Make sure all your units are consistent.

4. Write the function of interest or choose the fundamental equation or method to use to solve the problem.

5. Solve the problem.
   a. Solve equations algebraically for the variable of interest and make any algebraic substitutions BEFORE plugging in numerical values.
   b. Substitute numerical values for the known variables and calculate. Be careful about negative signs and entering parentheses into your calculator!

6. Evaluate your answer.
   a. Does it make sense?
   b. Does it answer what is being asked in the original question?

Understand that this is an "organic" process where the steps may not be completely separate from one another!

Physics Process Guides

Problem Solving Guide: Energy

1. **Understand the problem.**
   What are you trying to find out? What variable represents this?

   What about the problem lends itself to being solved with the conservation of energy?

Physics Process Guide: Force Problems

2. **Draw a picture; if you are solving a forces problem, your picture should be a free body diagram.**
   Before you draw your FBD, make sure you determine what forces are acting:

   Is the force of gravity acting on the object?
   If so, in which direction will the gravitational force act?

   Is there a surface?
   If not, there will be no normal force. If so, in which direction will the normal force act?

   Is there tension?
   If so, in which direction will the tension force act?

   Is there friction?
   If not, why not? (Just to double check!)
   If so, is it static friction or kinetic friction?
   In which direction does the friction force act?

   Are there any other forces acting on the object?
   If so, what are they?

   Is the object accelerating?
   If not, remember this means that the forces add up to zero; make sure the vectors in your FBD would sum to zero!
   If so, in which direction?
   Make sure the sum of the vectors in your FBD (the net force) point in that direction!

Physics Process Guide: Force Problems

Draw your free body diagram; recall that all forces begin at the dot (your object) regardless of how they are applied.

- Label each force with a variable.
- Label all angles.
- Confirm that all forces in your FBD act ON the object of interest – forces applied BY the object of interest don't affect the motion of the object and therefore don't belong on the FBD.
- Near the FBD, but separate from it, identify the coordinate system.
- If the system is not in equilibrium, then near the FBD, but separate from it, identify the direction of the acceleration.

•

3. Identify knowns and unknowns.

List the variables you have assigned to each of the forces in your FBD. Write down anything you know about the values or the directions of the forces.

Physics Process Guide: Energy Problems

5. **Do the math.**
   Write down the work-energy theorem.

   Write an expression for W_{nc} that shows the work done by each of the non-conservative forces acting on the system. You should be working with variables; no numbers yet!

   Write an expression for E_i that shows the types of energy that the object has initially, then plug in mathematical definitions of each type of energy. You should still be working with variables; no numbers yet!

   Write an expression for E_f that shows the types of energy that the object has initially, then plug in mathematical definitions of each type of energy. You should still be working with variables; no numbers yet!

   Put your expressions for W_{nc}, E_i, and E_i into the work-energy theorem.

   Solve for the variable you want.

   Put in numbers:

Calculus Process Guides

Finding Derivatives: How to Get Started

To get started finding the derivatives of functions using the Chain Rule, look at the Big Picture first. Begin with the outer most layer and work your way to the inner most later. Here are some examples worked out. Keep practicing and maintain a positive attitude. You **CAN** do this!!!

Calculus Process Guides: Specific Situations

When finding the derivative of a function of the form
y = trig function(argument)

y' = derivative of the trig function(SAME ARGUMENT) times (the derivative of the argument).

Then, to simplify, rewrite by placing the derivative of the argument in front of the trig function.

So, if $y = \sin(f(x))$
then $y' = \cos(f(x))(f'(x))$
or $y' = f'(x)\cos(f(x))$

When finding the derivative of a function of the form $y = e^{f(x)}$

y' = the original function itself, $e^{f(x)}$*, times (the derivative of the exponent, f(x)).*

Then, we can rewrite by placing the derivative of the exponent in front of the base.

So, if $y = e^{f(x)}$
Then $y' = e^{f(x)}(f'(x))$
or $y' = f'(x)\, e^{f(x)}$

Calculus Process Guides: Example Problems

Example 1: $y = (5x^3 - 7x + 5)^{20}$

The **outer layer** is the **20th power**. The **inner layer** is $(5x^3 - 7x + 5)$.

Differentiate the 20th power first leaving the inner layer of $(5x^3 - 7x + 5)$ as is.

NOTE: The key is to leave the inner layer unchanged until after you have differentiated the outer layer. Then, you multiply by the derivative of the inner layer.

Then, differentiate $5x^3 - 7x + 5$.

$$y' = 20(5x^3 - 7x + 5)^{19}(15x^2 - 7)$$

Simplifying, we obtain

$$y' = 20(15x^2 - 7)(5x^3 - 7x + 5)^{19}$$

Example 5: $y = \sqrt{\sin(\ln(5x)}$

Before deciding on the outer layer, remove the square root symbol by replacing it with the ½ power. So,

$$y = (\sin(\ln(5x)))^{1/2}$$

The **first layer** is the **½ power**. The **second layer** is the **sine function**, the **third layer** is the **natural logarithmic function**, and the **fourth layer** is **$5x$**.

Differentiate the ½ power first, leaving the sine function, the natural logarithmic function, and the $5x$ unchanged.

Then differentiate the sine function leaving the natural logarithmic function and the $5x$ unchanged.

Then, take the derivative of the natural logarithmic function and leave the $5x$ as is.

Finally, take the derivative of $5x$.

$$y' = \frac{1}{2}(\sin(\ln(5x)))^{-1/2}[\cos(\ln(5x))][\frac{1}{5x} \cdot 5]$$

Simplifying, we obtain

$$y' = \frac{\cos(\ln(5x))}{2x\sqrt{\sin(\ln(5x))}}$$

Takeaways

- Help students access prior knowledge
- Give concrete instructions for abstract processes
- The goal:
  Assist students in learning how to apply skills beyond the immediate context
- What skills in your course do students need to employ in later courses?