Helping Students Make Connections with Prior Mathematical Knowledge

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Goals of this session

Our project:

- Objectives
- Origins
- Current status

Discussion:

- Other ideas
- Dissemination

Objectives of our project

- Connections and scaffolding
- References and reminders

Partial Table of Contents
(in a publication for students)

1. Accessing Prior Knowledge
2. Making Connections
3. Problem-Solving Processes

Origins of our project – 1

I Am STEM

Science Technology Engineering Math

National Science Foundation

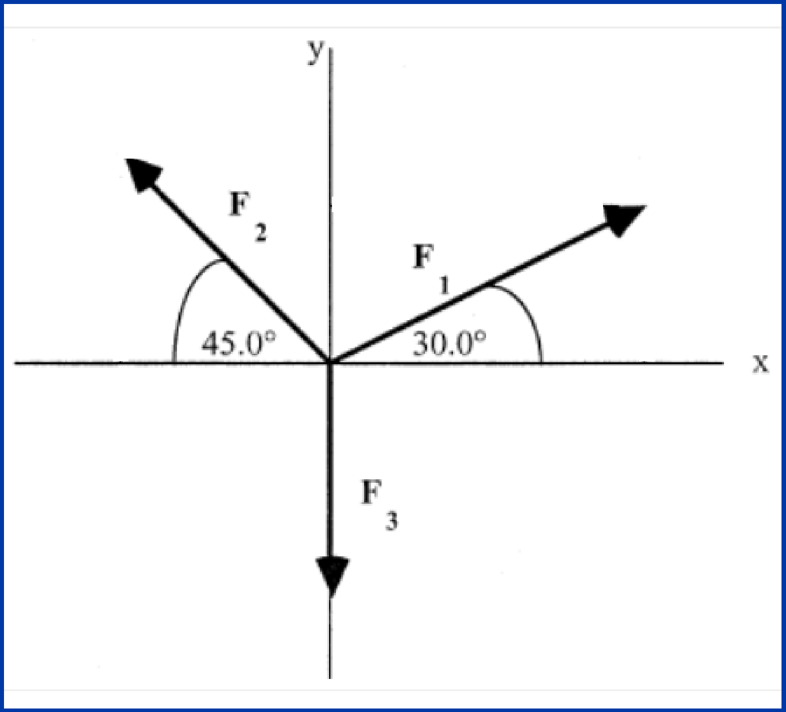
Georgia State University | Perimeter College

Identifying Students' Misconceptions

Determine whether the statements are true or false. If false, explain why and correct the error. If true, discuss why the statement is true.

1. $The\ solution\ to\ the\ equation\ x^2 = 25\ is\ x = 5$
2. $(x - 5)^2 = x^2 - 25$
3. $\sqrt{x^2 + y^2} = x + y$
4. $x^2 x^4 = x^8$
5. $(x^2)^4 = x^6$
6. $\log(x + y) = logx + logy$
7. $\frac{\frac{1}{2}xy^2 - xz}{v} = \frac{\frac{1}{2}y^2 - z}{v}$
8. $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$
9. $\frac{a+b}{a+c} = \frac{b}{c}$
10. $(4x)^2 = 4x^2$

Origins of our project – 2



Suppose a 1.8 kg object is subjected to the forces illustrated in the diagram.

The magnitudes of the forces are $\mathbf{F}_1$ = 22 N, $\mathbf{F}_2$ = 18 N, and $\mathbf{F}_3$ = 15 N.

What is the net force acting on the object?

Accessing Prior Knowledge

Multiplying vs. Factoring

Multiplying

$(x+3)^3$

$= (x+3)(x+3)(x+3)$

FOIL Method

$= (x+3)(x^2+6x+9)$

Distributive Property

$= x^3 + 6x^2 + 9x + 3x^2 + 18x + 27$

$= x^3 + 9x^2 + 27x + 27$

$= x^3 + 9x^2 + 27x + 27$

Factoring

$x^3 + 3^3$

same sign

x times x

x times 3

3 times 3

$(x+3)(x^2 - 3x + 9)$

opposite sign

always positive

$= (x+3)(x^2 - 3x + 9)$

Accessing Prior Knowledge

Things You Should Know From Algebra for Calculus

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ when $ax^2 + bx + c = 0$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ for the line through (x_1, y_1) and (x_2, y_2)

Equations of Lines: Point-Slope form: $y - y_1 = m(x - x_1)$
Slope-Intercept form: $y = mx + b$

Factoring Formulas: Difference of two squares: $x^2 - y^2 = (x - y)(x + y)$
Sum of two cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
Difference of two cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Special Product Formulas:
$(a - b)(a + b) = a^2 - b^2$
$(a + b)^2 = a^2 + 2ab + b^2$ **NOTE:** $(a + b)^2 \neq a^2 + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

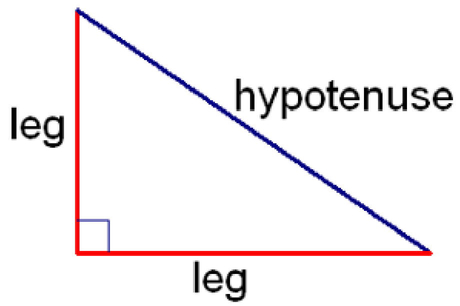
Adding Fractions: $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$ **NOTE:** $\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a+b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

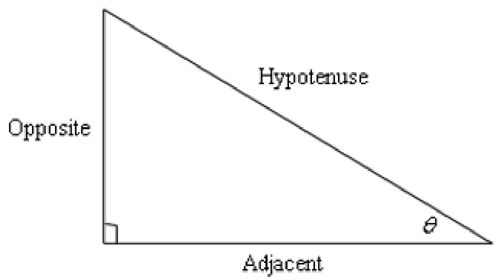
Subtracting Fractions: $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$

Things You Should Know From Trigonometry for Physics

Pythagorean Theorem: $(leg)^2 + (leg)^2 = (hypotenuse)^2$



Right Triangle Trigonometry:



"SOHCAHTOA" is a helpful mnemonic for remembering the definitions of the trigonometric functions sine, cosine, and tangent i.e., **sine** equals **opposite** over **hypotenuse**, **cosine** equals **adjacent** over **hypotenuse**, and **tangent** equals **opposite** over **adjacent**.

$$sin\theta = \frac{opposite}{hypotenuse}$$

$$cos\theta = \frac{adjacent}{hypotenuse}$$

$$tan\theta = \frac{opposite}{adjacent}$$

Making Connections

AVERAGE RATE OF CHANGE

Algebra

Find the slope of the line that passes through the points $(4, 1)$ and $(7, 16)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{16 - 1}{7 - 4}$$

$$= \frac{15}{3}$$

$$= \mathbf{5}$$

Precalculus/Calculus

For the function $f(x) = (x - 3)^2$, find the average rate of change between the following points:
$x = 4$ and $x = 7$.

$$\textbf{average rate of change} = \frac{f(b) - f(a)}{b - a}$$

$$f(x) = (x - 3)^2$$
$$f(7) = (7 - 3)^2 = 16$$
$$f(4) = (4 - 3)^2 = 1$$

$$\text{average rate of change} = \frac{f(7) - f(4)}{7 - 4}$$

$$= \frac{16 - 1}{7 - 4}$$

$$= \frac{15}{3}$$

$$= \mathbf{5}$$

Physics

If an object is dropped from a building, then the distance it has fallen after t seconds is given by the function $d(t) = 16t^2$. Find its average speed between 1 s and 5 s.

$$\textbf{average speed} = \frac{f(b) - f(a)}{b - a}$$

$$d(t) = 16t^2$$
$$d(5) = 16(5)^2 = 400$$
$$d(1) = 16(1)^2 = 16$$

$$\text{average speed} = \frac{400 - 16}{5 - 1}$$

$$= \frac{384}{4}$$

$$= \mathbf{96\ ft/s}$$

Linear Systems

Algebra

Solve the system of linear equations using the substitution method:

$$x - 7y = 9 \quad (1)$$
$$2y - x = 1 \quad (2)$$

Take equation (1) and isolate x.

$$x - 7y = 9$$
$$x = 9 + 7y$$

Now take equation (2) and substitute $9 + 7y$ for x.

$$2y - x = 1$$
$$2y - (9 + 7y) = 1$$
$$2y - 9 - 7y = 1$$
$$-5y - 9 = 1$$
$$-5y = 10$$
$$y = -2$$

Since,

$$x = 9 + 7y$$
$$x = 9 + 7(-2)$$
$$x = 9 - 14$$
$$x = -5$$

Solution: $\mathbf{(-5, -2)}$

Physics

Solve the system of linear equations using the substitution method:

$$F_T - m_1 g = m_1 a \quad (1)$$
$$m_2 g - F_T = m_2 a \quad (2)$$

Take equation (1)and isolate F_T.

$$F_T - m_1 g = m_1 a$$
$$F_T = m_1 a + m_1 g$$

Now take equation (2) and substitute $m_1 a + m_1 g$ for F_T.

$$m_2 g - F_T = m_2 a$$
$$m_2 g - (m_1 a + m_1 g) = m_2 a$$
$$m_2 g - m_1 a - m_1 g = m_2 a$$
$$m_2 g - m_1 g = m_2 a + m_1 a$$
$$g(m_2 - m_1) = a(m_2 + m_1)$$
$$g\left(\frac{m_2 - m_1}{m_2 + m_1}\right) = a$$

$$\boldsymbol{a = g\left(\frac{m_2 - m_1}{m_2 + m_1}\right)}$$

Solving Proportions

Algebra

Solve for x:

$$\frac{x}{36} = \frac{8}{9}$$

$$\frac{x}{36} = \frac{8}{9}$$

$$9x = 36 \bullet 8$$

$$x = \frac{36 \bullet 8}{9}$$

$$x = \mathbf{32}$$

Chemistry

What is the mass of oxygen in 3600 grams of water?

Let mO_2 be the mass of oxygen in 3600 grams of water. Note that gram molecular weight of water H_2O is 2*(gram atomic weight of Hydrogen) + 1*(gram atomic weight of Oxygen) = 2*1 + 16 = 18.

Since the mass of oxygen is proportional to the total mass of water, you can write the proportion

$$\frac{mO_2}{3600} = \frac{16}{18}$$

$$mO_2 = \frac{3600 \bullet 16}{18} = \mathbf{3200}$$

The mass of oxygen in 3600 grams of water is 3200 grams.

Methods and Processes

Finding Derivatives: How to Get Started

To get started finding the derivatives of functions using the Chain Rule, look at the Big Picture first. Begin with the outer most layer and work your way to the inner most later. Here are some examples worked out. Keep practicing and maintain a positive attitude. You CAN do This!!!

Example 1: $y = (5x^3 - 7x + 5)^{20}$

The outer layer is the 20^{th} power. The inner layer is $(5x^3 - 7x + 5)$.

Differentiate the 20^{th} power first leaving the inner layer of $(5x^3 - 7x + 5)$ as is.
NOTE: **The key is to leave the inner layer unchanged until after you have differentiated the outer layer. Then, you multiply by the derivative of the inner layer.**

Then, differentiate $5x^3 - 7x + 5$.

$$y' = 20(5x^3 - 7x + 5)^{19}(15x^2 - 7)$$

Simplifying, we obtain

$$y' = 20(15x^2 - 7)(5x^3 - 7x + 5)^{19}$$

Example 3: $y = \ln(5x^6 + 7x^3)$

The outer layer is the natural logarithmic function and the inner layer is $5x^6 + 7x^3$.

Differentiate the natural logarithmic function first, leaving the inner layer of $5x^6 + 7x^3$ as is.

Then, differentiate $5x^6 + 7x^3$

$$y' = \frac{1}{5x^6 + 7x^3} \cdot 30x^5 + 21x^2$$

Simplifying, we obtain

$$y' = \frac{30x^5 + 21x^2}{5x^6 + 7x^3}$$

Example 4: $y = cos^2(x^3)$

Before deciding on the outer layer, remember that that notation $cos^2 x$ means that the entire function, $cosx$ is being squared, $(cosx)^2$, so it is important to rewrite the function first. So,

$$y = cos^2(x^3) \text{ becomes}$$

$$y = [cos(x^3)]^2$$

The first layer is the square. The second layer is the cosine function, and the **third layer is x^3.**

Differentiate the square first, leaving the cosine function and x^3 unchanged.

Then differentiate the cosine function leaving x^3 as is.

Finally, take the derivative of x^3.

$$y' = 2[\cos(x^3)]^1[-\sin(x^3)][3x^2]$$

Simplifying, we obtain

$$y' = -6x^2\cos(x^3)\sin(x^3)$$

Problem Solving Process

A General Problem-Solving Method

1. Understand the problem.
   a. Read the problem carefully.
   b. What are you looking for?
   c. What technique are you going to use?

2. Draw pictures or diagrams.

3. Identify knowns and unknowns.
   a. Assign symbols or variables for all quantities involved – known AND unknown.
   b. Make sure all your units are consistent.

4. Write the function of interest or choose the fundamental equation or method to use to solve the problem.

5. Solve the problem.
   a. Solve equations algebraically for the variable of interest and make any algebraic substitutions BEFORE plugging in numerical values.
   b. Substitute numerical values for the known variables and calculate. Be careful about negative signs and entering parentheses into your calculator!

6. Evaluate your answer.
   a. Does it make sense?
   b. Does it answer what is being asked in the original question?

Understand that this is an "organic" process where the steps may not be completely separate from one another!

Problem Solving Process

Problem Solving Guide: Kinematics in One Dimension

1. **Understand the problem.**
   What are the "events" in the problem? Is this a one-stage problem or a multi-segment problem?

   What are you trying to find out? What variable represents this?

   What do you know from the statement of the problem?

2. **Draw pictures.**
   Sketch of the situation from beginning to end:

   Label your diagram with the following:
   - x_0 or y_0
   - Where x or $y = 0$
   - An arrow showing the direction in which x or y is increasing
   - The locations of each event
   - Between each event, the directions of velocity and acceleration (as labeled arrows)

3. **Identify knowns and unknowns.**
   Decide how many stages/segments the problem has and use the appropriate number of columns. Cross off what you aren't using. If working in the vertical direction, scratch out the x and replace with y.

$x_0 =$	$x_1 =$	$x_2 =$	$x_3 =$
$v_0 =$	$v_1 =$	$v_2 =$	$v_3 =$
$t_0 =$	$t_1 =$	$t_2 =$	$t_3 =$
$a_{01} =$	$a_{12} =$	$a_{23} =$	

4. **Pick your physics.**
   What equations will you need:
   - Is there a kinematic equation that gets what you want to find out in one step? If not, which one can you use first to get information to use in a second equation?
   - If you have multiple segments, how are your knowns and unknowns interspersed – are you going to need to start from the beginning and work forwards or at the end and work backwards?

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_{01}(t_1 - t_0)^2$$
$$v_1 = v_0 + a_{01}(t_1 - t_0)$$
$$x_1 = x_0 + \frac{1}{2}(v_0 + v_1)\,(t_1 - t_0)$$
$$v_1{}^2 = v_0{}^2 + 2a_{01}(x_1 - x_0)$$

5. **Do the math.**
   - Write down the equation you are starting with.
   - Algebraically solve the for the desired variable.
   - Plug numbers in and find the result.
     - Think about how many significant figures your answer should have. Hold on to a guard digit if necessary.
     - If you take a square root, think carefully about whether the positive or negative root is appropriate.
   - If you have not yet found the variable you are looking for, place what you just found into your table of knowns and unknowns and start the cycle again.
   - Write your final answer with the correct number of significant figures and units.

6. **Does the answer make sense?**
   Think about how you could know whether your answer is reasonable or not. You do not need to write anything here, but you should be able to make a justification. Think about directions, whether velocities and distances in general make sense, etc.

   Justification:

“Roundtable” Discussion

What do *you* see about where your students struggle?

How do we make this the most useful to students?

How do we get this *to* the students?

Feel free to contact us!

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