Simplifying Trigonometric Expressions

Objective: To use algebra and fundamental identities to simplify a trigonometric expression

- You need to memorize the fundamental trigonometric identities on page 532 in your textbook.

- You need to be able to recognize rearrangements of fundamental identities. In particular, you often see rearrangements of Pythagorean Identities. For example,

$$\sin^2 x + \cos^2 x = 1 \quad \Longrightarrow \quad \sin^2 x = 1 - \cos^2 x$$
$$\sin^2 x + \cos^2 x = 1 \quad \Longrightarrow \quad \cos^2 x = 1 - \sin^2 x$$

- Simplifying trigonometric expressions often takes some trial and error, but the following strategies may be helpful.
    - Use algebra and fundamental identities to simplify the expression.
    - Sometimes, writing all functions in terms of sines and cosines may help.
    - Sometimes, combining fractions by getting a common denominator may help.
    - Sometimes, breaking one fraction into two fractions may help: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$
    - Sometimes, factoring may help.

Strategy	**Example**	**Approach**
Rewriting in terms of sine and cosine	$\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$ $= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}$ $= \sin x$	• $\tan x = \frac{\sin x}{\cos x}$ • $\sec x = \frac{1}{\cos x}$ • To divide by a fraction, multiply by the reciprocal of the denominator • Reduce the resulting product

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Strategy	Example	Approach
Factoring	$\cos x - \cos x \sin^2 x = \cos x\,(1 - \sin^2 x)$ $= \cos x \cdot \cos^2 x$ $= \cos^3 x$	• Factor out a common factor of $\cos x$ • Use the identity: $\cos^2 x = 1 - \sin^2 x$ • Use a property of exponents to multiply $\cos x$ and $\cos^2 x$
Getting a common denominator	$\sin x + \cos x \cot x = \sin x + \cos x \cdot \frac{\cos x}{\sin x}$ $= \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$ $= \frac{\sin^2 x + \cos^2 x}{\sin x}$ $= \frac{1}{\sin x}$ $= \csc x$	• $\cot x = \frac{\cos x}{\sin x}$ • Get a common denominator of $\sin x$ and add the two fractions • $\sin^2 x + \cos^2 x = 1$ • $\csc x = \frac{1}{\sin x}$
Splitting one fraction into two fractions	$\frac{\sec x - \cos x}{\sec x} = \frac{\sec x}{\sec x} - \frac{\cos x}{\sec x}$ $= 1 - \cos^2 x$ $= \sin^2 x$	• $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ • $\sec x$ divided by itself is 1 • $\sec x = \frac{1}{\cos x}$ so $\frac{\cos x}{\sec x} = \cos^2 x$ • $1 - \cos^2 x = \sin^2 x$

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Simplify the following expressions.

1) $\sin x \cot x$

2) $\frac{\sec x}{\csc x}$

3) $\frac{1-\sin^2 x}{\cos x}$

4) $\sin t - \sin t \cos^2 t$

5) $\cos x + \tan x \sin x$

6) $\sin^3 x + \sin x \cos^2 x$

7) $\frac{\csc x - \sin x}{\csc x}$

8) $\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x}$

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Solutions:

1) $\sin x \cot x = \sin x \cdot \frac{\cos x}{\sin x} = \cos x$

2) $\frac{\sec x}{\csc x} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x$

3) $\frac{1-\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$

4) $\sin t - \sin t \cos^2 t = \sin t \cdot (1 - \cos^2 t) = \sin t \cdot \sin^2 t = \sin^3 t$

5) $\cos x + \tan x \sin x = \cos x + \frac{\sin x}{\cos x} \cdot \sin x = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

6) $\sin^3 x + \sin x \cos^2 x = \sin x \cdot (\sin^2 x + \cos^2 x) = \sin x$

7) $\frac{\csc x - \sin x}{\csc x} = \frac{\csc x}{\csc x} - \frac{\sin x}{\csc x} = 1 - \sin^2 x = \cos^2 x$

8) $\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x \cdot (1+\sin x) + \cos x \cdot \cos x}{\cos x \cdot (1+\sin x)} = \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x \cdot (1+\sin x)} = \frac{\sin x + 1}{\cos x \cdot (1+\sin x)} = \frac{1}{\cos x} = \sec x$