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# Efficiency wages, unemployment, and environmental policy

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ABSTRACT

We study the incidence of pollution taxes and their impact on unemployment in an analytical general equilibrium efficiency wage model. We find closed-form solutions for the effect of a pollution tax on unemployment, factor prices, and output prices, and we identify and isolate different channels through which these general equilibrium effects arise. An effect arising from the efficiency wage specification depends on the form of the workers' effort function. Numerical simulations further illustrate our results and show that this efficiency wage effect can fully offset the sources-side incidence results found in models that omit it.

## 1. Introduction

The effects that environmental policies may have on labor markets, and whether and to what extent they kill jobs or create jobs, is of utmost importance to policymakers. Much popular aversion to environmental regulation comes from its perceived negative impact on jobs. Other distributional impacts of policy, like the sources-side and uses-side incidence, can depend on frictions in the labor market that yield unemployment. It is important for policymakers to understand the effect of environmental policies on unemployment and on both factor and output prices.

There are several ways to go about addressing the general question of how environmental policies affect labor markets and unemployment. Many papers empirically estimate the impact of specific environmental policies on employment, including Martin et al. (2014), Curtis (2018) and Colmer et al. (2018). Other papers use computable general equilibrium (CGE) models to quantify the large-scale effects that policies like an economy-wide carbon tax might have, including Böhringer et al. (2003), Hafstead et al. (2018), and Castellanos and Heutel (2019). A third approach uses analytical general equilibrium modeling, which can shed light on the mechanisms behind the effects that can be quantified through empirical or CGE models. Both Hafstead and Williams (2018) and Aubert and Chiroleu-Assouline (2019) introduce pollution policy and unemployment resulting from labor search frictions into an analytical general equilibrium model. Our paper follows this third approach.

The purpose of this paper is to study the effect of pollution taxes on unemployment and incidence using an analytical general equilibrium model where unemployment is endogenously generated via efficiency wages. Workers' effort is a function of the real wage and the economy's unemployment level. Pollution is modeled as a production input along with capital and labor. We find closed-form analytic solutions for the general equilibrium responses to a change in the pollution tax rate, including expressions for changes in the unemployment rate, factor prices (the sources-side incidence), output prices (the uses-side incidence), and worker effort. The model allows us to decompose the net effects into substitution effects, output effects, and effects from the efficiency wage specification. Lastly, we conduct numerical simulations using calibrated parameter values.

Our modeling approach dates back to the canonical tax incidence modeling of Harberger (1962). Like Agell and Lundborg (1992) and Rapanos (2006), our paper adds an efficiency wage theory of unemployment to the model, though those papers do not model pollution. Like Fullerton and Heutel (2007), our paper adds pollution and pollution taxes to the model, though that paper does not model unemployment.[1] We incorporate both efficiency wages and environmental policy into a Harberger-style analytical general equilibrium tax incidence model. Our paper is most similar to Hafstead and Williams (2018) and Aubert and Chiroleu-Assouline (2019), which both also model environmental policy and unemployment in an analytical general equilibrium setting. However, in both of those papers, unemployment arises from Diamond-Mortensen-Pissarides-style search frictions (Pissarides, 2000). In our

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[1] Other papers that use a similar methodology to incorporate pollution policy into analytical general equilibrium modeling include Gonzalez (2012), Fullerton and Monti (2013), Dissou and Siddiqui (2014), and Baylis et al. (2014).

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paper, unemployment arises from efficiency wage theory (Akerlof, 1982; Shapiro and Stiglitz, 1984).[2]

Our theoretical results add new insights to the tax incidence literature. We identify effects that have been found in previous studies of environmental taxes, like the output and substitution effects. For example, an output effect exists such that the pollution tax disproportionately burdens the factor that is used more intensively in the polluting sector. These effects differ, though, when there is endogenous unemployment generated through efficiency wages. Along with these standard effects, we identify an effect that is new to the environmental tax literature, which we call the *efficiency wage effect*. The magnitude and direction of this effect depend on the form of the workers' effort function. Generally, the less elastic the workers' marginal effort response to the real wage is, the less burden labor bears, and the smaller increase in unemployment. With more structure on the effort function, we show that the crucial parameters of the effort function are the elasticities of effort with respect to the real wage and to unemployment. When effort responds more strongly to the real wage, then the magnitude of the efficiency wage effect is larger, which alleviates the tax burden on labor. When effort responds more strongly to unemployment, then the magnitude of the efficiency wage effect is smaller.

This key result depends on the efficiency wage specification causing unemployment, and so it is missing from previous studies that model unemployment through other causes. Our efficiency wage specification is general enough to accommodate different causes of efficiency wages and different effort functions. A gift exchange or fair wage efficiency wage model like Akerlof (1982) will lead to effort being very responsive to the real wage, and we find that this implies that the efficiency wage effect will be large. A shirking and firing model like Shapiro and Stiglitz (1984) will lead to effort being very responsive to unemployment, and we find that this implies that the efficiency wage effect will be small. Thus, the structural origin of unemployment fundamentally affects how large of an effect the unemployment friction will have on standard tax incidence outcomes.

The calibrated numerical simulation results, based on a $40 per ton carbon tax, provide further insights into these effects. The disproportionate burden of the tax on labor from substitution effects is offset by the disproportionate burden on capital from the efficiency wage effect. Ignoring the efficiency wage effect, as in previous environmental tax incidence models, thus gets the sign of the sources-side incidence wrong. Because of the efficiency wage effect, the carbon tax burdens capital disproportionately higher than labor. The tax increases the unemployment rate by just under 1%; this effect is mainly driven by a substitution effect from the larger, untaxed clean sector, rather than substitution within the smaller, taxed dirty sector. Sensitivity analyses show that the effects on unemployment and on sources-side incidence depend on the effort function elasticities, the effect on sources-side incidence also depends on production elasticities, and the effect on uses-side incidence is relatively unaffected by these parameters. Both the analytical and the numerical results highlight the important role of the efficiency wage effect and the form of the effort function in the analysis of pollution tax incidence and unemployment effects.

The paper is organized as follows. Section 2 presents the model and derives a system of linearized equations. Section 3 presents and interprets the general solution, decomposing the net effect into separate effects. Section 4 calibrates and numerically simulates the model. The last section concludes.

[2] Furthermore, Hafstead and Williams (2018) do not provide analytical, closed-form solutions, just numerical simulations, and neither Hafstead and Williams (2018) nor Aubert and Chiroleu-Assouline (2019) include capital in their model.

## 2. Model

Our model is a two-sector, two-factor incidence model, in the spirit of Harberger (1962), with the addition of involuntary unemployment through an efficiency wage as in Agell and Lundborg (1992) and Rapanos (2006), and with the addition of pollution as in Fullerton and Heutel (2007, 2010). The model is linearized so that it can be solved analytically. We consider a competitive two-sector economy using two factors of production: capital and labor. Both factors are perfectly mobile between sectors. A third variable input is pollution, $Z$, which is only used in production of one of the goods (the "dirty" good). The constant returns to scale production functions are:

$$X = X(K_X, E_X)$$

$$Y = Y(K_Y, E_Y, Z)$$

where X is the "clean" good, Y is the "dirty" good, $K_X$ and $K_Y$ are the capital used in each sector, and $E_X$ and $E_Y$ are the effective labor, in efficiency units, used in each sector.

The effective labor in each sector is defined as the actual amount of labor $L$ times the effort level $e$:

$$E_X = e\left(\frac{W}{P}, U\right) \bullet L_X$$

$$E_Y = e\left(\frac{W}{P}, U\right) \bullet L_Y$$

where $e\left(\frac{w}{P}, U\right)$, the effort level of a representative worker, depends on the real wage rate $\frac{w}{P}$, and on the level of unemployment $U$.

This effort function is how we incorporate the efficiency wage theory of unemployment into our model. In structural models of efficiency wages, effort is an endogenously-determined optimal response of workers given the possibility of termination if caught shirking (Shapiro and Stiglitz, 1984) or norms of fairness (Akerlof, 1982). But here, the effort function is a reduced-form relationship between the wage, unemployment, and the level of effort. Our reduced-form effort function is identical to that of Rapanos (2006).[3]

Structural efficiency wage models predict that effort is positively related to the real wage $\left(\frac{w}{P}\right)$ and to the economy-wide level of unemployment, so we impose that the first derivatives $e_1$ and $e_2$ are positive.[4] The effort level is identical across the two sectors (since neither the real wage nor unemployment are sector-specific). $L_X$ and $L_Y$ are the labor used in each sector in terms of the number of workers. Linearizing the two equations defining effective labor gives us:

$$\widehat{E_X} = \widehat{e} + \widehat{L_X} \tag{1}$$

$$\widehat{E_Y} = \widehat{e} + \widehat{L_Y} \tag{2}$$

We adopt the "hat" notation where a variable with a hat represents a proportional change in the variable. That is, $\widehat{E_X} \equiv dE_X/E_X$, and likewise for the other variables.

Both representative firms face the same effort function $e$, and they set their wages $w$ to minimize the effective wage cost per worker $v \equiv w/e$. Formally, the optimization problem for the representative firm is:

$$\min_w v = \frac{w}{e\left(\frac{W}{P}, U\right)}$$

[3] The reduced-form effort function in Agell and Lundborg (1992) is slightly different; effort is a function of the relative wages across industries and the ratio of the wage to capital rental rate.

[4] Empirical support for this reduced-form relationship is found in Raff and Summers (1987) and Cappelli and Chauvin (1991).

The first-order condition is

$$e - e_1 \frac{w}{P} = 0$$

where $e_1$ is the first derivative of the effort function with respect to the real wage. This condition can be written as $\varepsilon_1 \equiv \frac{e_1 w}{ep} = 1$, meaning that the wage is set so that the elasticity of effort with respect to the real wage is one. The intuition is that the firm sets the wage just so that the value of the extra effort induced by a marginal increase in the wage ($e_1 \frac{w}{P}$) is equal to the effort level, thus minimizing the effective wage cost per worker.[5] Because we seek a linearized solution to the model, we totally differentiate this first-order condition and employ the assumption that $e_{12} = \frac{\partial^2 e}{\partial\left(\frac{w}{P}\right)\partial U} = 0$ to obtain:

$$e_2 dU = \frac{e_{11} w^2}{P^2}\left(\frac{dw}{w} - \frac{dP}{P}\right)$$

which can be rewritten, using the firm's first-order condition, as

$$\widehat{U} = \frac{e_{11} w^2}{e_2 U P^2}\left(\widehat{w} - \widehat{P}\right) = \frac{\frac{e_{11}}{e_1} \bullet \frac{w}{P}}{\frac{e_2}{e} \bullet U}\left(\widehat{w} - \widehat{P}\right) \#$$

$$\widehat{U} = \frac{\varepsilon_{11}}{\varepsilon_2}\left(\widehat{w} - \widehat{P}\right) \quad (3)$$

where $\varepsilon_{11} \equiv \left(\frac{e_{11}}{e_1}\right)\left(\frac{w}{P}\right)$, and $\varepsilon_2 \equiv \left(\frac{e_2}{e}\right)U$. Since $e_2 > 0$, we also have $\varepsilon_2 > 0$, which is the elasticity of effort with respect to unemployment. We assume concavity of the effort function with respect to the real wage $w/P$ to ensure an interior solution to the minimization problem, so $e_{11} < 0$, which implies that $\varepsilon_{11} < 0$.

The intuition behind eq. (3) is that in equilibrium the change in the unemployment rate must be related to the change in the real wage ($\widehat{w} - \widehat{P}$) according to the shape of the effort function and in particular the value of $\varepsilon_{11}$. This parameter is important throughout the analysis and arises in the closed-form solutions presented below. It is a measure of the concavity of the effort function with respect to the real wage. If it is close to zero, the effort function is close to linear in the real wage. In this case, equilibrium unemployment will not be very responsive to changes in the real wage, because changes in the wage will mainly affect effort. If $\varepsilon_{11}$ is large in absolute value, then the marginal effort with respect to the real wage ($e_1$) declines quickly as the wage increases.[6] In this case, unemployment is very responsive to the real wage, since changes in the wage do not affect effort very much.[7] As firms in two sectors set wages in order to affect the effort level of their workers, they will not necessarily offer the same wage to workers. In the appendix, we lay out the model when there is not only one single wage rate across the two sectors, and discuss the reasons for assuming one identical wage rate in our main model.

Totally differentiating the effort function $e = e\left(\frac{w}{P}, U\right)$ obtains

$$\widehat{e} = \widehat{w} - \widehat{P} + \varepsilon_2 \widehat{U} \quad (4)$$

From the definition of effective wage v, we have

$$\widehat{v} = \widehat{w} - \widehat{e} \quad (5)$$

The first five equations of our model describe the labor market and are identical to those in the efficiency wage model of Rapanos (2006). Following both Rapanos (2006) and Agell and Lundborg (1992), the wage and the effort level are identical across the two sectors. In principle, there could be a separating equilibrium where these differ across sectors so long as workers are indifferent between the wage-effort bundle in each sectors (i.e. a higher wage offsets a higher effort level in one sector). We ignore this possibility in our model and solutions since it adds unnecessary complications in the solutions without offering further insights. In Appendix A.IV., we present the extension to the model that includes the possibility of heterogeneous wage and effort.

The resource constraints are:

$$K_X + K_Y = \overline{K}$$

$$L_X + L_Y = \overline{L} - U$$

where $\overline{K}$ and $\overline{L}$ are the fixed total amounts of capital and labor in the economy.[8] All capital is fully employed. A fraction $U$ of the total labor stock is unemployed, because, as described above, the wage may be set above the market-clearing level by firms who want to affect workers' effort. Totally differentiating the resource constraints (noting that $\overline{K}$ and $\overline{L}$ remain fixed) yields

$$\widehat{K_X} \bullet \lambda_{KX} + \widehat{K_Y} \bullet \lambda_{KY} = 0 \quad (6)$$

$$\widehat{L_X} \bullet \lambda_{LX} + \widehat{L_Y} \bullet \lambda_{LY} = -\widehat{U} \bullet \lambda_{LU} \quad (7)$$

where $\lambda_{ij}$ denotes sector $j$'s share of factor $i$ ($\lambda_{KX} = \frac{K_X}{K}$). $\lambda_{LU}$ denotes the unemployment rate ($\lambda_{LU} = \frac{U}{L}$). Pollution $Z$ has no equivalent resource constraint. As in Fullerton and Heutel (2007), we start with a preexisting positive tax $\tau_Z$ on pollution.

When modeling producer behavior, we consider the producers responding to the price and quantity of effective labor rather than actual labor. Workers receive the actual wage $w$ times the number of hours worked $L_X$ or $L_Y$, though from the firms' perspective it is the price of a unit of effective labor $v$ and the quantities $E_X$ and $E_Y$ that matter. Producers of $X$ can substitute between factors in response to changes in the factor prices $p_K \equiv r(1 + \tau_K)$ and $p_E \equiv v(1 + \tau_E)$, where $\tau_K$ and $\tau_E$ are the *ad valorem* taxes on capital and effective labor. We will only consider a change in the pollution tax, not in any of the other taxes, so, $\widehat{p_K} = \widehat{r}$ and $\widehat{p_E} = \widehat{v}$. Since we do not model changes in other pre-existing tax rates, like the labor tax, we do not consider revenue recycling (i.e. using the pollution tax revenues to reduce the labor tax rate). Pollution tax revenues are returned lump-sum and assumed to not affect equilibrium prices.[9] The elasticity of substitution in production $\sigma_X$ is defined to capture this response to factor price changes:

$$\widehat{K_X} - \widehat{E_X} = \sigma_X(\widehat{v} - \widehat{r}) \quad (8)$$

where $\sigma_X$ is defined to be positive.

[5] As Rapanos (2006) points out, this first-order condition is equivalent to the traditional Solow condition (Solow, 1979)

[6] Rapanos (2006) describes the parameter $\varepsilon_{11}$ as "the rate at which workers get satisfied with real wages." (p. 481).

[7] Alternatively, rewriting Eq. (3) to solve for $\widehat{w}$, the firm's choice variable, yields: $\widehat{w} = \frac{\varepsilon_2}{\varepsilon_{11}}\widehat{U} + \widehat{P}$. This also demonstrates how the elasticity parameters mitigate how responsive equilibrium wage is to the unemployment rate.

[8] As is standard in Harberger-type incidence models, total resources are fixed, though they can be re-allocated across sectors in response to policy. These models are thus sometimes described as "medium-run" adjustment models.

[9] CGE models, like Hafstead et al. (2018) and Castellanos and Heutel (2019), consider revenue recycling.

Producers of $Y$ use three inputs: capital, effective labor, and pollution. Firms face no market price for pollution, just a tax on per unit of pollution, so $p_Z = \tau_z$ and $\widehat{p_Z} = \widehat{\tau}_z$.[10] We model firm $Y$'s behavior by assuming a constant elasticity of substitution (CES) production function, yielding:

$$\widehat{K_Y} - \widehat{Z} = \sigma_Y(\widehat{\tau}_Z - \widehat{r}) \tag{9}$$

$$\widehat{E_Y} - \widehat{Z} = \sigma_Y(\widehat{\tau}_Z - \widehat{v}) \tag{10}$$

where $\sigma_Y > 0$ is the elasticity of substitution in production. Eqs. (9) and (10) show how a change in any of the input prices affects the relative demand for the three inputs. The change in relative demand is a function of the change in relative prices and the substitution elasticity. A more complicated and general way of modeling production when the dirty sector has three inputs is to use Allen elasticities of demand, as in Fullerton and Heutel (2007). While that assumption is more general than CES, the resulting general solution is very long and complicated and does not add additional insight into the effect of the efficiency wage specification on outcomes. In the appendix, we present and analyze the full solution from the more general model, while here in the text we use the CES simplification.

Using the assumptions of perfect competition and constant returns to scale, we get

$$\widehat{p_X} + \widehat{X} = \theta_{XK}\left(\widehat{r} + \widehat{K_X}\right) + \theta_{XE}\left(\widehat{v} + \widehat{E_X}\right) \tag{11}$$

$$\widehat{p_Y} + \widehat{Y} = \theta_{YK}\left(\widehat{r} + \widehat{K_Y}\right) + \theta_{YE}\left(\widehat{v} + \widehat{E_Y}\right) + \theta_{YZ}\left(\widehat{Z} + \widehat{\tau}_Z\right) \tag{12}$$

Here $\theta_{YK} \equiv \frac{r(1+\tau_K)K_Y}{p_Y \cdot Y}$, $\theta_{YE} \equiv \frac{v(1+\tau_E)E_Y}{p_Y \cdot Y}$ and $\theta_{YZ} \equiv \frac{\tau_Z \cdot Z}{p_Y \cdot Y}$ are the share of sales revenue from $Y$ that is paid to capital, to effective labor, and to pollution (through the tax), respectively. Define $\theta_{XK}$ and $\theta_{XE}$ similarly to $\theta_{YK}$. (Note that $\theta_{XK} + \theta_{XE} = 1$ and $\theta_{YK} + \theta_{YE} + \theta_{YZ} = 1$.) Totally differentiate each sector's production function and substitute in the conditions from the perfect competition assumption to get

$$\widehat{X} = \theta_{XK}\widehat{K_X} + \theta_{XE}\widehat{E_X} \tag{13}$$

$$\widehat{Y} = \theta_{YK}\widehat{K_Y} + \theta_{YE}\widehat{E_Y} + \theta_{YZ}\widehat{Z} \tag{14}$$

The details of the derivation of eqs. (11) through (14) can be found in Fullerton and Heutel (2007, Appendix A).

Consumer preferences are modeled using $\sigma_u$, the elasticity of substitution between goods $X$ and $Y$.[11] The definition of this elasticity yields

$$\widehat{X} - \widehat{Y} = \sigma_u(\widehat{p_Y} - \widehat{p_X}) \tag{15}$$

Lastly, the price index $P$, which appears in the effort function, is defined to equal a weighted average of the output prices of the two goods, i.e. $P = p_X{}^{\eta} \cdot p_Y{}^{1-\eta}, (\eta < 1)$. Then the change in the price index can be written as

$$\widehat{P} = \eta\widehat{p_X} + (1-\eta)\widehat{p_Y} \tag{16}$$

The full model is eqs. (1) through (16). It contains just one exogenous policy variable ($\widehat{\tau}_Z$) and 17 endogenous variables. To solve it, we impose a normalization assumption by assuming that the price index $P$ is the numeraire and unchanged, so that $\widehat{P} = 0$.[12] Dropping $\widehat{P}$ from the model thus yields 16 equations with 16 unknowns ($\widehat{K_X}, \widehat{K_Y}, \widehat{E_X}, \widehat{E_Y}, \widehat{L_X}, \widehat{L_Y}, \widehat{Z}, \widehat{U}, \widehat{e}, \widehat{w}, \widehat{v}, \widehat{p_X}, \widehat{p_Y}, \widehat{r}, \widehat{X}, \widehat{Y}$).The model is solved with successive substitution, as described in Appendix A.I.

The graphs in Fig. 1 illustrate the intuition of the efficiency wage theory used in our model. On the left-hand side is the labor market, where supply is perfectly inelastic (since $\overline{L}$ is fixed). When labor demand is given by the curve $LD$, the equilibrium wage would be where demand and supply intersect. However, the right graph presents the effort function $e$ as a function of the wage $w$ on the x-axis. For a given effort function, the firm chooses the wage to maximize productivity per dollar paid. This is given by the tangent point of the effort function to a straight line starting at the origin, which yields the wage $w^*$ for the effort function $e = f(w, U)$. This wage is higher than the market-clearing wage, and thus causes unemployment $U$ (since quantity supplied at this wage, $LS^*$, exceeds quantity demanded at this wage, $LD^*$). Equilibrium is where the wage $w^*$ chosen at the tangent line to the effort function in the right graph for unemployment level $U$ also yields unemployment $U$ in the labor market on the left graph.

Suppose now that a pollution tax burdens the firms and therefore decreases labor demand to $LD'$. Without a change in the wage from $w^*$, unemployment would drastically increase. But the effort function also responds to unemployment, which changes the firm's optimal wage shown on the right graph. A new unemployment level yields a new optimal wage (from eq. 3). The new, post-pollution-tax equilibrium involves an unemployment level $U'$ such that the optimal wage $w^{*'}$ given $U'$ from the effort function (right graph) yields unemployment $U'$ in the labor market (left graph). As drawn in Fig. 1, the pollution tax reduces labor demand, lowers the wage, and increases unemployment. However, both the magnitude and the direction of these changes depend on model parameters. For example, the effort function might be such that the new wage $w^{*'}$ is higher than the original wage $w^*$. We explore these effects in both the analytical solutions and numerical simulations below.[13]

The simplified analysis in Fig. 1 omits several features of the model, including the two production sectors and the interaction between labor demand and capital demand. In the full multi-sector model, there are two different labor demands, one for each sector, and that is aggregated into one sector in the intuitive analysis presented in Fig. 1. The simplified analysis also omits several general-equilibrium effects that are present in the full model; for example, as the polluting sector is taxed, its size will contract, consumers will substitute into the other good, increasing its labor demand. These and other more complicated effects are captured in the full model. But the simplified analysis in Fig. 1 demonstrates that how workers' effort responds to changes in the wage and unemployment is crucial in determining the effects of the pollution tax. In an efficiency wage model, the wage is determined not only by the interaction of labor supply and demand. Rather, the wage affects both the workers' effort (internal margin) and the quantity of labor demanded (external margin). The inclusion of this margin has the potential to

[10] Modeling pollution as an input allows for a very general form of substitutability between pollution, capital, and labor. One can alternatively interpret the pollution input as an energy input. In the numerical calibration below, we calibrate pollution factor shares and other parameters based on data on energy factor shares.

[11] Like the original Harberger (1962) model, consumer demand is modeled solely through this elasticity. There is no consumer utility function or budget constraint since this is not a Walrasian general equilibrium model.

[12] This normalization implies that all price changes analyzed in the model are price changes relative to the price index $P$. An increase in the pollution tax $\tau_Z$ is actually an increase in the ratio $\frac{\tau_Z}{P}$. By contrast, Fullerton and Heutel (2007) normalize by setting the clean good price change $\widehat{p_X} = 0$, and Rapanos normalizes by setting $\widehat{p_Y} = 0$. Garnache and Mérel (2020) demonstrate that choosing the overall price index $P$ as numeraire is a more natural assumption that eliminates some counterintuitive cases.

[13] For more intuition on the microfoundations of the efficiency wage model, see Yellen (1984) or Weiss (2014).

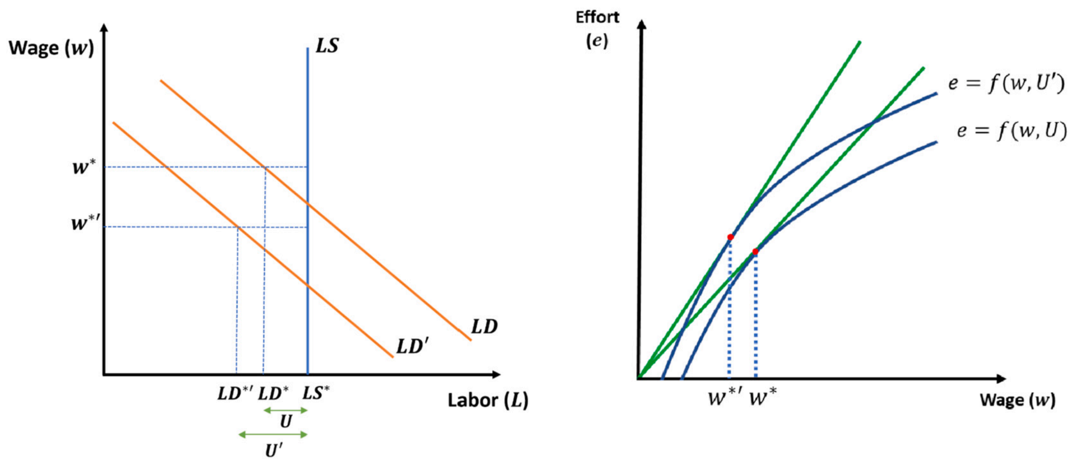


**Fig. 1. Graphical Intuition of Efficiency Wage Model** Notes: These graphs demonstrate the intuition behind how a change in the pollution tax affects wages and employment in an efficiency wage model. The left graph is the labor market, where labor supply is perfectly inelastic. The right graph presents workers' effort function $e$ as a function of the wage $w$ on the x-axis. The firms set the wage where the effort function is tangent to a ray from the origin, maximizing productivity per dollar paid, and thus creating unemployment. A pollution tax can reduce labor demand from $LD$ to $LD'$. Unemployment and thus the effort function change, and the equilibrium wage decreases from $w^*$ to $w^{*'}$.

affect the incidence results from pollution policy.

## 3. Solution

Our focus is on incidence and unemployment effects, so we are most interested in solutions for changes in factor prices ($\widehat{w}$ and $\widehat{r}$), output prices ($\widehat{p_X}$ and $\widehat{p_Y}$), and unemployment $\widehat{U}$.

We present three closed-form solutions. The first is $\widehat{U}$, which is the change in unemployment or the unemployment rate.[14] The second is $\widehat{w}-\widehat{r}$, which represents the sources-side incidence, i.e., the relative burden on labor versus capital. If $\widehat{w}-\widehat{r}$ is positive, then the wage increases more than the rental rate does (or decreases less), so the burden of the tax falls relatively more on capital than on labor. The third is $\widehat{p_Y}-\widehat{p_X}$, which represents the uses-side incidence, i.e. the relative burden on consumers of the dirty good versus consumers of the clean good. If $\widehat{p_Y}-\widehat{p_X}$ is positive, then the difference in these prices increases, so the burden of the tax falls relatively more on consumers of the dirty good versus consumers of the clean good. These solutions are:

$$\widehat{U}=\frac{\theta_{YZ}}{\varepsilon_2 D}\left\{\sigma_Y[A(1-\eta)+\eta_K(\gamma_K-\gamma_L)]+\sigma_u\theta_{XK}(\gamma_L-\gamma_K)+C\sigma_X(1-\eta)\right\}\widehat{\tau_Z} \tag{17}$$

$$\widehat{w}-\widehat{r}=\frac{\theta_{YZ}}{\varepsilon_{11}D}\left\{\begin{array}{c}\sigma_Y[(1-\eta)(A+\varepsilon_{11}B)+(\gamma_L-\gamma_K)(\varepsilon_{11}\eta_E-\eta_K)]\\+\sigma_u(\gamma_L-\gamma_K)(\theta_{XK}-\varepsilon_{11}\theta_{XE})+C\sigma_X(1-\eta)(1+\varepsilon_{11})-(1-\eta)M\end{array}\right\}\widehat{\tau_Z} \tag{18}$$

$$\widehat{p_Y}-\widehat{p_X}=\frac{\theta_{YZ}}{D}\left\{\sigma_Y[\gamma_L\gamma_K+\gamma_L\theta_{XK}+\gamma_K\theta_{XE}]+C\sigma_X-M\frac{\theta_{XK}}{\varepsilon_{11}}\right\}\widehat{\tau_Z} \tag{19}$$

These solutions use the following definitions and simplifications: $\gamma_L\equiv\frac{\lambda_{LY}}{\lambda_{LX}}$, $\gamma_K\equiv\frac{\lambda_{KY}}{\lambda_{KX}}$, $A\equiv\gamma_L\gamma_K+\gamma_L\theta_{YK}+\gamma_K(1-\theta_{YK})$, $B\equiv\gamma_K\gamma_L+\gamma_K\theta_{YE}+\gamma_L(1-\theta_{YE})$, $C\equiv\theta_{XK}\gamma_K+\theta_{XE}\gamma_L+1$, $\eta_K\equiv\theta_{XK}\eta+\theta_{YK}(1-\eta)$, $\eta_E\equiv\theta_{XE}\eta+\theta_{YE}(1-\eta)$, $M\equiv\frac{1}{\lambda_{KX}\lambda_{LX}}\left[(1-\lambda_{LU})+\varepsilon_{11}\left(1-\lambda_{LU}-\frac{\lambda_{LU}}{\varepsilon_2}\right)\right]$, and $D\equiv\sigma_u(\gamma_L-\gamma_K)(\theta_{XK}\theta_{YE}-\theta_{XE}\theta_{YK})+\sigma_Y(A\eta_E+B\eta_K)+C\sigma_X(\eta_E+\eta_K)-M\eta_K/\varepsilon_{11}$.

All three expressions are linear functions of the change of the pollution tax $\widehat{\tau_Z}$, since the model is linearized and $\widehat{\tau_Z}$ is the only exogenous policy variable. These expressions can be decomposed into several effects that can be separately analyzed.[15] In the following subsections, we decompose each expression into terms representing several intuitive effects, in the spirit of Mieszkowski (1967): an output effect, two substitution effects (one from the clean sector and one from the dirty sector), and an effect that we call the efficiency wage effect. The output effect is represented by the terms that include the elasticity of substitution in utility, $\sigma_u$. The clean sector substitution effect is represented by the terms that include the elasticity of substitution in production in the clean sector, $\sigma_X$. The dirty sector substitution effect is represented by the terms that include the elasticity of substitution in production in the dirty sector, $\sigma_Y$. Finally, the efficiency wage effect is represented by the terms that include $M$.[16] While assigning these names to each separable term, the efficiency wage effect that we identify does not capture all the channels of the pollution tax's impact related to workers' effort. The effort elasticity parameters $\varepsilon_{11}$ and $\varepsilon_2$ are also in the coefficients in front of eqs. (17) and (18), as well as in the denominator $D$. We should interpret the terms including $M$ as the identifiable part of the complex effect of the efficiency wage, while bearing in mind that the assumption of efficiency wage changes the sizes of all three other effects compared to the previous literature.

### 3.1. *Efficiency wage effect*

Our main result is the interpretation of an effect that we call the efficiency wage effect. This effect of course is absent in previous models without an efficiency wage or endogenous unemployment. It is present in previous models with efficiency wage-driven unemployment, for instance Rapanos (2006) derives a similar effect that he calls the "unemployment effect." But in those papers, the effect arises not from a

[14] $U$ is the level of unemployment, so $\widehat{U}$ is defined as the percentage change in the level of unemployment. But since the total labor force is fixed, $\widehat{U}$ is also the percent change in the unemployment rate. It is not a percentage point change. For example, if the baseline unemployment rate is 4%, then $\widehat{U}=0.1$ is a ten-percent increase in that baseline rate, to 4.4%.

[15] Throughout the analysis below, we assume that the denominator $D$ is positive, which it is in all of the numerical simulations. A sufficient but not necessary condition ensuring that $D$ is positive is $\varepsilon_{11}>-1$.

[16] Since these results are so complicated, we also consider a simpler model that does not include capital. This model is presented in Appendix A.II. While the results are simpler than those from the main model, it cannot be used to analyze sources-side incidence or to see how substitution between labor and capital affects unemployment.

pollution tax (since there is no pollution in those models) but rather from factor income taxes.

This efficiency wage effect is absent from the equation for unemployment, eq. (17) (there is no term with $M$ in that equation). That may seem counterintuitive since of course the efficiency wage component of the model must affect unemployment. However, the coefficient $\frac{1}{\varepsilon_2}$ in front of eq. (17) captures this relationship. The substitution and output effects are scaled by this coefficient, which shows how the form of the effort function translates these effects into unemployment. When workers' effort is more responsive to unemployment, $\varepsilon_2$ is large, so the equilibrium effect on unemployment is smaller in magnitude, all else equal. The intuition for this effect is clearer in the simplified model in Appendix A.II, where the effect on unemployment is also scaled by $\frac{1}{\varepsilon_2}$.

The efficiency wage effect is its own term in the expressions for the sources-side and uses-side incidence; it is represented by the terms with $M$ in it. As defined earlier, $\equiv \frac{1}{\lambda_{KX}\lambda_{LX}}\left[(1-\lambda_{LU})+\varepsilon_{11}\left(1-\lambda_{LU}-\frac{\lambda_{LU}}{\varepsilon_2}\right)\right]$, which is strictly positive if $\varepsilon_{11} > -1$. The efficiency wage effect in the expression for sources-side incidence (eq. 18) is $-\frac{\theta_{YZ}}{\varepsilon_{11}D}(1-\eta)M$ and so is the same sign of $M$. If workers' marginal effort with respect to the real wage does not decline too fast as the wage increases (i.e. $\varepsilon_{11} > -1$), then the efficiency wage effect on $\widehat{w}-\widehat{r}$ is strictly positive, meaning that a pollution tax disproportionately burdens capital. The uses-side efficiency wage effect from eq. (19) is $-\frac{\theta_{YZ}\theta_{XK}}{\varepsilon_{11}D}M$. Under the same assumption that $\varepsilon_{11} > -1$, this effect is strictly positive, which means the dirty good price increases more than the clean good price, and the uses-side incidence falls more on consumers of the dirty good.

To further interpret this effect, we can impose a functional form on the worker's effort function:

$$e\left(\frac{W}{P},U\right)=\phi\left(\frac{W}{P}\right)^{\alpha}+\psi U^{\beta}$$

where $\phi > 0$, $0 < \alpha < 1$, $\psi < 0$, and $\beta < 0$.[17] These parameter restrictions ensure that $e_1 > 0$, $e_2 > 0$, $e_{12} = 0$, $e_{11} < 0$, and $e_{22} < 0$. The elasticity of effort with respect to the wage is $\varepsilon_1 = \frac{\alpha\phi\left(\frac{w}{P}\right)^{\alpha}}{\phi\left(\frac{w}{P}\right)^{\alpha}+\psi U^{\beta}}$. The first-order condition that this elasticity is one amounts to $\phi(\alpha-1)\left(\frac{w}{P}\right)^{\alpha} = \psi U^{\beta}$. The elasticity of effort with respect to unemployment is $\varepsilon_2 = \frac{\beta\psi U^{\beta}}{\phi\left(\frac{w}{P}\right)^{\alpha}+\psi U^{\beta}} = \frac{\beta(\alpha-1)}{\alpha} > 0$. The concavity of the effort function with respect to wage is $\varepsilon_{11} = \alpha - 1 < 0$. Under this functional form assumption, $M = \frac{\alpha}{\lambda_{KX}\lambda_{LX}}\left(1-\left(1+\frac{1}{\beta}\right)\lambda_{LU}\right)$. This expression is strictly positive, and its magnitude depends on both the unemployment rate $\lambda_{LU}$ and the elasticities of the effort function. These elasticities ultimately depend on the values of $\alpha$ and $\beta$, which represent the responsiveness of effort with respect to the real wage and to unemployment, respectively.

We explore how the magnitude of $M$, and thus of the efficiency wage effect on both the sources-side and uses-side incidence, depends on these parameters. First, $\frac{\partial M}{\partial \alpha} = \frac{1}{\lambda_{KX}\lambda_{LX}}\left(1-\left(1+\frac{1}{\beta}\right)\lambda_{LU}\right)$. This derivative is strictly positive (since $\beta < 0$). The efficiency wage effect becomes larger in magnitude ($M$ increases) as workers' effort becomes more responsive to the real wage ($\alpha$ increases). Second, $\frac{\partial M}{\partial \beta} = \frac{\lambda_{LU}}{\lambda_{LX}\lambda_{KX}}\frac{\alpha}{\beta^2}$. This derivative is strictly positive. The efficiency wage effect becomes smaller in magnitude ($M$ decreases) as workers' effort becomes more responsive to unemployment ($\beta$ decreases).[18] These two derivatives demonstrate how the source of the efficiency wage matters greatly to incidence effects. A high $\alpha$ means effort is very responsive to the real wage, which is likely to be true in a gift exchange or fair wage efficiency wage model like Akerlof (1982). A high $\beta$ means effort is very responsive to unemployment, which is likely to be true in a shirking and firing model like Shapiro and Stiglitz (1984). If a fair wage model is more accurate, then the efficiency wage incidence effect is large, whereas if a shirking and firing model is more accurate, then the efficiency wage incidence effect is smaller.

We provide intuition for this key result that the efficiency wage effect on both sources-side and uses-side incidence is larger when effort is more responsive to the real wage and is smaller when effort is more responsive to unemployment. When effort is very responsive to the real wage, then the efficiency wage effect on sources-side incidence, which increases the relative burden on capital, is large, because in equilibrium the wage must rise high enough to provide the incentive for worker effort. Thus, workers' effort being highly responsive to the wage benefits workers by forcing the wage to increase. When effort is very responsive to the unemployment rate, then the efficiency wage effect on sources-side incidence is small, because in equilibrium the effort can be incentivized through unemployment rather than through the wage. Thus, workers' effort being highly responsive to the unemployment rate hurts workers by decreasing the wage.

The intuition is similar for the efficiency wage effect on uses-side incidence. When effort is very responsive to the real wage, then the efficiency wage effect on uses-side incidence, which increases the relative burden on consumers of the dirty good, is large, because the increase in the wage necessary to induce equilibrium effort is partially passed through to output prices and disproportionately to the dirty good price. When effort is very responsive to the unemployment rate, then the efficiency wage effect on uses-side incidence is small, because effort need not be induced through a change in the wage but instead can be induced through equilibrium unemployment, and less of a price change passes through to output prices.

We also explore how the unemployment rate affects the efficiency wage effect: $\frac{\partial M}{\partial \lambda_{LU}} = \frac{-\alpha}{\lambda_{KX}\lambda_{LX}}\left(1+\frac{1}{\beta}\right)$. This is positive if and only if $-1 < \beta < 0$, and negative if and only if $\beta < -1$. All else equal, one might predict that a larger baseline unemployment rate will increase the magnitude of the efficiency wage effect. But if effort is very responsive to unemployment ($\beta$ is large in absolute value) then this might not be the case.

While the efficiency wage effect is represented by the term with $M$ in the expression for sources-side incidence, the entire expression for sources-side incidence (eq. 18) is scaled by the factor $\frac{1}{\varepsilon_{11}}$. This scale factor also appears in the corresponding expression in the simpler model in Appendix A.II. As in that model, this factor shows that all of the effects on the sources-side incidence depend on the magnitude of $\varepsilon_{11}$. If it is large in absolute value, then workers' marginal effort is highly responsive to the wage, so that all of the effects on the wage (and thus on the sources-side incidence) are dampened.

### 3.2. Output effect

The remaining effects in eqs. (17), (18), and (19) are the output effect and two substitution effects. These are standard effects found in the tax incidence literature dating back to Harberger (1962) and Mieszkowski (1967). Here, we focus on how the inclusion of pollution and unemployment modifies these effects.

[17] A more natural functional form assumption is Cobb-Douglas, but a Cobb-Douglas effort function does not satisfy the assumption that $e_{12} = 0$, and the first-order condition that $\varepsilon_1 = 1$ demands that effort is linear in the real wage.

[18] Because $\beta < 0$ and the elasticity of effort with respect to unemployment is proportional to the absolute value of $\beta$, a lower value of $\beta$ (more negative) represents more elastic effort.

In both eqs. (17) and (18), the terms that include $\sigma_u$, the substitution elasticity of demand between the two goods $X$ and $Y$, represent an output effect. Through the output effect, the pollution tax disproportionately affects the dirty sector — because the dirty sector is the only sector that uses pollution as an input — and reduces its output in a way that depends on consumer preferences via $\sigma_u$. Less output means less demand for all inputs, but particularly the input used intensively in that sector.

The output effect caused by a one-unit change in the pollution tax ($\widehat{\tau_Z}$) on unemployment $\widehat{U}$ is $\frac{\theta_{YZ}}{\varepsilon_2 D}\{\sigma_u\theta_{XK}(\gamma_L-\gamma_K)\}$. This term is negative whenever $\gamma_K > \gamma_L$, which holds whenever the dirty sector $Y$ is relatively capital-intensive.[19] The dirty sector being capital-intensive means that the pollution tax will impose a larger burden on capital than on labor, which translates to a decrease in unemployment, captured in this term.

In the expression for sources-side incidence $\widehat{w}-\widehat{r}$, eq. (18), the output effect $\frac{\theta_{YZ}}{\varepsilon_{11} D}\{-\sigma_u(\gamma_L-\gamma_K)(-\theta_{XK}+\varepsilon_{11}\theta_{XE})\}$ is positive whenever $\gamma_K > \gamma_L$. If the dirty sector is relatively capital-intensive ($\gamma_K > \gamma_L$), then this output effect will decrease the price of capital relative to the wage ($\widehat{w}-\widehat{r} > 0$). The magnitude of this effect is proportional to the substitution elasticity of demand between the two goods, $\sigma_u$.

There is no output effect on the uses-side incidence $\widehat{p_Y}-\widehat{p_X}$; the relative factor intensities do not affect uses-side incidence, only sources-side incidence.

### 3.3. Clean sector substitution effect

Next, we identify two kinds of substitution effects. In eqs. (17), (18), and (19), the terms that include $\sigma_X$, the substitution elasticity of input demand between capital and labor for the clean ($X$) sector, are what we call the clean sector substitution effect. This captures the response of the clean sector to the change of relative input prices. Because the model is general equilibrium and total factor quantities (capital and labor) across sectors are fixed, the effect of substitutability within the clean industry impacts the incidence of a tax levied only on the dirty industry.

In the expression for the change in unemployment $\widehat{U}$ (eq. 17), the clean sector substitution effect is $\frac{\theta_{YZ}}{\varepsilon_2 D}\{C\sigma_X(1-\eta)\}$. This term is unambiguously positive. An increase in the pollution tax unambiguously increases unemployment through the clear sector substitution effect. Similarly, for the sources-side incidence (eq. 18), the clean sector substitution effect is $\frac{\theta_{YZ}}{\varepsilon_{11} D}\{C\sigma_X(1-\eta)(1+\varepsilon_{11})\}$. This term is unambiguously negative, so when the pollution tax increases, this effect decreases $\widehat{w}-\widehat{r}$ and places more burden of the tax on labor.

The clean sector substitution effect's impact on both unemployment $\widehat{U}$ and sources-side incidence $\widehat{w}-\widehat{r}$ arises from the same intuition. The tax increase is an overall distortion to the economy. While the total amount of capital employed is fixed, the total amount of labor employed varies because of endogenous unemployment. The overall distortion from the pollution tax thus exacerbates the tax wedge affecting unemployment, increasing overall unemployment and disproportionately burdening labor income (due to the link between unemployment and labor income from the effort function).

The clean sector substitution effect on the uses-side incidence $\widehat{p_Y}-\widehat{p_X}$ is $\frac{\theta_{YZ}}{D}\{C\sigma_X\}$ which is always positive. An increase in the pollution tax burdens consumers of the dirty good more than it burdens consumers of the clean good through this effect.

The clean sector substitution effect's magnitude on all three outcomes is scaled by the magnitude of $\sigma_X$. The easier it is for the clean sector to substitute between capital and labor (larger $\sigma_X$), the larger is the size of each of the effects described above.[20]

### 3.4. Dirty sector substitution effect

The other substitution effect comes from substitutability among inputs in the dirty sector. It is represented by the terms that contain the dirty sector's substitution elasticity, $\sigma_Y$. The dirty sector substitution effect on the change in unemployment in eq. 17 is $\frac{\theta_{YZ}}{\varepsilon_2 D}\sigma_Y[A(1-\eta)+\eta_K(\gamma_K-\gamma_L)]$. This effect contains two parts. The first, $A(1-\eta)$, is strictly positive, so this part of the effect increases the unemployment rate. The second part, $\eta_K(\gamma_K-\gamma_L)$, is of the same sign as $\gamma_K-\gamma_L$, and so it is positive whenever the dirty sector is capital-intensive, and it is negative whenever the dirty sector is labor-intensive. When the dirty sector is capital-intensive, then the pollution tax unambiguously increases the unemployment rate via the dirty sector substitution effect, but when the dirty sector is labor-intensive, then the dirty sector substitution effect contains offsetting terms on unemployment.

The dirty sector substitution effect on sources-side incidence in eq. 18 is $\frac{\theta_{YZ}}{D}\sigma_Y\left[(1-\eta)\left(\frac{A}{\varepsilon_{11}}+B\right)+(\gamma_L-\gamma_K)\left(\eta_E-\frac{\eta_K}{\varepsilon_{11}}\right)\right]$. The second half of this effect, $(\gamma_L-\gamma_K)\left(\eta_E-\frac{\eta_K}{\varepsilon_{11}}\right)$, is of the same sign as $\gamma_L-\gamma_K$, and so is positive when the dirty sector is labor-intensive. The first half of the effect is of ambiguous sign, depending on the sign of $\frac{A}{\varepsilon_{11}}+B$. Like the dirty sector substitution effect on unemployment, the dirty sector substitution effect on sources-side incidence can have ambiguous sign.

However, the dirty sector substitution effect on uses-side incidence in eq. 19, $\frac{\theta_{YZ}}{D}\sigma_Y[\gamma_L\gamma_K+\gamma_L\theta_{XK}+\gamma_K\theta_{XE}]$, is unambiguously positive. This effect increases the burden of the pollution tax disproportionately for consumers of the dirty good.

Appendix A.III presents results from a more general model that does not impose the CES assumption about production in the dirty sector; as a result the dirty sector substitution effect is much more complicated. It depends on the relative substitutability among inputs (for example, whether labor or capital is a better substitute for pollution), which is missing under the CES assumption.

### 3.5. Other outcomes

The focus of our model is the effect of the pollution tax on unemployment, sources-side incidence, and uses-side incidence, which are given in eqs. 17–19. Two other outcomes may also be of interest: the effect on worker effort and on pollution. In a standard tax incidence model, the sources-side burden on workers is fully captured by the change in the wage. Here, workers' burden is more complicated, since the tax affects the wage, unemployment, and worker effort, all of which contribute to worker welfare.[21] The effect on effort is:

$$\widehat{e}=\frac{(\varepsilon_{11}+1)\theta_{YZ}}{\varepsilon_{11} D}\{\sigma_Y[A(1-\eta)+\eta_K(\gamma_K-\gamma_L)]+\sigma_u\theta_{XK}(\gamma_L-\gamma_K)+C\sigma_X(1-\eta)\}\widehat{\tau_Z} \tag{20}$$

Eqs. 3 and 4 show that the equilibrium change in effort is just a multiple of the equilibrium change in unemployment: $\widehat{e}=\varepsilon_2\left(\frac{1+\varepsilon_{11}}{\varepsilon_{11}}\right)\widehat{U}$.

[19] $\gamma_K > \gamma_L$ implies $\frac{\lambda_{KY}}{\lambda_{KX}} > \frac{\lambda_{LY}}{\lambda_{LX}}$, which implies $\frac{K_Y}{K_X} > \frac{L_Y}{L_X}$.

[20] A similar effect is found in Rapanos (2006). For example, the first term in equation 34 in Rapanos (2006) is the clean sector substitution effect on sources-side incidence, and it also is scaled by the substitution elasticity in consumption between the two goods (denoted by $\sigma_D$ in his model).

[21] Our model does not have an explicit utility function that can be used to measure worker welfare. Fullerton and Ta (2020) is an analytical general equilibrium that does have a utility function, and their section 10 considers welfare implications. Bartik (2015) and Kuminoff et al. (2015) discuss the welfare implications of unemployment caused by environmental regulations.

Since $\varepsilon_2\left(\frac{1+\varepsilon_{11}}{\varepsilon_{11}}\right)<0$, if $\varepsilon_{11} > -1$ (always true under the effort function form $e\left(\frac{w}{P}, U\right) = \phi\left(\frac{w}{P}\right)^{\alpha} + \psi U^{\beta}$), the sign of the change in effort $\hat{e}$ is always opposite of the sign of the change of unemployment $\hat{U}$. These two effects on worker welfare move in opposite directions; an increase in unemployment always coincides with a decrease in effort (among those with jobs). Eqs. 3 and 4 also show that the change in the wage is a multiple of the change in effort: $\hat{e} = (1+\varepsilon_{11})\hat{w}$. When $\varepsilon_{11} > -1$, the wage and the effort move in the same direction so have opposite effects on worker welfare; an increase in the wage coincides with an increase in effort.[22]

Since $\hat{e} = \varepsilon_2\left(\frac{1+\varepsilon_{11}}{\varepsilon_{11}}\right)\hat{U}$, the effect of the pollution tax on effort can be decomposed into the substitution effects and output effect that appear in eq. 17 for unemployment. For example, the clean sector substitution effect is $\frac{(\varepsilon_{11}+1)\theta_{YZ}}{\varepsilon_{11}D}\{C\sigma_X(1-\eta)\}$, which is unambiguously negative. An increase in the pollution tax unambiguously decreases workers' effort through this effect. The output effect on the effort level $\hat{e}$ is $\frac{(\varepsilon_{11}+1)\theta_{YZ}}{\varepsilon_{11}D}\{\sigma_u\theta_{XK}(\gamma_L-\gamma_K)\}$, which is positive whenever $\gamma_K > \gamma_L$ and $\varepsilon_{11} > -1$. If the dirty sector is capital-intensive and workers' marginal effort with respect to the real wage does not decline too fast as the wage increases, workers' effort will increase when the pollution tax increases.

Another outcome of interest is the effect of the pollution tax on pollution itself, $Z$. Unfortunately, a closed-form solution for $\hat{Z}$ is too complicated to be able to interpret. In Appendix A.I, we present intermediate steps in the solution method, including an equation (eq. A.2) in which the change in pollution $\hat{Z}$ can be expressed in terms of other endogenous variables. Instead of an analytical solution, the effect of the pollution tax on pollution will be considered below using numerical simulations.

## 4. Numerical simulations

Here we numerically simulate the model by assigning parameter values calibrated from data and taken from the previous literature. Ours is a simple two-sector, two-input model, not a CGE model, so the purpose of these simulations is not to pin down plausible quantitative values for the magnitudes of these effects. Rather, the purpose is to explore how the net effects are decomposed into the effects identified in the previous section and how sensitive the magnitudes are to various parameter values. We begin by presenting base-case simulations decomposed into the effects from the analytical model. Then we vary parameter values, including the effort function elasticities.

### 4.1. Calibration

We use the 2017 Integrated Industry-Level Production Account (KLEMS) data provided by the U.S. Bureau of Economic Analysis for the calibration of the factor share and factor intensity parameters.[23]

First, we use the energy inputs (in millions of dollars) as a measurement of the pollution input $Z$ in our dirty sector. There are very few places with prices on pollution, so we do not calibrate pollution based on market-based pollution policies; instead we interpret the pollution input as an energy input. The KLEMS data contains 64 major industries. We rank them based on their ratios of energy inputs to gross outputs, and we assign the top 16 energy-intensive industries as the dirty sector and the remaining industries as the clean sector. The dirty sector includes utilities (with energy inputs at 17.63% of output), rail transportation (10.08%), and truck transportation (9.39%). The 47 clean industries range from accommodation (energy inputs at 2.63% of output) to insurance carriers and related activities (0.06%). This assignment implies that the dirty sector makes up about 30% of gross outputs. We let the weight of the price of $X$ on the price index $P$, $\eta = 0.7$, mirroring the fact that the clean sector is 70% of income.

Second, the shares of each sector's revenue paid to labor, capital, and energy are measured using the ratios of compensation to labor, capital, and energy to the outputs of each sector. The clean sector is more labor-intensive, with about 61% of its revenue paid to labor, so $\theta_{XK} = 0.39$ and $\theta_{XE} = 0.61$. The dirty sector is more capital-intensive and pays about 7% of its revenue to energy inputs, so we have $\theta_{YZ} = 0.07$, $\theta_{YK} = 0.56$ and $\theta_{YE} = 0.37$.

Third, we use the different factor intensities of the two sectors and their share of gross output to calculate each sector's share of capital and labor. Sector X's share of capital is $\lambda_{KX} = 0.62$ and sector Y's is $\lambda_{KY} = 0.38$, showing that even though the dirty sector Y is capital-intensive, it still uses a smaller share of the economy's capital because it only accounts for 30% of the economy. We set the unemployment rate $\lambda_{LU}$ to be 0.04 to roughly coincide with the average U.S. monthly unemployment rate (4.35%) in 2017. Thus, we get $\lambda_{LX} = 0.76$, $\lambda_{LY} = 0.20$. These imply that $\gamma_L = 0.26$ and $\gamma_K = 0.61$.

Fourth, we use unity for the elasticity of substitution between capital and labor in the clean sector ($\sigma_X = 1$) and the elasticity of substitution in consumption between the clean and dirty goods ($\sigma_u = 1$), following Fullerton and Heutel (2007). For the elasticity of substitution in production in the dirty sector $\sigma_Y$, we use 0.5 based on Fullerton and Heutel (2010).[24]

Finally, we found no source for the parameter values related to the effort function, $\varepsilon_{11}$ and $\varepsilon_2$. When we impose the functional form described earlier in the text, $e\left(\frac{w}{P}, U\right) = \phi\left(\frac{w}{P}\right)^{\alpha} + \psi U^{\beta}$, these parameters are $\varepsilon_{11} = \alpha - 1$ and $\varepsilon_2 = \frac{\beta(\alpha-1)}{\alpha}$. So, for the base case, we arbitrarily assume $\alpha$ to be 0.5 and $\beta$ to be $-0.5$, implying that $\varepsilon_{11} = -0.5$ and $\varepsilon_2 = 0.5$. Table 1 summarizes the base-case parameter values.

The exogenous policy choice variable is the change in the pollution tax $\hat{\tau}_Z$. We model the change in the price of energy under a carbon tax set at the social cost of carbon (SCC). The federal Interagency Working

**Table 1**
Base case parameter values.

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $\theta_{XK}$ | 0.39 | $\lambda_{KX}$ | 0.62 |
| $\theta_{XE}$ | 0.61 | $\lambda_{KY}$ | 0.38 |
| $\theta_{YK}$ | 0.56 | $\lambda_{LX}$ | 0.76 |
| $\theta_{YE}$ | 0.37 | $\lambda_{LY}$ | 0.20 |
| $\theta_{YZ}$ | 0.07 | $\lambda_{LU}$ | 0.04 |
| $\gamma_K$ | 0.61 | $\gamma_L$ | 0.26 |
| $\eta$ | 0.7 | $\sigma_X$ | 1 |
| $\varepsilon_{11}$ | $-0.5$ | $\sigma_u$ | 1 |
| $\varepsilon_2$ | 0.5 | $\sigma_Y$ | 0.5 |

*Note*: These values are calibrated based on data and on the previous literature as described in the text.

[22] This relationship holds for the change in the wage $\hat{w}$, relative to the numeraire, but not necessarily for the relative change in the wage compared to the rental rate, $\hat{w} - \hat{r}$, presented in equation 18.

[23] U.S. Bureau of Economic Analysis, "Production Account Tables, 1998–2017," https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems (accessed December 12, 2019).

[24] Fullerton and Heutel (2010) model production in the dirty sector using Allen elasticities instead of CES (see our Appendix A.III.), and they use $e_{KE} = 0.5$, $e_{KZ} = 0.5$, and $e_{EZ} = 0.3$ for the three cross-price Allen elasticities. This indicates that capital is a slightly better substitute for pollution than is labor ($e_{KZ} > e_{EZ}$). With CES, though, these Allen elasticities must all be equal to each other, so we choose 0.5 for their value ($\sigma_Y$). In Table 4 below we will consider alternate values that do not assume CES. In Fullerton and Heutel (2010), there is no effective labor $E$, just labor $L$, so we assume that their $e_{KL}$ is equal to our $e_{KE}$, etc.

Group on Social Cost of Greenhouse Gases provides an updated estimate of the SCC based on new versions of three IAM models (DICE, PAGE, and FUND) in 2016. We adopt an estimate of $40 per metric ton of $CO_2$ based on the report (Interagency Working Group on Social Cost of Greenhouse Gases, 2016). Then we calculate the weighted average energy price with and without a carbon tax at $40 per metric ton $CO_2$. The calculation is based on the fuel price calculator provided by Hafstead and Picciano (2017), and we use the 2015 energy price and industrial sector energy usage data provided by the U.S. Energy Information Administration.[25] In 2015, the energy generated from coal, petroleum, and natural gas is 1.38, 8.25, and 9.43 quadrillion British thermal units (BTU), respectively. The average percentage increase of prices for coal (all types), petroleum products, and natural gas is 264%, 25%, and 50%, respectively. Weighted by the energy usage amount, we get that the $40 carbon tax increases the energy price by 35% on average. Therefore, we present simulation results with $\widehat{\tau_Z} = 0.35$. This choice of $\widehat{\tau_Z}$ allows us to compare our model's results to other models that consider a carbon tax set at the SCC.[26]

### 4.2. Results

We first present results under the base-case parameterization. In Table 2, and all of the numerical simulation tables, we present the effects of a 35% increase in the pollution tax on unemployment ($\widehat{U}$), the sources-side incidence ($\widehat{w} - \widehat{r}$), and the uses-side incidence ($\widehat{p_Y} - \widehat{p_X}$), which are the three main results from our model. Table 2 also presents the effect of the pollution tax on effort ($\widehat{e}$), and the subsequent tables also present the effect on effort and on pollution ($\widehat{Z}$). The last row of Table 2 (row 5) presents the net effect of the tax, and rows 1 through 4 decompose this net effect into the four effects discussed earlier.

From the theoretical results, there is no efficiency wage effect in the expression for unemployment and workers' effort, and there is no output effect in the expression for uses-side incidence (so these entries in Table 2 are blank).

The net effect of the 35% increase in the pollution tax on unemployment is to increase unemployment by 0.81% (a percent change in the unemployment rate, not a percentage-point change). This is small, because the dirty (taxed) sector is just 30% of the overall economy, and pollution is just 7% of the value of its inputs, and the tax rate increase is just 35%. The increase in unemployment is mainly driven by the clean sector substitution effect (0.76% increase) versus the dirty sector substitution effect (0.30% increase). Even though the dirty sector is the taxed sector, substitution among inputs in the clean sector has a larger effect on unemployment. This is because the clean sector is the larger sector (70%), and in general equilibrium, its substitution possibility is more important for employment than is the dirty sector's substitution. The output effect is negative since the dirty (taxed) sector is capital-intensive.

**Table 2**
Base case simulation results.

| Row | | $\widehat{U}$ | $\widehat{w} - \widehat{r}$ | $\widehat{p_Y} - \widehat{p_X}$ | $\widehat{e}$ |
|---|---|---|---|---|---|
| 1 | Output Effect | −0.25% | 0.44% | – | 0.12% |
| 2 | Clean Sector Substitution Effect | 0.76% | −0.38% | 1.27% | −0.38% |
| 3 | Dirty Sector Substitution Effect | 0.30% | −0.31% | 0.29% | −0.15% |
| 4 | Efficiency Wage Effect | – | 0.60% | 0.78% | – |
| 5 | Net Effect | 0.81% | 0.35% | 2.35% | −0.41% |

*Note:* This table presents the simulated effects on unemployment, sources-side incidence, and uses-side incidence of a $40 per metric ton carbon tax (a 35% increase in the pollution tax) under the base case parameter values (listed in Table 1).

For the sources-side incidence, the efficiency wage effect plays a significant role. Both dirty and clean sector substitution effects serve to increase the relative burden on labor ($\widehat{w} - \widehat{r} < 0$). From these two effects alone, the wage relative to the capital rental rate decreases by 0.7%. The output effect offsets these effects somewhat, again since the dirty sector is capital-intensive. But the efficiency wage effect reverses the sign and completely offsets the substitution effects and decreases the relative burden on labor. The sources-side incidence goes from favoring capital to favoring labor.

For the uses-side incidence (the relative burden on output prices), we see a positive sign from all three effects; each puts more of the burden on consumers of the dirty good than on consumers of the clean good. Ignoring the efficiency wage effect would miss about 30% of this net effect.

The net effect of a pollution tax on workers' effort level is that they work 0.41% less hard, which implies a small utility gain if effort is costly. Just like the effect on unemployment, this net effect is dominated by the clean sector substitution effect. Finally, Table 2 does not present the base-case effect of the pollution tax increase on pollution, because that change cannot be decomposed into the different effects. The net effect of the 35% pollution tax increase is to decrease pollution by 19%.

We compare our base-case results to those from CGE papers that simulate the effects of carbon taxes on unemployment. In Hafstead et al. (2018), unemployment is generated through search frictions, not efficiency wages. According to their Figs. 2 and 3, a $40 per metric ton carbon price with lump-sum rebate results in a 30% emission reduction and a 0.3 percentage-point change in the unemployment rate. Given their 5% base steady-state unemployment rate, the new unemployment rate is 5.3%, so the percent change is 6%.[27] Castellanos and Heutel's (2019) CGE model generates unemployment through a wage curve. They find that a $35 per ton carbon tax increases unemployment by 4.4% and decreases emissions by 30%. Our $40 per metric ton carbon tax results in a roughly 19% pollution reduction and 0.81% increase in the unemployment rate, which is from our base 4% to 4.0324% (= 4 times 1.0081%). Thus, for roughly the same pollution tax increase, those CGE models find a decrease in pollution about twice as large as ours, and an increase in unemployment about five to six times as large as ours.

Several explanations could account for this difference in the magnitudes of the results. First, in those other models, unemployment is generated differently than in our model. In Hafstead et al. (2018), unemployment is generated via search frictions, and in Castellanos and Heutel (2019), unemployment is generated via a wage curve. In our model, it is generated via efficiency wages. Second, those models are multisector calibrated CGE models, while ours is a two-sector analytical model. Third, our model is linearized, so the 35% tax rate change that we model may create non-linearities. Fourth, our results could be sensitive to the choice of the effort function parameters $\varepsilon_{11}$ and $\varepsilon_2$, for which we were unable to find a calibration source. We investigate this in the sensitivity analysis below.

The base-case results depend on the base-case parameters, so we next

[25] U.S. Energy Information Administration, "U.S. industrial sector energy use by source, 1950–2018," https://www.eia.gov/energyexplained/use-of-energy/industry.php (accessed December 12, 2019).

[26] Our model is linear, so the effects of a smaller pollution price change are scaled linearly.

[27] Hafstead et al. (2018) also find that a roughly $15 per metric ton carbon price with lump-sum rebate induces a 15% emission reduction and a roughly 3% increase in the unemployment rate (percent not percentage-point). Hafstead and Williams (2018) also model unemployment through search frictions, though their model is a two-sector general equilibrium model rather than a CGE model. They find that a $20 per ton carbon tax increases unemployment by 3% (5% to 5.16%) and decreases emissions by 13.6%.

**Table 3**
Sensitivity analysis – varying effort function elasticities.

| Row | $\varepsilon_{11}$ | $\varepsilon_2$ | $\widehat{U}$ | $\widehat{w}-\widehat{r}$ | $\widehat{p_Y}-\widehat{p_X}$ | $\widehat{Z}$ | $\widehat{e}$ |
|---|---|---|---|---|---|---|---|
| 1 | −0.1 | 0.1 | 1.08% | 0.46% | 2.21% | −19.22% | −0.97% |
| 2 | −0.1 | 0.5 | 0.22% | 0.42% | 2.22% | −19.22% | −1.00% |
| 3 | −0.1 | 0.9 | 0.12% | 0.42% | 2.22% | −19.22% | −1.00% |
| 4 | −0.5 | 0.1 | 3.67% | 0.49% | 2.33% | −18.99% | −0.37% |
| 5 | −0.5 | 0.5 | 0.81% | 0.35% | 2.35% | −18.96% | −0.41% |
| 6 | −0.5 | 0.9 | 0.46% | 0.34% | 2.35% | −18.95% | −0.41% |
| 7 | −0.9 | 0.1 | 5.00% | 0.50% | 2.39% | −18.87% | −0.06% |
| 8 | −0.9 | 0.5 | 1.16% | 0.32% | 2.43% | −18.80% | −0.06% |
| 9 | −0.9 | 0.9 | 0.66% | 0.29% | 2.43% | −18.79% | −0.07% |

*Note:* This table presents the simulated effects on unemployment, sources-side incidence, uses-side incidence, and pollution of a $40 per metric ton carbon tax (a 35% increase in the pollution tax) for different values of the effort function elasticities. Their base-case values are used in row 5. All the other parameters are kept at their base case values (listed in Table 1).

conduct sensitivity analysis over parameter values. First, we vary the effort function elasticity parameters $\varepsilon_{11}$ and $\varepsilon_2$. These results are presented in Table 3, which presents the outcomes when all of the parameters are at the base case, except for these two parameters. In Table 3 and the remaining tables, we also present the resulting change in pollution, $\widehat{Z}$.

In Table 3, unemployment increases the least when the elasticity of marginal effort with respect to wage ($\varepsilon_{11}$ in absolute value) is small and the elasticity of effort with respect to unemployment ($\varepsilon_2$) is large. The explanation is that if $\varepsilon_{11}$ is large in absolute value, workers' marginal reduced effort increases quickly as the wage drops. This restrains the magnitude of the wage dropping relative to capital price because the reduced wage will cause extra loss of productivity due to a quickly decreased effort level. If $\varepsilon_2$ is large, workers are more sensitive to unemployment and work much harder, then their extra productivity will offset the rising cost of energy and there will be less increase in unemployment. In row seven, where the magnitude of $\varepsilon_{11}$ is highest and $\varepsilon_2$ is smallest, we see the largest increase in unemployment of 5%. This is about the same magnitude change found for the same carbon tax increase in Hafstead et al. (2018).

The uses-side incidence always falls disproportionately on consumers of the dirty good and is not much affected by the effort function elasticities. Likewise, the fall in pollution is largely unaffected by these elasticities: a 35% increase in the tax rate yields a pollution reduction of about 19%. The effort level always decreases when varying effort function elasticities, and its change correlates more strongly with $\varepsilon_{11}$. The effort decreases the least when the elasticity of marginal effort with respect to wage ($\varepsilon_{11}$ in absolute value) is large.

Next, in Table 4, we investigate the effect of substitution elasticities in the dirty sector. Rather than simply varying the CES elasticity $\sigma_Y$, we employ the more complicated model of dirty sector production from Appendix A.III, where production is modeled using Allen elasticities of substitution. We vary the Allen cross-price elasticities in Table 4. We keep the elasticity between labor and capital, $e_{KE}$, equal to its base-case value of 0.5, and we vary the other two cross-price elasticities $e_{KZ}$ and $e_{EZ}$ to vary among 0, 0.5, and 1. All of the other parameters are kept at their base case values, except that the own-price elasticities $e_{KK}$, $e_{EE}$, and $e_{ZZ}$ must also vary with the cross-price elasticities. To demonstrate, we also include in the third column of Table 4 the resulting value of the own-price elasticity $e_{ZZ}$.

In Table 4, unemployment always increases with the 35% increase in the carbon tax, and it increases the most when capital is a better substitute for pollution relative to labor ($e_{KZ} > e_{EZ}$). The value of $\widehat{w}-\widehat{r}$ varies across different parameter values, and it is small or even negative when $e_{KZ} > e_{EZ}$. The change in pollution $\widehat{Z}$ is always negative, but its magnitude varies considerably. The pollution tax is much more effective in reducing pollution when inputs are strong substitutes. When $e_{ZZ}$ is large in absolute value (as in the last row), then the change in pollution is large in absolute value. The percent change in pollution in these rows from the 35% pollution tax increase is similar to that found in Hafstead et al. (2018). The effort level decreases the most in row 7 when capital is a much better substitute for pollution than is labor. The effort decreases the least in row 3 when labor is a much better substitute for pollution than is capital.

Lastly, in Table 5, we hold the factor substitution elasticities and the effort function elasticities fixed at their base-case values and consider the impact of changes in factor intensities. We vary the value of $\gamma_K-\gamma_L$ from —0.35 to 0.55; this measures the capital intensity of the dirty sector ($\gamma_K-\gamma_L$ is positive if the dirty sector is more capital intensive than the clean sector). We maintain the assumption that the clean sector is 70% of income, and we set the ratio of total capital to labor in the economy to be 0.45/0.55 to be roughly consistent with the base case.

As the dirty sector becomes more capital-intensive (as $\gamma_K-\gamma_L$ increases), the increase in unemployment declines, capital bears an

**Table 5**
Sensitivity analysis – varying factor intensities.

| Row | $\gamma_K-\gamma_L$ | $\widehat{U}$ | $\widehat{w}-\widehat{r}$ | $\widehat{p_Y}-\widehat{p_X}$ | $\widehat{Z}$ | $\widehat{e}$ |
|---|---|---|---|---|---|---|
| 1 | −0.35 | 1.10% | −0.11% | 2.59% | −19.11% | −0.55% |
| 2 | −0.25 | 1.06% | −0.04% | 2.57% | −19.09% | −0.53% |
| 3 | 0 | 0.95% | 0.13% | 2.50% | −19.04% | −0.48% |
| 4 | 0.25 | 0.86% | 0.29% | 2.41% | −18.97% | −0.43% |
| 5 | 0.35 | 0.82% | 0.35% | 2.36% | −18.95% | −0.41% |
| 6 | 0.55 | 0.75% | 0.46% | 2.27% | −18.90% | −0.38% |

*Note:* This table presents the simulated effects on unemployment, sources-side incidence, uses-side incidence, and pollution of a $40 per metric ton carbon tax (a 35% increase in the pollution tax) for different values of relative factor intensities. All the other parameters are kept at their base case values (listed in Table 1). Their base-case values (rounded to the nearest hundredth) are used in row 5.

**Table 4**
Sensitivity analysis – varying dirty sector substitution elasticities.

| Row | $e_{KZ}$ | $e_{EZ}$ | $e_{ZZ}$ | $\widehat{U}$ | $\widehat{w}-\widehat{r}$ | $\widehat{p_Y}-\widehat{p_X}$ | $\widehat{Z}$ | $\widehat{e}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0.66% | 0.60% | 2.32% | −1.85% | −0.33% |
| 2 | 0 | 0.5 | −2.64 | 0.49% | 0.88% | 2.28% | −8.43% | −0.25% |
| 3 | 0 | 1 | −5.29 | 0.32% | 1.15% | 2.24% | −14.99% | −0.16% |
| 4 | 0.5 | 0 | −4 | 0.99% | 0.08% | 2.39% | −12.31% | −0.49% |
| 5 | 0.5 | 0.5 | −6.64 | 0.81% | 0.35% | 2.35% | −18.96% | −0.41% |
| 6 | 0.5 | 1 | −9.29 | 0.64% | 0.63% | 2.31% | −25.57% | −0.32% |
| 7 | 1 | 0 | −8 | 1.31% | −0.44% | 2.46% | −22.66% | −0.65% |
| 8 | 1 | 0.5 | −10.64 | 1.13% | −0.16% | 2.42% | −29.37% | −0.57% |
| 9 | 1 | 1 | −13.29 | 0.96% | 0.12% | 2.38% | −36.04% | −0.48% |

*Note:* This table presents the simulated effects on unemployment, sources-side incidence, uses-side incidence, and pollution of a $40 per metric ton carbon tax (a 35% increase in the pollution tax) for different values of the substitution elasticities $e_{KZ}$ and $e_{EZ}$. Their base-case values are used in row 5. All the other parameters are kept at their base case values (listed in Table 1).

increasing share of the burden ($\widehat{w} - \widehat{r}$ increases), and workers' effort level decreases less. Varying capital intensities yields only minor variation in the relative change in output prices and the change in pollution.

The calibration is based on US data, but one could also apply this model to other countries with different parameters. For example, China has a proportionately larger manufacturing sector than the US. China's manufacturing industry contributes about 40.5% of its GDP in 2017 and employs 28.1% of its workers.[28] In the US, the manufacturing sector takes up only 19.1% of the GDP, which is only half of China's, with the service sector being the largest contributor to GDP (80%). The employed US population in manufacturing sector is only 19.7%. Without carefully recalibrating our simulation using industry-specific data, we can roughly take the manufacturing sector as the dirty sector to find that $\gamma_K - \gamma_L$ for China is around $-0.2$, which is more labor-intensive than the US. This means Row 2 of Table 5 will more closely reflect the impact on China's economy.

In summary, the purpose of these simulations is not to pin down point estimates of the pollution tax's effects (this is not a CGE model), but rather to explore how the net effects are decomposed into different channels and to explore how sensitive the effects are to parameter values. From the decomposition (Table 2), we learn that the efficiency wage effect has a substantial influence on the sources-side incidence of the tax. From the sensitivity analyses (Tables 3 through 5), we learn that the net effect on unemployment is highly sensitive to the elasticities of the effort function but relatively insensitive to substitution elasticities in production or factor shares. The sources-side incidence is highly sensitive to both substitution elasticities in production and factor shares, while the uses-side incidence is generally insensitive to any of these parameters. The effect of the pollution tax on pollution only depends on the substitution elasticities in production.

## 5. Conclusion

We use an analytical general equilibrium model with unemployment generated through efficiency wages to analyze the effect of a pollution tax on unemployment and on sources-side and uses-side incidence. We decompose the general equilibrium impact of the tax on unemployment and incidence into several effects, including an output effect, substitution effects, and an effect that we call the efficiency wage effect, which has not been previously identified in the environmental tax incidence literature. The efficiency wage effect reduces the tax's burden on labor. The magnitude of this efficiency wage effect depends crucially on how workers' effort responds to both the real wage and unemployment. A key theoretical result is that when workers are more responsive to the real wage, the efficiency wage effect is larger, and when workers are more responsive to unemployment, the efficiency wage effect is smaller.

We further illustrate our results through calibrated numerical simulations. At the base-case parameterization, the new efficiency wage effect offsets the substitution and output effects on the sources-side incidence. Ignoring the efficiency wage effect, the burden of a pollution tax increase falls mostly on labor, while including it, the burden falls mostly on capital. In other words, this new effect completely overturns the direction of the sources-side incidence of a pollution tax. On unemployment, the output effect reduces unemployment since the dirty sector is capital-intensive, but it is dominated by substitution effects that increase unemployment. The magnitudes of the effects on unemployment and on sources-side incidence depend greatly on the structure of the effort function, though the magnitude of the uses-side incidence is largely independent of that. The uses-side incidence always falls disproportionately on consumers of the dirty good. The effect on pollution reduction only varies drastically when we change the substitutability between the three inputs of dirty sector. A pollution tax increase is most effective when both labor and capital are strong substitutes for pollution.

We employ a parsimonious model to interpret the intuition behind our results, so there are many ways in which the model could be extended by relaxing various assumptions. For example, further work could consider other effort functions, including one that depends on the wage to rental rate ratio (Agell and Lundborg, 1992), or could include heterogeneity among workers (Fullerton and Monti, 2013). We do not consider the benefit of pollution reduction and its incidence or effect on unemployment. We do not consider the effects of different choices of revenue recycling, for example using pollution tax revenue to reduce the pre-existing labor tax rate. Future work could consider including various labor market policies, such as the minimum wage or pre-existing labor taxes.

Nevertheless, our results provide theoretical insights into the impact of environmental policy on labor markets that could inform policymakers. A key takeaway is that the effect of policy on unemployment depends on *how* unemployment is generated in the economy. Our model is an efficiency wage model, rather than a search-and-matching model (Hafstead and Williams, 2018; Aubert and Chiroleu-Assouline, 2019) or another model of unemployment. But our model nests several different structural causes of efficiency wages. Under a fair wage model, worker effort may respond greatly to the real wage, while under a shirking and firing model, worker effort may respond greatly to unemployment. We show that how effort responds is a crucial determinant of how overall unemployment will be affected by a pollution tax, as well as its incidence.

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## Credit author statement

Garth Heutel: Conceptualization, Methodology, Validation, Formal analysis, Writing- Original Draft, Writing – Review & Editing, Supervision.

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## Appendix A. Appendix

### A.1. Solution method

We begin by eliminating through successive substitution several of the endogenous variables from the system of equations. Output quantities $\widehat{X}$ and $\widehat{Y}$ can be eliminated with eqs. (13) and (14); effort and the effective wage $\widehat{e}$ and $\widehat{v}$ can be eliminated with eqs. (4) and (5); and the effective labor levels $\widehat{E_X}$ and $\widehat{E_Y}$ can be eliminated with eqs. (1) and (2). Then, capital and labor used in each sector ($\widehat{K_X}, \widehat{K_Y}, \widehat{L_X}, \widehat{L_Y}$) can be eliminated with eqs. (6), (7), (9), and (10), after substitution in for the variables that had already been eliminated. That leaves six remaining endogenous variables – $\widehat{Z}$, $\widehat{U}$, $\widehat{w}$, $\widehat{r}$, $\widehat{p_X}$, and

[28] National Bureau of Statistics of China, "China Statistical Yearbook 2019," http://www.stats.gov.cn/tjsj/ndsj/2019/indexch.htm

$\widehat{p_y}$ – and the following six equations:

$$\widehat{U} = \frac{\varepsilon_{11}}{\varepsilon_2}\widehat{w} \tag{A1}$$

$$(\gamma_L - \gamma_K)\left(\widehat{Z} + \sigma_Y \widehat{\tau_Z}\right) - (1 + \gamma_L(1 - \sigma_Y \varepsilon_{11}))\widehat{w} + (\gamma_K \sigma_Y + \sigma_X)\widehat{r} = \left[\varepsilon_2(1 + \gamma_L - \sigma_X) - \frac{\lambda_{LU}}{\lambda_{LX}}\right]\widehat{U} \tag{A2}$$

$$\widehat{p_X} = \theta_{XK}\widehat{r} - \theta_{XE}\varepsilon_{11}\widehat{w} \tag{A3}$$

$$\widehat{p_Y} = \theta_{YK}\widehat{r} - \theta_{YE}\varepsilon_{11}\widehat{w} + \theta_{YZ}\widehat{\tau_Z} \tag{A4}$$

$$0 = \eta\widehat{p_X} + (1 - \eta)\widehat{p_Y} \tag{A5}$$

$$\sigma_u(\widehat{p_Y} - \widehat{p_X}) = -(\theta_{XK}\gamma_K + \theta_{XE}\gamma_L + 1)\widehat{Z} + (\theta_{XK}\gamma_K + \theta_{YK})\sigma_Y\widehat{r} + \theta_{XE}\left[(1 + \gamma_L)\varepsilon_2 - \frac{\lambda_{LU}}{\lambda_{LX}}\right]\widehat{U} + [\theta_{XE}(\gamma_L + 1) - (\theta_{XE}\gamma_L + \theta_{YE})\sigma_Y\varepsilon_{11}]\widehat{w} - (\theta_{XK}\gamma_K + \theta_{YK} + \theta_{XE}\gamma_L + \theta_{YE})\sigma_Y\widehat{\tau_Z} \tag{A6}$$

We then successively solve for the remaining variables.

### A.2. Model without Capital

We consider a competitive two-sector economy using only one factor of production: labor, which is perfectly mobile between sectors. The second variable input, pollution, is only used in the production of the dirty good (sector $Y$). This simpler model allows us to more easily some of the effects found in the more complicated general solutions presented in the main text.

The constant-returns-to-scale production functions become:

$$X = X(E_X)$$

$$Y = Y(E_Y, Z)$$

The labor market equations are the same as eqs. (1)–(5) in the original model. There is now only one resource constraint, which is on labor and is the same as eq. (7). Producers of $Y$ choose between labor and pollution. The elasticity of substitution in production $\sigma_Y$ is defined to capture this response to factor price changes:

$$\widehat{Z} - \widehat{E_Y} = \sigma_Y(\widehat{v} - \widehat{\tau}_z) \tag{A7}$$

where $\sigma_Y$ is defined to be positive.

Using the assumptions of perfect competition and constant returns to scale, we get

$$\widehat{p_X} + \widehat{X} = \widehat{v} + \widehat{E_X} \tag{A8}$$

$$\widehat{p_Y} + \widehat{Y} = \theta_{YE}\left(\widehat{v} + \widehat{E_Y}\right) + \theta_{YZ}\left(\widehat{Z} + \widehat{\tau_Z}\right) \tag{A9}$$

Totally differentiate each sector's production function and substitute in the conditions from the perfect competition assumption to get

$$\widehat{X} = \widehat{E_X} \tag{A10}$$

$$\widehat{Y} = \theta_{YE}\widehat{E_Y} + \theta_{YZ}\widehat{Z} \tag{A11}$$

The consumer side is the same as in our original model, represented by eqs. (15) and (16). We also normalize the overall price level so that $\widehat{P} = 0$, and we drop that variable out of the system.

The full model is eqs. (1)–(5), (7), (15), (16) from the main text model, and (A7) through (A11). It contains one exogenous policy variable ($\tau_Z$), 13 equations and 13 endogenous variables ($\widehat{E_X}, \widehat{E_Y}, \widehat{L_X}, \widehat{L_Y}, \widehat{Z}, \widehat{U}, \widehat{e}, \widehat{w}, \widehat{v}, \widehat{p_X}, \widehat{p_Y}, \widehat{X}, \widehat{Y}$). The model is solved with successive substitution similar to the method described in Appendix A.I.

The results are as follows.

$$\widehat{U} = \frac{(1 - \eta)\theta_{YZ}\widehat{\tau_Z}}{\varepsilon_2((1 - \eta)\theta_{YE} + \eta)} \tag{A12}$$

$$\widehat{w} = \frac{(1 - \eta)\theta_{YZ}\widehat{\tau_Z}}{\varepsilon_{11}((1 - \eta)\theta_{YE} + \eta)} \tag{A13}$$

$$\widehat{p_Y} - \widehat{p_X} = \frac{\theta_{YZ}\widehat{\tau_Z}}{(1 - \eta)\theta_{YE} + \eta} \tag{A14}$$

$$\widehat{e} = \frac{(\varepsilon_{11} + 1)(1 - \eta)\theta_{YZ}\widehat{\tau_Z}}{\varepsilon_{11}((1 - \eta)\theta_{YE} + \eta)} \tag{A15}$$

Note that there is no $\sigma_X$, $\sigma_Y$, or $\sigma_u$ in the expressions, which means there is no clean sector substitution effect, dirty sector substitution effect, or output effect in an economy with no capital. Therefore, eqs. A12 through A15 fully capture the efficiency wage effect of the pollution tax on the change of unemployment, wage, relative output prices, and workers' effort. To interpret the results, we need to take a closer look at the term $\frac{(1-\eta)\theta_{YZ}}{(1-\eta)\theta_{YE}+\eta}$. The term $\eta$ represents the weight of the clean good's price in the overall price level, or the share of the clean sector in the economy. Therefore, if the overall revenue of the economy is 1 unit, then the compensation to effective labor is $(1-\eta)\theta_{YE}+\eta\theta_{XE}=(1-\eta)\theta_{YE}+\eta$, since all the clean sector's revenue is paid to labor ($\theta_{XE}=1$). The compensation to pollution or energy is $(1-\eta)\theta_{YZ}$. Then eq. A12 becomes

$$\widehat{U}=\frac{\textit{revenue paid to energy}}{\textit{revenue paid to labor}}\frac{\widehat{\tau_Z}}{\varepsilon_2}>0$$

which is very straightforward. The effect of an increase in the pollution tax on unemployment is determined by the share of revenue paid to energy compared to labor in the economy, but its effect will be restrained by the elasticity of workers' effort with respect to unemployment ($\varepsilon_2$). An increase in the pollution tax will increase unemployment more for a more energy-intensive economy. If workers' effort is more sensitive to unemployment ($\varepsilon_2$ is large), then their extra productivity will offset the rising cost of energy and there will be less increase in unemployment. In the solution to the full model in the text (eq. 17), the overall effect is also scaled by $\frac{1}{\varepsilon_2}$ for the same reason, though the terms inside the bracket are much more complicated.

Similarly,

$$\widehat{w}=\frac{\textit{revenue paid to energy}}{\textit{revenue paid to labor}}\frac{\widehat{\tau_Z}}{\varepsilon_{11}}<0$$

The effect on the wage is determined by the share of revenue paid to energy compared to labor, restrained by the rate at which workers get satisfied with the wage ($\varepsilon_{11}$). The more energy-intensive the economy is, the carbon tax increase will lead to lower wages to compensate for the rising costs on energy. If $\varepsilon_{11}$ is large in absolute value, workers' marginal effort declines quickly as the wage increases, or equivalently, as the wage decreases the marginal reduced effort increases quickly. This restrains the magnitude of the wage dropping, because the reduced wage will cause an increasing loss of productivity. This effect is also seen in the analogous solution to the full model (eq. 18), which is scaled by $\frac{1}{\varepsilon_2}$.

Likewise,

$$\widehat{p_Y}-\widehat{p_X}=\frac{\theta_{YZ}\widehat{\tau_Z}}{(1-\eta)\theta_{YE}+\eta}=\frac{\textit{revenue paid to energy}}{\textit{revenue paid to labor}}\frac{\widehat{\tau_Z}}{1-\eta}>0$$

The increase of the dirty good price relative to the clean good price is proportional to the ratio of revenue paid to energy compared to labor, whose effect will be restrained by the share of the dirty sector in the economy $(1-\eta)$.

Lastly,

$$\widehat{e}=\frac{(\varepsilon_{11}+1)(1-\eta)\theta_{YZ}\widehat{\tau_Z}}{\varepsilon_{11}((1-\eta)\theta_{YE}+\eta)}=\left(\frac{\varepsilon_{11}+1}{\varepsilon_{11}}\right)\frac{\textit{revenue paid to energy}}{\textit{revenue paid to labor}}\widehat{\tau_Z}$$

Since $\varepsilon_{11}<0$, $\widehat{e}$ is negative as long as $\varepsilon_{11}>-1$, consistent with Cobb-Douglas effort. The effect of a pollution tax increase on workers' effort is determined by the share of revenue paid to energy compared to labor, factored by $\left(\frac{\varepsilon_{11}+1}{\varepsilon_{11}}\right)$. If $\varepsilon_{11}$ is large in absolute value, workers' marginal effort declines quickly as the wage increases, then workers' equilibrium effort will become only slightly lower. If $\varepsilon_{11}$ is closer to zero, which means the effort is closer to a linear function of wage, then workers' equilibrium effort will become much smaller. In other words, since the policy reduces the wage ($\widehat{w}<0$) workers will work less hard, but "how much less" depends on their effort elasticity to wage. Curiously, the effect on effort is independent of effort's responsiveness to unemployment, $\varepsilon_2$, even though unemployment is also changed by the pollution tax. In the main model in the paper, the elasticity $\varepsilon_2$ affects effort through its effect on the equilibrium change in the wage, but that effect is missing in this simpler model.

These results help us tease out the meaning of the efficiency wage effect: the weight of energy or pollution expenditures in the economy adjusted by the workers' response to the changing real wage and unemployment rate due to the tax. However, this model cannot be used to analyze sources-side incidence or to see how substitution between labor and capital affects unemployment, which is why the more complicated model with capital is the focus of this paper.

### *A.3. Model with Allen elasticities in Dirty Sector*

Instead of assuming a CES production function in the dirty sector, we can be more general by modeling production using Allen elasticities of substitution $e_{ij}$. This elasticity is positive for two substitutes and negative for two complements, and the own price Allen elasticity must always be negative. We assume that cross-price Allen elasticities are always positive, so that any two inputs are substitutes for each other. The magnitudes of the Allen elasticities determine which inputs are better substitutes. For example, if $e_{KZ}>e_{EZ}$, then capital is a better substitute for pollution than is labor.

Following Fullerton and Heutel (2007) (see their Appendix A for the derivation), we arrive at two equations describing the dirty sector's production decisions:

$$\widehat{K_Y}-\widehat{Z}=\theta_{YK}(e_{KK}-e_{ZK})\widehat{r}+\theta_{YE}(e_{KE}-e_{ZE})\widehat{v}+\theta_{YZ}(e_{KZ}-e_{ZZ})\widehat{\tau_Z} \quad (9')$$

$$\widehat{E_Y}-\widehat{Z}=\theta_{YK}(e_{EK}-e_{ZK})\widehat{r}+\theta_{YE}(e_{EE}-e_{ZE})\widehat{v}+\theta_{YZ}(e_{EZ}-e_{ZZ})\widehat{\tau_Z} \quad (10')$$

All the other equations remain the same as in the general model. This is a more general case of our original model that can greatly complicate the solutions. The eqs. (9′) and (10′) simplify to eqs. (9) and (10) from the main model when all of the cross-price elasticities $e_{ij}$ are equal to each other and equal $\sigma_Y$.[29]

[29] Karney (2016) shows that production can also be characterized by Morishima elasticities rather than Allen elasticities, and his equations 19 and 20 demonstrate how Morishima elasticities can also be transformed into CES production.

Solving the model, we get the closed form solutions:

$$\widehat{U} = \frac{\theta_{YZ}}{\varepsilon_2 D'}\left\{\begin{array}{c} A[-\theta_{YK}(1-\eta)(e_{KK}-e_{ZK})+\eta_K(e_{KZ}-e_{ZZ})] \\ +B[\theta_{YK}(1-\eta)(e_{KE}-e_{ZK})-\eta_K(e_{EZ}-e_{ZZ})] \\ +\sigma_u\theta_{XK}(\gamma_L-\gamma_K)+C\sigma_X(1-\eta) \end{array}\right\}\widehat{\tau_Z} \quad (17')$$

$$\widehat{w}-\widehat{r} = \frac{\theta_{YZ}}{\varepsilon_{11}D'}\left\{\begin{array}{c} -A[\theta_{YK}(1-\eta)(e_{KK}-e_{ZK})-(\eta_K-\varepsilon_{11}\eta_E)(e_{KZ}-e_{ZZ})-\theta_{YE}(1-\eta)\varepsilon_{11}(e_{KE}-e_{ZE})] \\ -B[-\theta_{YK}(1-\eta)(e_{KE}-e_{ZK})+(\eta_K-\varepsilon_{11}\eta_E)(e_{EZ}-e_{ZZ})+\theta_{YE}(1-\eta)\varepsilon_{11}(e_{EE}-e_{ZE})] \\ -\sigma_u(\gamma_L-\gamma_K)(-\theta_{XK}+\varepsilon_{11}\theta_{XE})+C\sigma_X(1-\eta)(1+\varepsilon_{11})-(1-\eta)M \end{array}\right\}\widehat{\tau_Z} \quad (18')$$

$$\widehat{p_Y}-\widehat{p_X} = \frac{\theta_{YZ}}{D'}\left\{\begin{array}{c} -\theta_{XE}\theta_{YK}[A(e_{KK}-e_{ZK}+e_{ZZ}-e_{KZ})+B(e_{ZK}-e_{KE}+e_{EZ}-e_{ZZ})] \\ -\theta_{XK}\theta_{YE}[A(e_{ZE}-e_{KE}+e_{KZ}-e_{ZZ})+B(e_{EE}-e_{ZE}+e_{ZZ}-e_{EZ})] \\ +C\sigma_X-M\dfrac{\theta_{XK}}{\varepsilon_{11}} \end{array}\right\}\widehat{\tau_Z} \quad (19')$$

These solutions use the same constants defined in the main solution, except that here the denominator is $D' \equiv A[-\theta_{YK}(e_{KK}-e_{ZK})\eta_E+\theta_{YE}(e_{KE}-e_{ZE})\eta_K]+B[\theta_{YK}(e_{KE}-e_{ZK})\eta_E-\theta_{YE}(e_{EE}-e_{ZE})\eta_K]+\sigma_u(\gamma_L-\gamma_K)(\theta_{XK}\theta_{YE}-\theta_{XE}\theta_{YK})-M\frac{\eta_K}{\varepsilon_{11}}+C\sigma_X(\eta_K+\eta_E)$.

The efficiency wage effect, the output effect, and the clean sector substitution effect are all identical in these equations to what they were in the original model's equations. The dirty sector substitution effect here is different; it is all of the terms that contain the Allen elasticities of substitution $e_{ij}$. This effect in each outcome is long and complicated and difficult to interpret, which is why in the main model we chose to employ the CES assumption. The relative magnitudes of the various Allen elasticities affects the sign and magnitude of this effect.

The dirty sector substitution effect can be simplified under an additional assumption. The simplifying assumption is that the two sectors have equal factor intensities; that is, $\gamma_K=\gamma_L\equiv\gamma$. Then we have $A=B=(1+\gamma)\gamma$ and $C=\gamma+1$. This eliminates the output effect. It also greatly simplifies the complicated dirty sector substitution effect. The solutions under this assumption are:

$$\widehat{U} = \frac{\theta_{YZ}}{\varepsilon_2 D}(1+\gamma)\{-\gamma[\theta_{YK}(1-\eta)(e_{KK}-e_{KE})+\eta_K(e_{EZ}-e_{KZ})]+\sigma_X(1-\eta)\}\,\widehat{\tau_Z}$$

$$\widehat{w}-\widehat{r} = \frac{\theta_{YZ}}{\varepsilon_{11}D}\left\{\begin{array}{c} -\gamma(1+\gamma)[(1-\eta)(-2e_{KE}(1-\theta_{YZ}))-\theta_{YZ}(e_{KZ}+e_{EZ})+(\eta_K-\varepsilon_{11}\eta_E)(e_{EZ}-e_{KZ})] \\ +(1+\gamma)\sigma_X(1-\eta)(1+\varepsilon_{11})-(1-\eta)M \end{array}\right\}\widehat{\tau_Z}$$

$$\widehat{p_Y}-\widehat{p_X} = \frac{\theta_{YZ}}{D}\left\{\begin{array}{c} -\gamma(1+\gamma)[\theta_{XE}\theta_{YK}(e_{KK}-e_{KE}+e_{KZ}+e_{EE})] \\ +(1+\gamma)\sigma_X-M\dfrac{\theta_{XK}}{\varepsilon_{11}} \end{array}\right\}\widehat{\tau_Z}$$

The dirty sector substitution effect on unemployment $\widehat{U}$ is $-\frac{\theta_{YZ}}{\varepsilon_2 D}(1+\gamma)\gamma[\theta_{YK}(1-\eta)(e_{KK}-e_{KE})+\eta_K(e_{EZ}-e_{KZ})]$. We can sign the following parts: $-\frac{\theta_{YZ}}{\varepsilon_2 D}(1+\gamma)\gamma<0$ and $\theta_{YK}(1-\eta)(e_{KK}-e_{KE})<0$. Therefore, as long as $e_{EZ}-e_{KZ}<0$, this effect is positive. If capital is a better substitute for pollution than is labor ($e_{EZ}-e_{KZ}<0$), then an increase in the pollution tax increases unemployment through this effect. However, if labor is a better substitute for pollution than is capital ($e_{EZ}-e_{KZ}>0$), we cannot say with certainty whether it increases or decreases the unemployment through this effect.

The dirty sector substitution effect on $\widehat{w}-\widehat{r}$ is $-\frac{\theta_{YZ}}{\varepsilon_{11}D}\gamma(1+\gamma)[(1-\eta)(-2e_{KE}(1-\theta_{YZ}))-\theta_{YZ}(e_{KZ}+e_{EZ})+(\eta_K-\varepsilon_{11}\eta_E)(e_{EZ}-e_{KZ})]$. If capital is a better substitute for pollution than is labor ($e_{EZ}-e_{KZ}<0$), then this effect is strictly negative, so the pollution tax imposes more burden on labor.

When it comes to the uses-side incidence, the dirty sector substitution effect is $-\frac{\theta_{YZ}}{D}\gamma(1+\gamma)[\theta_{XE}\theta_{YK}(e_{KK}-e_{KE}+e_{KZ}+e_{EE})]$. The sign of this term is determined by $e_{KK}-e_{KE}+e_{KZ}+e_{EE}$. Since $e_{KK}$ and $e_{EE}$ are negative, one simple case is that if capital and labor are better substitutes than are capital and pollution ($e_{KE}>e_{KZ}$), then the dirty sector substitution effect on $\widehat{p_Y}-\widehat{p_X}$ is positive, which means the price of the dirty good increases more than the clean good through this effect.

### *A.4. Model with heterogeneous wage rates and effort levels*

In the main model, we assume that two sectors set an identical wage rate, and thus their workers have an identical effort level. Here we lay out the model with nonidentical wage rates and effort levels for each sector. Firms in the clean sector X and the dirty sector Y set the nominal wage at $w_X$ and $w_Y$, respectively. The effective labor in each sector is:

$$E_X = e\left(\frac{w_X}{P}, U\right)\bullet L_X$$

$$E_Y = e\left(\frac{w_Y}{P}, U\right)\bullet L_Y$$

Workers have the same effort function, $e\left(\frac{w}{P}, U\right)$, but they now perform at different effort levels $e_X$ and $e_Y$, due to the different wages. Eqs. (1) and (2) become:

$$\widehat{E_X} = \widehat{e_X}+\widehat{L_X} \quad (1'')$$

$$\widehat{E_Y} = \widehat{e_Y}+\widehat{L_Y} \quad (2'')$$

The firms in sector $j, j\in\{X, Y\}$, set their wage $w_j$ to minimize the effective wage $v\equiv w_j/e_j$. Formally, the optimization problem for the representative firm is:

$$\min_{w_j} v_j = \frac{w_j}{e\left(\frac{w_j}{P}, U\right)}, j \in \{X, Y\}$$

The first-order condition for sector *j* is

$$e_j - e^j_1 \frac{w_j}{P} = 0$$

where $e_j$ is the effort level of sector *j*, and $e_1{}^j$ is the derivative of workers' effort with respect to sector *j*'s real wage. Each sector has its own first-order condition and $\varepsilon_1 = 1$ still holds, but they now have nonidentical values for $\varepsilon_{11}$. For sector *j*, $\varepsilon_{11}{}^j \equiv \left(\frac{e^j_{11}}{e^j_1}\right)\left(\frac{w_j}{P}\right), j \in \{X, Y\}$. We also maintain the assumption that $e_{12} = \frac{\partial^2 e}{\partial\left(\frac{w_j}{P}\right)\partial U} = 0$. Therefore, for $\varepsilon_2{}^j \equiv \left(\frac{e_2}{e_j}\right)U, j \in \{X, Y\}$. Eq. (3) is now two equations:

$$\widehat{U} = \frac{\varepsilon_{11}{}^X}{\varepsilon_2{}^X}\left(\widehat{w_X} - \widehat{P}\right) \quad (3'a)$$

$$\widehat{U} = \frac{\varepsilon_{11}{}^Y}{\varepsilon_2{}^Y}\left(\widehat{w_Y} - \widehat{P}\right) \quad (3'b)$$

To allow multiple optimal wages, we no longer assume concavity of the effort function with respect to the real wage *w/P*. Instead, we only have $e_{11}{}^j < 0$, and thus $\varepsilon_{11}{}^j < 0$, at the equilibria. Totally differentiating the effort function for each sector, we obtain

$$\widehat{e_X} = \widehat{w_X} - \widehat{P} + \varepsilon_2{}^X \widehat{U} \quad (4'a)$$

$$\widehat{e_Y} = \widehat{w_Y} - \widehat{P} + \varepsilon_2{}^Y \widehat{U} \quad (4'b)$$

From the definition of effective wage *v*, we have

$$\widehat{v_X} = \widehat{w_X} - \widehat{e_X} \quad (5'a)$$

$$\widehat{v_Y} = \widehat{w_Y} - \widehat{e_Y} \quad (5'b)$$

The rest of the model is the same as the main one, so the model with nonidentical wage contains in total 19 equations with one exogenous variable ($\widehat{\tau_Z}$) and 19 unknowns ($\widehat{K_X}, \widehat{K_Y}, \widehat{E_X}, \widehat{E_Y}, \widehat{L_X}, \widehat{L_Y}, \widehat{Z}, \widehat{U}, \widehat{e_X}, \widehat{e_Y}, \widehat{w_X}, \widehat{w_Y}, \widehat{v_X}, \widehat{v_Y}, \widehat{p_X}, \widehat{p_Y}, \widehat{r}, \widehat{X}, \widehat{Y}$) after imposing that $\widehat{P} = 0$.

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