



Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedcPollution and labor market search externalities over the business cycle[☆]John Gibson^a, Garth Heutel^{b,*}^a Department of Economics and Finance and Hunt Institute for Global Competitiveness, University of Texas at El Paso, United States^b Georgia State University and NBER, United States

ARTICLE INFO

Article history:

Received 26 April 2022

Revised 10 April 2023

Accepted 26 April 2023

Available online 29 April 2023

JEL classification:

E24

E32

Q58

Keywords:

Environmental policy

Labor market search

Business cycles

Pollution tax

Unemployment

ABSTRACT

We study the relationship between unemployment, environmental policy, and business cycles. We develop a dynamic stochastic general equilibrium real business cycle model that includes both a pollution externality and congestion externalities from labor market search frictions. These frictions generate unemployment on the extensive margin and affect hours worked on the intensive margin. We consider two policies to address the market failures: an emissions tax and a tax or subsidy on job creation. With both policies present, the efficient outcome can be achieved. When one policy is constrained or absent, we solve for the second best. The absence of a vacancy policy to address the congestion externalities substantially affects the value of the emissions tax, both in steady state and over the business cycle.

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1. Introduction

The effects of environmental policy on unemployment are at the center of the public debate on environmental policy – many argue that these policies are “job-killing regulations.” Since the Great Recession, the importance of business cycles has come to the forefront, highlighting the impact of cycles on unemployment and the environment. Standard neoclassical economic theory dictates that the inefficiencies caused by externalities can be eliminated through Pigouvian pricing, e.g. pollution taxes. Search-and-matching theories of unemployment involve externalities too, where both workers looking for jobs and employers looking to hire create congestion in the labor market. This labor market friction can manifest along the extensive margin – unemployment – or the intensive margin – hours worked. The interaction between multiple market failures – the congestion externalities from labor market search and the production externalities from pollution – can have important policy implications. The efficient Pigouvian response to a negative pollution externality may change when labor

[☆] Thanks to Yin Germaschewski, Marc Hafstead, Jay Harvath, Milton Marquis, and seminar participants at the ASSA, Econometric Society, AERE, Midwest Macro, and SEA conferences for comments, and to Ken Castellanos, Wenwen Li, Tejendra Pratap Singh, and Olanrewaju Yusuff for valuable research assistance.

* Corresponding author.

E-mail addresses: jdgibson@utep.edu (J. Gibson), gheutel@gsu.edu (G. Heutel).

market congestion externalities are present, modifying both the intensive and extensive margin for labor. Furthermore, the efficient level of these policies may vary over the business cycle.

The purpose of this paper is to study the efficiency effects of policy in an economy with unemployment, pollution, and business cycles. We begin with an analytical model of policy in the presence of pollution externalities and labor market search frictions. We then develop a dynamic stochastic general equilibrium (DSGE) model where fluctuations are driven by productivity shocks, as in the real business cycle (RBC) model. We add two features to the standard RBC model: a pollution externality and labor market search frictions that create unemployment. The pollution externality can be reduced through investment in a stock of abatement capital. The search frictions can affect both the extensive margin of employment and the intensive margin of hours worked. We allow for two policy options: an emissions tax and a tax or subsidy on job vacancy creation. We calibrate the model to the U.S. economy and carbon dioxide emissions and numerically solve it. We simulate both the first-best policy responses, where the government dynamically optimizes both policy tools, and second-best policies, where one policy is constrained.

A growing literature incorporates environmental policy in RBC models by modeling pollution as a byproduct of production that negatively affects productivity.¹ While labor is included as an input in most of these models, none include labor market search frictions or involuntary unemployment. A second literature incorporates involuntary unemployment via Diamond-Mortensen-Pissarides (DMP) labor market search frictions into RBC models.^{2,3} This literature does not consider pollution or pollution policy. A third literature studies the unemployment effects of environmental policy using DMP labor market search frictions, though without RBC fluctuations.⁴ Our model combines all three of these literatures. Our RBC model merges the pollution-generating production process of Heutel (2012) with the DMP search model of Atolia et al. (2018), thus adding cyclical fluctuations to a pollution and labor market search model like Aubert and Chiroleu-Assouline (2019).

Fewer papers consider the policy implications of the externalities generated by the DMP search model. Shi and Wen (1999) and Lu (2019) both consider various policies in addressing these externalities, including the minimum wage, unemployment insurance, and vacancy subsidies. In our paper, we focus on vacancy taxes or subsidies. Shi and Wen (1999) find that this is the most efficient policy, and we also find that this policy can bring about the first best. We consider dynamically-optimal policies, where the policies can vary with the business cycle; this is modeled in neither Shi and Wen (1999) nor Lu (2019).⁵ An enormous literature studies optimal environmental policy, and some of that literature considers the interactions between pollution externalities and other market failures. For example, Jaffe et al. (2005) model the interaction between pollution externalities and innovation externalities, and Kennedy (1994) models the interaction between pollution externalities and market power. We know of no other paper that considers the policy implications of the interaction between a pollution externality and congestion externalities arising from labor market search frictions.⁶

Our analytical model provides intuitions on the relationship between these multiple market failures and efficient Pigouvian pricing, which aids the interpretation of the simulation results from our numerical model. The pollution externality is a standard negative externality, internalized by a Pigouvian tax on the source of the externality (in the model, production). Labor search creates two offsetting congestion externalities. Each unemployed worker searching for employment reduces the probability of a match for all other unemployed workers but increases the probability of a match for all hiring firms. Each firm posting a job vacancy reduces the probability of a match for all other hiring firms but increases the probability of a match for all unemployed workers. Neither the worker nor the firm considers its impact on others' matching probabilities. The congestion externality caused by the unemployed worker leads to an inefficiently low level of employment, and the congestion externality caused by the hiring firm leads to an inefficiently high level of employment.

These two congestion externalities perfectly offset each other under a certain condition, the well-known "Hosios condition" (Hosios, 1990). If this condition is not met, then the inefficiency from the congestion externalities can be eliminated through a Pigouvian price on hiring. The efficient price instrument is either a subsidy to hiring or a tax on hiring, depending on whether equilibrium employment is inefficiently high or inefficiently low. We present closed-form expressions for the pair of policy instruments (production tax and hiring tax or subsidy) that induces efficiency. We also demonstrate the interaction between the two sets of externalities and the two policy instruments. For example, the magnitude of the pollution externality affects the level of the hiring tax or subsidy (which targets the congestion externalities).

Our calibrated DSGE model features both an extensive margin (employment vs. unemployment) and an intensive margin (hours worked) in the labor market, compared to previous models in the literature that examine only the extensive margin. By including the intensive margin, we are able to consider how variation in hours worked over the business cycle impacts other labor market dynamics within our model economy. Ultimately, we find that the inclusion of the hours worked margin

¹ For example, Fischer and Springborn (2011), Heutel (2012), Annicchiarico and Di Dio (2015), and Dissou and Karnizova (2016). See Fischer and Heutel (2013) for a literature review.

² This literature dates back to Andolfatto (1996) and Merz (1995), though we follow the more recent works of Atolia et al. (2018) and Atolia et al. (2019) and adapt a calibration strategy similar to Hagedorn and Manovskii (2008).

³ For a discussion of the basic DMP search and matching model, see Diamond (1982), Pissarides et al. (1985), Mortensen and Pissarides (1994), and Pissarides (2000). Rogerson et al. (2005) provides a literature review.

⁴ Hafstead and Williams (2018) and Aubert and Chiroleu-Assouline (2019) do so in analytical general equilibrium models, and Hafstead et al. (2022) does so in a computable general equilibrium model.

⁵ Blanchard and Gali (2010) consider a New Keynesian DMP model that includes monetary policy tools, which we do not include.

⁶ Both Hafstead and Williams (2018) and Aubert and Chiroleu-Assouline (2019) model DMP search frictions and environmental policy, but neither solves for optimal policy à la Pigou or Ramsey.

leads to some undesirable outcomes that are a poor fit to key real-world moments. Furthermore, the relationship between the planner's problem solution and the decentralized unregulated solution is puzzling in that the level of the (negative) pollution externality is higher for the planner. This is because with the inclusion of the hours margin, the congestion externalities from the search frictions, which lead to too much production and too much pollution, overwhelm the negative pollution externality. For these reasons, we drop the hours worked margin when conducting policy analysis. This demonstrates an important caveat in the application of the DMP model to policy analysis that has not yet been examined.

The calibrated model that we use for policy analysis finds that the unregulated decentralized equilibrium has higher pollution than the efficient outcome, as expected. Employment is also higher in the decentralized equilibrium, because the calibrated parameters in the wage-setting mechanism favor firms, resulting in a greater congestion externality from the firms than from the workers. Unemployment in the decentralized equilibrium is *too low* relative to the efficient outcome, and thus a tax on vacancy creation (rather than a subsidy to it) increases efficiency. This direction of the inefficiency (too little unemployment rather than too much) depends on the values of the two parameters crucial to the Hosios condition – the wage bargaining parameter and the matching function elasticity – which we demonstrate in sensitivity analysis. Unemployment is *too low* only on grounds of efficiency, not equity, and we show that moving to the efficient outcome disproportionately burdens workers relative to firms.

When both policy instruments – a vacancy tax and an emissions tax – are present, the efficient outcome can be achieved; this is the standard Pigouvian result. Our main results arise when one of the taxes is assumed to be unavailable. We solve for the second-best value of the remaining tax, which must be adjusted to “pick up the slack” and address both sets of externalities. Most importantly, when the vacancy tax is unavailable, the second-best level of the emissions tax is approximately 60 times greater than its first-best value at steady state. Furthermore, the cyclical properties of the two taxes differ, with the second-best emissions tax being significantly more volatile than the first-best emissions tax. Given that vacancy taxes are rarely if ever implemented, these results point to a potentially enormous policy implication for the design of emissions taxes.

The paper proceeds as follows. In [Section 2](#), we present a simple analytical model in which we do not consider business cycles but show through analysis of the steady-state equilibrium the main intuitive findings. Then in [Section 3](#) we present the full DSGE model. In [Section 4](#) we provide a discussion of our calibration strategy. In [Section 5](#) we present our results, and in [Section 6](#) we conclude.

2. Analytical model

In this section, we present steady-state results and policy responses for the canonical DMP labor search model that has been expanded to include a pollution externality.⁷ We include two possible policy instruments: a hiring tax or subsidy to internalize the search congestion externalities and a production tax or subsidy to internalize the pollution externality. In this section, we briefly describe the results and the intuitions; details and derivations are relegated to the appendix [Section A](#).

The economy consists of a continuum of homogeneous firms and a continuum of homogeneous workers. Time is continuous, and the size of the labor force is fixed at L . The unemployment rate, or the fraction of workers who are unemployed, is $1 - n$, where n is the employment rate. The vacancy rate, or the ratio of job vacancy postings to the total labor force, is v . Let $\Phi \equiv v/(1 - n)$ denote labor market tightness. The number of job matches m in a time period is a function of labor market tightness. The vacancy-filling rate, or the probability of a given job vacancy being filled, is defined as $m/v = q(\Phi)$. Let $q'(\Phi) \leq 0$, so that a higher labor market tightness (higher ratio of job vacancies to unemployed workers) leads to a lower probability of a vacancy being filled. Let the elasticity of q be between -1 and 0 and its absolute value be $\eta(\Phi)$ (i.e. $\eta(\Phi) \equiv -\frac{\Phi q'(\Phi)}{q(\Phi)}$). The value of $\eta(\Phi)$ is thus the negative of the elasticity of the vacancy-filling rate with respect to labor market tightness. The job-finding rate, or the probability of a given unemployed worker finding a job is $m/(1 - n) = f(\Phi) = \Phi q(\Phi)$, which is increasing in Φ , so that a higher labor market tightness leads to a higher probability of a worker finding a job. All workers and firms treat labor market tightness, Φ , as exogenous.

Each period an exogenous fraction of jobs, s , are destroyed ([Shimer, 2005](#)). In a given small time interval Δt , the number of workers who enter unemployment is $snL\Delta t$, and the number of workers who leave unemployment is $(1 - n)\Phi q(\Phi)L\Delta t$. Thus the change in the unemployment rate is $(1 - \dot{n}) = sn - \Phi q(\Phi)(1 - n)$, and the steady-state unemployment rate is

$$1 - n = \frac{s}{s + \Phi q(\Phi)}. \quad (1)$$

The value of a job's output is $p(1 - d)$, where p is the baseline exogenous productivity level, and $1 - d$ represents a pollution externality. All workers and firms treat the externality as exogenous. The hiring cost is $p(1 - d)c$, where c is also exogenous. The discount rate is r , the workers' bargaining power (or their share of the match surplus) is b , and the return to unemployment (or the value of leisure) is z .⁸

⁷ Specifically, we combine a pollution externality with the basic model of sections 1.1–1.5 of [Pissarides \(2000\)](#), the efficiency analysis of section 8.1 and the introduction of policy instruments in section 9.1–9.3.

⁸ As in [Pissarides \(2000\)](#), we assume that productivity (here $p(1 - d)$) is greater than or equal to the return to unemployment z , which ensures that the wage is greater than or equal to z , ensuring that workers stay in their jobs.

There are two policy variables, τ_h and τ_e . Both of these are modeled as a tax, but can be a subsidy if they take negative values. First, firms pay a one-time hiring tax τ_h , or receive a one-time hiring subsidy if $\tau_h < 0$, when a match is created. The amount of the tax is proportional to net productivity and equals $-\tau_h p(1-d)$. Second, firms pay an employment tax τ_e , or receive an employment subsidy if $\tau_e < 0$, for each filled job. The distinction is that the hiring tax τ_h is a one-time payment, while the employment tax τ_e occurs each period the match is operational.

In appendix Section A, we show how in the steady state, the model's equilibrium is characterized by three equations (Eqs. 1, A.1, and A.2). The core endogenous variables are the wage w , the unemployment rate $1-n$, and labor market tightness Φ . The solution depends on the values of the policy parameters τ_h and τ_e . To get an equation that will be more useful for policy analysis, substitute the wage from Eq. A.2 into Eq. A.1 to get

$$(p(1-d)-z)(1-b) - \frac{s+r+b\Phi q(\Phi)}{q(\Phi)} pc(1-d) - (1-b)(r+s)p\tau_h(1-d) - (1-b)\tau_e = 0. \quad (2)$$

Finally, consider the efficient allocation in this model, which in general will not coincide with the decentralized equilibrium because of the pollution and congestion externalities. The social planner maximizes a social welfare function that includes the value of output produced by the employed, the leisure value of the unemployed, and the cost of job search. Unlike the workers or the firms, the social planner recognizes that the pollution externality d is not exogenous but rather a function of total production. In particular, it is a function of the unemployment rate, $d(1-n)$, where $d'(1-n) < 0$, indicating that higher employment (i.e. lower unemployment) leads to higher damages d . Also, unlike the workers or firms, the social planner recognizes the congestion externalities from labor search by treating labor market tightness Φ as endogenous rather than fixed.

Appendix Section A shows that the solution to the planner's problem can be expressed as:

$$(p(1-d(1-n))-z)(1-\eta(\Phi)) + d'(1-n)p(n+c\Phi(1-n))(1-\eta(\Phi)) - \frac{s+r+\eta(\Phi)\Phi q(\Phi)}{q(\Phi)} pc(1-d(1-n)) = 0 \quad (3)$$

Eq. 3 is the key equation describing the efficient levels of employment and vacancies.

We can now compare Eq. 2, which describes the decentralized equilibrium, to Eq. 3, which describes the efficient allocation. Begin by making two simplifying assumptions. First, assume that $d'(1-n) = 0$, which is to say that the pollution "externality" d is in fact not an externality but merely an exogenous shift in productivity. Second, assume that $\eta(\Phi) = b$, i.e. that the absolute value of the elasticity of the matching function equals the worker's bargaining power. This is the Hosios condition (Hosios, 1990). When both assumptions hold, Eq. 2 coincides with Eq. 3 without any policy intervention (i.e. the hiring and employment taxes, τ_h and τ_e , are both zero). The unregulated equilibrium is efficient.

The first assumption amounts to eliminating the pollution externality. The second assumption (the Hosios condition) amounts to setting the parameters such that the two opposing congestion externalities exactly offset each other. The intuition is explored in Hosios (1990) and Pissarides (2000, p. 185–187). Roughly, when the surplus is divided according to $b = \eta(\Phi)$, each side (workers and firms) is rewarded in accordance with its social contribution.

When either of these two assumptions is not met, then the unregulated equilibrium is not efficient. But, with policy instruments τ_h and τ_e , the equilibrium can be made efficient. Dropping the two assumptions from the previous paragraphs, the equilibrium outcome is identical to the efficient outcome under the following two policy values:

$$\tau_{h,eff} = -(b-\eta(\Phi)) \frac{p(1-d(1-n))(1+c)-z}{(1-b)(r+s)p(1-d(1-n))} \quad (4)$$

$$\tau_{e,eff} = -d'(1-n)p(n+c\Phi(1-n)) \frac{1-\eta(\Phi)}{1-b} \quad (5)$$

These two equations describe the values of the policy variables that induce the efficient levels of employment and vacancies. Both of them are Pigouvian prices, internalizing the externalities. It is clearest to see this in Eq. 5, where the employment tax is proportional to the marginal pollution externality $d'(1-n)$. Since $d'(1-n) < 0$, the efficient employment tax $\tau_{e,eff}$ is positive. This Pigouvian tax internalizes the negative externality from each additional employed worker.⁹

The Pigouvian nature of the efficient hiring tax $\tau_{h,eff}$ is also evident. The efficient hiring tax has the opposite sign as $b-\eta(\Phi)$.¹⁰ If this expression is positive, then worker bargaining power is higher than the matching function elasticity, which leads to inefficiently high unemployment in the unregulated equilibrium. Thus, a hiring subsidy ($\tau_{h,eff} < 0$) remedies the inefficiency. If $b-\eta(\Phi)$ is negative, so that the workers' bargaining power is lower than the matching function elasticity, then unemployment is inefficiently low in the unregulated equilibrium. Thus, a hiring tax ($\tau_{h,eff} > 0$) remedies the inefficiency.

⁹ This analysis is similar to that done by Mortensen and Pissarides (2001) when solving for policy combinations that induce the first best; see their Eqs. 30 and 31 and tables 2 and 3. But, they impose the Hosios condition and do not include a pollution externality, and so they are not addressing the same sources of market failure as we are.

¹⁰ This relies on the assumption stated earlier that $p(1-d) \geq z$.

Eqs. 4 and 5 also demonstrate the interaction between the two sets of externalities and the two policies. The magnitude of the efficient hiring tax $\tau_{h,eff}$ depends on the level of the production externality $d(1-n)$. Specifically, the derivative of $\tau_{h,eff}$ with respect to $d(1-n)$ is the same sign as $b-\eta(\Phi)$.¹¹ Suppose $b-\eta(\Phi) > 0$, implying that equilibrium unemployment is too high and the efficient hiring policy is a subsidy ($\tau_{h,eff} < 0$). All else equal, a higher level of the externality $d(1-n)$ implies a lower magnitude of the hiring subsidy (i.e. $\frac{\partial \tau_{h,eff}}{\partial d(1-n)} > 0$). This is intuitive – the higher externality means there is less justification for subsidizing hiring (which increases the externality).

From Eq. 5, the efficient employment tax is scaled by $\frac{1-\eta(\Phi)}{1-b}$. When $b-\eta(\Phi) > 0$, this fraction is greater than one. The inequality $b-\eta(\Phi) > 0$ implies that the equilibrium level of unemployment is too high, and the efficient hiring policy $\tau_{h,eff}$ is a subsidy. Thus, when hiring is subsidized, the efficient employment tax $\tau_{e,eff}$ is greater than marginal external damages. The subsidy to hiring, which addresses the search congestion externality, exacerbates the pollution externality. So, the employment tax, which addresses the pollution externality, must increase to compensate.

Our model is similar to the models in Hafstead and Williams (2018) and Aubert and Chiroleu-Assouline (2019), both of which analytically model labor market search frictions alongside a pollution externality. Like this model, neither of those papers' models consider productivity shocks or business cycles, though our paper's DSGE model considers them. More importantly, neither Hafstead and Williams (2018) nor Aubert and Chiroleu-Assouline (2019) consider Pigouvian pricing policies to minimize the inefficiencies from the two sources of market failure. Instead, both study the effect of exogenous (not optimized) pollution policy on labor markets, and neither consider policies to target the labor search externalities. However, these other papers include features absent from our paper, including two productive sectors (one polluting and one non-polluting) in Hafstead and Williams (2018), and heterogeneous workers (low-skill and high-skill) in Aubert and Chiroleu-Assouline (2019). Aubert and Chiroleu-Assouline (2019) demonstrate the importance of the Hosios condition on the distributional effects of a pollution tax; see their equation 17.

The analytical model provides intuition about the relationship between the two sets of externalities and the efficient policy responses. But it does so through several simplifying assumptions, like excluding capital, ignoring an intensive-margin hours worked decision, not explicitly modeling pollution or abatement, and not modeling productivity shocks. Furthermore, our analysis of this model concerns only the steady state and not any dynamic features, and it only solves for first-best taxes and not second-best taxes. To address these other issues, we turn next to a DSGE model. The DSGE model will not provide closed-form analytical solutions like Eqs. 4 and 5, so instead we will calibrate and numerically simulate the model.

3. Numerical model

In this section, we build a dynamic stochastic general equilibrium (DSGE) model that incorporates both labor search frictions and a production process that generates harmful emissions into a real business cycle (RBC) model. Just as in the analytical model presented above, the search frictions and emissions result in a decentralized equilibrium with both congestion and pollution externalities. In the next two subsections, we detail the core elements of our model: the labor market and the production process. Then, in the following three subsections, we characterize the equilibrium behavior across the following three model environments: (i) the decentralized competitive equilibrium, (ii) the social planner's problem, and (iii) the Ramsey-optimal dynamic equilibrium. We compare all model specifications to see how inefficiencies in the labor market impact model variables, with a specific focus on the level and cyclicalities of the policy response. As we will describe and justify below, when we present the results on Ramsey-optimal policies, we will use a simpler model that omits the hours worked margin, since the simpler model actually performs better on several dimensions than the more complex model with endogenous hours worked.

3.1. Labor market

Time is discrete, and the economy is populated by a representative household that contains employed and unemployed agents. Employed agents supply labor to the representative firm, while unemployed agents search for work. While our analytical model differs from our DSGE model in terms of timing, household structure, and the presence of a representative firm, many elements of the labor market are very similar across both models. The number of new employment matches m_t formed in period t is given by:

$$m_t = \gamma_1 (1 - n_t)^{\gamma_2} v_t^{1-\gamma_2} \quad (6)$$

where v_t and $1 - n_t$ denote vacancies and unemployment in time t , respectively. Given the matching function in Eq. 6, labor market tightness, Φ_t , the vacancy-filling rate, q_t , and the job-finding rate, f_t , can be defined as in the analytical model. We also retain exogenous separations s and the evolution of employment is:

$$n_{t+1} = (1 - s)n_t + m_t \quad (7)$$

¹¹ $\frac{\partial \tau_{h,eff}}{\partial d(1-n)} = \frac{z(b-\eta(\Phi))}{(1-b)(r+s)p(1-d(1-n))^2}$.

Our model also includes the margin of hours worked, in addition to the employment decision. Hours worked for an employed worker in the representative household in period t is denoted by l_t . The utility function is

$$u_t = \ln c_t + n_t \xi_1 H(1 - l_t) + (1 - n_t) \xi_2 \quad (8)$$

This formulation is adapted from [Andolfatto \(1996\)](#). The first term is consumption utility, the second term is leisure utility from employed household members, and the third term is leisure utility from unemployed household members. $H(1 - l_t)$ is the leisure function based on leisure hours $1 - l_t$ (the time endowment minus labor hours supplied). Unlike in [Andolfatto \(1996\)](#), we do not include a margin for labor search for the unemployed, so their leisure is a constant ξ_2 . The two parameters ξ_1 and ξ_2 can represent the different efficiencies in home production between employed and unemployed household members, or different utilities of leisure between the employed and unemployed. As in [Andolfatto \(1996\)](#) we set $H(1 - l_t) \equiv \frac{(1 - l_t)^{1 - \xi_3}}{(1 - \xi_3)}$. At our parameter values, the leisure utility of the employed is lower than that of the unemployed ($\xi_1 H(1 - l_t) < \xi_2$).¹²

3.2. Production and pollution

The representative firm produces potential output using the following production technology:

$$y_t = \theta_t k_t^\alpha (n_t l_t)^{1 - \alpha} \quad (9)$$

where k_t is capital, n_t is employment, and l_t is hours worked, which are rented or hired from households. θ_t is an exogenous total factor productivity shock, which follows:

$$\ln \theta_{t+1} = \rho_\theta \ln \theta_t + \epsilon_{t+1}, \quad \epsilon \sim N(0, \sigma_\theta) \quad (10)$$

The total output produced in a given period, Y_t , depends positively on the level of potential output, y_t , and negatively on the stock of pollution present at the end of the period, x_{t+1} ,

$$Y_t = [1 - d(x_{t+1})] \cdot y_t \quad (11)$$

where $d(\cdot)$ is a monotonically increasing damage function with values between 0 and 1 that measures the output loss from pollution. We assume a quadratic form for this function: $d(x) = d_2 x^2 + d_1 x + d_0$. Producing output generates emissions, e_t :

$$e_t = g(A_t) \cdot h(Y_t) \quad (12)$$

where $h(\cdot)$ determines the relationship between output and unabated emissions, and $g(A_t)$ is a function that maps the current amount of pollution abatement capital, A_t , to the level of emissions abated. The function $g(A_t)$ takes values between zero and one. When $g(A_t) = 1$, no emissions are abated, and when $g(A_t) = 0$, all emissions are abated. Thus, $1 - g(A_t)$ represents the fraction of emissions abated. The functional form is $g(A_t) = 1 - \theta_1 A_t^{\theta_2}$, based on the abatement function in DICE. The functional form for the relationship between emissions and unregulated output is $h(y_t) = y_t^{1 - \gamma}$, where $1 - \gamma$ is the short-run elasticity of emissions with respect to output.

Pollution abatement capital A_t is a stock variable that evolves according to:

$$A_{t+1} = \phi \cdot A_t + I_{A_t} \quad (13)$$

where ϕ is the decay rate of the pollution abatement capital stock, and I_{A_t} is the amount of investment in pollution abatement. Modeling abatement as a stock, rather than a flow variable, accounts for the fact that abatement spending is often in the form of medium-term or long-term investments (like scrubbers built on power plants) that are not easily reversible. Unabated emissions feeds back on output through its effect on the stock of pollution x_t , which evolves according to

$$x_{t+1} = \lambda x_t + e_t + e_t^{\text{row}} \quad (14)$$

where λ is pollution's decay rate and e_t^{row} represents emissions from the rest of the world.

3.3. Decentralized problem

In this subsection, we specify the problems faced by the representative household and firm and characterize the decentralized competitive equilibrium. We allow for two policy variables: (i) an emissions tax, τ_{et} , paid by the firm on each unit of current emissions and (ii) a tax on vacancy creation, τ_{vt} , paid by the firm as a fraction of their total vacancy posting costs $v_t G$. Either of these taxes could be negative, in which case they would be a subsidy. These two policies are analogous to the two policies introduced in the analytical model and they are intended to address the pollution and congestion externalities, respectively.¹³ We refer to the case with $\tau_{et} = \tau_{vt} = 0 \forall t$ as the unregulated equilibrium.

¹² As in [Andolfatto \(1996\)](#)'s parameterization, both of these leisure utilities are negative, though the leisure function H is increasing and concave.

¹³ The tax on vacancy creation is analogous to the hiring tax τ_h , while the emissions tax is analogous to the employment tax τ_e .

3.3.1. Household

The representative household chooses current consumption and next period's capital in order to maximize its present discounted value of expected lifetime utility, treating the wage, w_t , rental rate on capital, r_t , and the job-finding rate, f_t , as exogenous.¹⁴

$$V(k_t, n_t) = \max_{c_t, k_{t+1}} \left[\ln c_t + n_t \xi_1 \frac{(1 - l_t)^{1-\xi_3}}{(1 - \xi_3)} + (1 - n_t) \xi_2 + \beta E\{V(k_{t+1}, n_{t+1})\} \right]$$

s.t.

$$c_t + k_{t+1} - (1 - \delta)k_t \leq r_t k_t + w_t n_t l_t + \tau_{et} e_t + \tau_{vt} v_t G + \pi_t + T_t \quad (15)$$

$$n_{t+1} = (1 - s)n_t + f_t(1 - n_t) \quad (16)$$

β denotes the subjective discount factor. Eq. 15 is the household's budget constraint. The left-hand side denotes expenditures on consumption and capital investment, while the right-hand side denotes the sources of income. Tax revenues T_t are rebated lump-sum to the household. Eq. 16 is the evolution of employment from the perspective of the household and it reflects the fact that the household does not internalize how their search activity impacts the job-finding rate, f_t . The first-order and envelop conditions can be manipulated to yield:

$$\beta E_t \left\{ \frac{1}{c_{t+1}} [r_{t+1} + 1 - \delta] \right\} = \frac{1}{c_t} \quad (17)$$

$$V_{n_t} = \frac{w_t l_t}{c_t} - \xi_2 + \xi_1 \frac{(1 - l_t)^{1-\xi_3}}{(1 - \xi_3)} + (1 - s - f_t) \beta E_t \{V_{n_{t+1}}\} \quad (18)$$

3.3.2. Firm

The representative firm maximizes the present discount value of expected lifetime profits, treating factor and output prices and the productivity shock as exogenous. The firm also faces the exogenous policy variables τ_{et} and τ_{vt} . The firm's profit function is

$$\pi_t = Y_t - I_{At} - \tau_{et} g(A_t) h(Y_t) - r_t k_t - w_t n_t l_t - (1 + \tau_{vt}) v_t G \quad (19)$$

The first two terms term on the right-hand side of Eq. 19 are the firm's output net of abatement investment, and the third term is the firm's tax bill on unabated emissions. The remaining three terms are the firm's capital rental bill, wage bill, and tax-inclusive vacancy expenditures.

The firm's problem can be written as the following dynamic program.

$$J(n_t, A_t; \theta_t) = \max_{k_t, A_t, v_t} \left[\pi_t + \beta E \left\{ \frac{c_t}{c_{t+1}} J(n_{t+1}, A_{t+1}; \theta_{t+1}) \right\} \right]$$

s.t.

$$n_{t+1} = (1 - s)n_t + q_t v_t \quad (20)$$

and equations 9 through 13 and 19 from above

Though there is just one representative firm, we model pollution's effect on output as an externality by assuming that the firm treats x_t as exogenous. The firm chooses how much capital to rent, how much to invest in abatement, and how many vacancies to post, taking the wage, w_t , rental rate, r_t , and vacancy-filling rate, q_t , as exogenous.¹⁵ Eq. 20 is the evolution of labor from the perspective of the firm, and it shows that the firm does not internalize how their choices affect the vacancy-filling rate q_t .

The firm's problem yields the following important equations:

$$r_t = [1 - \tau_{et} g(A_t) h'(Y_t)] (1 - d(x_{t+1})) \alpha \theta_t \left(\frac{k_t}{n_t l_t} \right)^{\alpha-1} \quad (21)$$

$$\beta E_t \left\{ \frac{c_t}{c_{t+1}} (\phi - \tau_{et+1} g'(A_{t+1}) h(Y_{t+1})) \right\} = 1 \quad (22)$$

$$q_t \beta E_t \left\{ \frac{c_t}{c_{t+1}} J_{n_{t+1}} \right\} = (1 + \tau_{vt}) G \quad (23)$$

¹⁴ As described further below, firms and workers bargain over labor contracts that will specify both the wage, w , and the number of hours worked, l .

¹⁵ The firm discounts future periods using the stochastic discount factor, $\beta E \left\{ \frac{c_t}{c_{t+1}} \right\}$, consistent with household preferences.

$$J_{n_t} = \left[1 - \tau_{et} g(A_t) h'(Y_t) \right] (1 - d(x_{t+1})) (1 - \alpha) \theta_t \left(\frac{k_t}{n_t} \right)^\alpha - w_t l_t + (1 - s) \beta E_t \left\{ \frac{c_t}{c_{t+1}} J_{n_{t+1}} \right\} \quad (24)$$

where Eq. 21 states that the rental rate on capital equals the marginal product of capital less the costs imposed by the need for additional abatement or through the tax on unabated emissions. Similarly, Eqs. 22 and 23 are marginal conditions on abatement investment and vacancy creation respectively, while Eq. 24 is the marginal value of an additional worker to the firm.

3.3.3. Nash bargaining

Wages and hours worked are set through repeated Nash bargaining over the marginal surplus generated by a match. While it is possible to allow households to determine hours worked (e.g., add l as a choice variable in the household's problem), we choose to include the determination of hours as part of the bargain between workers and firms. As such, it is best to think of the bargain as determining a specific labor contract that specifies both the hourly wage and the number of hours worked. This results in two first-order conditions from the bargaining problem.

The first is the surplus sharing rule:

$$b \tilde{J}_{n_t} = (1 - b) V_{n_t} \quad (25)$$

where b is workers' bargaining power.¹⁶

Additionally, the first-order condition for the choice of hours yields:

$$b \frac{\partial V_n}{\partial l} + (1 - b) \frac{V_n}{\tilde{J}_n} \frac{\partial \tilde{J}_n}{\partial l} = 0 \quad (26)$$

This condition can be manipulated, with the value functions being substituted, to yield

$$\xi_1 (1 - l_t)^{\xi_3} = \frac{1}{c_t} (1 - \alpha)^2 (1 - d(x_{t+1})) \theta_t \left(\frac{k_t}{n_t l_t} \right)^\alpha [1 - \tau_{et} g(A_t) (h'(Y_t) + Y_t h''(Y_t))] \quad (27)$$

Then, both conditions can be manipulated to yield the following wage equation:

$$w_t = b \left([1 - \tau_{et} g(A_t) h'(Y_t)] (1 - d(x_{t+1})) (1 - \alpha) \theta_t \left(\frac{k_t}{n_t l_t} \right)^\alpha + \Phi_t (1 + \tau_{vt}) \frac{G}{l_t} \right) + (1 - b) \frac{c_t}{l_t} \left[\xi_2 - \frac{\xi_1}{1 - \xi_3} (1 - l_t)^{1 - \xi_3} \right] \quad (28)$$

where the first term in parentheses is the marginal product of a new worker less the costs imposed by additional abatement or the emissions tax, and the second term is the asset value of an employment match reflecting the vacancy costs saved by having an additional employed worker. The last term reflects the worker's disutility from work expressed in goods.

The decentralized equilibrium is fully characterized by the following set of equations: 11,14,17,20–23,27,28, the definitions of q_t and Φ_t , and the goods market clearing condition:

$$c_t + k_{t+1} - (1 - \delta) k_t + A_{t+1} - \phi A_t + v_t G = Y_t \quad (29)$$

These equations determine the 12 core endogenous variables of our model: c_t , k_t , n_t , l_t , A_t , Y_t , v_t , q_t , Φ_t , r_t , w_t , x_t .

3.4. Planner's problem

Due to the presence of the pollution and congestion externalities, the solution to the decentralized problem presented above will not in general coincide with the efficient solution of the planner's problem where all externalities have been internalized.¹⁷ Comparisons of the unregulated decentralized equilibrium and the planner's allocation highlight the inefficiencies arising from both externalities.

The social planner chooses consumption, investment in abatement capital, employment, and vacancies to maximize the present discounted value of expected household utility, internalizing all externalities. For example, the planner's problem recognizes how emissions from production affects future output, and how vacancies affect the number of job matches. See appendix Section B for the equations describing the planner's problem.

¹⁶ To ensure that the terms from the household's and firm's problems are measured in consistent units (utils), we use \tilde{J}_n , which is J_n weighted by the marginal utility of consumption, as an argument in the bargaining problem.

¹⁷ In the main paper, we only consider efficiency as our optimization criteria. However, in the appendix we present results highlighting the equity implications across policies.

3.5. Ramsey efficient dynamic tax problem

In [Section 3.3](#), we treat the emissions tax τ_{et} and the vacancy creation tax τ_{vt} as exogenous. However, we are interested in how the government could use these taxes to correct the market failures and reach the efficient allocation. To this end, we specify the government's problem as the Ramsey efficient dynamic tax problem. This problem is identical to the decentralized equilibrium described in [Section 3.3](#), but with the additional choice over τ_{et} and τ_{vt} subject to all of the constraints from the firm's and the household's optimization problems. Details of this optimization problem are in [appendix Section C](#).

When considering the Ramsey efficient dynamic tax problem, we will simplify the model by omitting the hours margin, l_t . Removing the hours margin simply requires one to set $l_t = 1$ in all equations from the full model; labor hours are now supplied inelastically by all employed workers. As described further in [Section 5.2](#) below, this simplification is partially motivated by the fact that the full model with hours worked provides a poor fit to key business cycle statistics related to the labor market. A further motivation is that, without this simplification, reaching the first-best outcome will require two tax policies to address the labor market frictions – one for each margin – in addition to the pollution tax to address that externality. We want to avoid this needless complication and focus instead on our comparison between a single labor market policy and a single pollution policy.

This simplification allows us to focus on the emissions tax, τ_{et} , and the vacancy creation tax (or subsidy), τ_{vt} . If the government can choose both taxes, then it can correct both market failures and reach the efficient allocation from the planner's problem (as we will show in the simulations below). By solving this problem, we see the level and dynamics of the emissions tax and the vacancy creation tax needed to internalize both externalities. We refer to the taxes recovered this way as the “first-best” taxes. These are standard Pigouvian taxes. More interesting are cases where one tax is fixed, while the other tax is optimized. Taxes recovered this way are referred to as “second-best” taxes. That is, the second-best emissions tax is found when the government sets $\{\tau_{et}\}_{t=0}^{\infty}$ to maximize welfare given that τ_{vt} is fixed at some value (perhaps 0), and vice-versa for the second-best vacancy creation tax.

Relative to the prior literature examining the relationship between labor market search frictions and pollution externalities ([Hafstead and Williams \(2018\)](#), [Aubert and Chiroleu-Assouline \(2019\)](#)), our model is the first to examine efficient policies in a DSGE model with productivity shocks. Both of these prior papers are based on static models without a dynamic or stochastic element. [Hafstead et al. \(2022\)](#) is a CGE model that includes search frictions in a similar manner as our model does, though without autocorrelated productivity shocks.

Importantly, ours is a one-sector model, as opposed to others which include a polluting and non-polluting sector ([Hafstead and Williams \(2018\)](#)), high- and low-skilled workers ([Aubert and Chiroleu-Assouline \(2019\)](#)), or a multi-sector CGE model ([Hafstead et al. \(2022\)](#)). Our focus is on aggregate rather than sectoral employment effects, and on the relationship between the pollution externality and the search frictions. We maintain the one-sector assumption to more clearly demonstrate and highlight these aggregate effects. As noted by [Hafstead and Williams \(2018\)](#), empirical studies that ignore unregulated sectors can overestimate unemployment effects. The purpose of our paper is not to estimate the magnitude of these effects, though, but rather to investigate the policy response to the two frictions in our economy. Including two sectors in our model, or modeling high-skilled vs. low-skilled workers, would not add clarity to our main findings. We note the caveat that to estimate or simulate the magnitudes of employment effects across different sectors would require a more disaggregated model.

4. Calibration

We calibrate our decentralized model to replicate key features of the U.S. economy using both data and values reported in the existing literature. Each specification of our model is solved using standard linearization techniques¹⁸ and as we move across model specifications, say from the decentralized problem to the planner's problem, we keep all parameter values fixed at their calibrated levels. The next several subsections describe our calibration strategy; more details are in [appendix Section D](#). The calibrated parameter values and their sources are summarized in [Table 1](#).

4.1. RBC parameters

We set our model's period length to one quarter, and we follow the RBC literature and set the agent's subjective quarterly discount factor, β , equal to 0.983, matching the real interest rate from U.S. data. Also following convention, we set the quarterly depreciation rate of private capital, δ , to 0.025, thereby targeting an annual depreciation rate of approximately 10 percent. We set capital's share of income, α , equal to 0.36, following the RBC literature. The persistence in the productivity shock process, ρ_θ , and the standard deviation of the innovation in the shock process, σ_θ are set to 0.925 and 0.0109, respectively. These values were set so that our model generates a series for average labor productivity (output per worker) that matches the first-order autocorrelation and percent volatility (0.878 and 2.00, respectively) reported in [Shimer \(2005\)](#). Lastly, the policy variables, τ_e and τ_v are both set to 0 in the baseline decentralized economy.

¹⁸ [Christiano \(2002\)](#) discusses the linearization methods used to approximate our model.

Table 1
Parameter Values.

Parameter	Description	Value	Source
β	Discount factor	0.983	
δ	Capital depreciation rate	0.025	RBC literature
α	Capital's share of income	0.36	
ρ_θ	Productivity persistence	0.925	
σ_θ	Standard deviation of productivity innovation	0.0109	Match labor productivity data
λ	Pollution decay rate	0.9965	IPCC reported CO ₂ half-life
ϕ	Persistence of abatement capital	0.9974	Fisher and Narain (2003)
$1 - \gamma$	Elasticity of output w.r.t. emissions	0.6	Regressions (see appendix)
θ_1	Abatement cost function coefficient	0.2574	DICE 2016R2
θ_2	Abatement cost function exponent	0.3845	
d_0	Damage function constant	-0.0076	
d_1	Damage function linear coefficient	$8.10e - 6$	Calibrated from DICE 2016R2 (see appendix)
d_2	Damage function quadratic coefficient	$1.05e - 8$	
γ_2	Exponent in matching function	0.72	Shimer (2005)
ξ_3	Parameter in leisure function	2.00	Andolfatto (1996)
b	Labor bargaining power	0.05	Target wage elasticity w.r.t. productivity
G	Vacancy posting costs	0.725	
γ_1	Coefficient in matching function	0.65	
s	Separation rate	0.048	Target 5 moments (See Section 4.3)
ξ_1	Coefficient on leisure of employed in utility function	0.563	
ξ_2	Coefficient on leisure of unemployed in utility function	-0.104	

4.2. Environmental parameters

The pollution stock decay rate λ is calibrated based on the estimated half-life of atmospheric carbon dioxide.¹⁹ The Fifth Assessment Report of the IPCC, Working Group 1, provides a calculation that about 50% of a pulse of anthropogenic CO₂ leaves the atmosphere after 50 years.²⁰ Given our quarterly time scale, this implies a decay rate $\lambda = 0.9965$.²¹ Carbon emissions from the rest of the world (e_t^{row}) are assumed to be exogenous and constant. Since the United States emits about one-sixth of global carbon dioxide,²² we set $e_t^{row} = e^{row}$ to equal five times the steady-state value of domestic emissions e_t .

The relationship between baseline pollution and output is given by $h(y_t) = y_t^{1-\gamma}$. In appendix Section D.1, we describe a calibration strategy for $1 - \gamma$, the short-run elasticity of emissions with respect to output. We collect monthly data and run a simple set of regressions of the log of emissions on the log of output. We find a range of point estimates from about 0.4 to 0.9 (see Table D.1), and we use a value of 0.6 in the simulations. This is the value reported for the short-run (cyclical) greenhouse gas emissions elasticity with respect to output in the United States, from Cohen et al. (2018).²³

The emissions abatement function $g(A_t)$ is calibrated based on DICE, though DICE does not include a stock of abatement capital like our model does. Instead, in DICE, the fraction of emissions abated in each period can be chosen independently of any past level of abatement, where the cost of that abatement is given as a fraction of gross output. The calibrated function values from DICE imply that the cost to abate 50% of gross emissions equals 1.22% of gross output, while the cost to abate 100% of gross emissions equals 7.4% of gross output.²⁴ We use these two stylized facts to calibrate our abatement function. The function $g(A_t)$ gives the fraction of unabated baseline emissions that remain after abatement: $e_t = g(A_t)h(y_t)$. We set this functional form $g(A_t) = 1 - \theta_1 A_t^{\theta_2}$, based on the analogous abatement function in DICE. Therefore, for a given abatement capital stock A_t , the fraction of gross emissions abated is $\theta_1 A_t^{\theta_2}$.

If 1.22% of gross output is spent on abatement each period, then the steady-state level of the abatement capital stock is $\frac{1}{1-\phi} 0.0122\bar{Y}$, where ϕ is the abatement capital depreciation rate and \bar{Y} is steady-state gross output. According to one of

¹⁹ Like Fischer and Springborn (2011) and other RBC papers on environmental policy, we calibrate the model to carbon dioxide emissions, even though carbon dioxide is a long-lived stock pollutant and business-cycle-scale emissions fluctuations have only small effects on the stock. As this literature demonstrates, business cycle fluctuations matter for environmental policy because of their effects on the costs of abatement rather than on the benefits of abatement. While our calibration is based on carbon dioxide, a global stock pollutant, we could alternatively calibrate based on local or flow pollutants like particulate matter.

²⁰ See Box 6.1 in Ciais et al. (2014).

²¹ This exponential decay rate specification simplifies a much more complicated set of processes that govern the movement of CO₂ in the atmosphere, including its interaction with the land and the ocean. Though about 50% leaves the atmosphere within 50 years, up to 15 to 40% can remain after 1000 years. "This is why the concept of a single, characteristic atmospheric lifetime is not applicable to CO₂." (Ciais et al. (2014), p. 473)

²² <https://www.epa.gov/ghgemissions/global-greenhouse-gas-emissions-data#Country>

²³ This is comparable to the value of 0.696 used in Heutel (2012), which was based on regressions using older data. The Heutel (2012) value was based on data on just CO₂ emissions, rather than all greenhouse gases, and it used data from 1981 to 2003, rather than 1990 through 2014 as in Cohen et al. (2018). The data used in Heutel (2012) are monthly, while the data in Cohen et al. (2018) are annual. Doda (2014) finds a higher elasticity, 1.01, using annual data on CO₂ emissions from 1950 - 2011.

²⁴ The DICE model allows for the abatement costs to decrease over time to capture growth in abatement technology. We abstract from growth and calibrate abatement cost based on its initial value. DICE's abatement cost calibration is based on the EMF-22 report (Clarke and Weyant, 2009).

our stylized facts from the DICE calibration, this level of the abatement capital stock should yield abatement of 50% of gross emissions: $g(\frac{1}{1-\phi}0.0122\bar{Y}) = 0.5$. Likewise, based on the other stylized fact, $g(\frac{1}{1-\phi}0.074\bar{Y}) = 0$. We use these two equations, combined with the functional form for the abatement function $g(A_t) = 1 - \theta_1 A_t^{\theta_2}$ to calibrate the parameters θ_1 and θ_2 . The value of θ_2 is independent of the values of either ϕ or \bar{Y} , and it equals 0.3845.²⁵ The value of θ_1 depends on both ϕ and \bar{Y} . For the abatement capital depreciation rate ϕ , we calibrate it based on the value used in Fisher and Narain (2003). They use a depreciation rate of 10% per decade, implying a quarterly depreciation rate of 0.26% (which means $\phi = 0.9974$).²⁶ For the steady-state gross output \bar{Y} , which is determined endogenously, we use the value from the decentralized steady-state where there is no investment in abatement. Together, these values yield $\theta_1 = 0.2574$.²⁷

In DICE, damages are a function of temperature rather than of atmospheric carbon concentrations. A dynamic climate model describes how carbon concentrations affect temperature over time. Our model does not include temperature or a carbon cycle model, so we approximate DICE's damage function by setting damages to be a function of the pollution stock x . Details of the calibration strategy are in appendix Section D.2. We use the same functional form and calibration strategy as in Heutel (2012), though based now on the most recent version of DICE (2016R2). The damage function $d(x)$ is quadratic: $d(x) = d_2 x^2 + d_1 x + d_0$, where $d_2 = 1.05e - 8$, $d_1 = 8.10e - 6$, and $d_0 = -0.0076$, and the units of the pollution stock x are gigatons of carbon (GtC). These parameters imply that at the current level of atmospheric carbon stock (851 GtC, from DICE 2016R2), the damages from climate change amount to 0.69% of gross output. A doubling of the current carbon stock (to 1702 GtC) would increase that to 3.66% of gross output.²⁸

4.3. Labor market parameters

We follow Christiano et al. (2016) and Atolia et al. (2019) and set vacancy posting costs, G , to a low value so that total spending on vacancy creation averages between 1 and 2 percent of total output. For the full model with hours worked, we follow Andolfatto (1996) and set $\xi_3 = 2$ based on conditions regarding labor supply elasticity. Next, we simultaneously set the coefficient in the matching function, γ_1 , the remaining utility parameters, ξ_1 and ξ_2 , and the separation rate, s , to match the following targets; an unemployment rate of 7.5 percent in steady state, a value of leisure in steady state near 0.4, a mean job-finding rate of 0.59, and a mean labor market tightness of 0.72.²⁹ Once the model is simplified by setting $l_t = 1$, this strategy yields the following parameter values: $\gamma_1 = 0.65$; $\xi = 0.8515$; $s = 0.048$.

The two crucial parameters related to the Hosios condition are the exponent in the matching function, γ_2 , and workers' bargaining power, b . In the spirit of Hagedorn and Manovskii (2008), we set workers' bargaining power to a low value to target the elasticity of wages with respect to average labor productivity near 0.45, yielding $b = 0.05$. Some other papers simply assume symmetric bargaining ($b = 0.5$, e.g. Hafstead and Williams (2018)), or assume that the bargaining parameter is equal to the matching elasticity, satisfying the Hosios condition (e.g. Shimer (2005)). Neither of these assumptions is empirically-based. The exponent in the matching function, γ_2 , is derived in the prior literature by estimating the matching function (Eq. 6) using data on the number of unemployed individuals reported by the Bureau of Labor Statistics, the number of active vacancies reported in the Conference Board Help Wanted Index, and estimates of the job-finding rate from the sample period 1951–2003. Following Shimer (2005), we set this value at 0.72 (see appendix Section D.3 for additional details regarding the labor market calibration).

Under this base-case calibration, the workers' bargaining power ($b = 0.05$) is significantly lower than the coefficient on unemployment in the matching function ($\gamma_2 = 0.72$). The analytical model tells us that the Hosios condition is not satisfied and that the equilibrium number of job vacancies is higher than the efficient level, implying that a vacancy creation tax ($\tau_{vt} > 0$) increases efficiency. We explore the policy implications of this in the simulations below, and we also conduct sensitivity analysis over both of these parameters.

5. Results

First, we compare the unregulated decentralized equilibrium to the planner's solution to show how the pollution and congestion externalities distort the equilibrium allocation. Next, we explore the model component of the variable hours margin, and show that this addition to the model actually results in a model that performs worse than a simpler model with inelastic intensive-margin labor supply. For the remaining results we use that simpler model. Then, we consider the first-best policy response of the government, using both the emissions tax and the vacancy creation tax to internalize all externalities and bring the decentralized economy in-line with the planner's solution. Lastly, we present our main results,

²⁵ Solving both equations for θ_1 and setting both expressions equal, both ϕ and \bar{Y} drop out, and we get $2^{\frac{1}{\theta_2}} = \frac{0.074}{0.0122}$, which gives $\theta_2 = 0.3845$.

²⁶ Fisher and Narain (2003) modify DICE by including an abatement capital stock, based on the specification in Kolstad (1996) (who treats the abatement capital decay rate as a parameter value that varies).

²⁷ Since the abatement capital stock, A_t , is in arbitrary units, just like output Y_t , there is no intuition or interpretation of this parameter value.

²⁸ The functional form of the damage function $d(x)$ does not ensure a value that is always between 0 and 1, as required by the model. We verify that, for all simulations and parameter values, the damage function does indeed take interior values.

²⁹ The first two targets are based on average values for unemployment and leisure hours worked while the second two targets follow from Atolia et al. (2019).

Table 2
Decentralized vs Planner Outcomes.

	Mean ^a		%Volatility ^b		
	Decentralized	Planner	Data	Decentralized	Planner
RBC Variables^c					
Output	1.18	1.27	1.58	1.82	1.82
Consumption	0.89	1.00	0.77	0.32	0.34
Investment	0.25	0.27	4.59	3.55	3.72
Labor Market Variables^d					
Unemployment	0.075	0.20	9.71	0.83	1.48
Vacancies	0.054	0.0025	9.13	2.82	8.67
Labor Market Tightness	0.714	0.0122	18.37	3.44	9.47
Hours Worked	0.389	0.48	-	0.32	0.22
Environmental Variables^e					
Emissions	1.11	1.14	1.23	0.60	0.60
Pollution	1894.4	1956.6	0.0532	0.0012	0.0013
Abatement Capital	0	0.00013	NA	0	1.89

^a All model statistics are from 100 simulations, each 300 quarters in length. Reported values are averages of our sample statistics across these simulations. The values reported for "Mean" are the average value of the simulated series. These numbers are quantitatively very close to the variables' steady state value.

^b Percent volatility is measured as the percent standard deviation of the cyclical component of the log of x relative to the percent standard deviation of the cyclical component of log output (statistic for output is presented in levels).

^c Data on output, consumption, and investment comes from the National Income and Product Accounts tables and covers the period of 1947–2019. Specifically, Y is Real Gross Domestic Product, c is Real Personal Consumption Expenditures, and inv is Real Gross Private Domestic Investment (all in billions of chained 2012 dollars). The cyclical component of all RBC variables (both data and model simulations) is found by HP-filtering with a smoothing parameter of 1,600.

^d The data values reported in this section come from Shimer (2005) and have been normalized by the percent volatility of output. Following Shimer (2005), the cyclical component of all labor market related variables (both data and model simulations) is found by HP-filtering with a smoothing parameter of 10^5 .

^e Emissions data are from the source described in the calibration section of the text. Pollution stock data are from NOAA's Global Monitoring Laboratory.

from the second-best tax problem, where the government is allowed to optimize one tax instrument while the other is fixed at zero.

5.1. Decentralized versus Planner's problem

Before we compare the unregulated decentralized equilibrium to the planner's solution, we verify that our decentralized model generates moments consistent with those from the existing literature and the data. Table 2 presents the mean and the percent volatility relative to output for several variables, under the decentralized solution, the planner's solution, and from the data. The first panel of Table 2 shows that the mean and percent volatility of output are 1.18 and 1.82, respectively, under the unregulated decentralized equilibrium. (its relative percent volatility is 1 by construction).³⁰ The mean and relative volatility of consumption are 0.89 and 0.32, and the mean and relative volatility of investment are 0.25 and 3.55. These moments reflect realistic shares of consumption and investment in output and are consistent with consumption smoothing and investment that is significantly more volatile than output. This is shown by comparing the percent volatilities from the decentralized model (in the fourth column) to the percent volatility in the data (in the third column). We do not present mean values from the data, since the units of the variables in our model are arbitrary. The second panel of Table 2 provides the means and relative volatilities of unemployment, vacancies, labor market tightness, and hours worked, which are targets in our calibration strategy. While both vacancies and labor market tightness are more volatile than output in the decentralized economy, unemployment is found to be smoother than output. High labor market volatility is consistent with U.S. data, and the inability of the baseline model to generate additional volatility in unemployment points to a weakness of the baseline specification that includes hours worked.³¹ The third panel of Table 2 shows that the relative volatility of emis-

³⁰ We report the statistics for output inclusive of environmental damages, Y_t , rather than gross output, y_t . This is more appropriate to compare to the data moments, which include whatever climate damages are actually occurring. In practice, the choice over these two output measures will make no difference in this table, since the mean damages from climate are less than one percent of output, and the pollution stock has such a long half-life that it is virtually invariant over the business cycle.

³¹ Matching the exact level of volatility found in the U.S. data has been difficult across a broad class of models. See Atolia et al. (2018) and Atolia et al. (2019) for more details on the difficulties matching labor market volatility using RBC search models.

sions is 0.6, compared to a value about twice as large from the data. Abatement capital is constant at 0 in the unregulated decentralized economy, since there is no incentive to abate, and therefore has no relative volatility.³²

Fig. 1 presents time paths for key model variables recovered from both the decentralized and planner's problems. To generate these simulations we take 100 draws for the innovation in the total factor productivity process using a random number generator with a fixed seed. Given that the parameters governing the productivity process, ρ_θ and σ_θ , are the same across model specifications, the same series for total factor productivity is used when generating the time paths for both the decentralized and planner's problems. With the exogenous productivity series fixed across both specifications, observed differences in the simulated time paths are used to draw comparisons between the decentralized and planner's solutions. The first four panels of Fig. 1 show how the time paths of output, consumption, vacancies, and labor market tightness deviate between the decentralized and planner's problems. The congestion externality results in an oversupply of vacancies. The impact of this congestion can also be seen from the fifth panel of Fig. 1 which shows that the unemployment rate is significantly higher under than planner's solution than in the decentralized economy. However, output is lower under the decentralized case than under the planner's solution. This is because hours worked (sixth panel) is much higher in the planner's solution. With both margins (hours worked and employment) available to bargain over, the decentralized solution results in too much employment, but too little hours worked per employed worker. Surprisingly, the pollution level is higher under the planner's solution than the decentralized – there is an inefficiently low level of pollution in the unregulated economy. This is despite the fact that there is no abatement at all under the decentralized solution. This is because the congestion externality dominates the pollution externality. This point will be revisited when we consider the government's first- and second-best policy options in the next few subsections.

We return to Table 2 to compare the first and second moments of key outcome variables from the decentralized solution to the planner's solution, to gauge the effect that the two frictions have on the economy. While the traditional RBC variables are only moderately impacted by the change in model specification, we find significant differences in labor market variables. These large differences in means come from the congestion externality, which causes firms to create more vacancies than is efficient. Because the Hosios condition is not met, and in fact the bargaining parameter b is so much lower than the match function elasticity γ_2 , the net congestion externality results in a vast oversupply of employment; the efficient unemployment rate (20%) is more than double the unregulated decentralized unemployment rate (7.5%).

As for environmental variables, the third panel of Table 2 shows that mean emissions increase slightly, from 1.11 to 1.14 as we move from the decentralized problem to the planner's problem. Again, this is despite the fact that pollution is a negative externality – emissions increase because the labor market congestion externality dominates the pollution externality. While the volatility of emissions and pollution is similar across specifications, abatement capital, which is now positive under the planner's solution, varies more than output.

5.2. Model simplification - Inelastic labor supply

When we consider the Ramsey optimal first- and second-best tax problems, we use a simpler model that omits the hours margin, so that labor is supplied inelastically ($l_t = 1, \forall t$) for those who are employed. As described further below, this simplifying assumption is made for three reasons. First, it allows us to focus on our primary research question, emissions policy in the presence of frictional labor markets, with the most parsimonious model. Second, assuming that employed agents supply labor inelastically allows our preference specification to collapse into one with constant disutility of work, which is commonly found within the labor search literature. And third, the second-moment properties of the model with endogenous hours worked fail to match the data in multiple dimensions, raising questions regarding the appropriateness of this model specification.

As mentioned above, by including both an hours-worked (intensive) margin and an employment (extensive) margin in the labor market, the congestion externality affects both and thus requires two policies to be fully internalized (in addition to the policy for the pollution externality). That is, achieving the first best requires both a tax or subsidy on vacancies (to address the extensive margin) and a tax or subsidy on hours worked (to address the intensive margin). Because we want to focus on the relationship between the congestion externality and the pollution externality, the inclusion of a third tax instrument would complicate our analysis without providing additional insight into our main research question.

Along with simplifying our policy analysis, shutting down the hours margin can also be supported by an appeal to the literature. Specifically, setting $l_t = 1, \forall t$ implies that our utility specification collapses to:

$$u(c_t, n_t) = \ln c_t - \xi n_t \quad (30)$$

This specification is consistent with that presented in Shimer (2010) and used in Atolia et al. (2019).

And finally, it is important to consider the empirical fit of our model including the hours margin. If this specification fails to capture key details of the U.S. economy, then an alternative specification that removes endogenous hours worked may be preferred. To this end, we find that the model including the hours margin produces many unexpected and puzzling moments. For instance, as we showed earlier, the level of emissions under the planner's solution is actually higher than under the decentralized solution, despite pollution being a negative externality. This puzzling finding follows from the fact that the

³² We were unable to find a data analog for the volatility of abatement capital. The Pollution Abatement Costs and Expenditures (PACE) survey was administered annually from 1973 to 1994, but then only periodically since then (1999 and 2005), so it is not suitable for calculating these statistics.

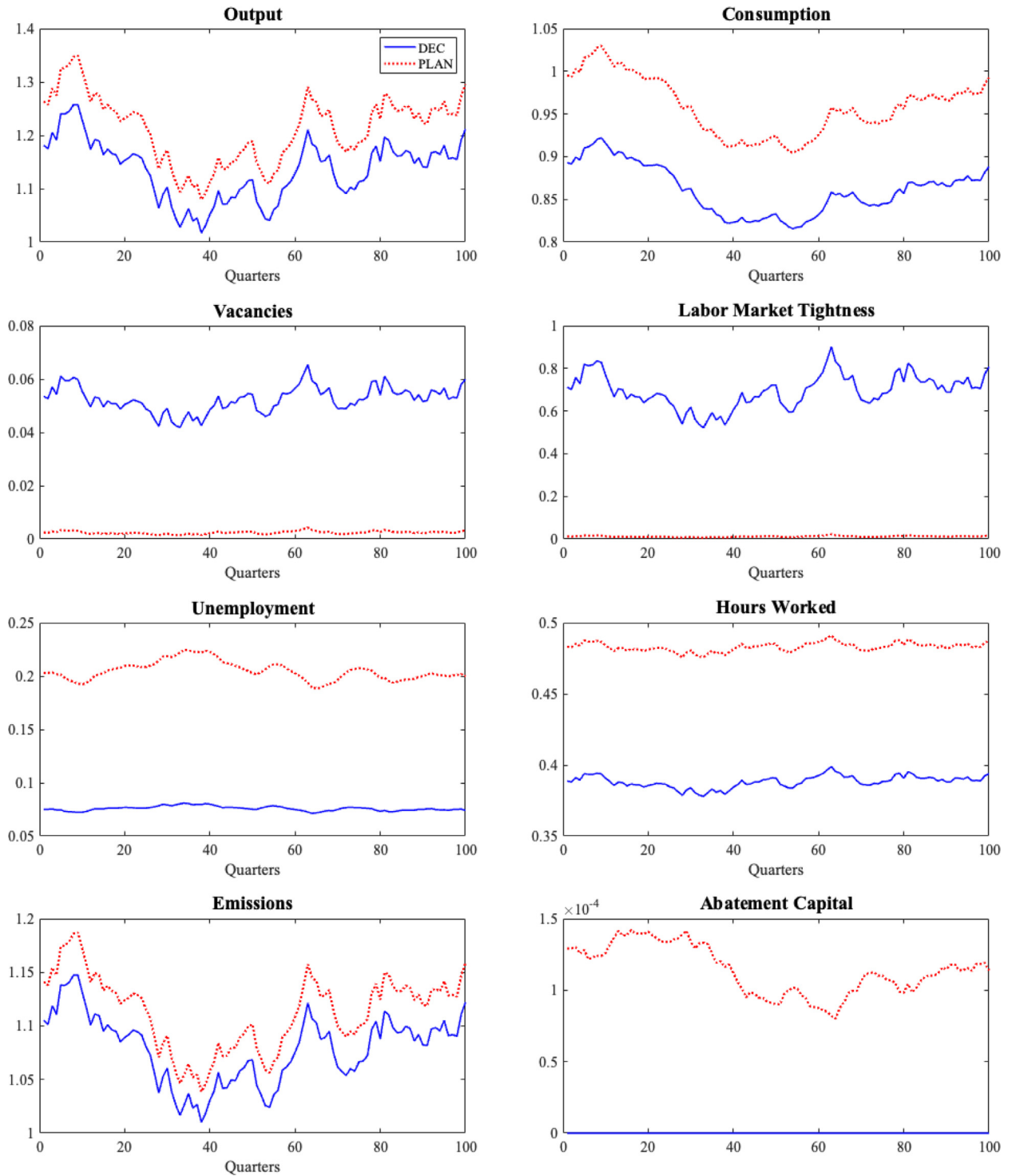


Fig. 1. Business Cycle Simulations - Decentralized Vs. Planner. Notes: All simulations are from randomly generated draws of productivity innovations for 100 periods.

pollution externality is swamped by the inefficiencies in the labor market that arise due to both labor search frictions (e.g., inefficient level of vacancy creation) and endogenous hours worked (e.g., inefficient hours worked per employee). Also, we find that this model produces weak volatility in labor market variables, with unemployment being smoother than output. This is inconsistent with empirical evidence and suggests that the inclusion of hours worked hinders the model's ability to generate reasonable levels of volatility in labor market variables.

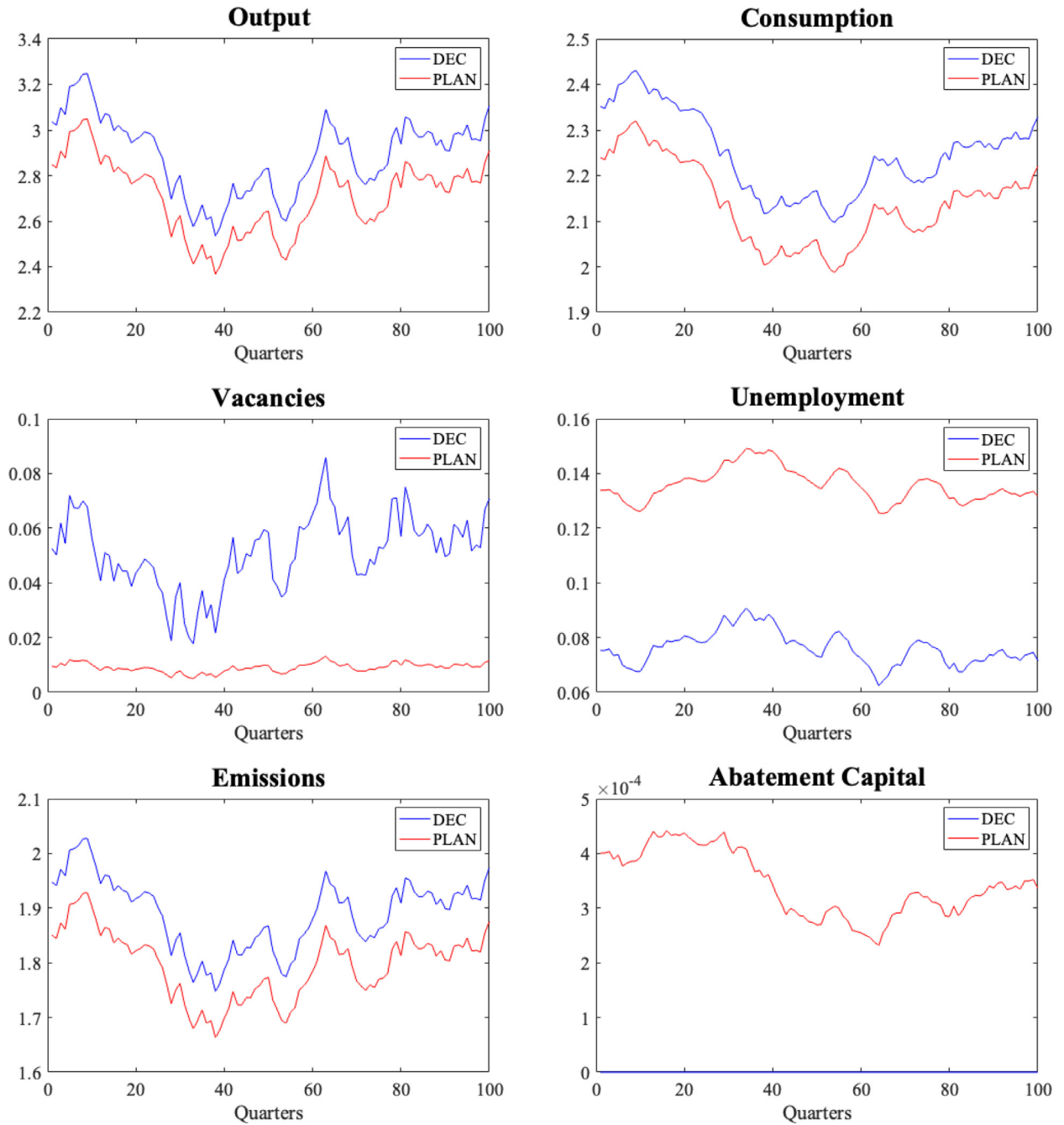


Fig. 2. Business Cycle Simulations - Decentralized Vs. Planner - No Hours Margin. Notes: All simulations are from randomly generated draws of productivity innovations for 100 periods.

To compare results, Fig. 2 and Table 3 replicate Fig. 1 and Table 2 under the simpler model. Fig. 2 shows that the simpler model is still characterized by a high level of vacancies and lower unemployment in the decentralized solution relative to the planner's. But now, the decentralized solution involves a higher pollution level than the planner's solution, as is expected. Table 3 demonstrates that several of the second moments under the simpler model fit the empirical data better, including the relative volatility in labor market variables. These volatilities are much higher in the model without the hours margin and closer to their values in the data, though still lower. More simulation results under both the decentralized and planner's solutions are provided in appendix Section E.1 (for the decentralized problem) and appendix Section E.2 (for the planner's problem).

Table 3
Decentralized vs Planner Outcomes – No Hours Margin.

	Mean ^a		%Volatility ^b		
	Decentralized	Planner	Data	Decentralized	Planner
RBC Variables^c					
Output	3.04	2.85	1.58	1.34	1.33
Consumption	2.36	2.24	0.77	0.36	0.39
Investment	0.64	0.60	4.59	3.19	3.42
Labor Market Variables^d					
Unemployment	0.075	0.134	9.71	2.61	1.51
Vacancies	0.053	0.0095	9.13	8.95	6.44
Labor Market Tightness	0.702	0.071	18.37	10.96	7.49
Environmental Variables^e					
Emissions	1.949	1.852	1.23	0.60	0.61
Pollution	3338.7	3172.3	0.0532	0.0013	0.0013
Abatement Capital	0.00	0.0004	NA	0	1.74

^a All model statistics are from 100 simulations, each 300 quarters in length. Reported values are averages of our sample statistics across these simulations. The values reported for “Mean” are the average value of the simulated series. These numbers are quantitatively very close to the variables’ steady state value.

^b Percent volatility is measured as the percent standard deviation of the cyclical component of the log of x relative to the percent standard deviation of the cyclical component of log output (statistic for output is presented in levels).

^c Data on output, consumption, and investment comes from the National Income and Product Accounts tables and covers the period of 1947–2019. Specifically, Y is Real Gross Domestic Product, c is Real Personal Consumption Expenditures, and inv is Real Gross Private Domestic Investment (all in billions of chained 2012 dollars). The cyclical component of all RBC variables (both data and model simulations) is found by HP-filtering with a smoothing parameter of 1,600.

^d The data values reported in this section come from Shimer (2005) and have been normalized by the percent volatility of output. Following Shimer (2005), the cyclical component of all labor market related variables (both data and model simulations) is found by HP-filtering with a smoothing parameter of 10^5 .

^e Emissions data are from the source described in the calibration section of the text. Pollution stock data are from NOAA’s Global Monitoring Laboratory.

5.3. First-Best tax problem

We now consider the solution under the efficient Ramsey tax, where the government sets an emissions tax and a vacancy tax or subsidy to internalize both externalities. Under the first-best taxes, the equilibrium coincides with that of the planner (and thus all outcomes under the first-best taxes are identical to those under the planner’s problem reported in Table 3). The steady-state value of the efficient emissions tax is $\tau_e = 8.8461e^{-4}$, which is in arbitrary units of the DSGE model. Because of the calibrated values of the parameters in the Hosios condition, vacancies are higher in the unregulated equilibrium than in the efficient planner’s problem. Thus, a tax (not a subsidy) on vacancies is efficient. The steady-state value of the efficient vacancy creation tax is $\tau_v = 11.09$. This corresponds to about a 1,100% ad valorem tax rate on vacancies; for every dollar that the firm spends on vacancy posting costs, it pays 11 dollars in the tax. This enormous tax rate reflects the large difference between the efficient and the decentralized unemployment rates, which in turn is caused by the large difference in the bargaining parameter $b = 0.05$ and the match function exponent $\gamma_2 = 0.72$.³³

This large value for the efficient vacancy tax rate may seem implausible and call into question the calibration of the two parameters in the Hosios condition. We address this later by conducting sensitivity analysis over these two parameters. But we also note here that this tax value is an overlooked implication of the canonical search-and-matching labor model, under our empirically-based parameter values and under a Pigouvian/Ramsey-style efficient policy framework. If the tax seems implausibly high, the appropriate response is not to choose alternate parameter values that are not supported by the data or by the literature, but rather to examine what aspects of the model or of the normative framework contribute to this result.

Fig. 3 presents impulse response functions (IRFs) for each of the two first-best taxes in response to a one-standard-deviation productivity shock. The purpose of this figure is to explore the impact of the first-best policies on the business cycle properties of the model, that is, how policies respond to short-run changes in productivity. The emissions tax increases by about 0.5% after the shock, gradually decaying. The unregulated contemporaneous increase in emissions from a positive productivity shock is muted under efficient dynamic taxes, a result that is also found in Heutel (2012). The vacancy tax rate decreases by about 1.5%, indicating that the increase in vacancies posted after the productivity shock is muted. These

³³ The impact of the vacancy creation tax on the share of output directed to vacancy creation is not as extreme as this large tax rate might imply. After-tax spending on vacancy creation rises from approximately 1.45% of GDP in the baseline decentralized equilibrium to approximately 3.36% under the efficient tax.

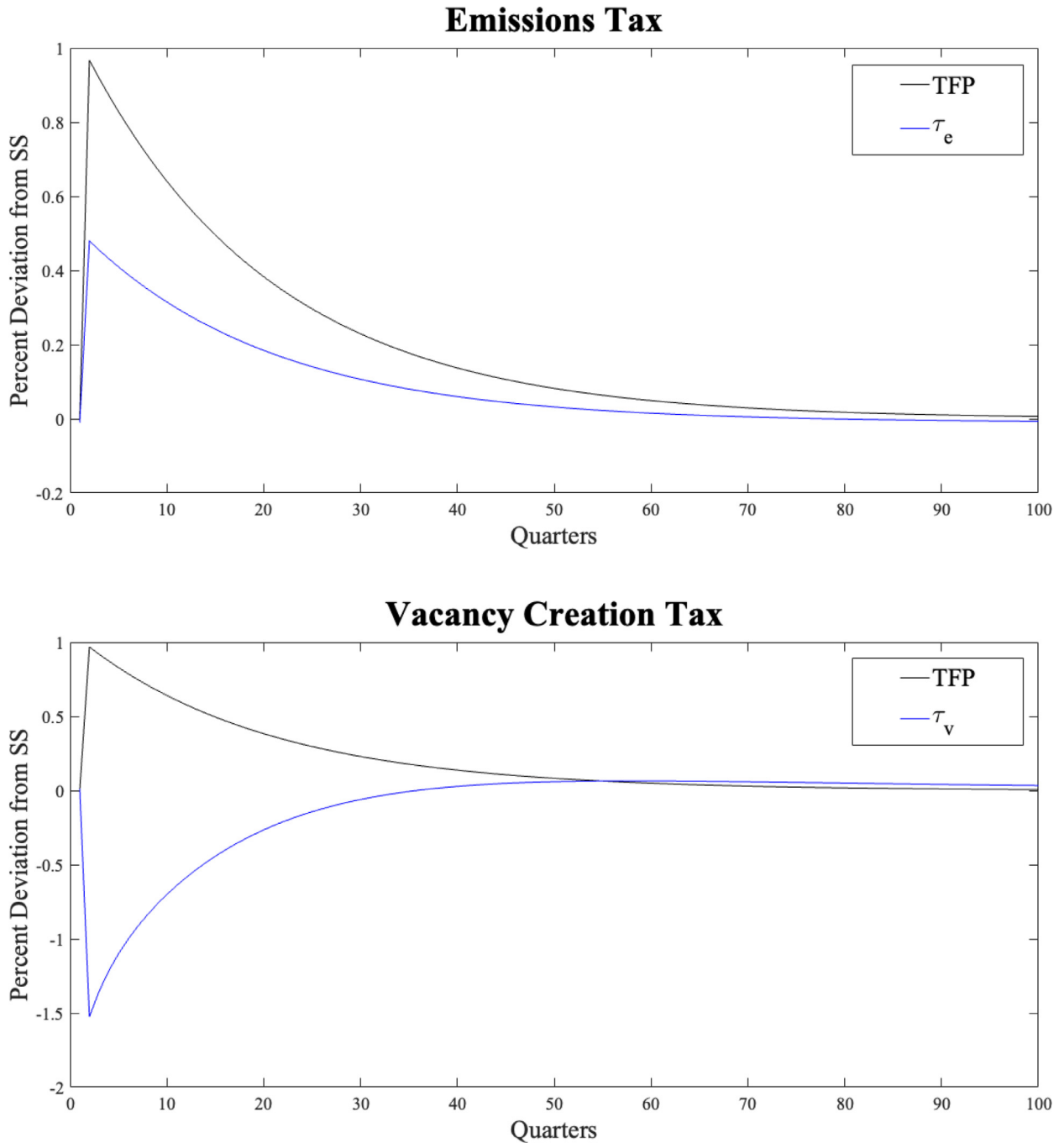


Fig. 3. Impulse Response Functions - First-Best Taxes. Notes: All simulations are from a one-unit one-time productivity innovation.

properties of the two taxes are also demonstrated in appendix [Section E.3](#) in [Fig. E.5](#), which simulates them under a business cycle. The emissions tax is procyclical, and the vacancy creation tax is countercyclical.

When productivity increases, the negative pollution externality is exacerbated, so the efficient emissions tax rises to overcome this. However, when productivity increases, the labor search congestion externality is lessened; the efficient vacancy tax is reduced to accommodate this. It is not obvious *ex ante* that the congestion externality decreases after a positive productivity shock. The congestion externality is actually the net externality of the pair of congestion externalities – one caused by the firm and one by the workers. Both of these congestion externalities increase following a positive productivity shock, but the one that leads to too little employment increases by more than the one that leads to too much employment. That scenario yields to the observed countercyclical behavior of the efficient vacancy tax.

Table 4
Second-Best Outcomes – No Hours Margin.

	Mean ^a		%Volatility ^b	
	Emissions Tax	Vacancy Tax	Emissions Tax	Vacancy Tax
RBC Variables^c				
Output	2.99	2.85	1.37	1.33
Consumption	2.34	2.24	0.36	0.39
Investment	0.62	0.60	3.19	3.42
Labor Market Variables^d				
Unemployment	0.086	0.134	3.05	1.51
Vacancies	0.036	0.0095	11.11	6.44
Labor Market Tightness	0.419	0.072	13.40	7.49
Environmental Variables				
Emissions	1.60	1.85	0.58	0.61
Pollution	2744.1	3172.4	0.0013	0.0013
Abatement Capital	0.342	0	2.04	NA

^a All model statistics are from 100 simulations, each 300 quarters in length. Reported values are averages of our sample statistics across these simulations. The values reported for “Mean” are the average value of the simulated series. These numbers are quantitative very close to the variables’ steady state value.

^b Percent volatility is measured as the percent standard deviation of the cyclical component of the log of x relative to the percent standard deviation of the cyclical component of log output (statistic for output is presented in levels).

^c For all RBC variables (both data and model simulation) the cyclical component is found by HP-filtering with a smoothing parameter of 1,600.

^d For all labor market related variables (both data and model simulation) the cyclical component is found by HP-filtering with a smoothing parameter of 10^5 (as in [Shimer, 2005](#)).

5.4. Second-Best tax problem

[Table 4](#) presents the mean and percent volatility of several variables under the solutions with the second-best emissions tax and the second-best vacancy tax. Focusing on the RBC variables, the second-best emissions tax results in mean values that are very close to that found in the unregulated decentralized economy, while the second-best vacancy tax results in mean values that are almost identical to the mean values recovered from the planner’s problem. From looking at the RBC variables alone, it appears that the second-best vacancy tax is more efficient than the second-best emissions tax.

The next two panels of [Table 4](#) show that the two second-best taxes have different effects on different parts of the economy. The second-best vacancy tax, which targets the labor market, results in mean labor market variables that are almost identical to the planner’s problem values. The second-best emissions tax brings the unemployment and vacancy rates somewhat closer to their efficient values, but not all the way there. For the environmental variables, the second-best emissions tax actually results in an overshoot – lower emissions and higher abatement capital than the efficient level. Because the second-best emissions tax is also being asked to address the large congestion externalities, it must overshoot in addressing the pollution externality. The second-best vacancy tax reduces emissions indirectly by reducing employment; there is still no spending on abatement capital without an emissions tax.

In [Figs. 4](#) and [5](#), we explore how the steady-state levels of the second-best taxes respond to the level of the constrained tax. While in [Table 4](#), the constrained tax is set to zero, these figures consider other non-zero values of the constrained tax. [Fig. 4](#) presents the results when the vacancy creation tax is constrained, and the emissions tax responds optimally. The x-axis of [Fig. 4](#) is the constrained level of the vacancy creation tax. It is centered at about 11.09, which is its first-best steady-state level. At that level, the optimized emissions tax is about $\tau_e = 8.8461e^{-4}$, its first-best steady-state level. However, when the vacancy creation tax is constrained to be higher, the second-best emissions tax is somewhat lower. This reflects the fact that both taxes are addressing market failures that go in the same direction (too much production). When one tax is higher than first-best, the other tax is lower. When the vacancy tax approaches zero (i.e., when this policy is absent), the second-best emissions tax is much higher than its first-best level. In fact, it is about 60 times greater than its first-best level when the vacancy tax is zero.

[Fig. 5](#) repeats this exercise, but it considers the second-best vacancy creation tax in response to a constrained emissions tax. Again, when one tax is higher than its first-best level (here the emissions tax), the second-best level of the other tax (here the vacancy creation tax) is lower than its first-best level. In contrast to the results in [Fig. 4](#), the second-best steady-state level of the vacancy creation tax does not vary by all that much as the constrained level of the emissions tax varies. Even with an emissions tax constrained at zero, the second-best vacancy creation tax is less than 1% greater than its first-best level. Thus, while second-best considerations affect the values of both taxes, the second-best adjustment to the emissions tax is much greater than the second-best adjustment to the vacancy tax.

Next, we consider the dynamic properties of the second-best taxes and how they compare to those of the first-best taxes. The right side of [Table 4](#) shows the volatilities for various variables under the second-best taxes. There is not much difference in the RBC variables’ volatilities. The vacancy tax yields much lower volatility for the labor market variables than

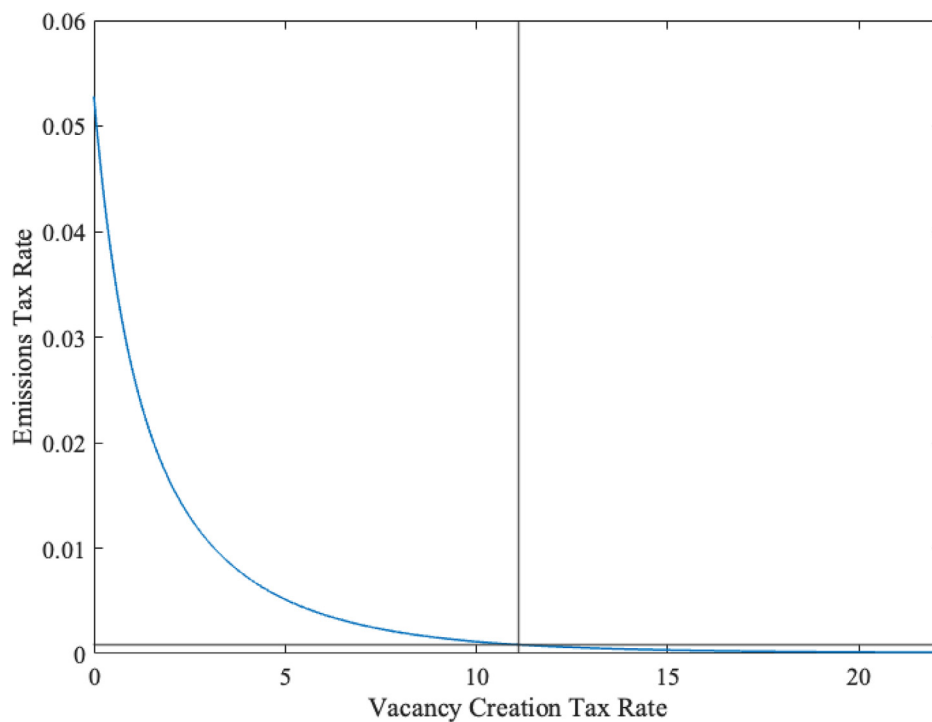


Fig. 4. Second-Best Emissions Tax. Notes: This figure is generated by solving for the (second-best) optimal emissions tax while varying the vacancy creation tax from 0 to twice its (first-best) optimal level.

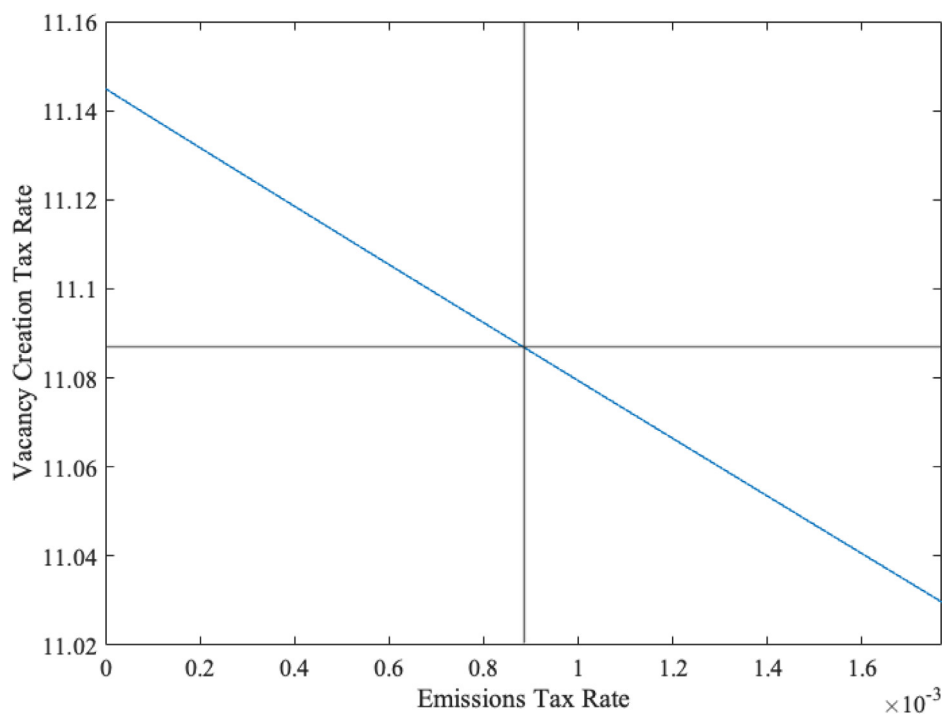


Fig. 5. Second-Best Vacancy Tax. Notes: This figure is generated by solving for the (second-best) optimal vacancy creation tax while varying the emissions tax from 0 to twice its (first-best) optimal level.

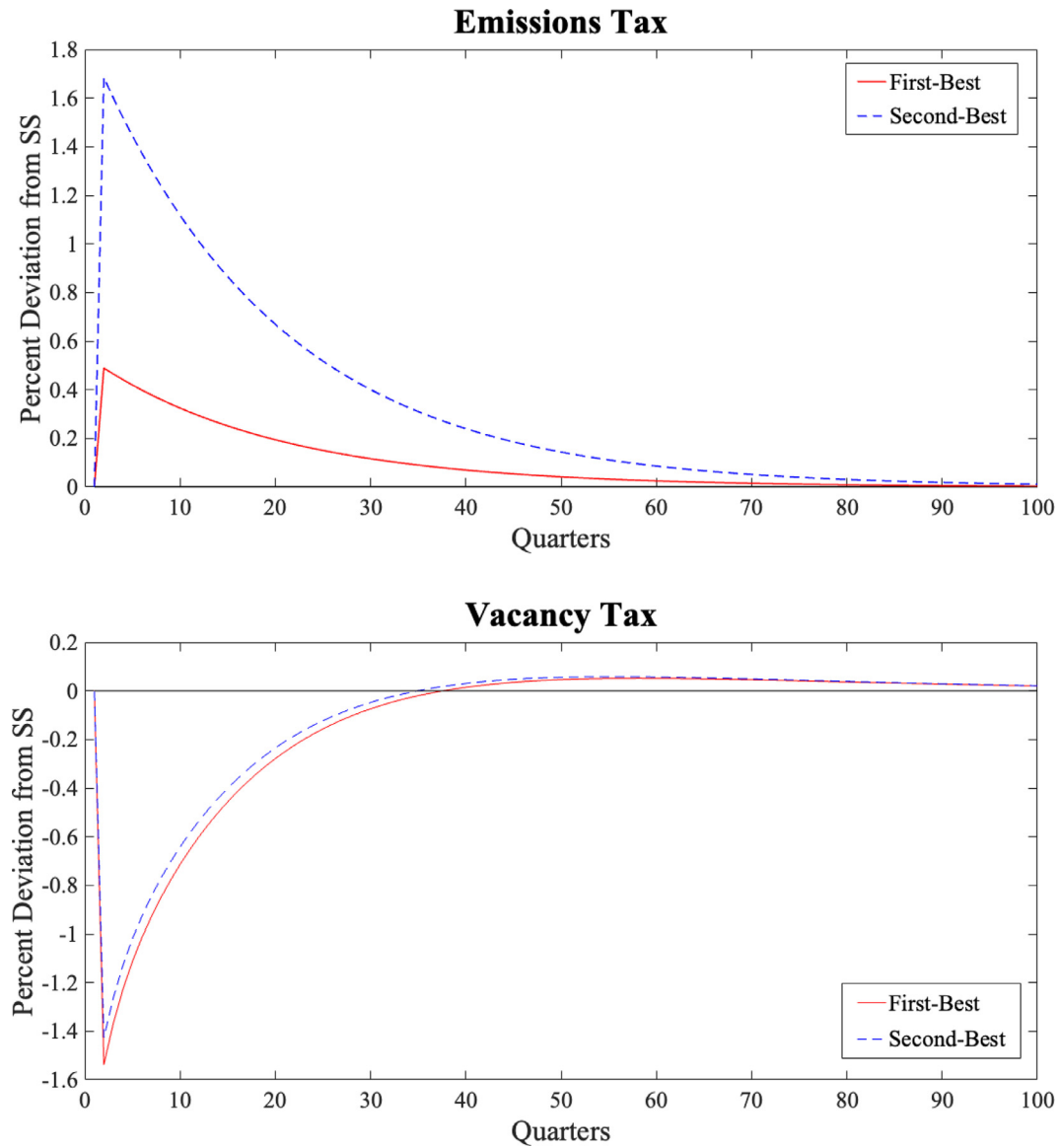


Fig. 6. Impulse Response Functions - First- vs Second-Best Taxes. Notes: All simulations are from a one-unit one-time productivity innovation.

does the emissions tax, and the volatilities match those of the planner's problem. There is not much difference in volatilities across the taxes for the environmental variables, except that there is no abatement without an emissions tax and thus no volatility in abatement capital.

Fig. 6 presents IRFs for both taxes' first-best values and second-best values when the other tax is constrained to zero. Like Fig. 3 for the first-best policies, Fig. 6 explores the implications of policy, in this case the second-best policies, on the business cycle properties of the model. The top panel shows that both the first-best and the second-best emissions taxes are procyclical, but the first-best tax is less so. The second-best emissions tax must account for both the pollution externality and the labor search congestion externalities. On the one hand, because the first-best vacancy tax is countercyclical (from Fig. 3), when the emissions tax is second-best and has to "pick up the slack" for the missing vacancy tax, it trades off between the procyclicality desired from the emissions tax and the countercyclicality desired from the vacancy tax. This would tend to make the second-best emissions tax less procyclical than the first-best tax. On the other hand, at their steady states, the second-best emissions tax is much larger than the first-best emissions tax. That means that the bulk of the "picking up the slack" from the missing vacancy tax is handled at the steady state, and less is needed to adjust over the business cycle. Thus the second-best emissions tax on net ends up being more procyclical than the first-best tax.

The bottom panel shows that both the second-best and first-best vacancy creation tax are countercyclical, with the second-best vacancy tax recovering to its pre-shock level slightly before the first-best. The similarity between the vacancy

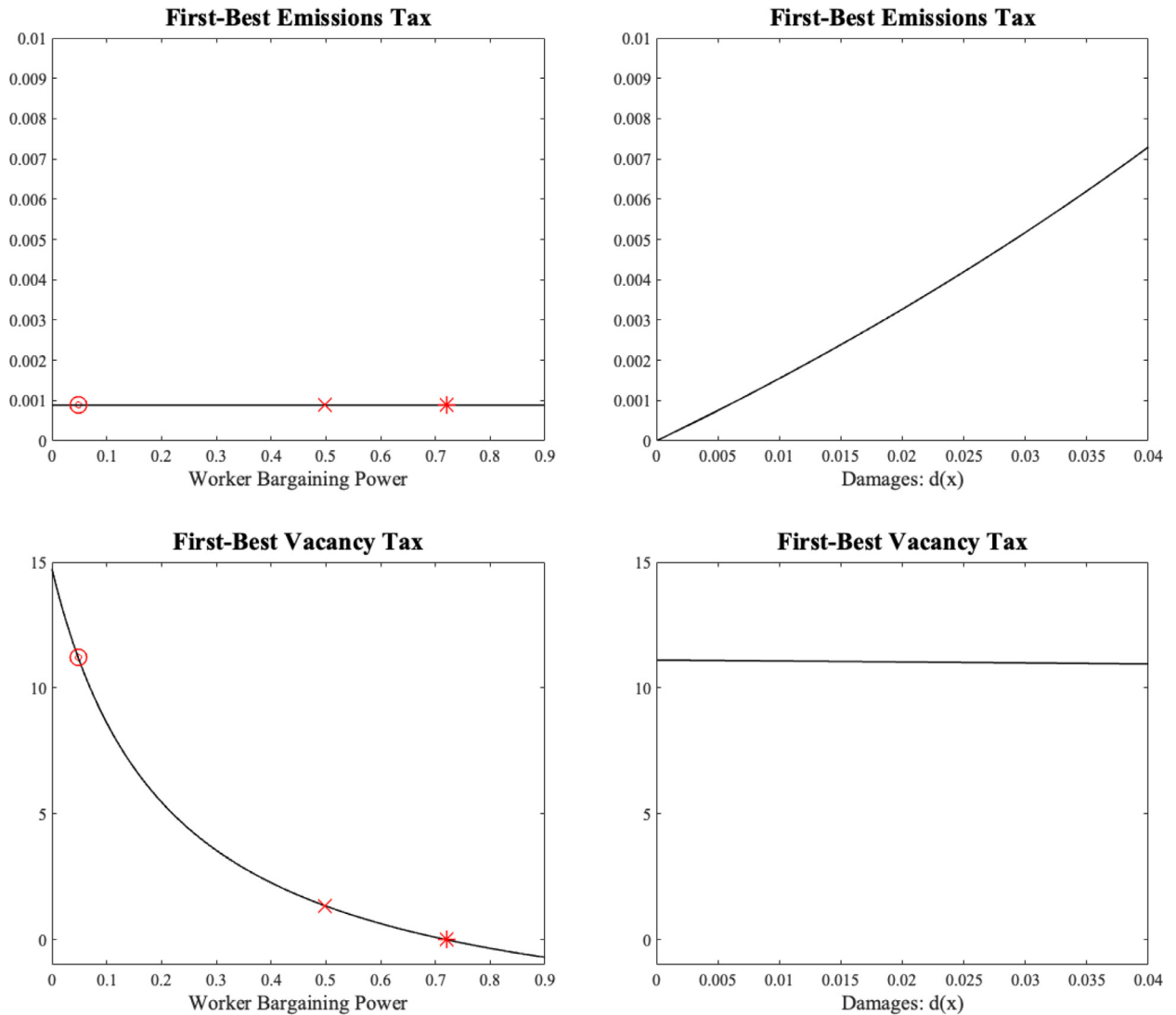


Fig. 7. Sensitivity Analysis - First-Best Taxes. Notes: Simulations of the first-best emissions and vacancy creation taxes for different values of worker bargaining power and pollution externality. The “+”, “x”, and “o” in the emissions tax plots mark the value of emissions tax under the Hosios condition, symmetric Nash bargaining, and our preferred calibration, respectively.

taxes again reflects the fact that the second-best vacancy tax brings the equilibrium closer to the planner's problem solution than does the second-best emissions tax. The large difference in the dynamic properties of the first-best and second-best emissions taxes and the minimal difference for the vacancy taxes can also be seen in Fig. E.6 of Appendix E.3, which simulates business cycle properties of the taxes.

We can also measure the welfare effect of the market failures by calculating the compensating variation for the unregulated decentralized economy, that is, the amount of additional consumption the representative household must be given in the unregulated decentralized equilibrium to make her as well off (equal lifetime utility) as the representative household under the planner's allocation. In doing this calculation, we initialize all state variables to their unregulated decentralized steady-state values and subject both systems to the same sequence of productivity shocks. As such, this welfare measure accounts for both business cycle fluctuations and the transition from the unregulated decentralized equilibrium to the equilibrium of the planner's problem.³⁴ The compensating variation of the decentralized equilibrium is 1.26%; that is, agents in

³⁴ Initializing all state variables to the same level when computing welfare effects is necessary in our model since the change in specifications has a dramatic effect on the steady-state level of some variables (namely pollution and unemployment). Steady-state utility under the second-best emissions tax is actually slightly higher than the steady-state utility under the first-best taxes. This result is explored further in appendix Section Appendix F, where we present transition dynamics across policies.

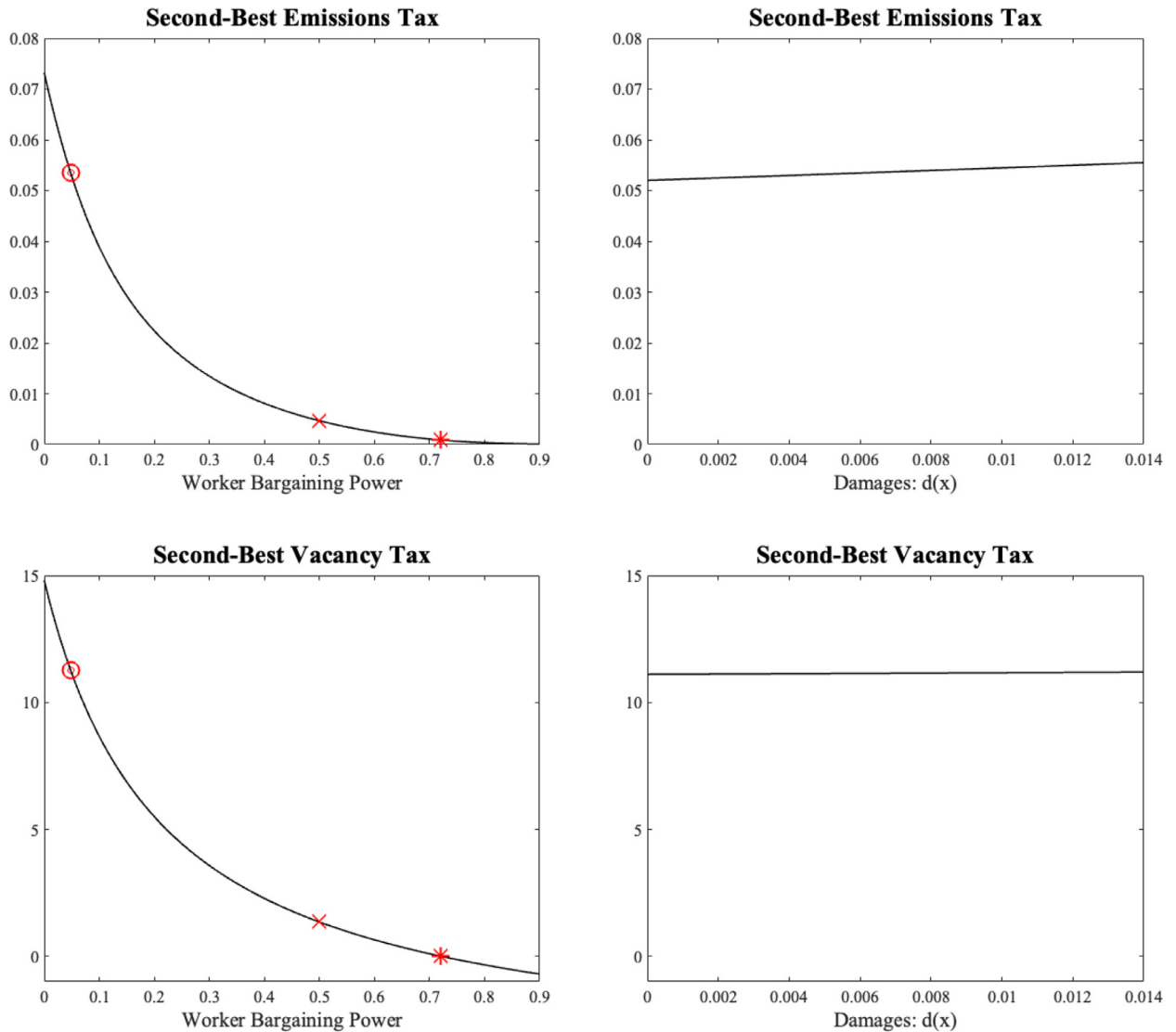


Fig. 8. Sensitivity Analysis - Second-Best Taxes. Notes: Simulations of the second-best emissions and vacancy creation taxes (when the other tax is fixed at zero) for different values of worker bargaining power and pollution externality. The “o”, “x”, and “*” in the emissions tax plots mark the value of emissions tax under the Hosios condition, symmetric Nash bargaining, and our preferred calibration, respectively.

the decentralized economy would have to be given 1.26% more consumption each period in order to be made indifferent between the decentralized economy and the planner's economy that corrects the externalities.

The welfare implications of the second-best taxes are calculated using the same methodology as we used for welfare implications of the decentralized economy: we calculate the compensating variation for each policy relative to the planner's problem. For the second-best emissions tax, this compensating variation is 0.85%, and for the second-best vacancy tax, it is practically 0. Therefore, while implementing either of the second-best taxes improves welfare relative to the unregulated decentralized equilibrium, the second-best vacancy tax is much more efficient than the second-best emissions tax. This reinforces our findings that the second-best vacancy tax comes closer to matching the planner's solution than does the second-best emissions tax.

Our results come from a model in which abatement is a stock variable. Short run adjustments, such as shocks to total factor productivity, result in the accumulation of abatement capital that mitigates the impact of emissions both today and in future periods. However, environmental DSGE models and integrated assessment models typically model abatement as a flow cost, explicitly modeling the fraction of current emissions that are abated and the amount of current output that must be forgone to achieve this (see for example [Annicchiarico and Di Dio \(2015\)](#) or [Dissou and Karnizova \(2016\)](#)). Appendix [Section G](#) provides additional details and results from an alternative specification of our model where abatement is

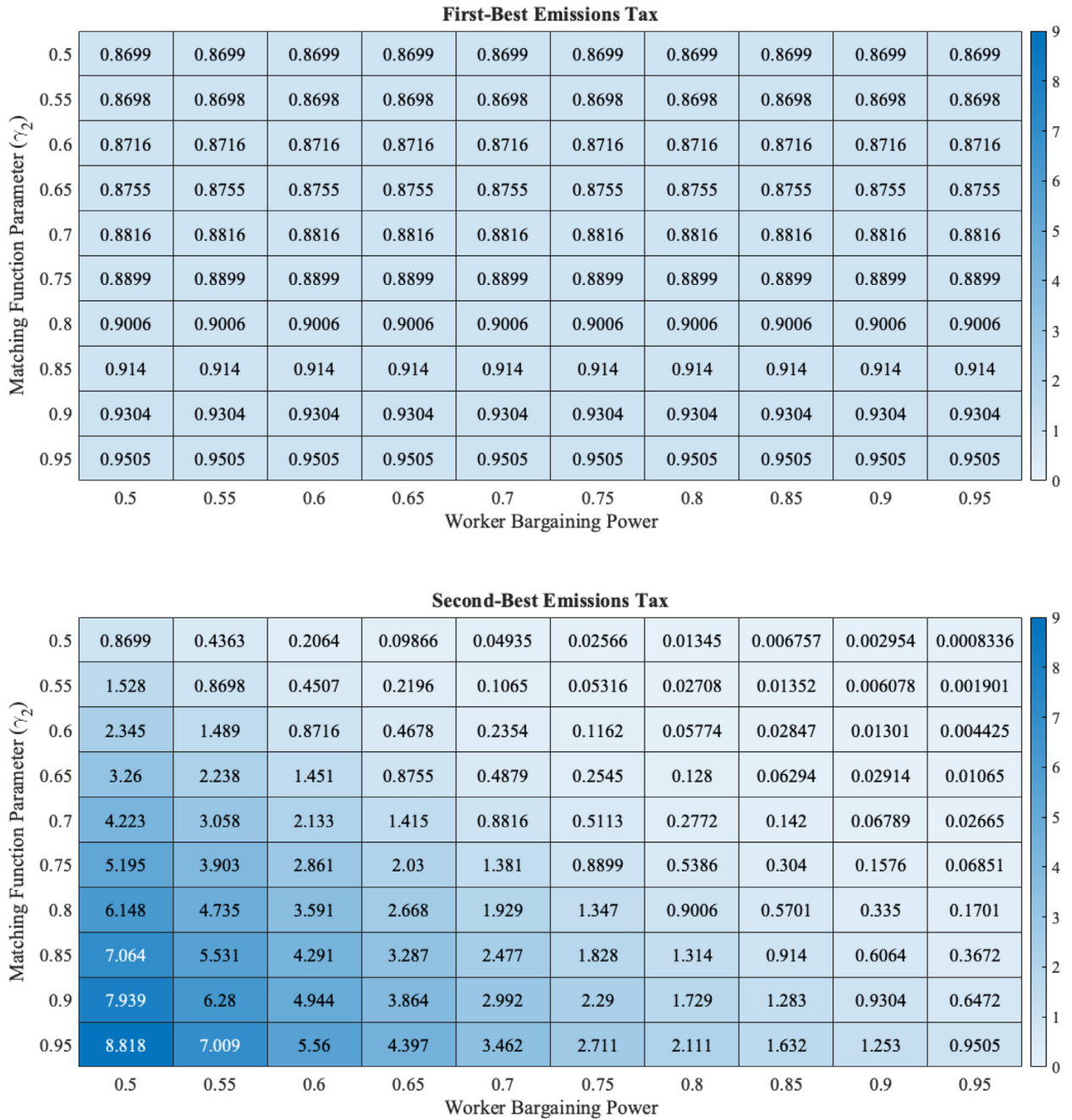


Fig. 9. Emissions Tax Heat Map. Notes: Simulations of the first and second-best emissions taxes for different combinations of worker bargaining power (b) and matching parameter (γ_2). To increase the readability of the table, the reported values are τ_e scaled by 1,000.

modeled as a flow cost. The results presented in appendix Section G are consistent with the results presented above with abatement as a stock variable.

5.5. Sensitivity analysis

We consider how sensitive the results are to our calibrated parameter values. In particular, we focus on the set of parameters that lead to the externalities. The magnitude of the labor market search externality is governed by the parameters from the Hosios condition – the worker's bargaining power b and the elasticity of the matching function γ_2 . The magnitude of the pollution externality is governed by the damage function, $d(x)$. In Figs. 7 and 8 we consider the sensitivity of

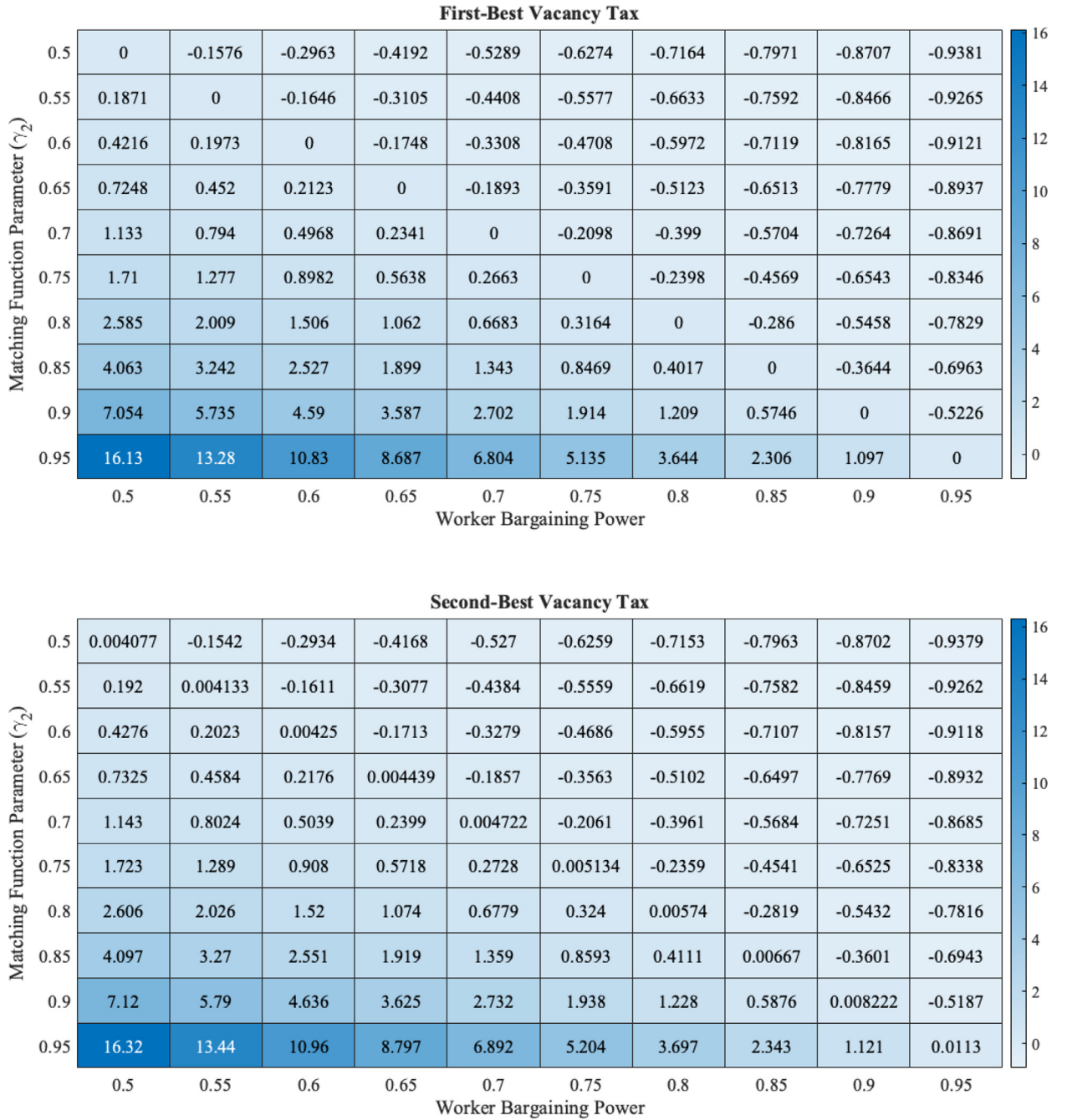


Fig. 10. Vacancy Tax Heat Map. Notes: Simulations of the first and second-best vacancy creation taxes for different combinations of worker bargaining power (b) and matching parameter (γ_2).

the first and second-best emissions and vacancy taxes to variations in either worker's bargaining power b or the damage function $d(x)$. We choose to vary worker bargaining power because it can be argued that this parameter is known with less confidence than the matching function elasticity, γ_2 , which we keep fixed at its calibrated level.³⁵

³⁵ Once the elasticity of the matching function is calibrated, worker bargaining power is usually determined in one of three ways. It is either set equal to the matching function elasticity parameter to satisfy the Hosios condition, set equal to 0.5 for symmetric Nash bargaining, or set to a low value to target the elasticity of wages with respect to average labor productivity as in Hagedorn and Manovskii (2008). As described above, we follow this third approach within our baseline calibration as it allows for an empirically justified value for worker bargaining power.

In Fig. 7, we present results for the first-best emissions and vacancy taxes under different parameter values. The left two panels show how the two taxes vary with the bargaining parameter b , and the right two panels show how they vary with pollution damages $d(x)$. As we vary bargaining power from 0, where workers receive none of the match surplus, to 1, where workers receive all of it, the first-best emissions tax barely changes. On the other hand, the first-best vacancy tax is very sensitive to b , as expected based on the model. The vacancy tax decreases as b increases, reflecting the fact from the efficient tax expression in Eq. 4 that the magnitude of the net congestion externality is proportional to $b - \gamma_2$, as in the Hosios condition. When b equals γ_2 the efficient vacancy tax is zero, which is marked by a “*” in the figure. When b is greater than γ_2 , then the efficient vacancy tax becomes negative, i.e. a vacancy subsidy, since the direction of the externality flips. Assuming symmetric Nash bargaining implies an efficient vacancy tax of 1.35 (marked by an “x” in the figure), which is substantially smaller than the value of 11.21 under our preferred calibration (marked by an “o” in the figure). This wide range of policies is not a “failure” of our sensitivity analysis, but rather it reinforces the findings from the model that the Hosios condition is a crucial component in calculating the efficient labor policy. As discussed earlier, we calibrate both b and γ_2 based on the data and the literature, rather than being chosen ad-hoc to satisfy the Hosios condition or symmetric bargaining. In contrast, the right two panels show that while the first-best emissions tax varies with pollution damages, $d(x)$, the first-best vacancy tax is little changed. Specifically, as the damages from pollution increase, the efficient emissions tax increases also. This reflects the efficient Pigouvian tax as captured in Eq. 5.

In Fig. 8, we present results for the second-best emissions and vacancy taxes under different parameter values. We now see that both the second-best emissions tax and the second-best vacancy tax are highly sensitive to variation in worker bargaining power, b . The second-best emissions tax decreases by about an order of magnitude as b increases from 0 to about 0.5. The second-best vacancy tax becomes a subsidy when b falls below γ_2 (0.72). Fig. 8 also highlights the value of these taxes when b equals 0.72 (Hosios), 0.5 (symmetric Nash), and 0.05 (preferred calibration). The large observed differences in the second-best taxes highlight the policy relevance of the labor market congestion externality. When we vary emissions damages in the right-side panels, both the second-best emissions tax and the second-best vacancy tax vary, though by a relatively small amount. The scale of the second-best emissions tax is much higher than the first-best emissions tax. In the second best, the emissions tax must address both sets of externalities, so it is higher than in models with only the pollution externality.³⁶

To further investigate the role played by the labor market search externality Figs. 9 and 10 report the first and second-best emissions and vacancy taxes while varying both worker bargaining power, b , and the matching function elasticity, γ_2 . The top panel of Fig. 9 shows that there is little variation in the first-best emissions tax with either parameter value. In contrast, the bottom panel of Fig. 9 shows substantial variation in the second-best emissions tax, with the highest values occurring when worker bargaining power is low and the matching function elasticity is high (e.g., when the gap is widest)³⁷. The two panels of Fig. 10 show that the first and second-best vacancy taxes respond similarly to changes in these parameters, with the highest values observed when worker bargaining power is low and the matching function elasticity is high.

Additionally, appendix Section H presents results from simulations that explore the equity implications of our model. Up to this point, the only social optimization criterion considered is efficiency, and the calibrated parameters imply that the efficient allocation involves a much higher unemployment rate than the unregulated equilibrium. Appendix Section H shows that the efficient allocation disproportionately benefits firms relative to workers. In that section we also perform an analysis where we add a parameter representing disutility from unemployment, which creates a social cost to unemployment beyond merely foregone production. With that disutility, the efficient level of unemployment is much lower.

6. Conclusion

We develop a dynamic stochastic general equilibrium model that includes autocorrelated productivity shocks, congestion externalities arising from labor market search frictions, an extensive and intensive margin to the labor market, and a pollution externality to examine the relationship between environmental policy and unemployment over the business cycle. The inclusion of an intensive margin of hours worked in the labor market leads to a model that performs worse than one without such a margin. In our benchmark calibrated model, the level of employment is inefficiently high both because of labor search congestion externalities and the pollution externality. When the government sets both an emissions tax and a vacancy creation tax, the efficient allocation is achieved. We also consider second-best taxes, where only one of those two policies is available. This second-best emissions tax is approximately 60 times larger than the first-best emissions tax and is much more volatile in response to productivity shocks.

Like other DSGE models, ours uses many simplifying assumptions to yield our main results. The model could be extended in many ways by exploring the importance of these assumptions. For example, one extension to this paper would be to incorporate multiple sectors rather than just a representative firm, as in Dissou and Karnizova (2016). Specifically, understanding the connection between environmental policies and the allocation of labor across output and “green technology” sectors is of interest. This concept of multiple productive sectors could also be extended to allow for a segmented labor market, at which point the model would be capable of addressing the reallocation of workers following a large policy shock.

³⁶ This result is similar to the second-best emissions tax found in Barrage (2020), where distortions from pre-existing taxes, rather than another market failure, make the efficient tax differ from marginal external damages.

³⁷ Note that the values reported in both panels of Fig. 9 are scaled by 1,000 to improve readability

The optimal policies that we model only address total social efficiency and ignore distributional or equity issues. We find that the efficient equilibrium features a higher level of unemployment than the unregulated equilibrium; thus increasing efficiency likely comes at the cost of reducing equity. A social welfare function that includes equity objectives would likely yield a different set of first-best policies.

Our research has important policy implications. Policy makers are concerned over both efficiency and equity, and many models of optimal environmental policy ignore distributional concerns like the effects of policies on unemployment. Furthermore, even absent any concerns about equity and distributional outcomes, we show how the incorporation of labor frictions and unemployment into a model of optimal environmental policy can affect efficiency, given that the labor frictions create distortions. We show how the inclusion of a more realistic representation of the labor market, with an intensive margin, can actually lead to a poorer performance overall of labor market dynamics and generate peculiar results due to the interaction of multiple market distortions. The Great Recession has made clear the fact that policy makers care about the cyclical effects of policies, and it is important to consider how various policies, like environmental and labor policies, interact over the business cycle.

Technical Appendix

In this appendix we provide additional details of the analytical model and planner's and Ramsey tax sub-problems from our DSGE model. We also provide additional details regarding our calibration, and we highlight additional results, including discussions regarding transitional dynamics, sensitivity analysis, and alternative welfare criteria.

Appendix A. More Details of Analytical Model

Let J equal the present discounted value of expected profit from an occupied job, and V equal the present discounted value of expected profit from a vacant job. In equilibrium the Bellman equation for the asset value of a vacancy is $rV = -p(1-d)c + q(\Phi)(J - p(1-d)\tau_h - V)$, where the left-hand-side is the capital cost (with exogenous discount rate r) and the right-hand-side is the rate of return on the vacancy. In equilibrium, there are no rents from vacant jobs so $V = 0$. The asset value of an occupied job is $rJ = p(1-d) - \tau_e - w - sJ$, where w is the wage. Solving these yields the job creation condition:

$$p(1-d) - w - \tau_e + p(1-d)(r+s) \left[-\tau_h - \frac{c}{q(\Phi)} \right] = 0 \quad (\text{A.1})$$

Let U equal the present discounted value of expected income of an unemployed worker, and W equal the present discounted value of expected income of an employed worker. The asset value of being unemployed is given by $rU = z + \Phi q(\Phi)(W - U)$, where z is the return to the worker during job search, for example the value of leisure. The asset value of a job for a worker is given by $rW = w + s(U - W)$.

The equilibrium wage is determined through Nash bargaining, where b is the bargaining power of the worker. The wage maximizes $(W - U)^b (J - p(1-d)\tau_h - V)^{1-b}$, since $W - U$ is the surplus from a match going to the worker and $J - p(1-d)\tau_h - V$ is the surplus going to the firm. Substituting in and solving yields the wage equation:

$$w = (1-b)z + b[(1+c\Phi - (r+s)\tau_h)p(1-d) - \tau_e] \quad (\text{A.2})$$

The social welfare function that the social planner seeks to maximize is

$$\Omega = \int_0^\infty e^{-rt} [p(1-d(1-n))n + z(1-n) - p(1-d(1-n))c\theta(1-n)] dt \quad (\text{A.3})$$

and the social planner is constrained by the unemployment evolution equation $\dot{(1-n)} = sn - \Phi q(\Phi)(1-n)$. This welfare function, used here in the analytical model, is not identical to the social welfare function used later in the DSGE model. This social welfare function is taken from the canonical search-and-matching model from [Pissarides \(2000\)](#), while the social welfare function for the DSGE model is based on those typically used in DSGE models. While the two functions are not identical to each other, they both represent the same fundamental trade-offs between consumption and leisure and between the present and the future. The social planner's choice variables are unemployment $1-n$ and labor market tightness Φ , and let the co-state variable be μ . The Euler conditions are

$$-e^{-rt} [p(1-d(1-n)) - z + p(1-d(1-n))c\Phi + d'(1-n)(pn + pc\Phi(1-n))] + [s + \Phi q(\Phi)]\mu - \dot{\mu} = 0 \quad (\text{A.4})$$

$$-e^{-rt} pc(1-n)(1-d(1-n)) + \mu(1-n)q(\Phi)[1 - \eta(\Phi)] = 0 \quad (\text{A.5})$$

Use [Eq. A.5](#) to solve for μ , take its time derivative $\dot{\mu}$, and substitute both into [Eq. A.4](#). This yields [Eq. 3](#) in the text.

Appendix B. Planner's Problem

The social planner chooses consumption, capital investment, vacancy creation, and abatement capital investment, in order to maximize the representative household's present discounted value of expected lifetime utility. The planner enters each period with a predetermined quantity of employed workers, capital, abatement capital, pollution, and knowledge of the current aggregate productivity in the economy. The optimization problem is constrained by an aggregate resource constraint and the aggregate evolution of employment in the economy and can be written as the following dynamic program:

$$\begin{aligned} V(k_t, n_t, x_t, A_t; \theta_t) = & \max_{c_t, l_t, v_t, k_{t+1}, x_{t+1}, A_{t+1}} \ln c_t + n_t \xi_1 \frac{(1-l_t)^{1-\xi_3}}{(1-\xi_3)} + (1-n_t)\xi_2 \\ & + \beta E\{V(k_{t+1}, n_{t+1}, x_{t+1}, A_{t+1}; \theta_{t+1})\} \\ \text{s.t.} \\ c_t + k_{t+1} - (1-\delta)k_t + A_{t+1} - \phi A_t + v_t G \leq & Y_t \end{aligned} \quad (\text{B.1})$$

$$n_{t+1} = (1-s)n_t + \gamma(1-n_t)^{\gamma_2} v_t^{1-\gamma_2} \quad (\text{B.2})$$

$$x_{t+1} = \lambda x_t + g(A_t)h(Y_t) + e_t^{\text{row}} \quad (\text{B.3})$$

Equation (B.1) denotes the economy's aggregate resource constraint. The left-hand side includes consumption expenditures, capital investment, abatement investment, and the economy's expenditures on vacancy creation, with G denoting per-vacancy posting cost. The right-hand side denotes the total output less abatement.

Solving this problem yields the following four dynamic Euler equations:

$$\beta E\{V_{k_{t+1}}\} = \frac{1}{c_t} \quad (\text{B.4})$$

$$\beta E\{V_{x_{t+1}}\} = \frac{\xi_1(1-l_t)^{-\xi_3}}{g(A_t)h'(y_t)(1-\alpha)(1-d(x'))\theta\left(\frac{k}{nl}\right)^\alpha} - \left(\frac{1}{c_t}\right)\left[\frac{1}{g(A_t)h'(y_t)}\right] + \left(\frac{1}{1-\alpha}\right)\left[\frac{d'(x_{t+1})}{1-d(x_{t+1})}\right]\xi_1(1-l_t)^{-\xi_3}n_t l_t \quad (\text{B.5})$$

$$\beta E\{V_{A_{t+1}}\} = \frac{1}{c_t} \quad (\text{B.6})$$

$$\beta E\{V_{N_{t+1}}\} = \frac{1}{c_t} \left[\frac{G}{\gamma_1(1-\gamma_2)\Phi_t^{-\gamma_2}} \right] \quad (\text{B.7})$$

where,

$$\begin{aligned} V_{k_{t+1}} &= \frac{1}{c_t} \left[\left(\frac{\alpha}{1-\alpha}\right)c_t \xi_1(1-l)^{-\xi_3} \left(\frac{k_t}{n_t l_t}\right)^{-1} + 1 - \delta \right] \\ V_{A_{t+1}} &= \frac{1}{c_t} \left[\phi + \frac{g'(A_t)h(y_t)}{g(A_t)h'(y_t)} \left[\frac{c_t \xi_1(1-l_t)^{-\xi_3}}{(1-\alpha)(1-d(x_{t+1}))\theta_t\left(\frac{k_t}{n_t l_t}\right)^\alpha} - 1 \right] \right] \\ V_{x_{t+1}} &= \eta \left[\frac{\xi_1(1-l_t)^{\xi_3}}{g(A_t)h'(y_t)(1-\alpha)[1-d(x_{t+1})]\theta_t\left(\frac{k_t}{n_t l_t}\right)^\alpha} - \frac{1}{c_t} \left[\frac{1}{g(A_t)h'(y_t)} \right] \right] \\ V_{N_{t+1}} &= \left(\frac{\xi_1}{1-\xi_3}\right)(1-l_t)^{1-\xi_3} - \xi_2 + \frac{1}{c_t} \left[\frac{(1-s-\gamma_1\gamma_2\Phi_t^{1-\gamma_2})G}{\gamma_1(1-\gamma_2)\Phi_t^{-\gamma_2}} \right] + \xi_1(1-l_t)^{-\xi_3} l_t \end{aligned}$$

Eq. B.4 is the dynamic Euler on capital accumulation, while Eq. B.5 is the dynamic Euler on emissions and pollution. Equations similar to (B.4) and (B.5) appear in Heutel (2012); here they depend on the current and future levels of employment in the economy. Eqs. B.6 and B.7 are unique to our model and defines the planner's optimal choices of both abatement capital and vacancy creation. Inspection of Eqs. B.4–B.7 and the corresponding envelope conditions show that there is a feedback relationship between emissions and employment, with the current and future emissions impacting vacancy creation and current and future employment impacting the emissions decision.

Appendix C. Ramsey Efficient Dynamic Tax Problem

The Ramsey efficient dynamic tax problem for a model economy can be written as the following constrained optimization (here the hours margin l_t is omitted):

$$\begin{aligned} & \max_{\{c_t, k_{t+1}, n_{t+1}, x_{t+1}, A_{t+1}, v_t, r_t, w_t, q_t, Y_t, \text{ and } \tau_{e_t} \text{ or } \tau_{v_t} \text{ (or both)}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [\ln c_t - \xi n_t] \\ & \text{s.t., for all } t \\ & \beta E_t \left\{ \left(\frac{1}{c_{t+1}} \right) [r_{t+1} + 1 - \delta] \right\} = \frac{1}{c_t} \end{aligned} \quad (\text{C.1})$$

$$r_t = \alpha (1 - d(x_{t+1})) \theta_t \left(\frac{k_t}{n_t} \right)^{\alpha-1} [1 - \tau_{e_t} g(A_t) h'(Y_t)] \quad (\text{C.2})$$

$$\beta E_t \left\{ \frac{c_t}{c_{t+1}} \left[\left(\frac{1-\alpha}{\alpha} \right) r_{t+1} \left(\frac{k_{t+1}}{n_{t+1}} \right) - w_{t+1} + (1-s) \frac{(1+\tau_{v_{t+1}})G}{q_{t+1}} \right] \right\} = \frac{(1+\tau_{v_t})G}{q_t} \quad (\text{C.3})$$

$$\beta E_t \left\{ \frac{c_t}{c_{t+1}} (\phi - \tau_{e_{t+1}} g'(A_{t+1}) h(Y_{t+1})) \right\} = 1 \quad (\text{C.4})$$

$$n_{t+1} = (1-s)n_t + v_t q_t \quad (\text{C.5})$$

$$w_t = (1-b)\xi c_t + b \left[\left(\frac{1-\alpha}{\alpha} \right) r_t \left(\frac{k_t}{n_t} \right) + \left(\frac{v_t}{1-n_t} \right) (1+\tau_{v_t})G \right] \quad (\text{C.6})$$

$$c_t + k_{t+1} - (1-\delta)k_t + A_{t+1} - \phi A_t + v_t G \leq Y_t \quad (\text{C.7})$$

$$x_{t+1} = \lambda x_t + g(A_t) h(Y_t) + e^{\text{ROW}} \quad (\text{C.8})$$

$$Y_t = (1-d(x_{t+1})) \theta_t k_t^\alpha n_t^{1-\alpha} \quad (\text{C.9})$$

$$q_t = \gamma_1 (1-n_t)^{\gamma_2} v_t^{-\gamma_2} \quad (\text{C.10})$$

All remaining endogenous variables are chosen by the government in order to maximize the representative household's present discounted value of expected lifetime utility, taking as constraints the equations from the decentralized problem. Notice that when we specify the government's choice variables we list $\{\tau_{e_t}\}_{t=0}^{\infty}$ or $\{\tau_{v_t}\}_{t=0}^{\infty}$ (or both), indicating that we will consider the following three scenarios: (i) the government recovers the first-best allocation by optimally choosing both $\{\tau_{e_t}\}_{t=0}^{\infty}$ and $\{\tau_{v_t}\}_{t=0}^{\infty}$; (ii) the government recovers a second-best allocation by optimally choosing $\{\tau_{e_t}\}_{t=0}^{\infty}$, with τ_{v_t} held fixed; (iii) the government recovers a second best allocation by optimally choosing $\{\tau_{v_t}\}_{t=0}^{\infty}$, with τ_{e_t} held fixed.

Appendix D. Additional Calibration Details

D1. Emissions elasticity calibration

We calibrate the relationship between emissions and output, $e_t = h(y_t) = y_t^{1-\gamma}$, using monthly and quarterly data on U.S. carbon dioxide emissions and GDP. The emissions data are monthly and come from the Energy Information Administration; they include carbon dioxide emissions from all sources of energy consumption starting in 1973.³⁸ For most specifications, we use emissions data that we seasonally adjust, using the Census Bureau's X-13ARIMA-SEATS seasonal adjustment program.³⁹ Quarterly GDP data through are available from the Federal Reserve Economic Data (FRED) website. Monthly estimates of GDP from 1980 through 2016 are from the dataset provided by Jarociński and Karadi (2020).⁴⁰ In all the following analysis, we will only use data starting in 1980, since that is the first year for which we have monthly GDP data.

Fig. D.1 plots the time series of GDP and emissions, using the quarterly GDP data through 2020 Q3 and aggregating the monthly emissions data to the quarterly level. Business cycles are apparent in the GDP series, including the Great Recession starting in 2007 and the sudden dip induced by the COVID-19 pandemic. The seasonally-adjusted emissions data are noisier,

³⁸ Data are available here: <https://www.eia.gov/totalenergy/data/monthly/#environment>.

³⁹ Details of this program are available here: <https://www.census.gov/data/software/x13as.html>. The seasonal adjustment is performed using the EViews software.

⁴⁰ These data are available here: <https://www.aeaweb.org/doi/10.1257/mac.20180090.data>. Interpolating GDP to a monthly frequency is based on the methodology described in Stock and Watson (2000).

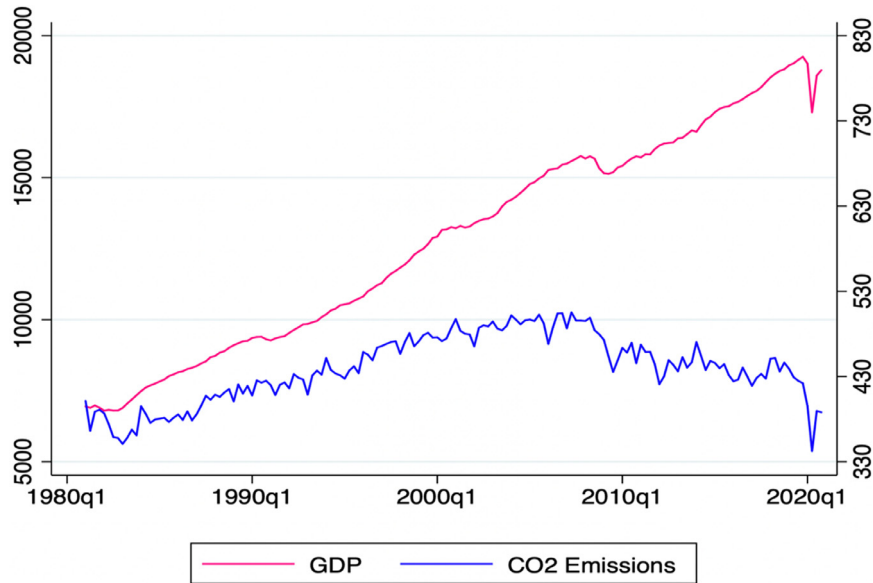


Fig. D.1. Carbon Dioxide Emissions and GDP.

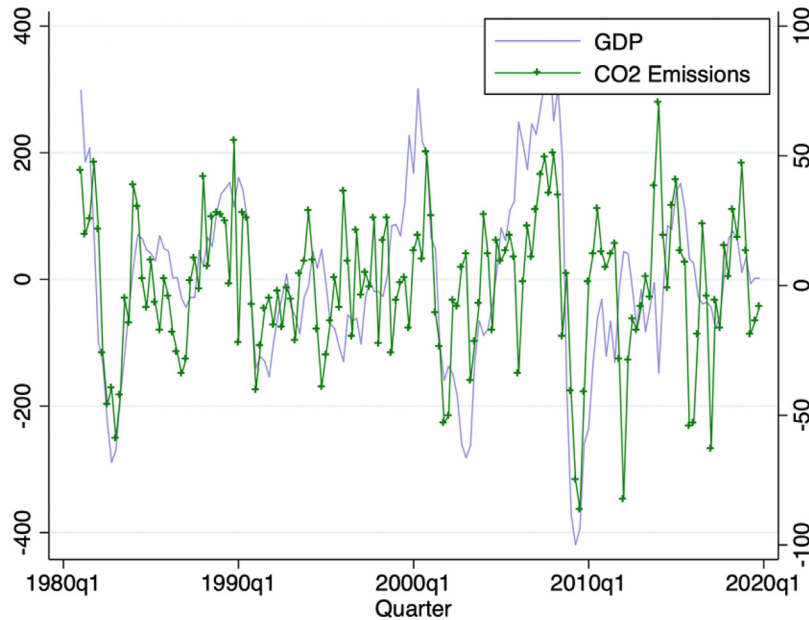


Fig. D.2. Carbon Dioxide Emissions and GDP, Cyclical Components.

but it appears to be the case that when GDP falls, so do emissions. The long-run trend of the carbon intensity of the economy decreasing is apparent, especially starting after the Great Recession.

Fig. D.2 plots the cyclical components of the GDP and emissions series. We use the quarterly data and detrend using the Hodrick-Prescott filter. The smoothness parameter used in the filtering is $\lambda = 1600$. We only plot through 2020 Q1 here, since including the quarters under the COVID-19 pandemic creates an enormous outlier. As with the plot of the levels in Fig. D.1, the cyclical components plotted in Fig. D.2 demonstrate the correlation between output and emissions. During recessions, emissions fall, and during expansions, emissions rise.

We turn to regression analysis to quantify the cyclical correlation between emissions and GDP. Table D.1 presents the regression results. Each column is a different specification, though all columns represent regressions where the dependent variable is the natural log of the cyclical component of emissions and the independent variable of interest is the natural log of the cyclical component of GDP. Thus the reported coefficients are estimates of the elasticity parameter $1 - \gamma$. The first

Table D.1
Emissions Elasticity Regressions .

	Seasonal ARIMA	Seasonally Adjusted	HP detrend	BK detrend	CF detrend	BW detrend
Panel A: Monthly						
Ln (GDP)	0.641*** (0.129)	0.482*** (0.158)	0.790*** (0.162)	0.933*** (0.114)	0.686*** (0.00581)	0.589*** (0.195)
Observations	416	428	429	333	429	429
Panel B: Quarterly with Pandemic						
Ln (GDP)	0.946*** (0.105)	1.195*** (0.0951)	1.090*** (0.202)	0.867*** (0.0804)	0.679*** (0.00567)	1.081*** (0.191)
Observations	147	159	160	64	160	160
Panel C: Quarterly without Pandemic						
Ln (GDP)	0.641*** (0.144)	0.840*** (0.156)	0.759*** (0.173)	0.844*** (0.078)	0.682*** (0.00492)	0.771*** (0.158)
Observations	143	155	156	60	156	156

Notes: Standard errors in parentheses. The dependent variable in each regression is the log of carbon dioxide emissions, adjusted or detrended as labeled. Panel 1 uses monthly data (only available through 2016), panel 2 uses quarterly data through 2020 Q3 (including the COVID-19 pandemic), and panel 3 uses quarterly data through 2020 Q1 (excluding the COVID-19 pandemic). Column 1 presents results from a seasonal ARIMA(1, 1, 1) \times (0, 1, 1)₁₂ regression of log of CO₂ (not seasonally adjusted) on log of GDP. Column 2 presents results from an ARIMA(1, 1, 2) regression of log of CO₂ (seasonally adjusted) on log of GDP. AR, MA, and cointegrating equation terms are not reported for either regression. Columns 3–6 present results from least squares regression of detrended emissions on detrended output allowing for Newey-West standard errors, where the series are detrended by either Hodrick-Prescott (column 3), Baxter-King (column 4), Christiano-Fitzgerald (column 5), or Butterworth (column 6) filters. Constant term is omitted from regressions using detrended series. *** p<0.01, ** p<0.05, * p<0.1

two columns are ARIMA regressions, while the remaining four columns are least squares regressions of data that have been detrended.

We present results both at the monthly level, in Panel A, and at the quarterly level. The monthly data only extend from 1980 through 2016, though the quarterly data are available through 2020 Q3. For the quarterly regressions we present one specification that includes the quarters under the start of the COVID-19 pandemic (through 2020 Q3, in Panel B), and one specification that excludes the COVID-19 pandemic (through 2020 Q1, in Panel C). Because those last two quarters, under the pandemic, contain such extreme outlier values, we present both specifications.

Column 1 presents a seasonal ARIMA(1, 1, 1) \times (0, 1, 1)₁₂ regression, where the left-hand-side emissions data have not been seasonally adjusted outside of the regression. Column 2 presents an ARIMA(1,1,2) regression, where the left-hand-side emissions data are seasonally adjusted using the Census X-13 program before the regressions. In all subsequent columns, the emissions data are seasonally adjusted this way before running the regressions. Columns 3 through 6 present regressions using different detrending algorithms. Column3/s regressions use the Hodrick-Prescott filter, column4/s use Baxter-King, column5/s use Christiano-Fitzgerald, and column6/s use Butterworth.⁴¹

In all regressions, we see a positive coefficient on emissions, indicating a procyclical relationship between emissions and output. This verifies what we can see in Figs. D.1 and D.2. The value of the coefficient ranges between 0.48 and 1.2, although in almost every specification it is less than 1, indicating that emissions are relatively inelastic to output. In Panel B, which includes the COVID-19 pandemic, the elasticities tend to be higher; only here do we see values greater than 1. Because of the range of estimated parameters, we choose an intermediate value of 0.6 to use in the simulations, consistent with the result found in Cohen et al. (2018).

D2. Damage function calibration

We calibrate the damage function $d(x)$, where x is the stock of carbon pollution in the atmosphere, and d is damages expressed as a fraction of gross output (so that net output equals gross output times $1 - d$). The calibration methodology is identical to that of Heutel (2012), though here we use the most updated version of the DICE model.

The calibration is as follows. We begin with a fixed, exogenous level of the atmospheric carbon stock, in GtC. For that fixed level, we run the climate model portion of DICE, in which the atmospheric carbon stock interacts over time with the ocean carbon stock, affects radiative forcing, and ultimately affects temperature. For a given temperature, the DICE model provides parameters for a damage function (which is quadratic in temperature). For the fixed, exogenous level of the atmospheric carbon stock, we run the climate model for 20 periods (100 years), and calculate temperature and thus climate damages in each period. We take a simple average of the climate damages over those 20 periods, and assign that damage fraction to be the damage associated with the given fixed, exogenous level of atmospheric carbon stock.

⁴¹ These specifications parallel those reported in Heutel (2012), though here we use more recent data.

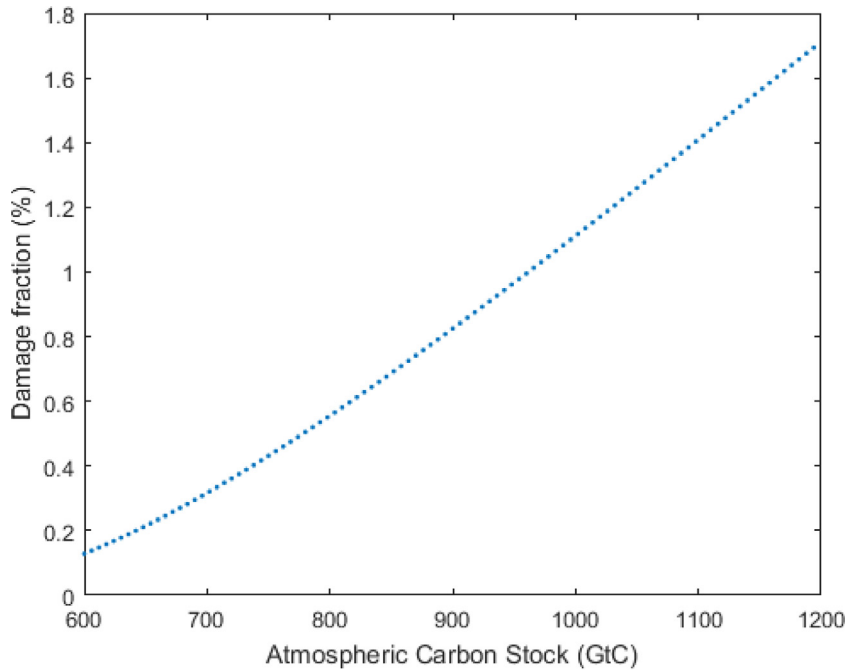


Fig. D.3. Damage Function Calibration.

We perform this simulation for 100 different values of this fixed, exogenous level of atmospheric carbon stock, ranging from 800 to 1600 GtC. The relationship between the carbon stock and damages is presented in Fig. D.3. We fit these points to a quadratic function of the carbon stock, yielding the damage function $d(x) = d_2x^2 + d_1x + d_0$, where $d_2 = 1.05e - 8$, $d_1 = 8.10e - 6$, and $d_0 = -0.0076$. The units of the carbon stock x in this calibrated function are GtC, while the units of the variable in our DSGE model are arbitrary. Therefore, in our code we re-scale our carbon stock variable to be consistent with the calibration. We set the steady-state-level of the carbon stock x to equal the current (as of DICE 2016R2) atmospheric carbon stock, 851 GtC.

D3. Labor market parameters

Several calibration strategies exist for models with DMP search frictions, with the primary difference being how vacancy postings costs, G , the disutility of work, ξ , and worker's bargaining power, b , are determined. Shimer (2005) presents a calibration strategy where ξ is set to target the replacement rate of unemployment insurance for the U.S. (approximately 40%) and worker's bargaining power is set equal to the elasticity of matches with respect to unemployment following Hosios (1990), the so-called Hosios condition. The flow cost of vacancy creation, G , is not explicitly targeted under this strategy, but is instead left free to adjust so the model can reach other desired targets. Such a calibration strategy is not suitable for our purposes as it implicitly shuts down one of our primary channels of interest, the inefficient level of vacancy creation by firms in the decentralized economy. Specifically, adopting the Hosios condition in calibration is a well-known strategy for internalizing congestion externalities and achieving the first-best allocation within a decentralized labor market search model. As such, under this calibration firms in the decentralized economy will create the socially efficient level of vacancies.

Fortunately, another popular calibration scheme proposed by Hagedorn and Manovskii (2008) does not impose the Hosios condition. Instead of determining vacancy costs as a residual of the calibration exercise, the authors target this value directly. The authors state that vacancy costs are significantly lower in the data and, as such, they set G to match their empirical target. Furthermore, workers' bargaining power is set to target the elasticity of wages with respect to average labor productivity observed in the data. This alternative strategy results in a small match surplus that varies significantly at business cycle frequencies and has been shown to generate empirically realistic volatility in labor market variables.

Setting $b = 0.05$ results in an elasticity of wages with respect to average labor productivity of about 0.6. While this is larger than the target of 0.449 reported in Hagedorn and Manovskii (2008), this outcome is not unexpected. One reason for this difference is that we are working with an RBC search model which adopts a more conventional utility function that assumes agents are risk averse. In contrast, Hagedorn and Manovskii (2008) present their results using a traditional DMP search model with risk neutral agents. See Atolia et al. (2018) for a more thorough discussion on this issue.

Appendix E. Additional Results

E1. Decentralized problem

Fig. E.1 presents three panels of impulse response functions recovered from the unregulated decentralized economy. The first panel shows the response of output, consumption, and investment to a one-time, one-standard deviation innovation in total factor productivity. For scale, the impulse response function for total factor productivity is also included. The second and third panels of Fig. E.1 show the response for labor market and environmental variables, respectively. As described in the main text, the response of these variables is consistent with existing literature and serve to highlight the relatively high volatility of labor market variables in the decentralized economy and the fact that firms do not abate emissions in absence of corrective policies (e.g., an emissions tax).

Fig. E.2 presents the 100-period stochastic time paths recovered from the baseline decentralized economy. These stochastic time paths are created using a sequence of innovations drawn from a random number generator after setting the seed to 5. Just as before, the top panel presents the time paths for traditional real business cycle variables along with total factor productivity, and the second and third panels present the time paths for labor market and environmental variables.

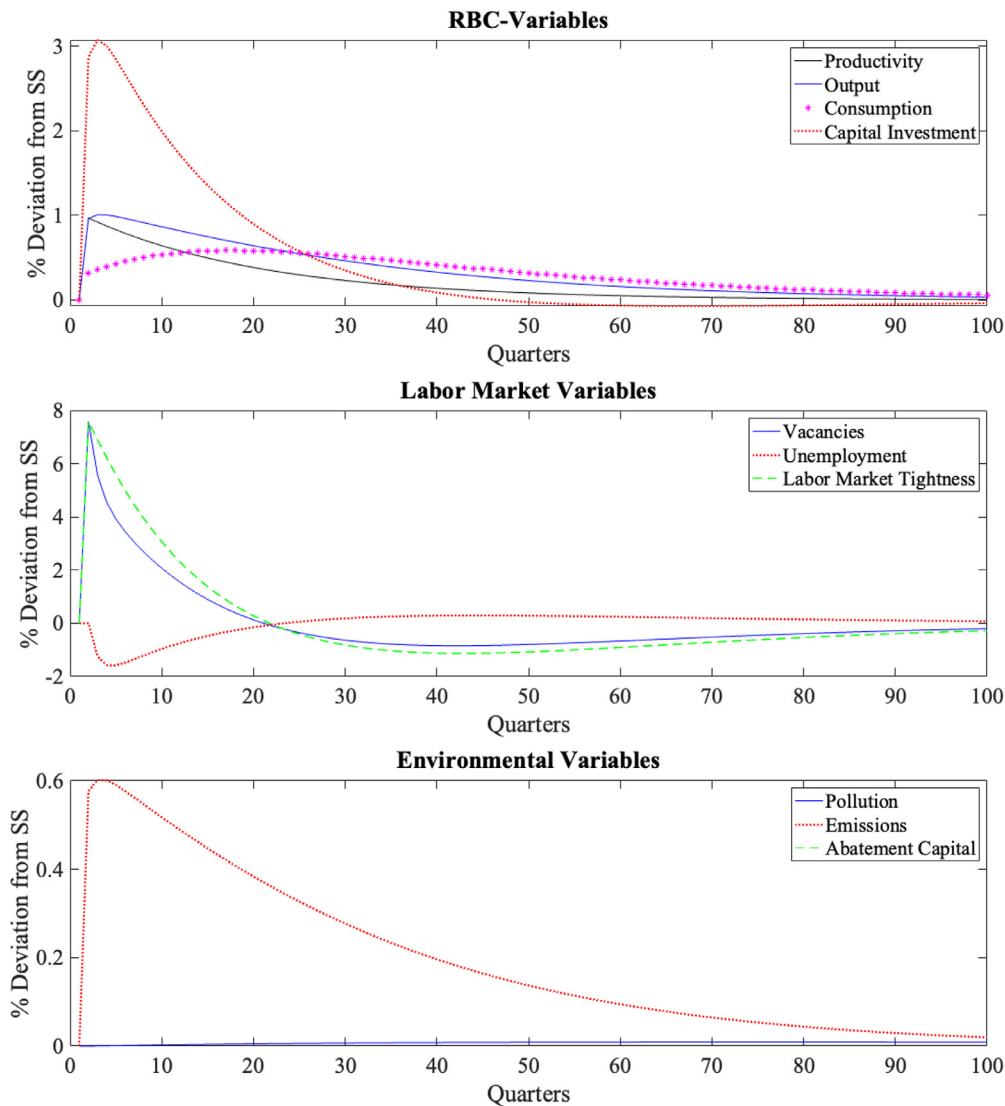


Fig. E.1. Impulse Response Functions - Decentralized Problem. Notes: All simulations are from a one-unit one-time productivity innovation.

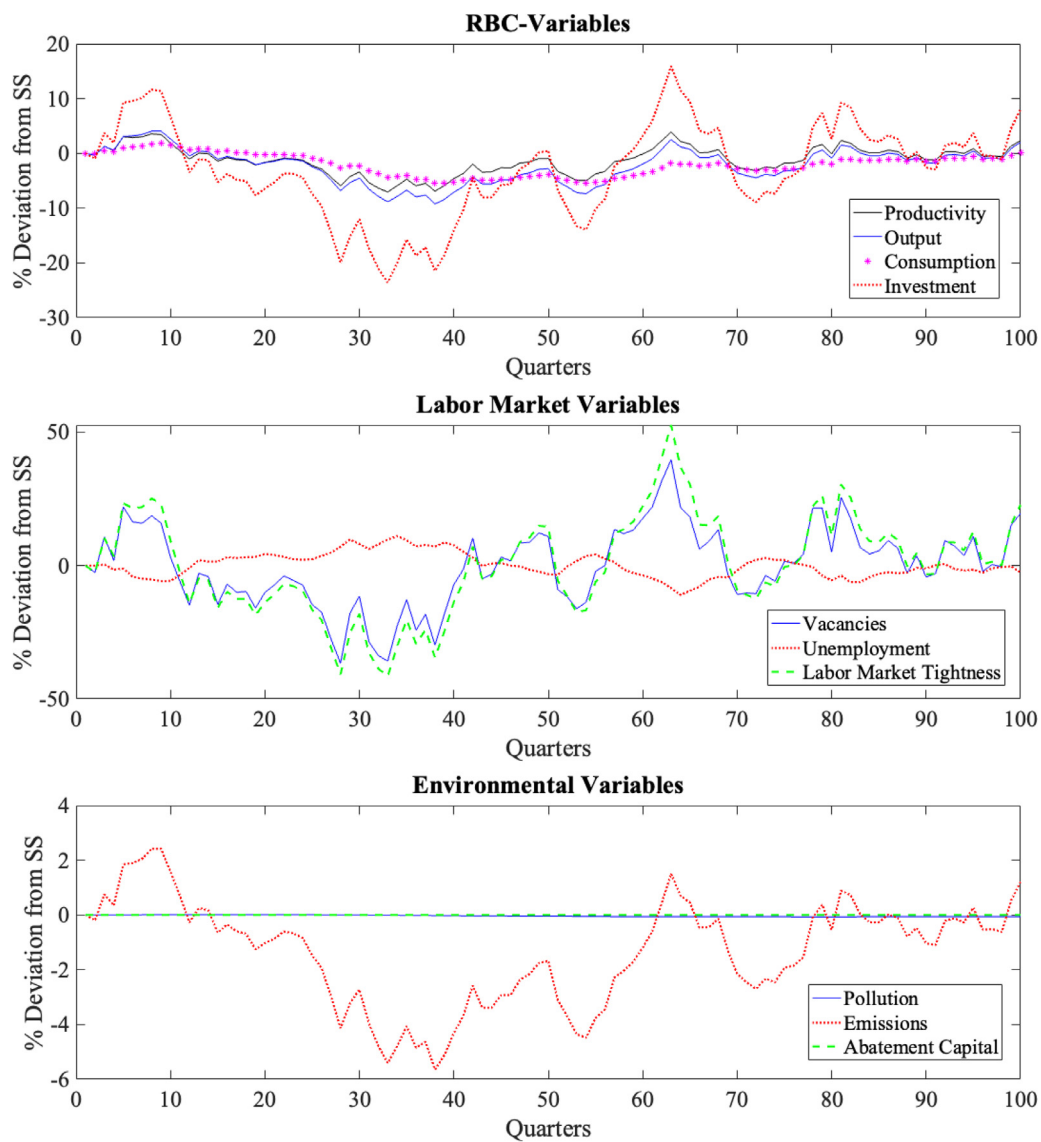


Fig. E.2. Business Cycle Simulations - Decentralized Problem. Notes: All simulations are from randomly generated draws of productivity innovations for 100 periods.

E2. Planner's problem

Figs. E.3 and E.4 present impulse response functions and time paths for the planner's problem. These figures are ordered in the same way and use the same exogenous paths for total factor productivity as Figs. E.1 and E.2 described above. The primary difference is the volatility in the fraction of emissions abated and the amount of resources (output) devoted to abatement spending in the planner's problem. Comparisons of the labor market plots also reveal differences in the volatility of labor market variables across model specifications.

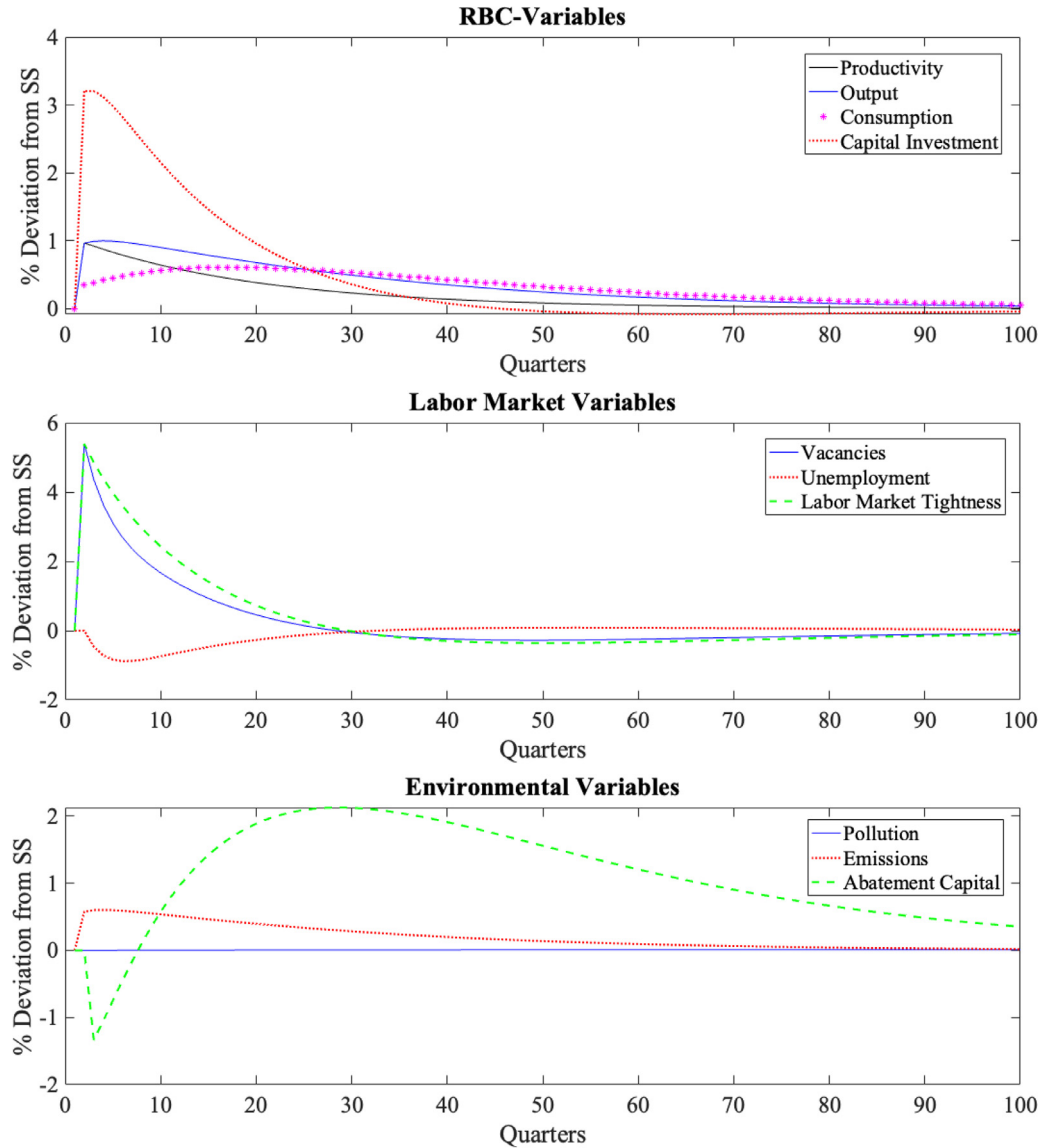


Fig. E.3. Impulse Response Functions - Planner's Problem. Notes: All simulations are from a one-unit one-time productivity innovation.

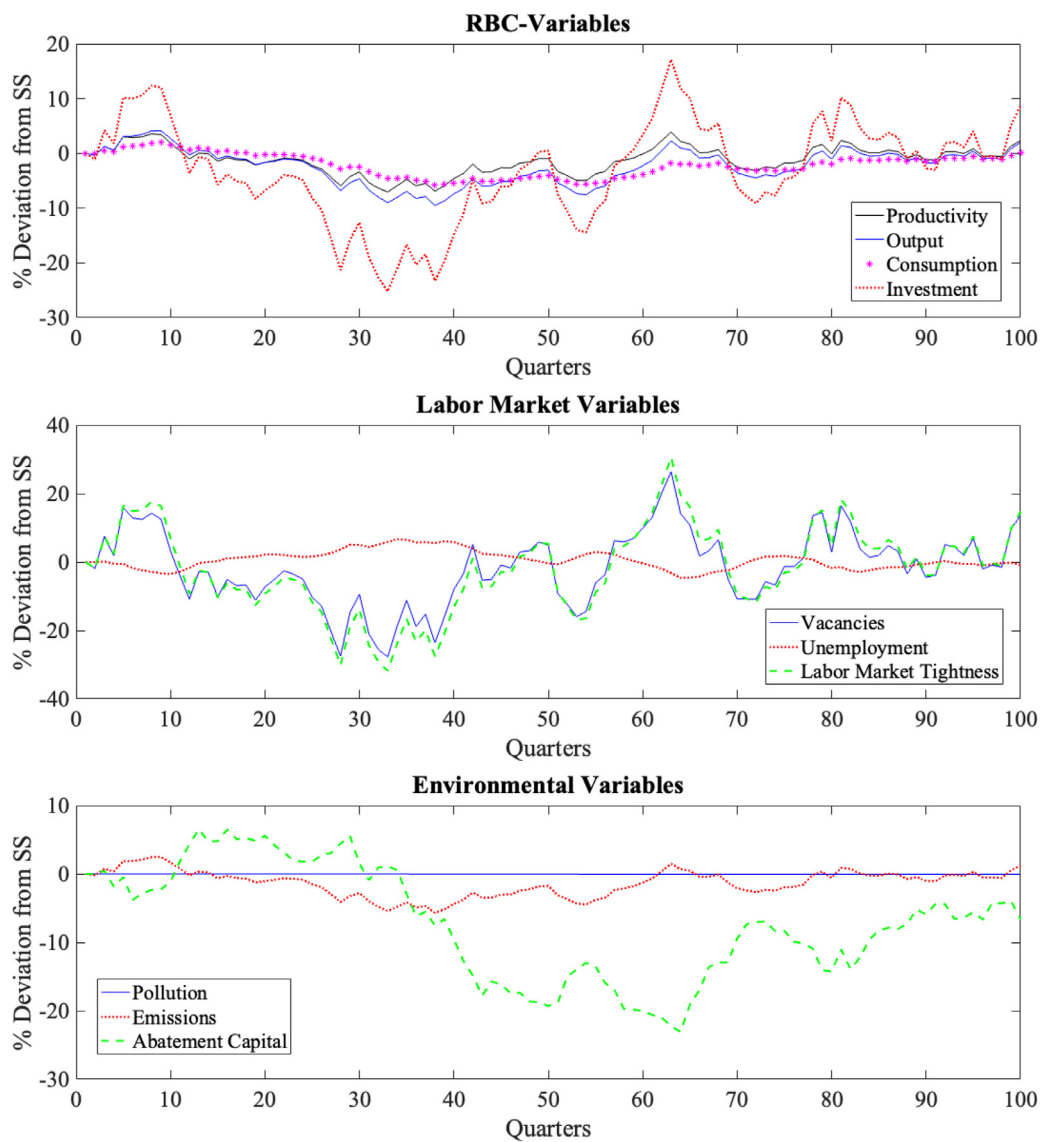


Fig. E.4. Business Cycle Simulations - Planner's Problem. Notes: All simulations are from randomly generated draws of productivity innovations for 100 periods.

E3. Ramsey efficient dynamic tax problem

Figs. E.5 and E.6 present time paths of the first-best and second-best Ramsey taxes, respectively.



Fig. E.5. Business Cycle Simulations - First-Best Taxes. Notes: All simulations are from randomly generated draws of productivity innovations for 100 periods.

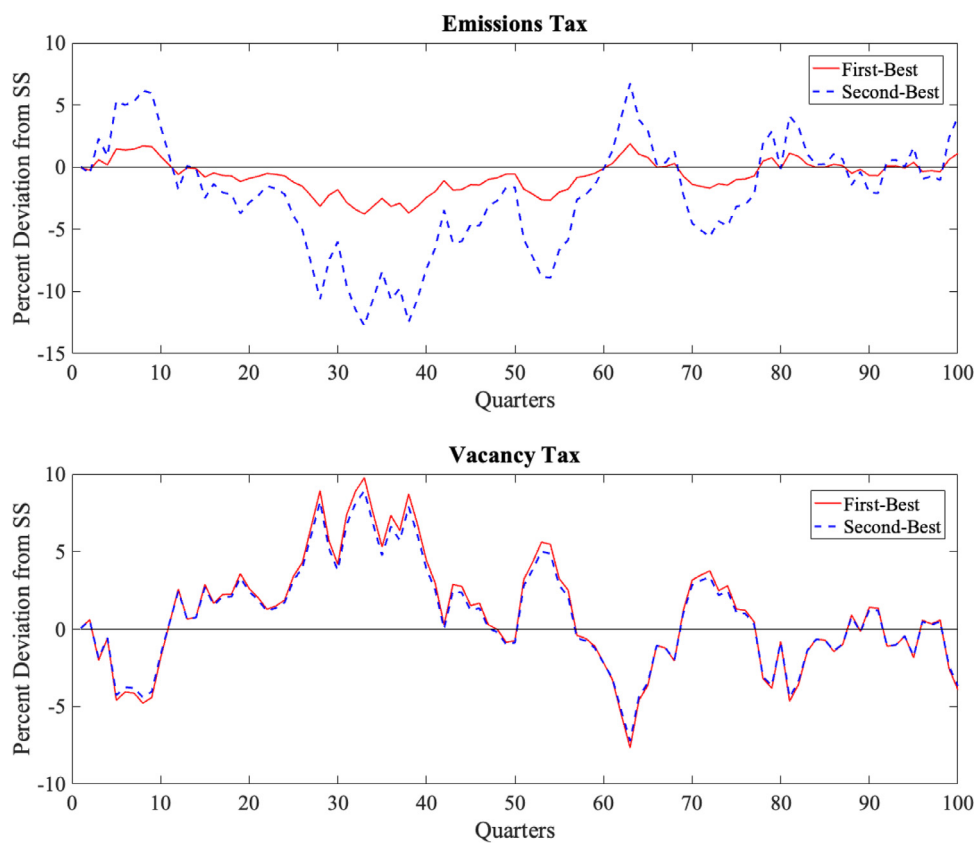


Fig. E.6. Business Cycle Simulations - First vs Second Best Taxes. Notes: All simulations are from randomly generated draws of productivity innovations for 100 periods.

Appendix F. Transition Dynamics

In this Appendix we present simulations of the transitions from one steady-state equilibrium to another. In particular, we run several simulations that all start at the steady state under the unregulated decentralized economy and have no TFP shocks. One simulation is the decentralized model without policy; under this simulation all variables simply remain at their steady-state values for all periods. Three other simulations employ the first-best emissions and vacancy taxes, the second-best emissions tax, and the second-best vacancy tax, respectively. In Fig. F.1, we present the levels of consumption,

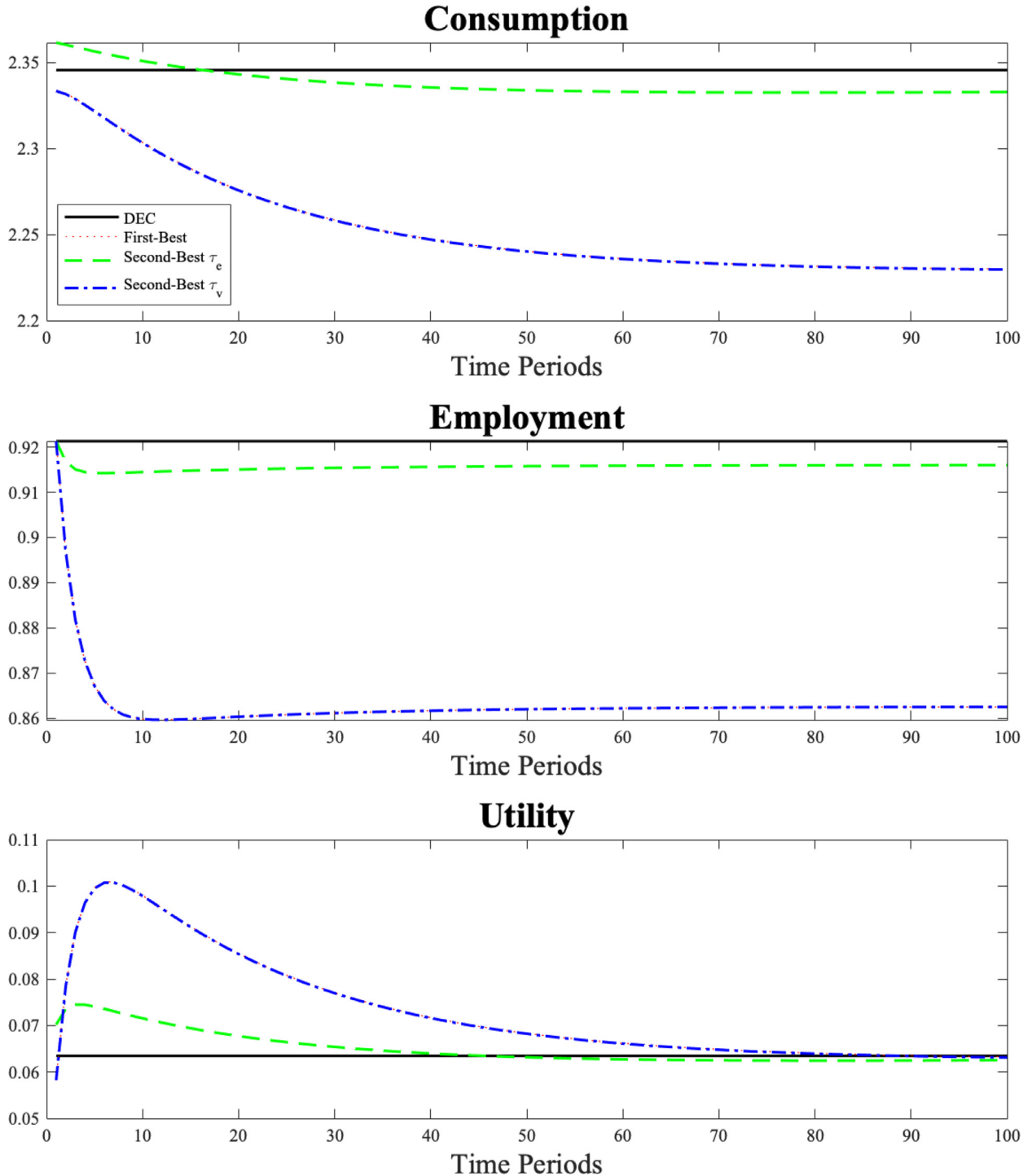


Fig. F.1. Transition Dynamics. Notes: All simulations begin at the decentralized (no policy) equilibrium steady state and feature no TFP shocks.

employment, and utility for each of the first 100 periods under each simulation. These simulations are run using a version of the model without a labor hours margin and with a flow cost of abatement (as in Appendix G).

As described in the text, when conducting welfare analysis by calculating compensating variation, rather than just comparing steady states across policies, we start all simulations at the same initial state and simulate the transition to the new steady state. If we just compared steady state welfare, we find that the steady-state welfare under the second-best vacancy tax is actually slightly higher than the steady-state welfare under the first-best taxes. The difference is so slight that it is difficult to see in the bottom panel of Fig. F.1 towards the end of the simulation as the steady state is approached. However, it is clear to see that through most periods of the transition, the first-best utility is higher than that of the second-best emissions tax, which is higher than that of the decentralized equilibrium. If instead we conducted welfare analysis by merely comparing welfare under the different steady states, this would not be an apples-to-apples comparison. The state variables, in particular pollution and employment, are drastically different across policies, so just comparing steady states amounts to unfairly starting the simulations at different points.

Fig. F.1 also clearly demonstrates that the second-best vacancy tax brings about an outcome much closer to the first best than does the second-best emissions tax. In Fig. F.1, the curves for the first best and the second-best vacancy tax are almost coincidental, while the second-best emissions tax is noticeably different. Under the second-best emissions tax, consumption is substantially higher than in the first best (which increases utility), but employment is also substantially higher (which reduces utility).

Appendix G. Alternative Specification with Abatement Spending

The numerical model presented in the main text allows for the accumulation of abatement capital. As such, short run changes in abatement spending can increase the stock of abatement capital available in both the current and future periods. However, abatement spending is often modelled as a flow cost that must be paid each period. In these models, the firm can choose what fraction of current emissions to abate, μ_t . The resource cost of this abatement, z_t , will be determined by μ_t and Y_t , as well as a function, $g(\cdot)$, that relates abatement to output cost. To put our model into this form, we must drop Eqs. 12 and 13 and replace them with:

$$e_t = (1 - \mu_t)h(Y_t) \quad (\text{G.1})$$

$$z_t = g(\mu_t)Y_t = [\theta_1\mu_t^{\theta_2}]Y_t \quad (\text{G.2})$$

where Eq. G.1 defines emissions under flow abatement cost and Eq. G.2 defines the output cost of current abatement.⁴²

Table G.1
Parameter Values (Flow Abatement Cost).

Parameter	Description	Value	Source
β	Discount factor	0.983	RBC literature
δ	Capital depreciation rate	0.025	
α	Capital's share of income	0.36	
ρ_θ	Productivity persistence	0.95	Match labor productivity data
σ_θ	Standard deviation of productivity innovation	0.01	
λ	Pollution decay rate	0.9965	IPCC reported CO ₂ half-life
$1 - \gamma$	Elasticity of output w.r.t. emissions	0.6	Regressions (see appendix)
θ_1	Abatement cost function coefficient	0.074	DICE 2016R2
θ_2	Abatement cost function exponent	2.600	
d_0	Damage function constant	-0.0076	Calibrated from DICE 2016R2 (see appendix)
d_1	Damage function linear coefficient	$8.10e - 6$	
d_2	Damage function quadratic coefficient	$1.05e - 8$	
γ_2	Exponent in matching function	0.72	Shimer (2005)
b	Labor bargaining power	0.05	Target wage elasticity w.r.t. productivity
G	Vacancy posting costs	0.8375	
γ_1	Coefficient in matching function	0.65	Target 5 moments (See Section 4.3)
s	Separation rate	0.048	
ξ	Coefficient on unemployment in utility function	0.8515	

Table G.1 presents the parameter values used for this specification with flow abatement cost. Comparison of Table G.1 with Table 1 of the main text reveals similar values for most parameters of the model. The notable exceptions are θ_1 and θ_2 , the two parameters of the abatement cost damage function.⁴³ Because the original DICE model includes

⁴² Modeling abatement as a flow cost will also require minor changes to the firm's profit function and the goods market clearing condition.

⁴³ There are small differences in the parameters of the TFP shock process (ρ_ϵ and σ_ϵ) between model specifications. These adjustments were made so that the alternative specification with flow abatement spending targets the same percent volatility and first-order autocorrelation of average labor productivity as the main specification. Also, while some of the labor market parameters differ between Tables G.1 and 1, these differences are due to the presence of the hours margin in Table 1, not abatement capital.

Table G.2
Decentralized vs Planner Outcomes – Abatement Spending.

	Mean ^a		%Volatility ^b		
	Decentralized	Planner	Data	Decentralized	Planner
RBC Variables ^c					
Output	3.04	2.85	1.58	1.34	1.33
Consumption	2.35	2.24	0.77	0.36	0.39
Investment	0.64	0.60	4.59	3.19	3.42
Labor Market Variables ^d					
Unemployment	0.075	0.134	9.71	2.61	1.52
Vacancies	0.053	0.010	9.13	8.96	6.45
Labor Market Tightness	0.702	0.071	18.37	10.97	7.51
Environmental Variables ^e					
Emissions	1.948	1.826	1.23	0.60	0.60
Pollution	3338.6	3128.9	0.0532	0.0013	0.0013
Fraction Abated	0.00	2.65	NA	0.00	0.095
Abatement Spending	0.00	1.6766e-5	NA	0.00	1.09

^a All model statistics are from 100 simulations, each 300 quarters in length. Reported values are averages of our sample statistics across these simulations. The values reported for “Mean” are the average value of the simulated series. These numbers are quantitatively very close to the variables’ steady state value.

^b Percent volatility is measured as the percent standard deviation of the cyclical component of the log of x relative to the percent standard deviation of the cyclical component of log output (statistic for output is presented in levels).

^c Data on output, consumption, and investment comes from the National Income and Product Accounts tables and covers the period of 1947–2019. Specifically, Y is Real Gross Domestic Product, c is Real Personal Consumption Expenditures, and inv is Real Gross Private Domestic Investment (all in billions of chained 2012 dollars). The cyclical component of all RBC variables (both data and model simulations) is found by HP-filtering with a smoothing parameter of 1,600.

^d The data values reported in this section come from [Shimer \(2005\)](#) and have been normalized by the percent volatility of output. Following [Shimer \(2005\)](#), the cyclical component of all labor market related variables (both data and model simulations) is found by HP-filtering with a smoothing parameter of 10^5 .

^e Emissions data are from the source described in the calibration section of the text. Pollution stock data are from NOAA’s Global Monitoring Laboratory.

abatement as a flow, this calibration is more straightforward than for our model in the main text. Here, we simply use the same two parameters from the DICE model. We also find that the modeling choice for abatement (flow cost versus capital stock) has little to no impact on the second moment properties of model. Specifically, [Table G.2](#) presents the mean and percent volatility of key variables for both the decentralized and planner’s solution to the model with flow abatement cost. Comparing [Table G.2](#) with [Table 3](#) in the main text reveals that these second moments are nearly identical. The similarity in results carry over to our policy analysis where we find that the first-best emissions and vacancy taxes under flow abatement cost are $8.7657\text{e-}04$ and 11.0877 , respectively (versus $8.8461\text{e-}4$ and 11.09 in the main text). The qualitative findings related to second-best taxes are also consistent with the main text. We find that the second-best emissions tax rises significantly from its first best level to 0.0296 while the second-best vacancy tax is very close to its first-best level at 11.145 (versus 0.0546 and 11.0872 in the main text). While the level of the second-best emissions tax differs between the two model specifications, we still observe a substantial increase in the tax rate relative to first best, suggesting the that main findings of the paper carry over to this alternative specification with flow abatement cost.

Appendix H. Equity Concerns and Alternate Welfare Criteria

The welfare analyses of the preceding sections assume that the welfare criterion defining the first best considers only efficiency. Equity or distributional concerns are ignored. Also, in the preceding analyses the only cost of unemployment is the foregone production of the idle workforce. The analyses ignore the substantial psychological costs to the unemployed, costs to society that exceed the opportunity cost of idleness.

We consider two extensions to the numerical model that address these issues. First, we address equity concerns. Our model has one representative household and one representative firm, so the only distributional outcome we can examine is the differential outcomes between these two representatives (i.e. we cannot look at distributional outcomes across income or race).⁴⁴ Furthermore, because the representative firm is owned by the representative household, the delineation between these two agents is muddled. Nevertheless, we proceed by analyzing the difference in outcomes for the firm and for the worker to provide an initial assessment of distributional outcomes of our policies.

⁴⁴ Other DSGE models do incorporate heterogeneous agents, including [Iacoviello \(2005\)](#), [Kaplan et al. \(2018\)](#), and [Hohberger et al. \(2020\)](#).

Table H.1
Distributional Outcomes .

	Household Utility	Firm Profit	Unemployment Rate
Unregulated Decentralized	0.0634 (99%)	0.0149 (12%)	7.87%
Planner's Problem	0.0642 (100%)	N/A	13.74%
First-Best Taxes	0.0642 (100%)	0.1220 (100%)	13.74%
Second-Best Emissions Tax	0.0675 (105%)	0.0380 (31%)	8.39%
Second-Best Vacancy Tax	0.0635 (99%)	0.1216 (100%)	13.74%

Notes: This table presents steady-state values for household utility, firm profit, and unemployment under five equilibrium scenarios under base-case parameter values. Household utility and firm profit are in arbitrary units; the percents in parentheses below these values are the percentage relative to the first best. Firm profit is undefined in the planner's problem since input prices are undefined. The representative household owns the representative firm and receives its profits as income.

All of the results in this appendix are from a slightly modified version of the model, in which the hours worked margin is ignored and abatement expenses are modeled as a flow expense rather than a capital stock (as in [Appendix G](#)).

In [Table H.1](#), we present the steady-state values of the household's utility, the firm's profit, and the unemployment rate, under several equilibrium outcomes. The first row presents these three values under the decentralized equilibrium with no policies ($\tau_e = \tau_v = 0$). The second row is from the planner's problem, which maximizes total welfare. The last three rows are from decentralized equilibria with non-zero policy values. The third row is with the first-best emissions and vacancy taxes; this row is equivalent to the planner's problem. The fourth and fifth rows are with the second-best emissions and vacancy taxes, respectively. The utility and profit values are in arbitrary units, so we also present these values as percentages of the first-best values (0.0642 for utility and 0.1220 for profit). Profit is undefined in the planner's problem since input prices are undefined. These calculations are based on just the steady-state equilibria under each of the policies and do not include the transition paths (which are explored in [appendix Section F](#)).

Moving from the unregulated decentralized equilibrium to the first best only slightly increases household utility but increases firm profit by almost an order of magnitude. The unemployment rate nearly doubles. While total efficiency increases, the benefits accrue mainly to the firm. Though the household has slightly higher utility in the first best than in the unregulated equilibrium, the doubling of unemployment should not be ignored. In the model, holding consumption constant an increase in unemployment actually benefits the household since leisure increases. In the real world, this unemployment increase is bad for workers. Comparing the second-best emissions tax with the second-best vacancy tax shows that the latter is more efficient overall (both utility and profit nearly as high as their first-best levels), but the former is better for the household (higher utility and lower unemployment). This very back-of-the-envelope decomposition thus suggests a trade-off between efficiency and equity in policy design, pitting the more efficient policy (the vacancy tax) against the more equitable policy (the emissions tax).

Second, we consider the excess psychological costs of unemployment by employing alternate welfare criteria. A large literature documents the negative effect of unemployment on people, including reductions in self-reported life satisfaction ([Kassenboehmer and Haiken-DeNew, 2009](#)), decreases in mental health ([Marcus, 2013](#)), decreases in social participation ([Kunze and Suppa, 2017](#)), and increases in suicides ([Classen and Dunn, 2012](#); [Milner et al., 2013](#)). The worker's single-period utility equals $\ln c_t - \xi n_t$, where ξ represents the disutility of work. To capture the psychological costs, or disutility, from unemployment, we introduce an additional component to the utility function, $\kappa(1 - n_t)$. The single-period utility is now $\ln c_t - \xi n_t - \kappa(1 - n_t)$. In order to keep the equilibrium outcomes the same as their calibrated values in the decentralized case, we assume that the households behave according to the original utility function $\ln c_t - \xi n_t$. But the government sets policy so as to maximize utility inclusive of the unemployment disutility, $\ln c_t - \xi n_t - \kappa(1 - n_t)$. This distinction between the utility function that describes the agent's behavior and the utility function that describes optimal or efficient outcomes is analogous to the distinction between decision utility and experienced utility often used in behavioral welfare economics ([Kahneman and Sugden, 2005](#)).

In [Fig. H.1](#), we present values for the first- and second-best steady-state emissions and vacancy taxes for different values of the unemployment disutility parameter κ , ranging from 0 to 0.6. For comparison, the disutility of work parameter ξ is 0.85, so when $\kappa = 0.425$, the unemployment disutility is half as high as the disutility of work. The first row shows that as unemployment disutility κ gets higher, the first-best vacancy tax decreases by a lot and the first-best emissions tax increases by a little. When $\kappa = 0.425$, the first-best vacancy tax is about an order of magnitude smaller than its base-case value when $\kappa = 0$, while the first-best emissions tax is only about 10% greater than its base-case value. Even including disutility of unemployment with just a modest magnitude greatly reduces the tax on vacancies. The bottom row of [Fig. H.1](#) shows how the second-best taxes respond to κ . The second-best vacancy tax is almost identical to the first-best vacancy tax. But,

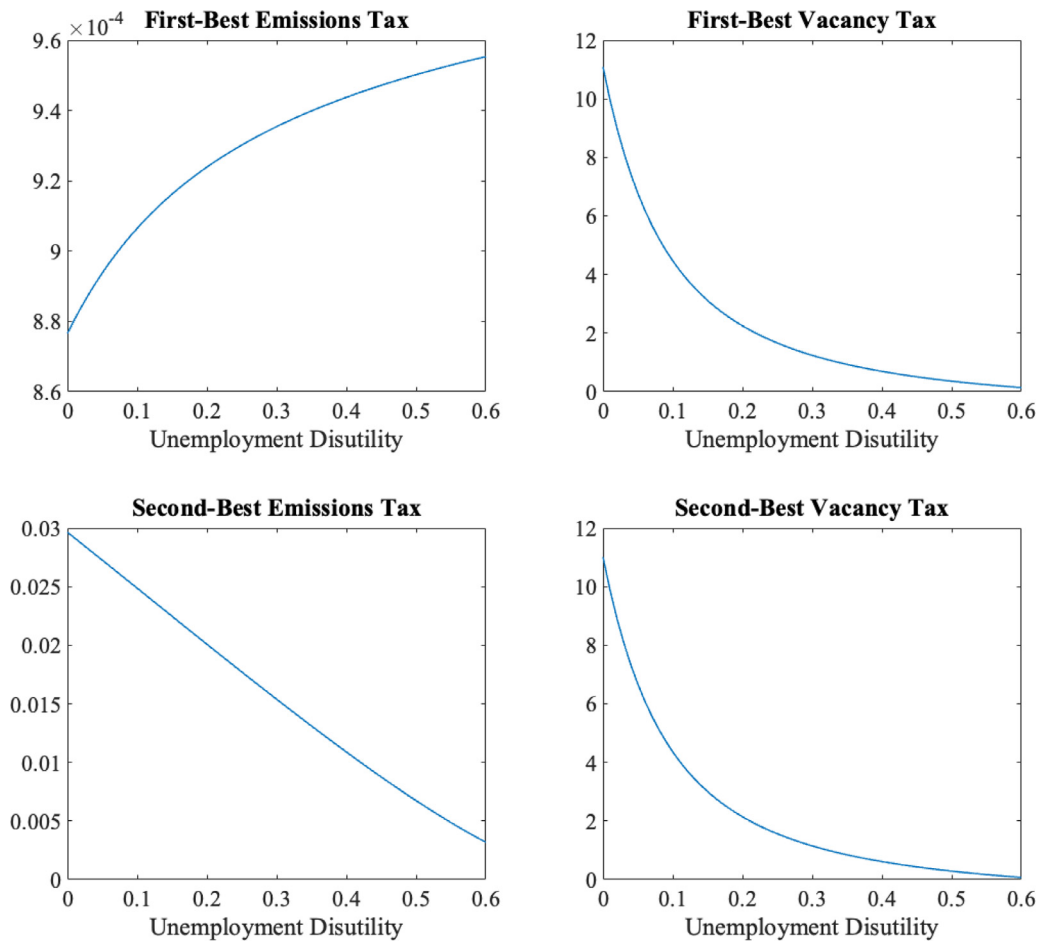


Fig. H.1. Unemployment Disutility. Notes: Simulations of the first- and second-best emissions and vacancy creation taxes for different values of the unemployment disutility parameter κ .

the second-best emissions tax is much more responsive to κ than is the first-best emissions tax. Increasing κ from 0 to 0.425 decreases the second-best emissions tax by more than 50%. This panel demonstrates an important trade-off between reducing emissions and avoiding adverse effects on workers.

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