

WORKSHEET 9/12/22
MATH 2331, FALL 2022

(1) Consider the following system of equations:

$$2x + 4y - 2z = -10$$

$$3x + 6y = -12$$

$$y + z = 3$$

- (a) What is the coefficient matrix A of the system?
- (b) What is the augmented matrix of the system?
- (c) Use Gauss–Jordan elimination to solve the system.
- (d) What is the reduced row echelon form of A (notation: $\text{rref}(A)$)?

(2) Given the reduced row-echelon form of the matrix, how many solutions does the corresponding system of linear equations have?

(a)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

(b)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 7 & 3 \end{array} \right]$$

(c)

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

(d)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

(e)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \end{array} \right]$$

(f)

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

(g)

$$\left[\begin{array}{c|c} 1 & 1 \\ 0 & 2 \\ 0 & 7 \end{array} \right]$$

(3) For which values of k is the system of linear equations

$$2x + 2y + kz = 3$$

$$kx + ky + 8z = k + 2$$

consistent? When it is consistent, for which values is there a unique solution, and for which values are there infinitely many?

(4) How large can the rank of a matrix be?

(5) What can you say about the number of solutions to a system of linear equations assuming the rank of its coefficient matrix is

- (a) equal to the number of rows?
- (b) less than the number of columns?
- (c) equal to the number of columns?