WORKSHEET 10/17/22
MATH 2331, FALL 2022

(1) Let $\mathfrak{B} = \{\vec{v}_1, \ldots, \vec{v}_m\}$ be a basis for $\mathbb{R}^m$. If $[T]_{\mathfrak{B}}$ is a diagonal matrix, what can you say about $T(\vec{v}_i)$?

(2) Is there a basis for $\mathbb{R}^2$ in which reflection over the line L is represented by a diagonal matrix?

(3) Is there a basis for $\mathbb{R}^2$ in which a 90 degree rotation is represented by a diagonal matrix?

In the remaining problems, $\vec{u}_1 = \frac{1}{2}(1,1,1,1)$, $\vec{u}_2 = \frac{1}{2}(1,1,-1,-1)$, $\vec{u}_3 = \frac{1}{2}(1,-1,1,-1)$, and $V = \mathrm{Span}(\vec{u}_1, \vec{u}_2, \vec{u}_3)$.

(4) Are the vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$ orthonormal?

(5) Can you find a vector $\vec{u}_4$ such that $\vec{u}_1, \ldots, \vec{u}_4$ are orthonormal?

(6) Suppose that $c_1\vec{u}_1 + c_2\vec{u}_2 + c_3\vec{u}_3 = \vec{0}$. What can you say about c_1, c_2, and c_3? If you're writing down a matrix, you're working too hard!

(7) Find a basis for V. Don't work too hard!

(8) Extend your basis from #7 to a basis $\mathfrak{B}$ for $\mathbb{R}^4$. Don't work too hard!

(9) Given a vector $\vec{x}$ in $\mathbb{R}^4$, what is $[\vec{x}]_{\mathfrak{B}}$? If you're writing down a matrix, you're working too hard!