

**WORKSHEET 10/24/22**  
**MATH 2331, FALL 2022**

(1) Find the inverse of the matrix  $\frac{1}{5} \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ .

(2) Find the inverse of the matrix  $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ .

(3) Based on your answers to #1 and #2, try to guess the inverses of these matrices:

$$\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

(4) Calculate  $\vec{v}^T \vec{w}$ , where  $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  and  $\vec{w} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ . What do you notice?

(5) Calculate  $Q^T Q$ , where  $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ .

(6) Based on your answer to #5, guess a formula for  $Q^T Q$  when the columns of  $Q$  are orthonormal.

(7) Based on your answer to #6, is  $Q$  invertible?

(8) Show that  $|\vec{x} - \vec{y}|^2 = |\vec{x}|^2 + |\vec{y}|^2$  if  $\vec{x}$  and  $\vec{y}$  are orthogonal. Does this remind you of anything?