Abstract—This letter proposes a novel approach to intent-expressive motion planning and intent estimation for agents/robots with uncertain discrete-time affine dynamics. In contrast to the more commonly considered stochastic settings, our intent-expressive trajectory planning approach is set-based and leverages the active model discrimination framework for designing optimal inputs to attain certain target/goal states, while allowing an observer/teammate to clearly infer the intended goal based only on observations of a partial trajectory before it has reached its target/goal state, despite worst-case uncertainties. Further, in tandem with the planning algorithm, we also propose an intent estimation algorithm that can be used by the observer/teammate to determine the intended goal from observations of a noisy, partial trajectory.

Index Terms—Model validation, fault diagnosis, estimation, identification.

I. INTRODUCTION

AUTONOMOUS teams of robots must often coordinate with each other and with humans to ensure efficient task satisfaction. In many cases, communication between the agents is imperative to allow for successful task completion since the interacting agents have limited access to the intent/state information of the other agents [1], and most traditional multi-agent systems assume the availability of these communication resources. However, in applications where communication constraints are present, the agents must still estimate and predict the behaviors of the teammates. Here, the trajectory/motion planning task must account for the expressiveness or “legibility” of the intended motion (as a form of non-verbal communication) to allow for successful “communicationless” coordination and to ensure safety.

Literature Review: Intent-expressive (or legible or anticipatory [2]–[6]) motion/trajectory planning has been mainly studied in the context of human-machine interactions. Specifically, the goal is for the agent/robot to move in such a way that the human trusts and understands. The mathematical conceptions of legibility and predictability are formally introduced in [4], [5] based on the principle of rational action and optimization techniques, while [7] presented improvements to the algorithms. Over the years, legible motion planning has remained a very active research area, but the literature primarily focuses on the probabilistic methods [4]–[6]. While significant progress has been made in this regard, these studies into legibility do not directly apply when the agent/robot model uncertainties are non-deterministic but bounded, or when the stochastic distributions are not known, and thus, legibility under this setting remains largely unexplored.

Hence, this letter will consider an alternative set-based approach to design intent-expression trajectories for minimizing the compatible set of intents under worst-case uncertainties. Specifically, our proposed approach is inspired by the literature on active model discrimination, e.g., [8]–[11], where the objective is to design a small excitation that has a minimal effect on the desired behavior of the system and guarantees the isolation of different fault models or the inference of intent models of other (adversarial) agents, as opposed to designing trajectories for one agent/robot model with multiple intended goals/destinations while conveying the intent to teammates/bystanders. On the other hand, passive model discrimination algorithms, including model (in)validation methods, can be used to infer the true models by exploiting the input-output data and system structure to eliminate incompatible models, e.g., [12], [13]. Notably, these approaches are set-based since they involve implicit or explicit computations of reachable state and/or output sets, with several extensions to closed-loop settings [14], [15], parametric uncertainties [9], [16] and nonlinear dynamics [17], [18]. More importantly, these approaches are applicable for non-deterministic models with bounded uncertainties, and offer appropriate tools for designing set-based algorithms for intent-expressive trajectory planning and intent estimation.

Contribution: In this letter, we propose novel intent-expressive/legible motion planning and intent estimation algorithms for discrete-time affine models with (non-stochastic) bounded noise. To our best knowledge, our proposed algorithms are the first set-based (i.e., approaches that consider the set of all possible realizations) intent-expressive motion planning methods, in contrast to most existing methods that consider stochastic uncertainties with known distributions. Our set-based approach leverages the active model discrimination framework in [8] for designing optimal inputs for a given number of intent models, characterized as target/goal states, such that these goal states are achieved while also ensuring that a partial output trajectory distinctly expresses the intended goal.
Thus, this letter also extends the set-based approach to single-goal motion planning in [19] that is based on passive methods for (fault) model inference.

Our proposed intent-expressive trajectory planning problem consists of 2 phases. The first phase deals with designing a set of optimal inputs over a shortened time horizon such that the partial output trajectories towards each goal state up to that point cannot be identical, while the second phase ensures that the worst-case cost-to-go to the intended goal state is strictly lower than the cost-to-go to any other goal. This formulation allows an observer/teammate to clearly infer the intent of the agent/robot based on observations of a partial trajectory despite worst-case uncertainties, and more importantly, before it has reached its target/goal state. In particular, we pose the planning problem as a bilevel optimization problem with semi-infinite constraints and then reformulate it into a mixed-integer linear program (MILP) that can be solved by off-the-shelf solvers such as [20].

In addition, we also propose an intent estimation algorithm that can be used by the observer/teammate to determine the intended goal state of the agent/robot based on the partially observed, noisy input-output trajectories, despite worst-case uncertainties. Specifically, the intent estimation problem is also posed as an optimization problem with semi-infinite constraints, which is then recast into a linear program (LP) by leveraging robust optimization.

II. PRELIMINARIES

A. Notations

Let \( x \in \mathbb{R}^n \) denote a vector and \( M \in \mathbb{R}^{n \times m} \) a matrix, with transpose \( M^\top \) and \( M \geq 0 \) denotes element-wise non-negativity. The vector norm of \( x \) is denoted by \( \| x \| \) with \( i \in \{ 1, \infty \} \), while \( \mathbf{0}, \mathbf{1} \) and \( \mathbb{I} \) represent the vector or matrix of zeros, the vector of ones and the identity matrix of appropriate dimensions. The \( \text{diag} \) and \( \text{vec} \) operators are defined for a set of matrices \( \{ M_i \}_{i=m}^n \) and matrix \( M \) as:

\[
\text{diag}_{i=m}^n(M_i) = \begin{bmatrix} M_m \cdots M_n \end{bmatrix}, \quad \text{vec}_{i=m}^n(M_i) = \begin{bmatrix} M_m \cdots M_n \end{bmatrix}^\top,
\]

\[
\text{diag}_{k=(i,j)}(M_k) = \begin{bmatrix} M_i & 0 \cdots 0 \\ 0 & M_j \end{bmatrix}, \quad \text{vec}_{k=(i,j)}(M_k) = \begin{bmatrix} M_i \\ M_j \end{bmatrix},
\]

\[
\text{diag}_N(M) = \mathbb{I}_N \otimes M, \quad \text{vec}_N(M) = \mathbb{I}_N \otimes M,
\]

where \( \otimes \) is the Kronecker product. The sets of positive and non-negative integers up to \( n \) are denoted by \( \mathbb{Z}_{\geq 0}^n \) and \( \mathbb{Z}_n^0 \), respectively. Note also the definition of Special Ordered Set of degree 1 (SOS-1) constraints in [20], [21].

B. Modeling Framework

Consider a discrete-time affine time-invariant model \( \mathcal{G} = (A, B_u, B_w, C, D_u, D_v, f, g) \) for an agent/robot with states \( x \in \mathbb{R}^n \), outputs \( z \in \mathbb{R}^m \), inputs \( u \in \mathbb{R}^m \), process noise \( w \in \mathbb{R}^m \) and measurement noise \( v \in \mathbb{R}^m \). The model evolves according to the following state and output equations:

\[
x(k + 1) = Ax(k) + Bu(k) + B_w w(k) + f, \quad (1)
\]

\[
z(k) = Cx(k) + Du(k) + D_v v(k) + g. \quad (2)
\]

The initial condition, denoted by \( x^0 = x(0) \), is constrained to a polyhedral set with \( c_0 \) inequalities:

\[
x^0 \in X_0 = \{ x \in \mathbb{R}^n : P_0 x \leq p_0 \}. \quad (3)
\]

Furthermore, \( u \) and \( x \) have to satisfy the following polyhedral input and state constraints with \( c_u \) and \( c_x \) inequalities:

\[
u(k) \in U = \{ u \in \mathbb{R}^{m_u} : P_u u \leq p_u \}, \quad (4)
\]

\[
x(k) \in X_i = \{ x \in \mathbb{R}^{m_x} : P_i x \leq p_i \}. \quad (5)
\]

On the other hand, the process noise \( w \) and measurement noise \( v \) are also polyhedrally constrained with \( c_w \) and \( c_v \) inequalities, respectively:

\[
w(k) \in W = \{ w \in \mathbb{R}^{m_w} : P_w w \leq p_w \}, \quad (6)
\]

\[
v(k) \in V = \{ v \in \mathbb{R}^{m_v} : P_v v \leq p_v \}. \quad (7)
\]

The agent/robot has several target/goal regions \( X_{g,i}, i \in \mathbb{Z}^+_N \) that are each polyhedrally constrained with \( c_{g,i} \) constraints,

\[
X_{g,i} = \{ x \in \mathbb{R}^n : P_{g,i} x \leq p_{g,i} \}, \quad \forall i \in \mathbb{Z}^+_N, \quad (8)
\]

and the goal of this letter is make its intent to reach a specific target/goal more apparent or legible by designing intent-expressive path/trajectory such that the final state, denoted by \( x_{f} = x(T_f) \), is within the intended goal region \( X_{g,i} \), i.e., \( x_{f} \in X_{g,i} \). In other words, the final state constraints are used to represent the different terminal goals or intents.

C. Time-Concatenated Models and Constraints

For each possible model \( \mathcal{G}_i \) corresponding to each intended (terminal) goal \( X_{g,i}, i \in \mathbb{Z}^+_N \), we denote the state, input, output and noise trajectories with the subscript \( i \), i.e., \( x_i(k), u_i(k), z_i(k), w_i(k) \) and \( v_i(k) \) for each \( i \in \mathbb{Z}^+_N \). Next, we will introduce time-concatenated notations. Given a time horizon \( T \in [T_f, T] \), where \( T \leq T_f \), the time-concatenated states, inputs, outputs and noises are defined as:

\[
x_{i,T} \triangleq \text{vec}_{k=0}^T(x_{i}(k)), s_{i,T} \triangleq \text{vec}_{k=0}^{T-1}(s_{i}(k)),
\]

for all \( s \in \{ z, u, w, v \} \). Thus, the time-concatenated dynamics is given by:

\[
x_{i,T} = \tilde{A}_{T} x_{i,0} + \tilde{B}_{u,T} u_{i,T} + \tilde{B}_{w,T} w_{i,T} + \tilde{f}_{T}, \quad (9)
\]

\[
z_{i,T} = \tilde{C}_{T} x_{i,T} + \tilde{D}_{u,T} u_{i,T} + \tilde{D}_{w,T} w_{i,T} + \tilde{g}_{T}, \quad (10)
\]

where the concatenated matrices/vectors are given in the Appendix. Next, we concatenate the input constraints in (4) across the time horizon \( T_f \),

\[
\tilde{P}_u u_{i,T_f} \leq \tilde{p}_u \quad (11)
\]

with \( \tilde{P}_u \triangleq \text{diag}_{T_f}[P_u] \) and \( \tilde{p}_u \triangleq \text{vec}_{T_f}[p_u] \), followed by the concatenation of the polyhedral state constraints in (5) and the final/goal state constraints (8) across the time horizons. Defining

\[
\tilde{P}_x,T \triangleq [\text{diag}_{T+1}(P_x) \, 0], \quad \tilde{p}_x,T \triangleq \text{vec}_{T+1}[p_x],
\]

\[
\tilde{P}_{x,T_i,j} \triangleq \begin{bmatrix} \text{diag}_{T_f-i+1}(P_{x,i}) & 0 \\ 0 & \tilde{P}_{g,i} \end{bmatrix}, \quad \tilde{p}_{x,T_i,j} \triangleq \begin{bmatrix} \text{vec}_{T_f-i}[p_{x,i}] \\ \tilde{p}_{g,i} \end{bmatrix},
\]

we can rewrite the polyhedral state constraints in (5) and (8) for the time horizons from 0 through \( T \) and from \( T+1 \) through

1Our goal is to enable the intent models to be inferred from observations from a (much) shorter time horizon \( T < T_f \) before the agent/robot arrives its goal in \( T_f \) time steps (i.e., intent-expressive motion planning).
of time horizon from 0 through T and from T + 1 through $T_f$, respectively, with

$$
\begin{align*}
\bar{H}_1, T \xi_i, T_j & \leq \bar{p}_1, T, \bar{P}_1, T_i, T_j, \\
\bar{H}_2, T, \xi_i, T_j & \leq \bar{p}_2, T, \bar{P}_2, T_i, T_j, \\
\bar{H}_3, T, \xi_i, T_j & \leq \bar{p}_3, T, \bar{P}_3, T_i, T_j,
\end{align*}
$$

for each model pair

$$
\forall \xi, \bar{\xi}, \bar{\eta}, \eta, \Delta, k \in \mathbb{Z}_{N_f}^{+}, \bar{\xi} \neq \eta, \Delta = \bar{\xi} = \eta, \Delta.
$$

Similarly, the initial state, input and noise constraints can be obtained as

$$
P_1, \bar{\xi}_i, T_j \leq p_1
$$

with $p_1 = \begin{bmatrix} P_0 & 0 \\ 0 & \text{diag}_{T_f}(P_w) \end{bmatrix}$ and $p_2 = \begin{bmatrix} p_0 \\ \text{vec}_{T_f}(P_w) \end{bmatrix}$.

### D. Pair-Concated Constraints

To ensure the outputs of the intent models are distinct from each other within a time horizon of $T \leq T_f$, we further introduce the model pair, which consists of two different models of $G_i$. Considering $N_g$ discrete-time affine models, i.e., $G_1, G_2, \ldots, G_{N_g}$, there are $Q = \binom{N_g}{2}$ model pairs and let the mode $q \in \{1, \ldots, Q\}$ denote the pair of models $(G_i, G_j)$. Next, we concatenate $u_{i, T}, \bar{u}_{i, T}$ and $z_{i, T}$ for each model pair $q = (\bar{q}, \hat{q})$ using $\hat{\xi}_{\bar{q}, q} = \hat{\xi}_{\bar{q}, q} = \hat{\xi}_{\bar{q}, q}$ for all $s \in \{u_r, \bar{u}_r, \bar{z}_r, z_r\}$, and define $\hat{\xi}_{\bar{q}, q} = \hat{\xi}_{\bar{q}, q} = \hat{\xi}_{\bar{q}, q} = \hat{\xi}_{\bar{q}, q}$.

Then, the pairwise initial state, input and noise constraints (including measurement noise) can be obtained as

$$
H_{\bar{q}, q} \xi_{\bar{q}, q} \leq h_{\bar{q}, q}
$$

with $H_{\bar{q}, q} \xi_{\bar{q}, q} = \text{diag}_{T_f}(\text{diag}[P_1, \text{diag}_{T_f}(P_r)])$ and $h_{\bar{q}, q} = \text{vec}_{T_f}(\text{vec}_{T_f}(P_r))$. Finally, the difference between pairwise outputs $\|\bar{z}_{\bar{q}, T} - z_{\bar{q}, T}\| \leq \delta_{\bar{q}}$ can be reduced to

$$
R\xi_{\bar{q}, q} \xi_{\bar{q}, q} \leq h_{\bar{q}, q} + S\xi_{\bar{q}, q}
$$

with $R = \begin{bmatrix} \bar{E}_T & \bar{F}_w, T & \bar{F}_v, T \\ -\bar{E}_T & -\bar{F}_w, T & -\bar{F}_v, T \\ \bar{E}_T & \bar{F}_w, T & \bar{F}_v, T \end{bmatrix}$ and $S = \begin{bmatrix} -\bar{F}_u, T & \bar{F}_u, T \\ -\bar{F}_u, T \end{bmatrix}$.

### III. Problem Formulation

We consider the intent-expressive trajectory planning and intent estimation problem for a given system in $(1)-(2)$ with a finite number of target state regions $X_{g, i}, i \in \mathbb{Z}_{N_f}^{+}$ in $(8)$. In the first intent-expressive trajectory planning problem, we look to find optimal inputs for the system to reach each of the goals at the end of a horizon, i.e., $x(T_f) \in X_{g, i}, \forall i \in \mathbb{Z}_{N_f}^{+}$ such that the observed output-input trajectories over a partial time horizon $T \leq T_f$ (typically, $T \ll T_f$) is obviously expressing which goal is intended, irrespective of any process or output noise terms. Then, in the second problem of intent estimation, we seek to infer the intended goal $X_{g, i}$ from only the observed input-output trajectories over a partial time horizon $T \leq T_f$.

These problems can be formalized as:

**Problem 1 (Intent-Expressive Trajectory Planning):** Given an affine model $G$ in $(1)-(2)$ with $N_g$ goals given by $(8)$, and state, input and noise constraints given by $(3)-(7)$, find a set of $N_g$ optimal input sequences $\{u_{T_f}^{\hat{i}, i}\}_{i=1}^{N_g}$ to minimize cost function $\sum_{i=1}^{N_g} J(u_{T_f}^{\hat{i}, i}) + \lambda \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} J(u_{T_f}^{\hat{j}, j})$ (where $u_{T_f}^{\hat{i}, i}$ are internal variables that represent inputs corresponding to deviations to other unintended goals) such that for all possible initial states $x_0$, process noise $w_T$ and measurement noise $v_T$, only one goal is intended, i.e., the output trajectories within $T \leq T_f$ corresponding to each goal have to differ by a threshold $\epsilon$ in at least one time instance, and the input costs to the intended goals $X_{g, *}$ are minimal, i.e., for each $\hat{\epsilon} \in \mathbb{Z}_{N_f}^{+}, J(u_{T_f}^{\hat{i}, i}) < J(u_{T_f}^{\hat{\epsilon}, \hat{\epsilon}})$ for all $\hat{\epsilon} \in \mathbb{Z}_{N_f}^{+}, \hat{\epsilon} \neq \hat{\epsilon}$. This intent-expressive trajectory planning problem can be captured by finding the optimal $\{u_{T_f}^{\hat{i}, i}\}_{i=1}^{N_g}$ that solves the optimization problem below:

$$
\begin{align*}
\min_{\{u_{T_f}^{\hat{i}, i}\}_{i=1}^{N_g}} & \sum_{i=1}^{N_g} J(u_{T_f}^{\hat{i}, i}) + \lambda \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} J(u_{T_f}^{\hat{j}, j}) \\
\text{s.t.} & \forall \hat{\epsilon} \neq \hat{\epsilon} \in \mathbb{Z}_{N_f}^{+}, \{u_{T_f}^{\hat{i}, i}\}, \hat{\epsilon} \neq \hat{\epsilon} : J(u_{T_f}^{\hat{i}, i}) < J(u_{T_f}^{\hat{\epsilon}, \hat{\epsilon}}), \\
& \forall \hat{\epsilon} \neq \hat{\epsilon} \in \mathbb{Z}_{N_f}^{+}, \hat{\epsilon} \neq \hat{\epsilon} : J(u_{T_f}^{\hat{i}, i}) < J(u_{T_f}^{\hat{\epsilon}, \hat{\epsilon}}), \\
& \forall x_0, w_T, v_T : (13) \text{ holds} \land (9), (10), (14) \text{ hold}
\end{align*}
$$

where $\lambda > 0$ is a very small constant.

The above problem can be viewed as consisting of 2 phases of planning. In the first phase, we find the first $T$ time steps of the inputs $\{u_{T_f}^{\hat{i}, i}\}_{i=1}^{N_g}$ that cause a separation of the observed output trajectories corresponding to each goal. Specifically, the second predicate in $(17c)$ ensures that for each pair of actuating inputs, there must exist at least one time instance $k \in \mathbb{Z}_{T_f-1}$, where the outputs of the model corresponding to each input are different. Then, given that each input from phase 1 is the optimal input required to reach a particular target state configuration while ensuring the separability condition is achieved, we propose that any deviation from the intended goal state to any other goal would require a greater control effort than maintaining the current trajectory. Thus, in the second phase, we find the inputs $\{u_{T_f}^{\hat{i}, i}\}_{i=1}^{N_g}$ for the rest of the time horizon from $T_f$ until $T_f - 1$ such that the model achieves the intended goals/targets, i.e., $x(T_f) \in X_{g, i}, \forall \hat{\epsilon} \in \mathbb{Z}_{N_f}^{+}, \hat{\epsilon} \neq \hat{\epsilon}$ while ensuring that the deviation to other unintended goals would involve greater control effort, which is encoded by the constraint in $(17b)$. Finally, the first predicate in $(17c)$
guarantees the satisfaction of state constraints (including the goal attainment) over the time horizon $T_f$.

Next, after applying the optimal intent-expressive input sequence over $T$ time steps to the real system and collecting the corresponding noisy output trajectory, we then solve the following intent estimation problem to determine which is the intended model/goal for the agent/robot.

**Problem 2 (Intent Estimation):** Given an observed input-output trajectory $\{u_{T_f}^\delta, z_{T_f}^\delta\}$, a set of intent models $\mathcal{G}^i$, $i \in \mathbb{Z}^+_N$ corresponding to goal regions $\mathcal{X}^i$, and a finite horizon $T_f$, find the optimal input sequences $\bar{u}_{i,T_f}$ for each intent model $\mathcal{G}^i$, $i \in \mathbb{Z}^+_N$ such that for all possible initial states $x_0$, process noise $w_{T_f}$, and measurement noise $v_{T_f}$, the goal region $\mathcal{X}^i$ is attained. The optimization problem for each $\mathcal{G}^i$, $i \in \mathbb{Z}^+_N$ can be formally stated as follows:

$$\bar{u}_{i,T_f} = \arg\min_{u_{T_f}} J(\bar{u}_{i,T_f})$$

s.t. \forall k \in \mathbb{Z}^+_T : (4) holds,

$$\forall k \in \mathbb{Z}^+_T : \left\{ \begin{align*}
\forall x_0, w_{T_f}, v_{T_f} : (9), (10), (14) \text{ hold} ,
\bar{z}_T = z_{T_f}^\delta, \bar{u}_T = u_{T_f}^\delta
\end{align*} \right\} \text{ : (13) holds.}$$

Then, find the model $i \in \mathbb{Z}^+_N$ with the smallest $J(\bar{u}_{i,T_f})$.

The solution to the above problem is then designated as the intended goal/model, which we will show in the following section to correspond to the true intended goal by virtue of the intent-expressive trajectory designed in Problem 1.

**IV. MAIN RESULT**

**A. Intent-Expressive Trajectory Planning**

First, we recast Problem 1 as a bilevel optimization problem. This problem can then be further reformulated into a single level optimization problem using the Karush-Kuhn-Tucker (KKT) conditions.

**Lemma 1 (Bilevel Optimization Formulation):** Given a separability index $\epsilon > 0$ and small constants $\lambda, \gamma > 0$, the intent-expressive trajectory planning problem in Problem 1 is equivalent to finding the optimal $\{u_{T_f}^\delta\}_{i=1}^{N_G}$ that is the solution to the following bilevel optimization problem:

$$\min_{\{u_{T_f}^\delta\}_{i=1}^{N_G}} \sum_{i=1}^{N_G} J(u_{T_f}^\delta) + \lambda \sum_{i=1}^{N_G} J(u_{T_f}^\delta) \quad (P_{Outer})$$

s.t. \forall i \in \mathbb{Z}^+_N : \{u_{T_f}^\delta, u_{T_f}^{\delta, i}\} \text{ satisfy (4)},

$$\forall i \in \mathbb{Z}^+_N : \left\{ \begin{align*}
\forall x_0, w_{T_f}, v_{T_f} : (9), (10), (14) \text{ hold} ,
\bar{z}_T = z_{T_f}^\delta, \bar{u}_T = u_{T_f}^\delta
\end{align*} \right\} \text{ : (13) holds,}$$

$$\forall q \in \mathbb{Z}^+_Q : \delta^\epsilon(u_{T_f}^\delta) \geq \epsilon, \quad \text{(19d)}$$

where $u_{T_f}^\delta$ is the first $m_uT$ elements of $u_{T_f}^\delta$ and $\delta^\epsilon(u_{T_f}^\delta)$ is the solution of the inner problem for separability.

$$\delta^\epsilon(u_{T_f}^\delta) = \inf_{\delta^\epsilon}$$

s.t. (15), (16) hold. \quad (P_{Sep})$$

**Proof:** Similar to [8], the separation condition in (17c) is equivalent to the constraint in (19d). Further, note that the state constraints is not needed in the inner problem since it is guaranteed to be satisfied by (19c). The rest of the optimization problem is the same as in Problem 1.

**Theorem 1 (Intent-Expressive Trajectory Planning):** Given a separability index $\epsilon > 0$ and small constants $\lambda, \gamma > 0$, the intent-expressive trajectory planning problem (Problem 1) is equivalent to finding the optimal $\{u_{T_f}^\delta\}_{i=1}^{N_G}$ that is the solution to the following mixed-integer optimization problem:

$$\min_{\{u_{T_f}^\delta\}_{i=1}^{N_G}} \sum_{i=1}^{N_G} J(u_{T_f}^\delta) + \lambda \sum_{i=1}^{N_G} J(u_{T_f}^\delta) \quad (P_{Did})$$

s.t. $\forall i, \tilde{x} \in \mathbb{Z}^+_N$ : $P_u u_{T_f}^{\delta, \tilde{x}} \leq \tilde{p}_u$,

$$J(u_{T_f}^\delta) \leq J(u_{T_f}^\delta) - \gamma, \text{ if } \tilde{z} \neq \tilde{x},$$

$$\sum_{i=1}^{N_G} \mu_{i,j}^\epsilon, P_i \leq \tilde{p}_u^{\delta, \tilde{x}},$$

$$\sum_{i=1}^{N_G} \mu_{i,j}^\epsilon, P_i \geq 0, \quad \text{(21)}$$

$$\forall q \in \mathbb{Z}^+_Q : \delta^\epsilon(u_{T_f}^\delta) \geq \epsilon,$$

$$0 = 1 - \mu_{q}^\epsilon 1,$$

$$0 = \sum_{i=1}^{N_G} \mu_{i,j}^\epsilon, H_i^q(i, m) + \sum_{j=1}^{N_G} \mu_{j,2}^\epsilon, R_i^q(j, m)$$

$$\forall m = 1, \ldots, n,$$

$$\sum_{i=1}^{N_G} \mu_{i,j}^\epsilon, h_i^q - h_i^q \leq 0, \mu_{i,j}^\epsilon \geq 0, \forall i = 1, \ldots, k,$$

$$\sum_{i=1}^{N_G} \mu_{i,j}^\epsilon, h_i^q - h_i^q \leq 0,$$

$$\forall q \in \mathbb{Z}^+_Q, \forall j \in \mathbb{Z}^+_N : SOS-1 : \{\mu_{i,j}^\epsilon, H_i^q(i, m) - h_i^q \},$$

$$\forall q \in \mathbb{Z}^+_Q, \forall j \in \mathbb{Z}^+_N : SOS-1 : \{\mu_{i,j}^\epsilon, R_i^q(j, m) - S_j(i, m) \}$$

with $u_{T_f}^\delta$ denoting the common subvector (first $m_uT$ elements) of $u_{T_f}^\delta$, $\forall \tilde{x} \in \mathbb{Z}^+_N$:

$$\tilde{p}_u^\epsilon \triangleq \left[ h_{x_{T_f},h_{x_{T_f}}}, \tilde{p}_x \right] \tilde{p}_x \triangleq \left[ h_{x_{T_f},h_{x_{T_f}}}, \tilde{p}_x \right] \tilde{p}_u \triangleq \text{diag}(P_{u_{T_f}} , \tilde{P}_u, \tilde{B}_{u_{T_f}} \tilde{B}_{u_{T_f}}),$$

where $P_{u_{T_f}}^\epsilon, \mu_{i,j}^\epsilon$ and $\mu_{j,2}^\epsilon$ are dual variables, while $H_i^q(j, m)$ and $R_i^q(j, m)$ are the $j$-th row of $H_i^q$ and $R_i^q$, respectively, $\eta = QT(c_0 + c_w + c_v)$ is the number of rows of $H_i^q$, $\kappa = 2QT(c_0 + c_w + c_v)$ is the number of rows of $H_i^q$, $\xi = 2QTp$ is the number of rows of $R_i^q$.

**Proof:** The constraint in (19c) can be recast into (21) by leveraging duality in the literature of robust optimization [22], where $\Pi_{u_{T_f}}^\epsilon$ are the corresponding dual variables. Further, following similar steps as in [8, Th. 1], the separability constraint in (19d) associated with $P_{Sep}$ is equivalent to (22), by introducing dual variables $\mu_{i,j}^\epsilon$ and $\mu_{j,2}^\epsilon$ and applying the KKT conditions for $P_{Sep}$.

**Remark 1:** Note that in our formulation, the separation of the output reachable sets is imposed/required within the time horizon of length $T$ (generally, $T$ is much smaller than $T_f$) to ensure that there will be a noticeable difference in the observed output trajectory towards the different targets/goals within that horizon, as is commonly done in active model discrimination problems, e.g., [8], [9]. Otherwise, there will be no means to infer the intended goal until after the agent/robot reaches the intended goal, which means that the trajectory is not intent-expressive or legible.

Further, we assume the cost function $J(\cdot)$ is convex and specifically, if the cost functions are chosen to be the 1-

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Given the observed input-output trajectories over a time horizon \( T \leq T_f \) and \( T_f \), we aim to find the intended goal.

**Theorem 2 (Intent/Model Estimation):** Given an observed input-output trajectory \( \{u_{i,T}^{obs}, x_{i,T}^{obs}\} \), a set of intent models \( G_i \), \( i \in \mathbb{Z}^+ \), for each model \( G_i \), let \( \hat{u}_{i,T_f}^* \) be a solution to the following linear programming problem:

\[
\hat{u}_{i,T_f}^* = \arg \min_{\hat{u}_{i,T_f}} \ J(\hat{u}_{i,T_f})
\]

\[
\text{s.t.} \quad \hat{P}_u \hat{u}_{i,T_f} \leq \hat{p}_u,
\]

\[
\Pi_1^T H_T + \Pi_2^T \hat{p}_x + \Pi_3^T \hat{h}_x \leq h_{x,T_f,i},
\]

\[
\Pi_1^T H_T + \Pi_2^T \hat{p}_x + \Pi_3^T \hat{h}_x = h_{x,T_f,i},
\]

\[
\Pi_1 \geq 0, \Pi_2 \geq 0,
\]

where \( \Pi_1, \Pi_2, \Pi_3 \) are dual variables, while \( \hat{P}_u, \hat{p}_x, h_x, h_{x,T_f,i} \) are the only observed states over a time horizon of length \( T = 4 \) for the purpose of intent estimation, i.e.,

\[
\begin{align*}
&z_1(k) = x(k) + n_x(k), \\
&z_2(k) = y(k) + n_y(k),
\end{align*}
\]

where \( n_x \) and \( n_y \) are the measurement noises in \( x \) and \( y \) directions. \( C_x \) and \( C_y \) are the damping coefficients in the two directions in \( \frac{\Delta T}{M} \). \( M \) is the mass of the rover in kg, and \( \delta T \) is the sampling time in s. We assume \( n_u(k), n_y(k) \in [-3000, 3000]N, \omega_x(k), \omega_y(k) \in [-50, 50]N, n_x(k), n_y(k) \in [-1, 1]m \) at the final time step \( T = 15 \). The resulting state space model is then given by:

\[
A = \begin{bmatrix}
1 & \delta T & 0 & 0 \\
0 & 0 & 1 & \delta T
\end{bmatrix}, \quad B_u = B_w = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad D_u = D_w = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad D_x = \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix}
\]

Furthermore, the initial state is given by \( X_0 = \{x, y, \omega_x, \omega_y\} : x = 5, y = 1, 1.8 \leq \omega_x \leq 2, 0.5 \leq \omega_y \leq 0.6 \) and the target/goal regions \( X_{g,i} \), \( i \in \mathbb{Z}^+ \) at the final time step \( T_f = 15 \) are described as follows:

\[
X_{g,1} = \left\{ \begin{array}{l}
x \leq 30, \quad y \leq 50, \\
45 \leq y \leq 50
\end{array} \right. \}
\]

\[
X_{g,2} = \left\{ \begin{array}{l}
x \leq 20, \quad y \leq 35, \\
35 \leq y \leq 40
\end{array} \right. \}
\]
The larger translucent boxes which overlay the darker ones show the (noisy) output sets in the first phase of motion (i.e., first T time steps). Here it can be seen that the output sets are completely separate, ensuring that an observer/bystander that can compute the optimal cost-to-go will be able to infer the intended goal/destination by comparing the optimal costs.

b) Intent Estimation: To demonstrate the capability to estimate the correct intent of the agent, assuming that \( X_{g,1} \) is the intended goal, we generate the observed intent-expansive (noisy) input-output trajectories for \( T = 4 \) using Theorem 1, as follows: \( u_T = 10^3 \times [3.0000, 0.4070, 0, 0] \), \( u_{T-1} = 10^3 \times [3.0000, 3.0000, 3.0000, 1.1172] \), and \( \epsilon^b = [4.2986, 1.4901, 6.1814, 1.7651, 8.1036, 3.5052, 10.4357, 6.8568] \).

Then, using the proposed Theorem 2 for intent estimation (with \( J() = \| \cdot \|_1 \)), we obtained the optimal inputs shown in Fig. 2, where \( u_1 = (u_{1,1}, u_{1,2}) \) and \( u_2 = (u_{2,1}, u_{2,2}) \) represent the optimal inputs for the agent/rover with minimum costs-to-go for the different target destinations/goals, i.e., \( X_{g,1} \) and \( X_{g,2} \), respectively. It can be observed that \( J(u_1) \) is lower than \( J(u_2) \), which implies that \( X_{g,1} \) is the intended goal. Further, since the first 4 time steps of the inputs are observed from the designed inputs from Theorem 1, they are identical for \( u_1 \) and \( u_2 \), which is consistent with the result.

VI. CONCLUSION

In this letter, we proposed a novel set-based approach to intent-expansive motion planning and intent estimation for agents/robots with uncertain affine dynamics to complement the large literature on probabilistic intent-expansive/legible motion planning with stochastic environments. Our planning approach leveraged the (set-based) active model discrimination framework to design optimal trajectories that can attain the intended goal states, and at the same time, clearly express the intended goal to an observer/teammate that only observes a partial trajectory before the goal state is reached. Moreover, we proposed a complementary intent estimation algorithm for the observer/teammate to infer the intended goal from observations of a partial trajectory, despite worst-case uncertainties. As future work, we will explore the combination of set-based and probabilistic legible motion planning algorithms to further improve intent-expansiveness and consider more general nonlinear agent/robot dynamics.