Combinatorics in Tarot

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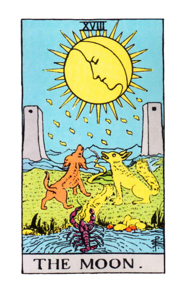
Tarot is a deck of cards used for games and fortune telling. To read one's fortune with Tarot cards, a reader presents a spread of n cards. We use combinatorial methods to calculate how many spreads of n Tarot cards exist. In the Tarot deck there are nine types of constellations comprising disjoint subsets of the deck. We write a Python program to count how many n-card spreads contain at least three cards from any one of these constellations and then find the probability that an n-card spread contains such a subset of cards.

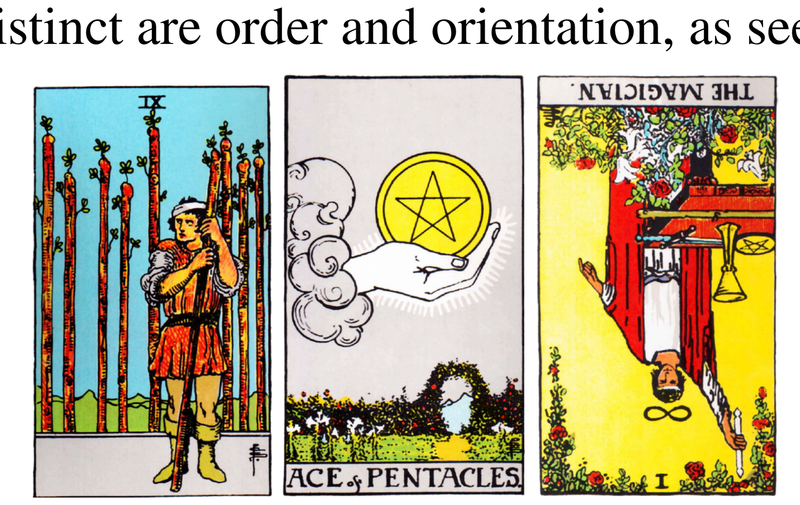
Terminology

The Tarot deck is comprised of 78 distinct cards divided into the major arcana and minor arcana. The Moon is an example of a **Major Arcana** card. Its name is at the bottom and number 0 through 21 at the top. The **Minor Arcana** cards are broken into 4 suits: pentacles, wands, swords, and cups. Each suit contains 14 cards: the court cards (King, Queen, Knight, and Page) and the number cards (numbered Ace and Two through Ten). To find a card's **Identifying Digit** decompose and sum the numbers until reaching a single digit. For example, the identifying digit for 19 is 1 because $\to 1+9 \to 10 \to 1+0 \to 1$. Knowing a card's identifying digit is crucial to placing it in a constellation.



Tarot Spreads and Constellations

A spread is a collection of cards presented during a reading. The cards below represent a three-card spread configuration. The features that make a spread distinct are order and orientation, as seen below.



When counting how many ways we can arrange n cards in a spread we proceed as follows. How many ways can we choose n cards from a deck of 78? How many ways can we order those n cards? How many ways can those n cards be oriented? We know the answer to these questions individually and just need to put it together:

$$\binom{78}{n} \cdot n! \cdot 2^n = \frac{78!}{n!(78-n)!} \cdot n! \cdot 2^n = \frac{78!}{(78-n)!} \cdot 2^n$$

Thus, this is our general form to find the total spreads possible with n cards.

In the Tarot deck there are nine constellations formed around the major arcana cards which have identifying digits from one to nine. The table below lists all the constellations and their cards. When a spread is presented there must be at least three cards from the constellation present for the reader to recognize it.

Tarot Constellation	Major Arcana	Minor Arcana	Total Cards
The Magician	1, 10, 19	10, Ace	11
The High Priestess	2, 11, 20	2	7
The Empress	3, 12, 21	3	7
The Emperor	4, 13, 22	4	7
The Hierophant	5, 14	5	6
The Lovers	6, 15	6	6
The Chariot	7, 16	7	6
Strength	8, 17	8	6
The Hermit	9, 18	9	6

Counting Constellations

Now that we know how many n-card spreads are possible, we wish to know how many of those spreads contain a constellation. We start with specific constellations and then generalize. For a constellation to not appear means that 0, 1, or 2 cards from that constellation are present. Thus the general formulas will find arrangements without the constellation and then subtract from total spreads to yield spreads with a specific constellation.

The Magician:

$$\left(\binom{78}{n} - \left[\binom{67}{n} + \binom{67}{n-1}\binom{11}{1} + \binom{67}{n-2}\binom{11}{2}\right]\right) 2^n \cdot n!$$

The High Priestess, The Empress, or The Emperor:

$$\left(\binom{78}{n} - \left[\binom{71}{n} + \binom{71}{n-1}\binom{7}{1} + \binom{71}{n-2}\binom{7}{2}\right]\right) 2^n \cdot n!$$

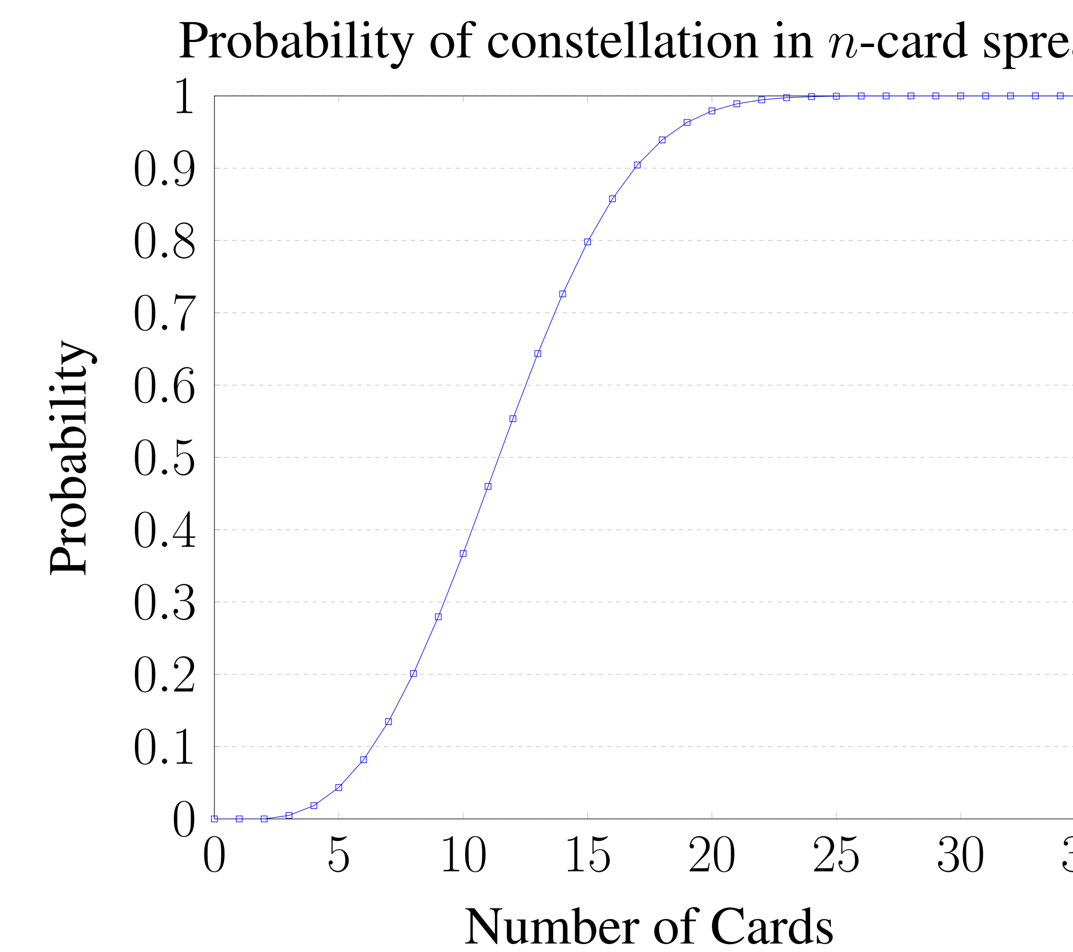
The Hierophant, The Lovers, The Chariot, Strength, or The Hermit:

$$\left(\binom{78}{n} - \left[\binom{72}{n} + \binom{72}{n-1}\binom{6}{1} + \binom{72}{n-2}\binom{6}{2}\right]\right) 2^n \cdot n!$$

The next question is how many spreads have at least one constellation? We use Python to write a counting program that uses 9 nested for loops to iterate through all possible combinations of having 0, 1, or 2 cards present from each of the 9 constellations. The variables $x_1, x_2, x_3, \ldots, x_9$ represent how many cards are present from each of the 9 constellations and x_0 represents how many cards are present from the remaining 16 cards not part of any constellation. The for loops index each variable through their possible values: $x_1, \ldots, x_9 \in \{0, 1, 2\}$, and $x_0 \in \{0, \ldots, 16\}$. Once the sum of the x_i equals n, the variables are used in the formula below which is added to the existing count of possible spreads so far. This formula counts all the ways a subset of the cards can be chosen from each specific constellation and arranged, as order and orientation matter.

$$\binom{11}{x_1}\binom{7}{x_2}\binom{7}{x_3}\binom{7}{x_4}\binom{6}{x_5}\binom{6}{x_6}\binom{6}{x_7}\binom{6}{x_8}\binom{6}{x_9}\binom{16}{x_0} \cdot n! \cdot 2^n$$

Combining these results from our Python program and the calculations of total spreads possible, we now compute the probability that a constellation is present in an n-card spread. For each n we simply divide the number of ways a constellation can appear by the total number of spreads; the resulting probabilities are plotted below.



Why is the probability 1 once we reach 35 cards? By the pigeonhole principle, we must have at least 19 of the 62 cards from the 9 constellations to guarantee 3 belong to the same constellation. Together with the 16 cards not part of any constellation, we must have 16 + 19 = 35 cards in the spread to guarantee the appearance of a constellation.

Future Research

Future research includes finding a general form to calculate the number of spreads with constellations for any n. We could also explore placing restrictions on the appearance of constellations. For example, how many spreads contain The Magician constellation, where the cards are placed consecutively and have no outside cards breaking up the constellation?

Acknowledgments

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