Laser Physics II (PH482/582-VT1 Spring 2026) Tentative Schedule:

#	Date		Text	Topics
1	Jan 12 (Mo)	**Module 1. Tunable Lasers**	LEO Ch11.1-11.4, Class Lecture	Tunable Lasers, Organic Dye Lasers
2	Jan 14 (We)		LEO Ch11.1-11.4, Class Lecture	Tunable Solid-State Lasers, Alexandrite & Ti-Sapph. Lasers, TM:II-VI Lasers(Class Homework 1, Due January 28
	Jan 19 (Mo)		MLK Holiday	No classes
3	Jan 21 (We)		LEO Ch11.1-11.4, Class Lecture	Tunable Solid-State Lasers, Alexandrite & Ti-Sapph. Lasers, TM:II-VI Lasers(Class Lecture)
4	Jan 26 (Mo)		Class Lecture	Color Center Lasers
5	Jan 28 (We)		Exam 1	Exam 1 over chapter 11
6	Feb 2 (Mo)	**Module 2. Semiconductor Lasers**	Ch 12.1-12.7 and class lecture	Semiconductor Lasers, Semiconductor Physics Background
7	Feb 4 (We)		Ch 12.1-12.7 and class lecture	Semiconductor Lasers, Homework 2, Due February 11
8	Feb 9 (Mo)		(Ch.12.8-12.15)	Semiconductor Lasers
9	Feb 11 (We)		(Ch.12.8-12.15) (Class Lecture, Ch.13)	Semiconductor Lasers. Ray Tracing in an Optical System Homework 3, Due Mar
10	Feb 16 (Mo)	**Module 3. Gaussian Beams and Optical Cavities**	(Class Lecture, Ch.13)	Semiconductor problem Solving. Ray Tracing in an Optical System
11	Feb 18 (We)		(Class Lecture or Ch.16.1-16.7)	Gaussian Beams
12	Feb 23 (Mo)		(Class Lecture or Ch.16.1-16.7)	Gaussian Beams
13	Feb 25 (We)		(Class Lect. Ch.15 and Ch.16.8-16.14)	Optical Cavities
14	Mar 2 (Mo)		(Class Lecture or Ch.15 and Ch.16.8-16.14)	Optical Cavities; Three and four Mirror Focused Cavities
15	Mar 4 (We)		(Ch.16.8-16.14)	Optical Cavities; Cavities for Producing Spectral Narrowing of Laser Output. Sample Problems for Test 2
	Mar 9 (Mo)		Spring Break	No classes
	Mar 11 (We)		Spring Break	No classes
16	Mar 16 (Mo)		Exam 2	Exam 2 over chapters 13-16
17	Mar 18 (We)	**Module 4. Nonlinear Optics. Wave propagation in a nonlinear medium. Frequency up and down conversion**	(LEO Ch. 17)	Optics of Anisotropic Media Homework #4 problems 17.2; 17.3; 17.4; 17.5 ch.17 LEO Due March 25.
18	Mar 23 (Mo)		(LEO Ch. 17)	Optics of Anisotropic Media
19	Mar 25 (We)		(LEO Ch. 19, 20)	Wave Propagation in Nonlinear Media Homework # 5 due April 6
20	Mar 30(Mo)		(LEO Ch. 19,20)	2nd Harm. Generation. Up and Down-Conversion, Optical Parametr. Amplificati
21	Apr 1 (We)		(LEO Ch. 19,20)	2nd Harm. Generation. Up and Down-Conversion, Optical Parametr. Amplificati
22	Apr 6 (Mo)		(LEO Ch. 18)	The Electro-Optics and Acousto-Optic Effects and Modulation of Light Beams Homework #6 due April 13 Homework #5 review
23	April 8 (We)	**Module 5. Detection of light**	(LEO Ch.21.1-21.6)	Detection of Optical Radiation
24	April 13 (Mo)		(LEO Ch.21.6-21.8)	Noise in Photodetectors Homework #7 due April 20
25	April 15 (We)		(LEO Ch.21.6-21.8)	Photodiode Arrays and CCDs; Sample problems for test 3
26	April 20 (Mo)		Exam 3	Exam 3 over chapters 18-22
27	April 22 (We)		Review for Final	Review for Final
28	April 27 (Mo)		FINAL GRADES	FINAL EXAM Over chapters 11-22 and class notes 4:15-6:45 pm at ESH3160

Laser Physics II

PH482/582-VTD (Mirov)

Semiconductor Lasers

Ch 12.1-12.15

Lectures 6-9

Spring 2026

C. Davis, “Lasers and Electro-optics”

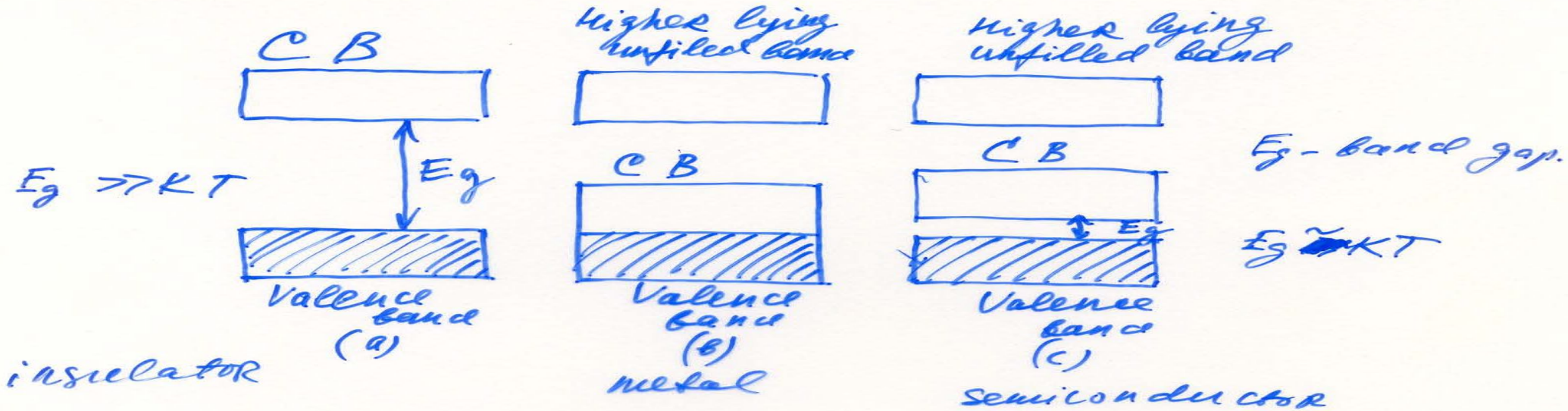
Semiconductor Lasers

- the most widely used of all lasers
- is manufactured in the largest quantities
- is of the greatest practical importance.
  - long & medium distance fiber optics communication.
  - compact disc players

Semiconductor Physics Background

- when atoms combine to form a solid the sharply defined energy states that we associate with free atoms, such as in a gas, broaden considerably, because of the large # of interactions between an atom and its neighbors. The result is a series of broad energy states, called bands.
- The energy states of the outermost $\bar{e}$ broaden and combine to produce a band of filled energy states - valence band.
- Excited energy states of the atoms broaden and combine to give rise to a series of excited energy bands. The lowest, empty energy band is called conduction band.
- The relative disposition of the valence and conduction bands determines whether the solid will be an insulator, a metal, or a semiconductor

Simple band structure (one-dimensional energy representation) for (a) an insulator, (b) a metal, (c) a semiconductor, where $E_g \sim KT$.



Semiconductors:

Silicon, Germanium } monocrystals

$GaAs$, $InSb$ } binary crystals III – V

$Al_x Ga_{1-x} As$ } ternary crystals $x \leq 1$

$In_x Ga_{1-x} As_y P_{1-y}$ } quaternary crystals.

The most important parameter that changes from one semiconductor system to the next is the band gap.

The primary process that leads to the light generation in semicond. involves the e^- falling from the CB to the valence Band and releasing a photon with frequency $\nu = \frac{E_g}{h}$; $\frac{c}{\lambda} = \frac{E_g}{h}$; $\lambda = \frac{ch}{E_g}$

or wavelength $\lambda = \frac{ch}{E_g}$

Material	Energy gap at 300k (eV)	$\lambda = \frac{ch}{E_g}$ (μm)
Germanium	0.66	1.88
Silicon	~~0.66~~ 1.12	~~1.88~~ 1.1
In Sb	0.17	7.3
In As	0.36	3.5
In P	1.35	0.92
Ga P	2.25	0.55
GaAs	1.42	0.87
$In_{0.8}Ga_{0.2}As_{0.34}P_{0.65}$	1.1	1.13
$In_{0.5}Ga_{0.5}P$	2.0	0.62
Al Sb	1.6	1.58
$Al_xGa_{1-x}As$	1.42-1.92	0.65-0.87
$In_{0.53}Ga_{047}As$	0.74	1.67

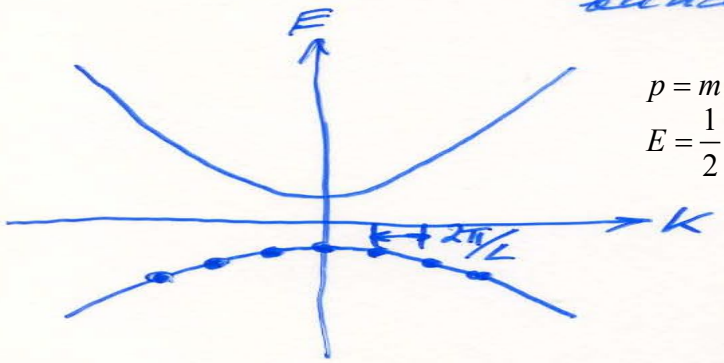
- To explain how electrons move within a band we must a) allow for the existance of at least one vacant state in the band.
  b) remember that because e behave like waves, like the modes of the black body radiation field they are quantized.

- In a cubic crystal of side "L" the allowed wave vectors of the $\bar{e}$ can only take values

$$K_i = \frac{2\pi n}{L} \qquad (i = x, y, z)$$

n = ± integer, or zero.

conclusion: there are discrete electron states available in the valence and conduction bands.

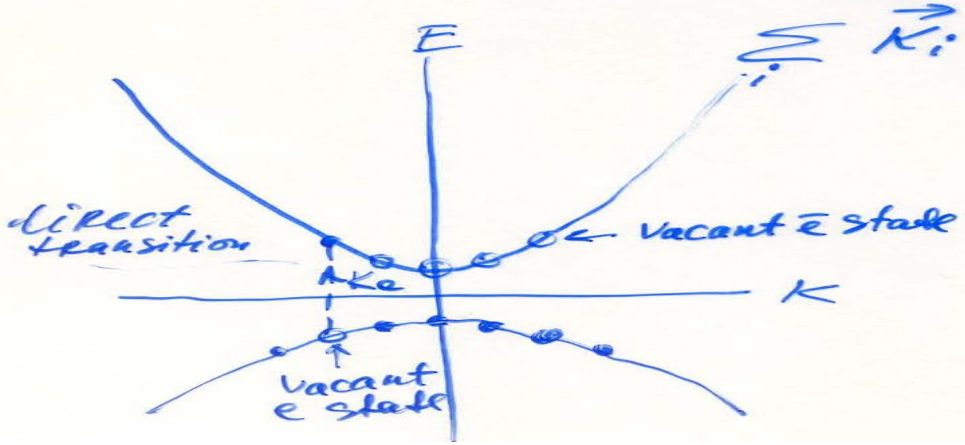


$$p = m^* v;\ p = h/\lambda = \hbar k$$
$$E = \frac{1}{2} m^* v^2 = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}$$

Parabolic energy band diagram showing the filled discrete states in the valence band at 0K.

- For a filled energy band, the vector sum of the momenta of all the electrons is zero

$$\sum_i \vec{K}_i = 0$$



Parabolic energy band diagram of a direct band gap semiconductor in which an $\bar{e}$ has been excited from the Val. Band to a vacant state in the CB leaving a vacancy in the valence band.

- the vacancy left in the valence band behaves very much like a positive charge called a <u>hole</u>.
- the wave vector of the e is $\vec{k}_e$, so to conserve total momentum of all the $\bar{e}$ (in V.B. & C.B.) the total momentum of the $\bar{e}$ in the valence band $\vec{k}_h = -\vec{k}_e$
- ⟹ the hole is not really located on the E-k diagram at the location of the electr. vacancy.
- if an electric field is applied to a semiconductor then electrical conduction occurs through motion of the cond. band e in the opposite direction to the applied field, and holes in the same direction as the field.

$$m_h^* = -m_e^* \quad \text{(valence band e)}$$

- Under the action of an applied field the vacancy in the valence band does not move in the simple fashion.
- Motion of electrons into and out of the vacancy leads to motion of vacancy in the opposite direction to the applied field, because the equivalent hole is moving in the same direction as the field.
- In describing basic properties of semiconductors we treat material as containing ⊖ $\bar{e}$ and ⊕ holes.

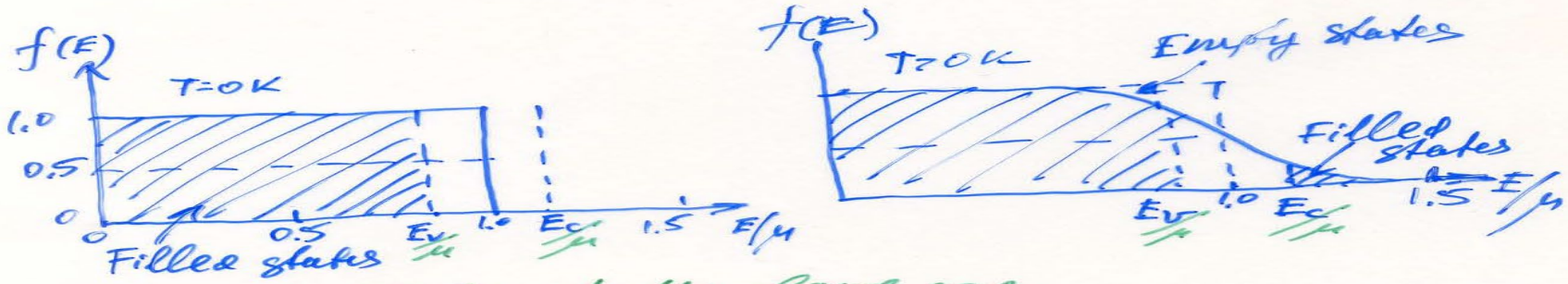
Carrier Concentrations

- In a very pure (intrinsic) semiconductor at T = 0°K there are some $\bar{e}$ in the CB and corresponding holes in the V.B.
- To determine their concentrations at temp. T we must find the probability that an $\bar{e}$ is thermally excited across the band gap.
- This probability is not a $e^{-E_g/KT}$ because $\bar{e}$ distribute themselves among available energy states according to Fermi-Dirac statistics.
- Electrons avoid each other to the extent that no two electrons can occupy exactly the same energy state.
- The probability an $\bar{e}$ will occupy an energy state of energy E_K is given

$$f(E_K) = \frac{1}{e^{(E_K-\mu)/KT}+1}$$

; μ – Fermi level (reference energy level)

for $E_K - \mu >> KT$ $\quad f(E_K) = e^{(\mu-E_K)/KT}$



- μ is in the center of the band gap.
- at the Fermi level $f(E) = \frac{1}{2}$; probability of a state at energy E being occupied.
- since there are no energy states in the band gap there will be no $\bar{e}$ in a state at energy μ.

The Fermi level can be considered to be a hypothetical energy level of an electron, such that at thermodynamic equilibrium this energy level would have a *50% probability of being occupied at any given time*.

- to calculate the $\bar{e}$ concentration we need, in addition to the probability of occupation of each mode, the actual # of states involved
- calculation is similar to the one for Black-body radiation.
- the # of states per unit volume for which the $\bar{e}$ waves have wave vectors within a small range dk at $k = |\vec{k}|$ is

$$\rho(k)dk = \frac{k^2}{\pi^2} dk$$

- the $\bar{e}$ have 2 different kind of states - spin states
- for an $\bar{e}$ in the CB the total energy

$$E_k = E_c + \frac{\hbar^2 k^2}{2 m_e^*}$$

- $$k = \sqrt{\frac{2m_e^*}{\hbar^2}} \sqrt{E_k - E_c}$$

- $$dk = \frac{1}{2} \sqrt{\frac{2m_e^*}{\hbar^2}} \frac{dE_k}{\sqrt{E_k - E_c}}$$

- The density of states $\rho(E_k)$, which tells how many $\bar{e}$ states exist in a range of energies between E_k and $E_k + dE_k$ is found from

$$\rho(E_k)dE_k = \rho(k)dk \Rightarrow \frac{k^2}{\pi^2} dk = \rho(E_k)dE_k$$

- $$\Rightarrow \boxed{\rho(E_k)dE_k = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} \sqrt{E_k - E_c}\, dE_k}$$

- the total # of $\bar{e}$ in the C.B is

$$n = \int_{E_c}^{\infty} p(E_k) f(E_k) dE_k$$

$$f(E_k) = \frac{1}{e^{(E_k - \mu)/kT} + 1} \; ; \quad p(E_k) dE_k = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} \sqrt{E_k - E_c}\, dE_k$$

- $n = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} e^{\mu/kT} \int_{E_c}^{\infty} \sqrt{E_k - E_c}\, e^{-(E_k/kT)}\, dE_k =$

$$= 2 \left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2} e^{\frac{(\mu - E_c)}{kT}}$$

- a similar result for the concentr. of holes in the V.B. can be derived if we note $f_h(E) = 1 - f(E)$

$$\Rightarrow f_h(E) = \frac{1}{e^{(\mu - E_k)/kT} + 1}$$

- hole concentration

$$p = 2 \left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2} e^{(E_v - \mu)/kT}$$

- noting that $E_c - E_v = E_g$

$$np = 4 \left(\frac{kT}{2\pi\hbar^2}\right)^3 (m_e^* m_h^*)^{3/2} e^{-\frac{E_g}{kT}} = K^2 e^{-\frac{E_g}{kT}}$$

Intrinsic and Extrinsic Semiconductors

- The conductivity of a semiconductor is determined by the concentration of conduction band $\bar{e}$ and valence band holes, and by ability of these charges to move through the crystal under the action of electric field, characterized by their mobilities μ_e and μ_h.

$$\sigma = n e \mu_e + p e \mu_h$$; e – magnit. of electronic charge.

- Conductivity is controlled by doping.
- On $10^{-7} \div 10^{4}\ S \cdot m^{-1}$ S ( ) unit for conductance $= \Omega^{-1}$

siemens

insulator $\sigma \leq 10^{-9}\ S \cdot m^{-1}$

conductor $\sigma > 10^{6}\ S \cdot m^{-1}$

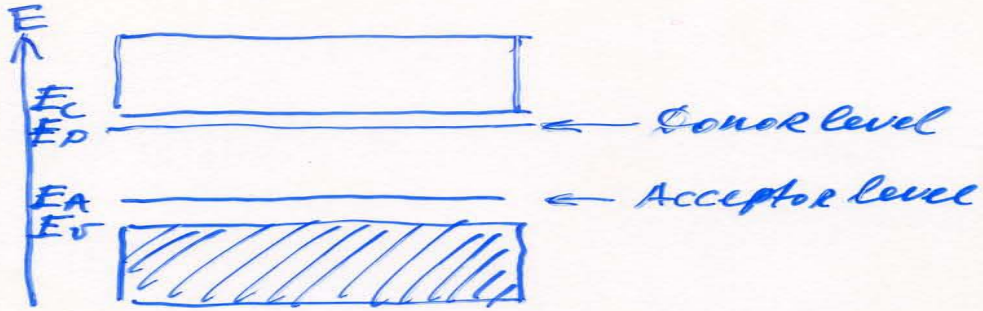
- intrinsic semiconductors $n = p$ (no impurities)
- extrinsic semiconductors – impurity dopants are added.

Examples:

1) GaAs n type. doping Si or Ge in Ga sites.
III group IV group V group.
Si or Ge have an additional $\bar{e}$ that is donated to the conduction band

2) GaAs p type. doping Zn, or Cd (group II) in Ga sites
Zn or Cd would like to accept $\bar{e}$

3) GaAs p type replacing As (group V) with Si or Ge (IV)



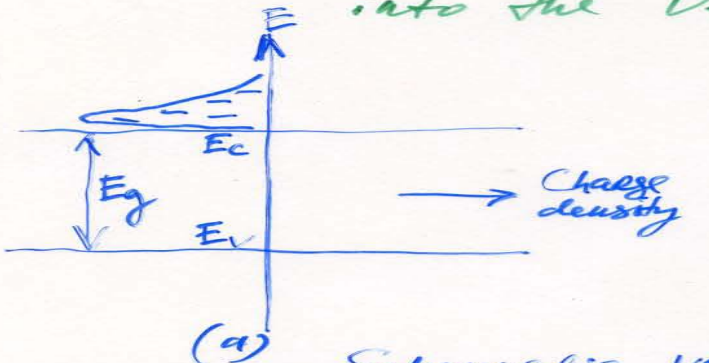
Schematic band diagram showing position of donor and acceptor levels in the band gap of a doped semiconductor

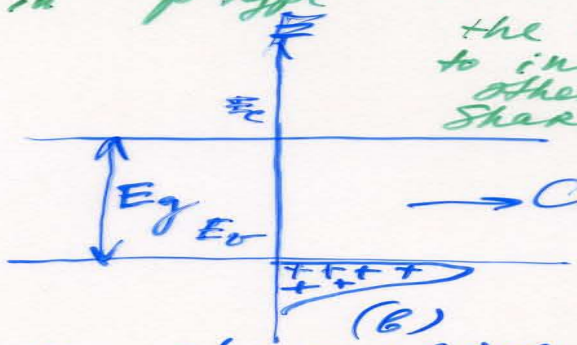
n type $n > p$

p type $p > n$

- when intrinsic semiconductor is doped the Fermi level moves from the center of the band gap → C.B. n doping
  → V.B. p doping.
- when doping is high enough Fermi level moves into the C.B. in n type material & degenerately into the V.B in p type material } doped material

the impurity atoms begin to interact with each other and their once sharp energy levels start to spread and merge with neighboring band.





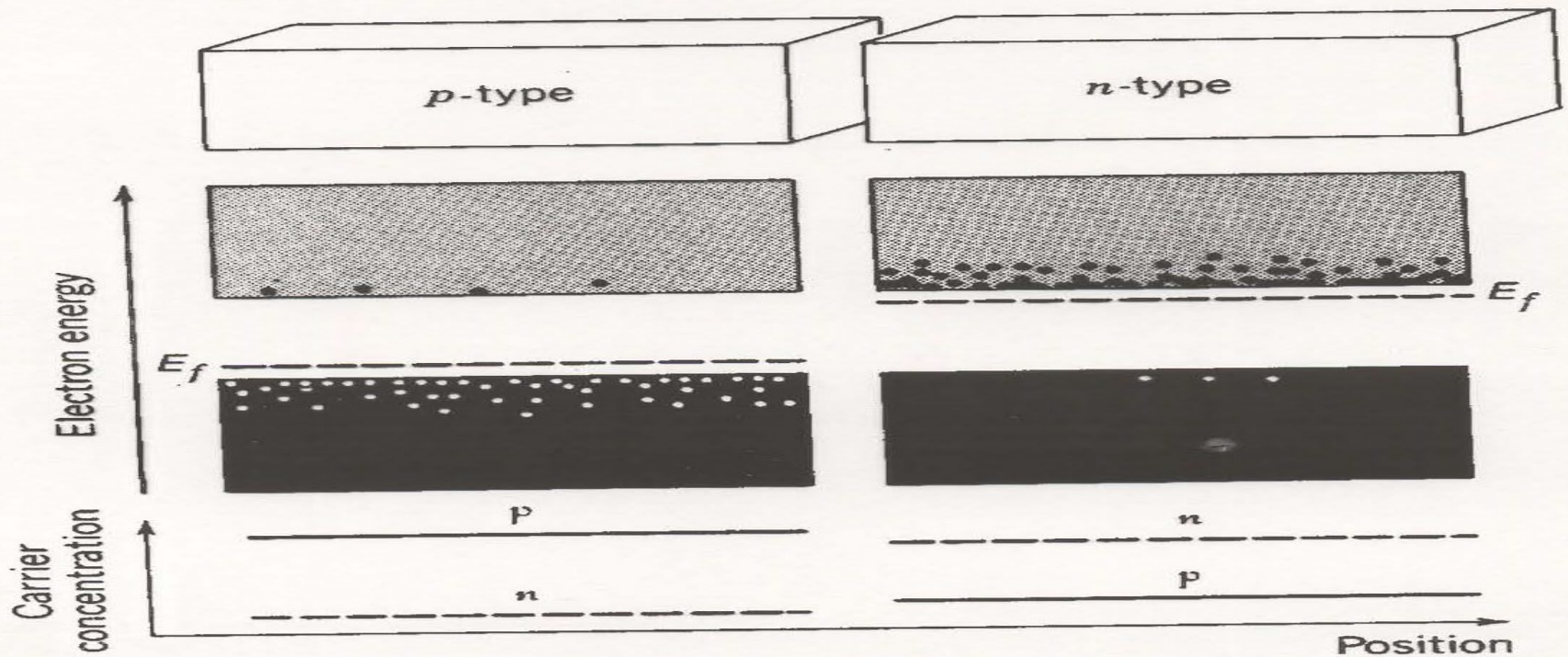
Schematic variation of carrier concentration in degenerately doped a) n type and b) p-type semiconductors.

- if we can superimpose the two halves (a) and (b) we would have a condition of population inversion.
- we do this by forward biasing a p-n junction.

The p-n Junction

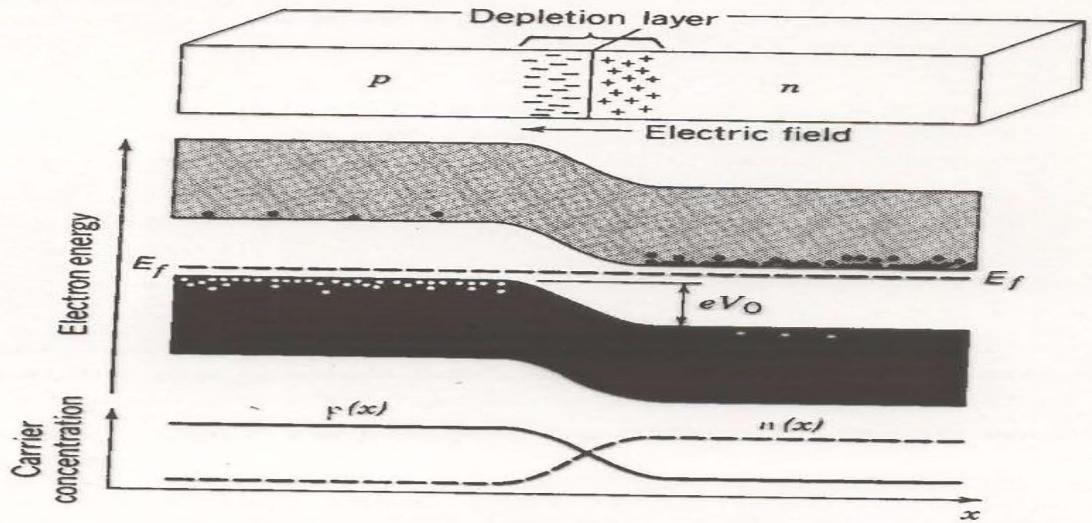
The p-n **junction is a homojunction between a p-type and an *n-type* semiconductor. It acts as a diode which can serve in electronics as a rectifier, logic gate, voltage regulator (Zener diode), and tuner (varactor diode); and in optoelectronics as a light-emitting diode (LED), laser diode, photodetector, and solar cell.**

A *p-n* junction consists of a p-type and an n-type section of the same semiconducting materials in metallurgical contact with each other. The *p*-type region has an abundance of holes (majority carriers) and few mobile electrons (minority carriers); the n-type region has an abundance of mobile electrons and few holes. Both charge carriers are in continuous random thermal motion in all directions.



When the two regions are brought into contact the following sequence of event takes place:

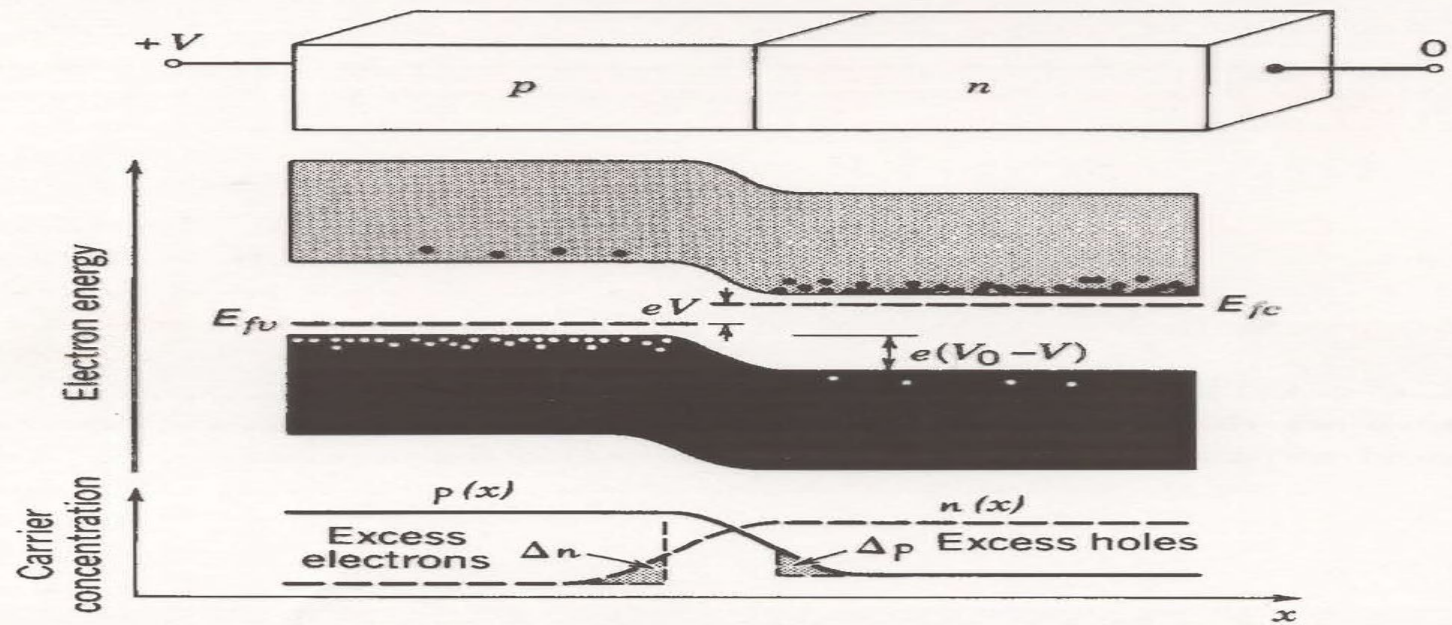
- Electrons and holes diffuse from areas of high concentration toward areas of low concentration. Thus electrons diffuse away from the n-region into the p-region, leaving behind positively charged ionized donor atoms. In the p-region the electrons recombine with the abundant holes. Similarly, holes diffuse away from the p-region, leaving behind negatively charged ionized acceptor atoms. In the n-region the holes recombine with the abundant mobile electrons. This diffusion process cannot continue indefinitely, however, because it causes a disruption of the charge balance in the two regions.
- As a result, a narrow region on both sides of the junction becomes almost totally depleted of *mobile* charge carriers. This region is called the **depletion layer**. It contains only the fixed charges (positive ions on the n-side and negative ions on the p-side. The thickness of the depletion layer in each region is inversely proportional to the concentration of dopants in the region.
- The fixed charges create an electric field in the depletion layer which points from the *n*-side toward the *p*-side of the junction. The **built-in field** obstructs the diffusion of further mobile carriers through the junction region.
- An equilibrium condition is established that results in a net built-in potential difference V_o between the two sides of the depletion layer, with the *n*-side exhibiting a higher potential than the *p*-side.
- The built-in potential provides a lower potential energy for an electron on the n-side relative to the p-side. As a result, the energy bands bend. In thermal equilibrium there is only a single Fermi function for the entire structure so that the Fermi levels in the *p*- and *n*- regions must align.
- No net current flows across the junction. The diffusion and drift currents cancel for the electrons and holes independently.



A p-n junction in thermal equilibrium at T>0K. The depletion layer, energy band diagram, and concentration mobile electrons n(x) and holes p(x) are shown as functions of position x. The buils build in potential differe V_o corresponds to an energy eV_o, where e is the magnitude of the electron charge.

The Biased Junction

- An externally applied potential will alter the potential difference between the *p*- and n-regions. This, in turn, will modify the flow of majority carriers, so that the junction can be used as a "gate."
- If the junction is **forward biased** by applying a positive voltage *V* to the *p*-region, its potential is increased with respect to the *n*-region, so that an electric field is produced in a direction opposite to that of the built-in field. The presence of the external bias voltage causes a departure from equilibrium and a misalignment of the Fermi levels in the *p*- and *n*-regions, as well as in the depletion layer. The presence of two Fermi levels in the depletion layer, E_{fc} and E_{fv} represents a state of quasi-equilibrium.
- The net effect of the forward bias is a reduction in the height of the potential-energy hill by an amount eV. The majority carrier current turns out to increase by an exponential factor $exp(eV/k_BT)$ so that the net current becomes $i = i_s\, exp(eV/k_BT) - i_s$, where i_s is a constant.
- The excess majority carrier holes and electrons that enter the n- and p- regions, respectively, become minority carriers and recombine with the local majority carriers. Their concentration therefore decreases with distance from the junction. This process is known as **minorirty carrier injection.**



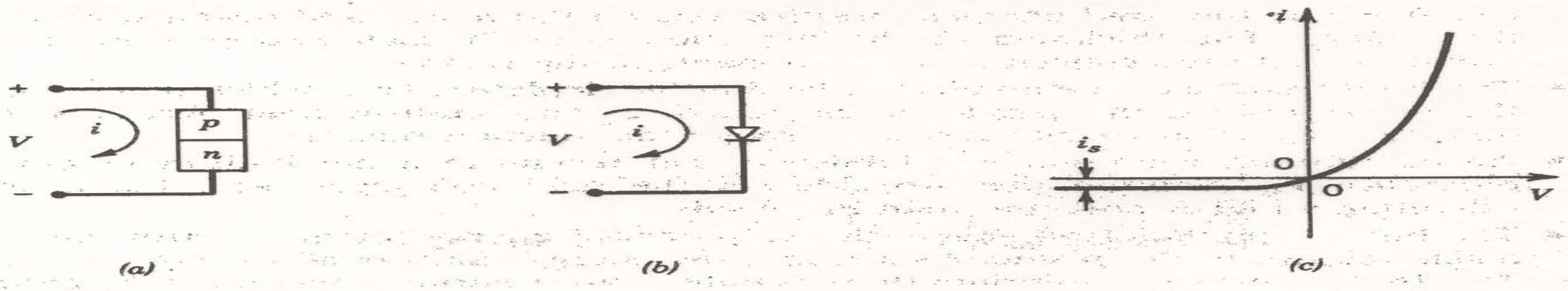
Energy-band diagram and carrier concentrations in a forward-biased p-n junction

Reverse Biased Junction

If the junction is **reverse biased** by applying a negative voltage V to the p-region, the height of the potential-energy hill is augmented by eV. This impedes the flow of majority carriers. The corresponding current is multiplied by the exponential factor $exp(eV/k_BT)$, where V is negative; i.e., it is reduced. The net result for the current is $i = i_s\ exp(eV/k_BT)\ -i_s$, so that a small current of magnitude $\sim i_s$ flows in the reverse direction when $|V| >> k_BT/e$.

A p-n junction therefore acts as a diode with a current-voltage $(i$-$V)$ characteristic

$$i = i_s\left[\exp\left(\frac{eV}{k_BT}\right) - 1\right]$$



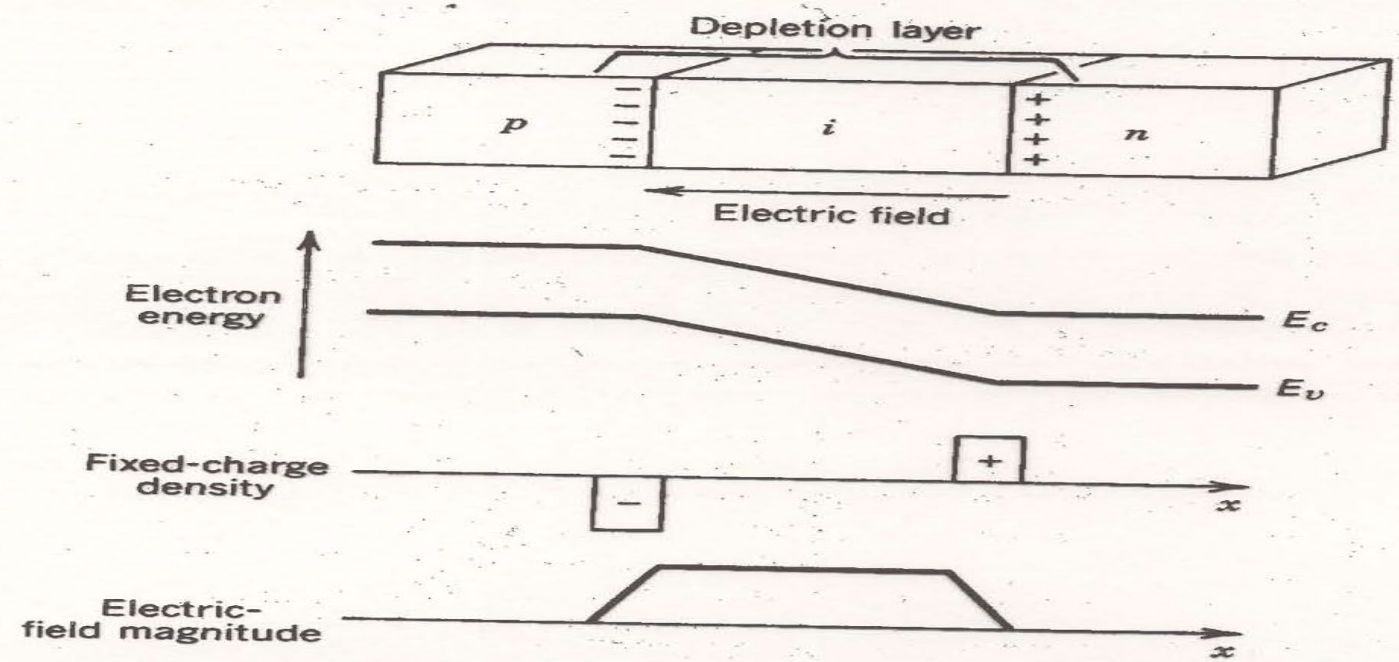
(a) Voltage and current in a p-n junction.
(b) Circuit representation of the p-n junction
(c) Current-voltage characteristic of the ideal p-n junction diode

The response of a p-n junction to a dynamic (ac) applied voltage is determined by solving the set of differential equations governing the processes of electron and hole diffusion, drift (under the influence of the built-in and external electric fields), and recombination. These effects are important for determining the speed at which the diode can be operated. They may be conveniently modeled by two capacitances, a **junction capacitance** and a **diffusion capacitance**, in parallel with an ideal diode. The **junction capacitance** accounts for the time necessary to change the fixed positive and negative charges stored in the depletion layer when the applied voltage changes. The thickness l of the depletion layer turns out to be proportional to $(Vo\ -V)^{1/2}$; it therefore increases under reverse-bias conditions (negative V) and decreases under forward-bias conditions (positive V).

Minority carrier injection in a forward-biased diode is described by the **diffusion capacitance**, which depends on the minority carrier lifetime and the operating current.

The p-i-n Junction Diode

A *p-i-n* diode is made by inserting a layer of intrinsic (or lightly doped) semiconductor material between a p-type region and an n-type region. Because the depletion layer extends into each side of a junction by a distance inversely proportional to the doping concentration, the depletion layer of the *p-i* junction penetrates deeply into the i-region. Similarly, the depletion layer of the *i-n* junction extends well into the i-region. As a result, the *p-i-n* diode can behave like a *p-n* junction with a depletion layer that encompasses the entire intrinsic region. The electron energy, density of fixed charges, and the electric field in a *p-i-n* diode in thermal equilibrium are illustrated below.

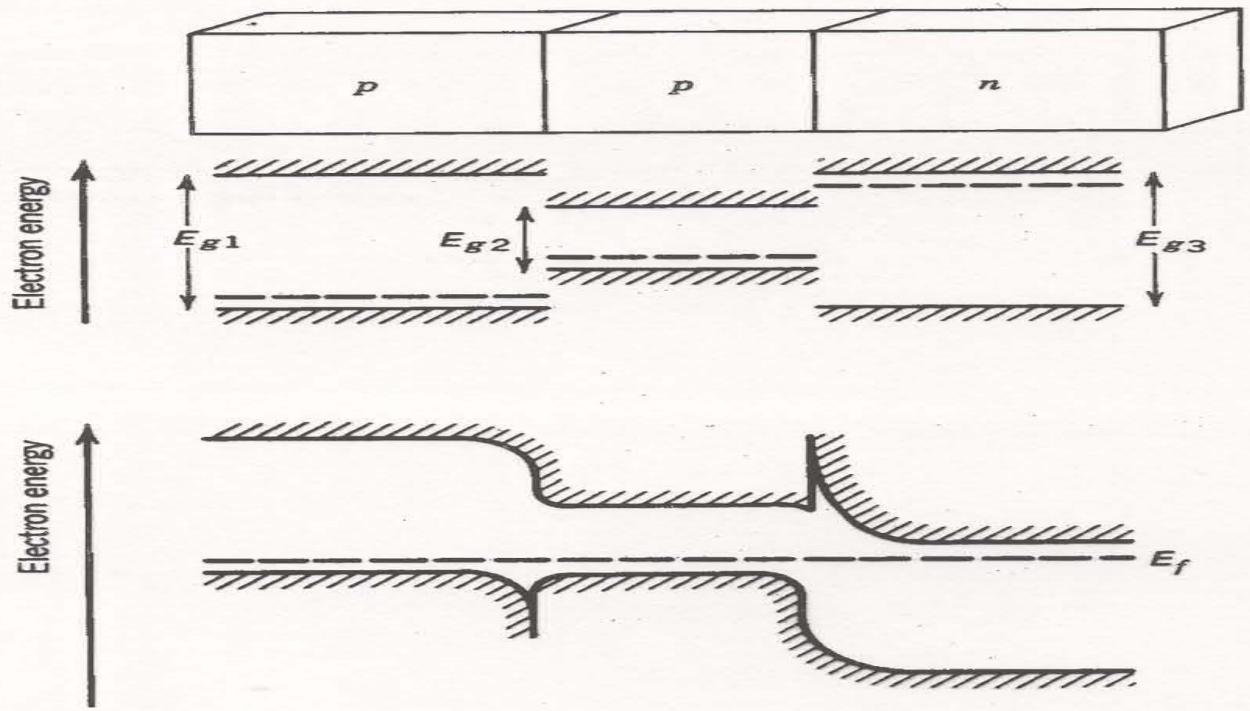


Electron energy, fixed charge energy, and electric field magnitude for p-I-n diode in thermal equilibrium

One advantage of using a diode with a large depletion layer is its small junction capacitance and its consequent fast response. For this reason, *p-i-n* diodes are favored over *p-n* diodes for use as semiconductor photodiodes. The large depletion layer also permits an increased fraction of the incident light to be captured, thereby increasing the photodetection efficiency

Heterojunctions

Junctions between different semiconductor materials are called heterojunctions. They provide substantial improvement in the performance of electronic and optoelectronic devices.

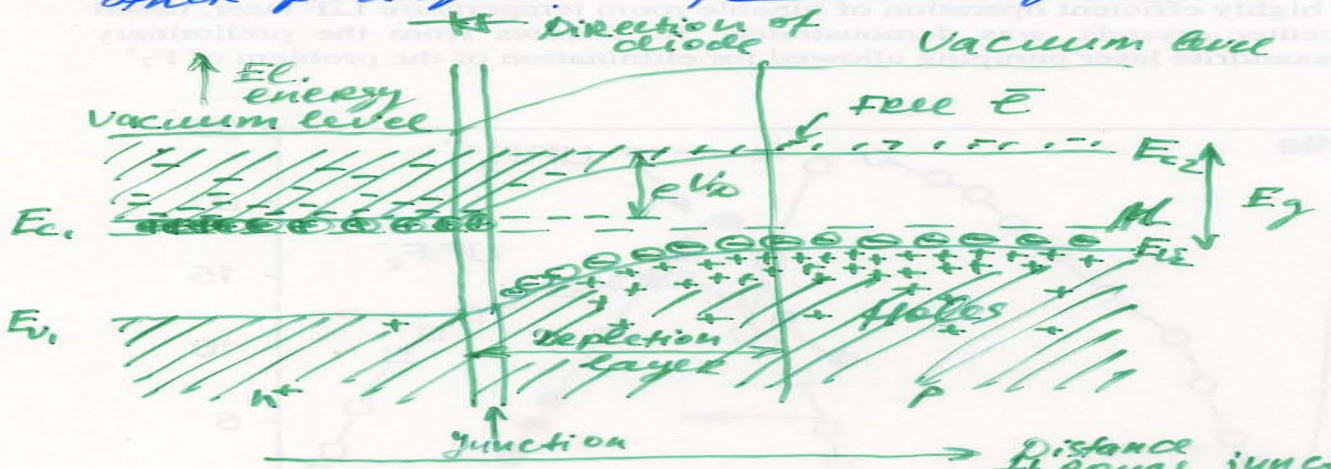


The *p-p-n* double heterojunction structure.

The middle layer is of narrower bandgap than the outer layers.
In equilibrium, the Fermi levels align so that the edge of the conduction band drops sharply at the *p-p* junction and the edge of the valence band drops sharply at the *p-n* junction. The ratio of the difference in conduction-band energies to the difference in valence-band energies is known as the band offset. When the device is forward biased, these jumps act as barriers that confine the injected minority carriers. Electrons injected from the n-region, for example, are prevented from diffusing beyond the barrier at the *p-p* junction. Similarly, holes injected from the p-region are not permitted to diffuse beyond the energy barrier at the *p-n* junction. This double heterostructure therefore forces electrons and holes to occupy a narrow common region. This is essential for the efficient operation of an injection laser diode

The p-n Junction

- the p-n junction is the interface region between two identical semiconductors, one of which is n-doped, the other p-doped. p-n homojunction.

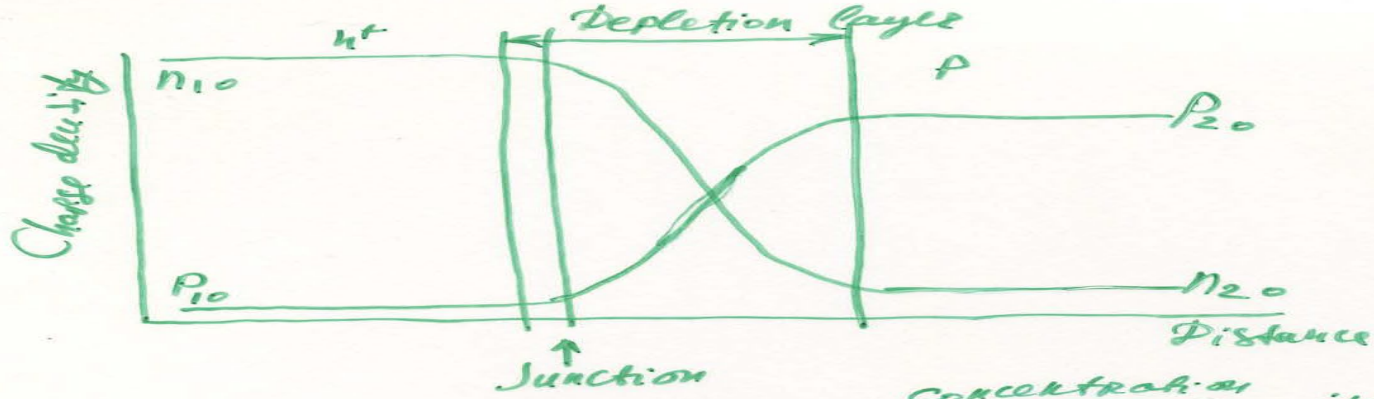


Band diagram of a p-n junction in equilibrium showing relative position of e and holes.

- the n type material is more heavily doped than p type.
- Fermi level is closer to the conduction band.
- the materials are designated n^+ and p to indicate this.
  $n_D \sim 10^{24}\ m^{-3}$ donor; $n_A \sim 10^{22} m^{-3}$.
- vacuum level is the energy to which ē would need to be raised to escape completely from semiconductor.
- in thermal equilibrium an ē that travels from n^+ to p must climb an energy hill eV_D, where V_D - diffusion potential.

$$eV_D = E_g - (E_{c_1} - \mu) - (\mu - E_{v_2})$$

- In a region called the depletion layer, which spans the junction, ē fall down the potential hill, while holes "fall" up the hill. (because they are positively charged their energy is lower in the p-side of the junction). $p \times n = const.$ The hill constitutes an energy barrier to the passage of e from $n^+ \Rightarrow p$ and holes from $p \Rightarrow n^+$

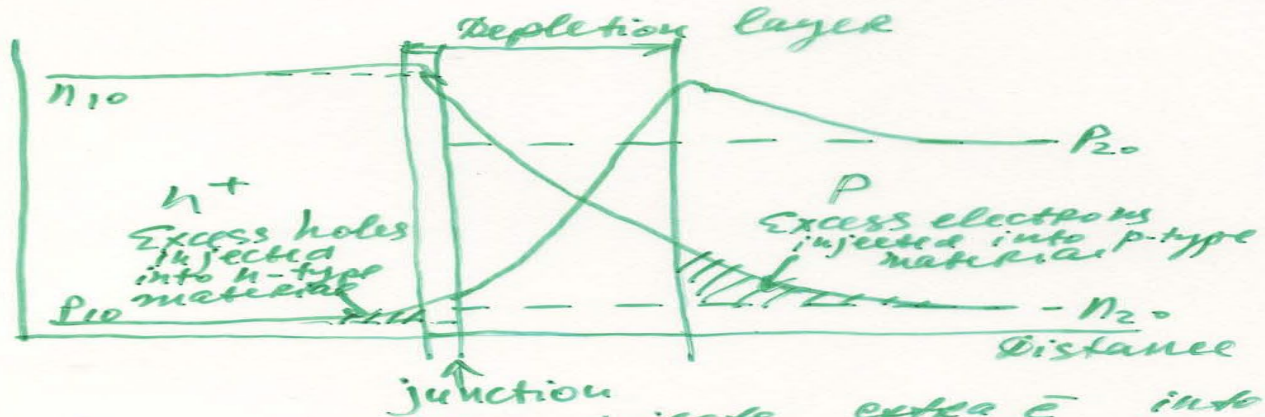


Schematic charge density variation of electrons & holes across a p-n junction in equilibrium

- Far from the junction concentration of n_{10} ; p_{10} in n^+ material
  n_{20} ; p_{20} in p material

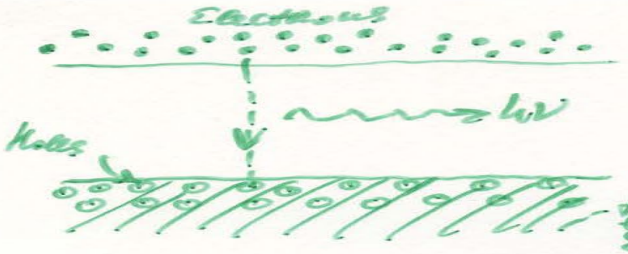
$n_{10} p_{10} = n_{20} p_{20} = n_i^2$ – equilibrium e and h concentration in undoped material.

- the probability factor that describes the likehood of a charge carrier being able to climb the hill is $e - e(V_0 - V)/KT$
- current – voltage relation $I = I_S (e^{eV/KT} - 1)$



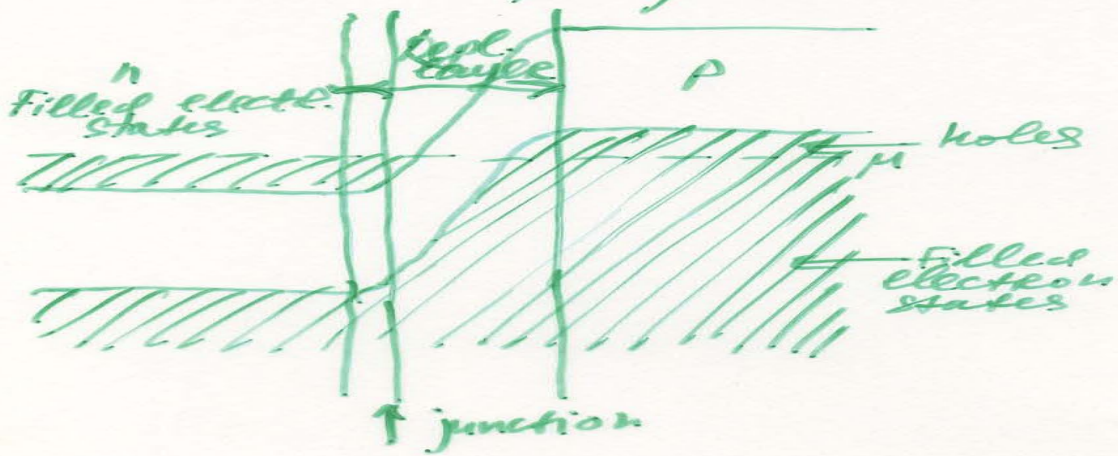
Charge density variation of electrons and holes across a forward biased p-n junction

- Current flow injects extra $\bar{e}$ into the p-type material, and a relatively small # of extra holes into the n-type material (p type is less heavily doped)
- to preserve electrical neutrality across the junction the concentr. of $\bar{e}$ in n^+ and n in p type falls to equal injected excess carriers.

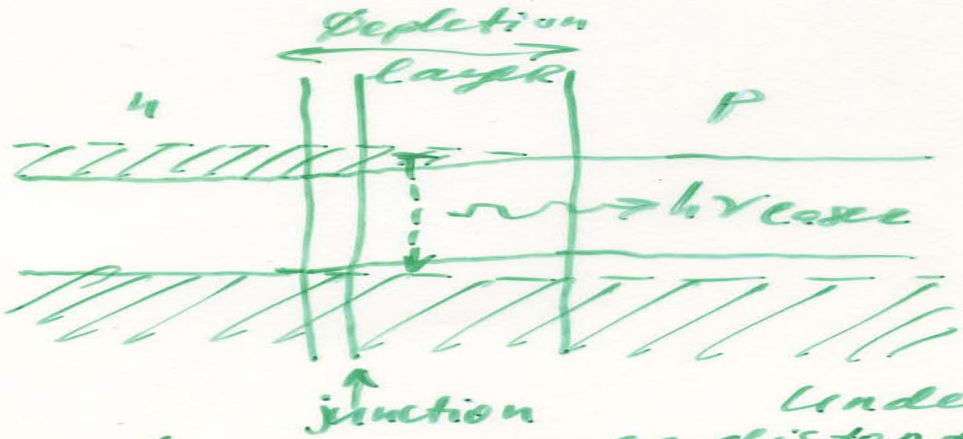


The excess carriers disappear by recombination that is accompanied by photon emission (Spontaneous Emission process).

The efficiency with which excess carriers recombine radiatively strongly determines the utility of the junction as a light emitting diode LED.



Band structure at the junction between two degenerately doped semiconductors at low temperature



Forward biased p-n junction between degenerately doped semiconductors showing schematically how a population inversion can be obtained.

Under forward bias the energy band diagram would distort to provide a region where filled electron states in the C.B. find themselves directly above empty electron states in the V.B. The resulting population inversion is ideal for laser oscillation.

Recombination and Luminescence

- the rate at which $\bar{e}$ and h recombine near a p-n junction is determined by the excess volume density of these carriers and their lifetimes.
- for excess electrons recombining in p-type material $\frac{d(\Delta n)}{dt} = -\frac{\Delta n}{\tau_p}$

where Δn is the excess electron volume density injected into the p-region and τ_p is the lifetime of these electrons in the p-type material.

- the actual rate of recombination depends on the local $e^{(n)}$ and hole (p) concentrations.
- $\frac{d(\Delta n)}{dt} = -\Delta n \left(\frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}}\right)$

τ_{rr}, τ_{nr} - lifetimes of radiative and nonradiative recombination.

- the fraction of recombinations that occurs radiatively is called the internal quantum efficiency, η_{int}

$$\eta_{int} = \frac{1}{1 + \frac{\tau_{rr}}{\tau_{nr}}}$$

- the overall recombination rate R_{rr} is

$$R_{rr} = r n p$$

where r is recombination rate constant ($m^3 s^{-1}$) characteristic of material.

- the generated optical power/unit volume is

$$\phi = r n p \eta_{int} (h\bar{\nu})$$

where $h\bar{\nu}$ - average photon energies of recombination.

- in an efficient LED most of the carriers that recombine do so radiatively.
- much of the current through the junction is carried by electrons, which generally have higher mobility than the holes.

  GaSe : $\mu_e = 0.8\ m^2 V^{-1} s^{-1}$; $\mu_h = 0.03\ m^2 V^{-1} s^{-1}$

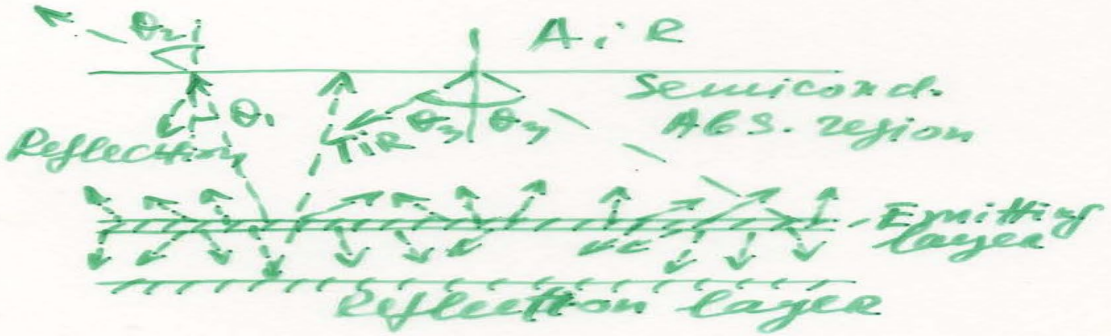
- the injection efficiency $\eta_i = \frac{\text{electron current}}{\text{total current}}$
- the optical power generated by an injected current I is:

$$\Phi = \eta_{int} \frac{I}{e} h\nu$$

- in a good LED with direct band gap material $\eta_{int} = 0.5$.
- most of the radiation is generated within a few diffusion lengths of the junction ~ a few tens of μm and depends on temperature and doping level.
- the probability of band to band recombination, which is the most desirable process for generating high energy photons is reduced in indirect band gap semiconductors (Si, Ge, GaP).

External Quantum Efficiency

- it is not sufficient to build an LED by using an active region where η_{int} is high.
- this will guarantee efficient photon generation, but unless these photons escape from the device useful light will not be generated.
- 7 four principal ways in which generated photons fail to escape from the active region:

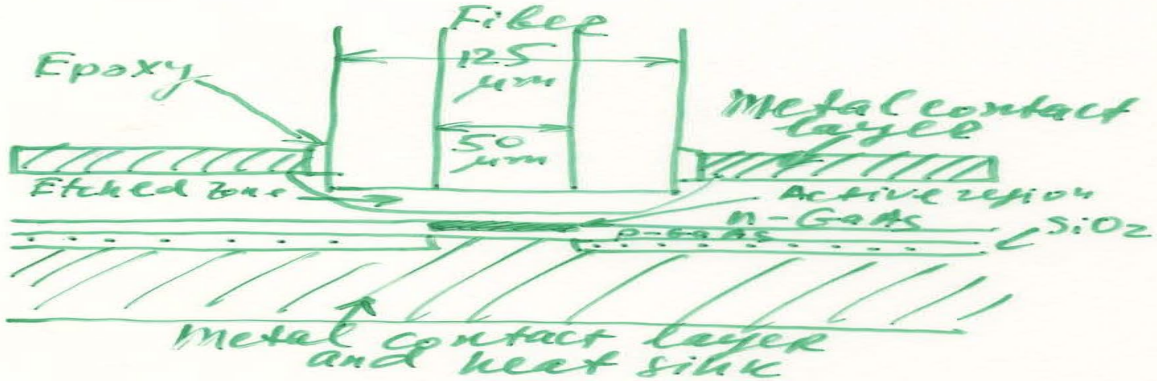


Schematic illustration of factors that affect the external quantum efficiency:

1) greatest importance of the reabsorption of photons in bulk material between the active layer and surface.
2) Reflectance losses at the outer Semiconductor air interface — AR coating
3) Total internal reflection at the outer interface
4) The half of the radiation that is not emitted towards the exit surface can be reduced with a reflective layer.

The external quantum efficiency η_{ext}

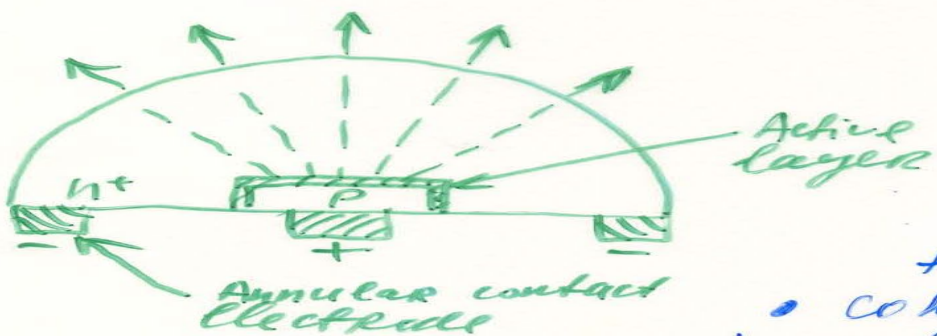
$$\eta_{ext} = \frac{\text{photons leaving exit interface}}{\#\text{ of carriers crossing junction}}$$



The most efficient way to ↑ η_{ext} is to build a planar structure in which an active layer is very close to the surface.

BURRUS-type surface emitting LED

A structure is produced in a series of steps involving the deposition of one layer of material on another and selective etching to produce the "etched-well" that allows the end of the fiber to be placed close to the active region.



Simple hemispherical LED.

- the most important mechanism of losses is reabsorption of emitted photons within the unexcited regions
- why? $h\nu > E_g$
- how to overcome? heterojunctions

to build composite structures in which the band gaps of the semic- regions near active layer are larger $\Rightarrow$ $h\nu < E_g$ – no reabsorption.

Heterojunctions

- Forward biased homojunction LEDs & LASERS are not optimal.
- their efficiency is characterized by the use of the η_{int} = $\frac{\#\text{ of band-to-band radiative recombins}}{\#\text{ of carriers crossing junction}}$

- $\frac{1}{\tau_p} = A + X$

  X – coeff for nonradiative recombin.
  A – Einstein coeff describing radiative recombin of e injected into p-type material with holes.

- $\eta_{int} = \frac{1}{1 + X/A}$

  ⇒ for efficient photon generation
  1) $A >> X$
  2) as many carriers as possible recombine soon after crossing the junction, and do not escape to travel into the bulk material far from the junction.
  3) e & h should be trapped in the region where recombination is desired

This trapping can be effected by the use of layered semiconductor structures that use heterojunctions.

Ternary and Quaternary Lattice Matched Materials

- it is possible to replace Ga atoms in GaAs with Al to make the ternary material $Ga_{1-x}Al_xAs$ without any significant change in the atomic arrangement of the lattice. upto $x = 0.37$ the ternary material remains a direct bandgap semiconductor.

  $E_g^{GaAs} = 1.42$ ev

  $E_g^{Ga_{0.63}Al_{0.37}As} = 1.92$ ev.

- it is possible to grow a layer of GaAlAs on GaAs or vice versa
- lattice-matched materials - no strain at the interface between them.
- the material between the GaAs and GaAlAs is called a heterojunction
- Either material can additionally be n- or p-doped.
- n-N, n-P, P-N, or P-P - heterojunctions. N & P refer to GaAlAs.
- Other important heterojunctions are

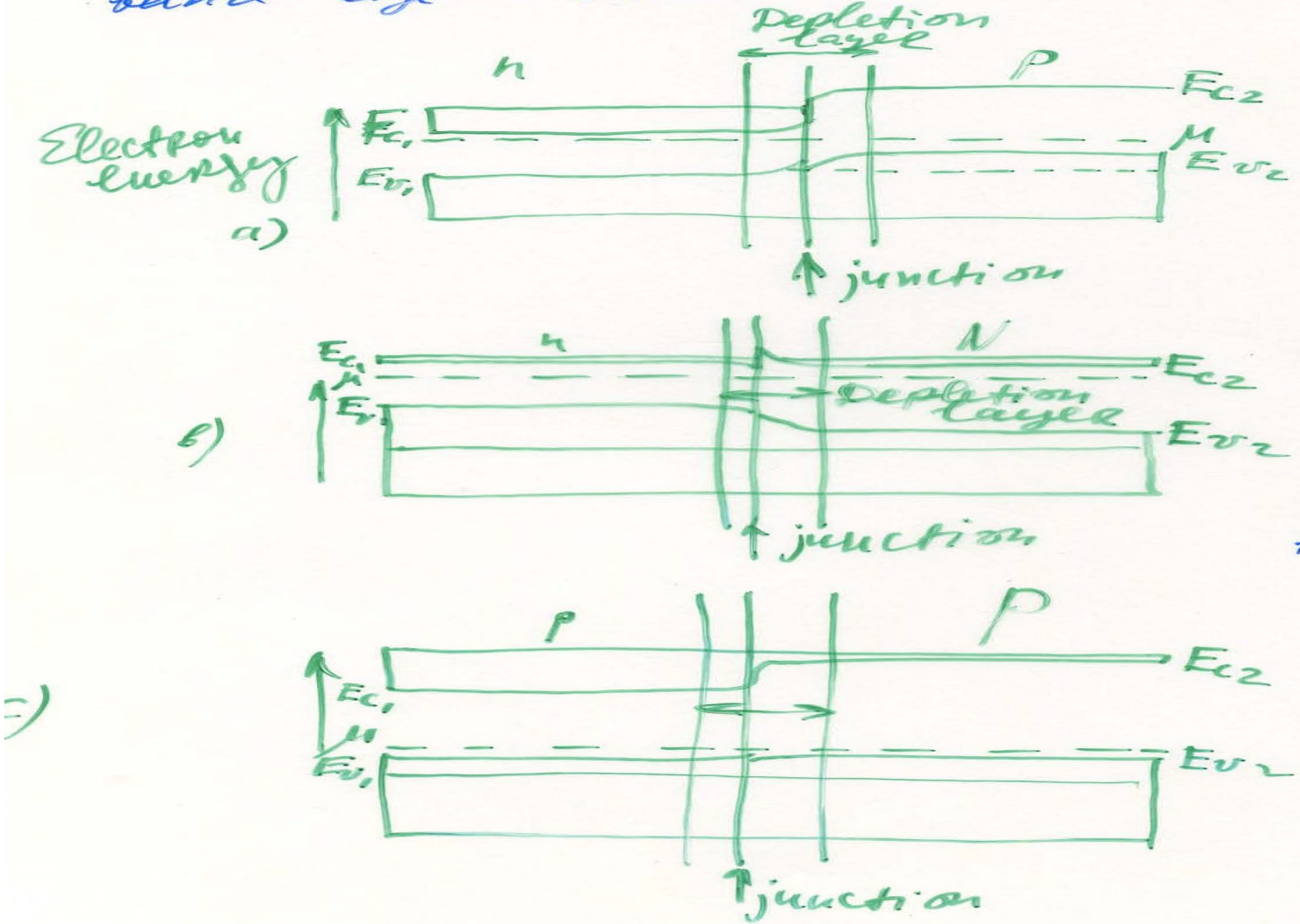
  $GaAs / In_x Ga_{1-x} As_{1-y} P_y$

  $InP / In_r Ga_{1-r} As_{1-s} P_s$

  GaSb $/ In_t Ga_{1-t} As_u Sb_{1-u}$

Energy Barriers and Rectification

- when junction between GaAs / GaAlAs is formed it results in energy barriers to the flow of e and holes. Why? – different energy gaps.
- the Fermi levels in the two materials line up across the junction so that deformation of the band edge occurs near the junction.

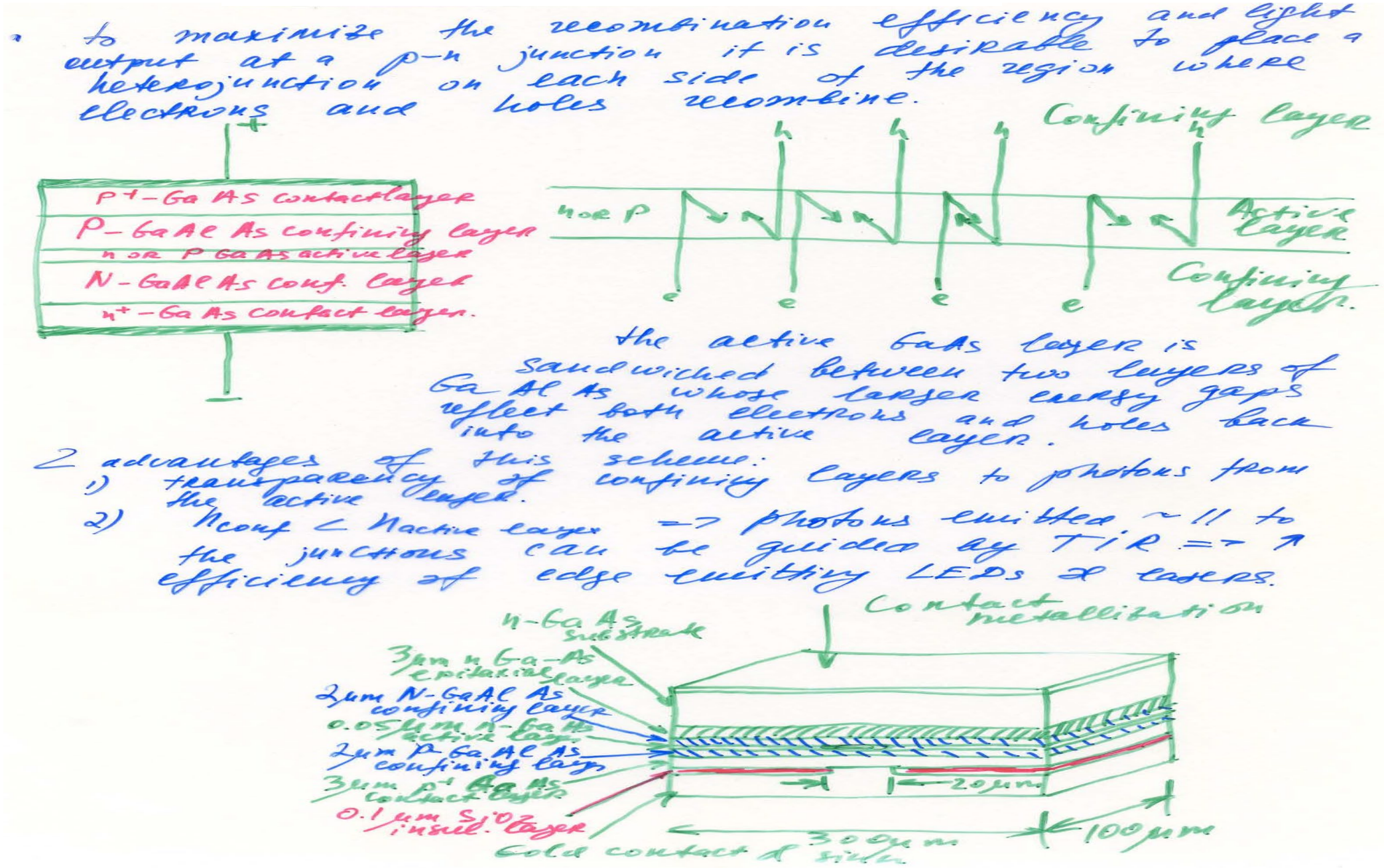


The n – P heterojunctions have rectification properties like a traditional diode

n – N & p – P can show a diode like behaviour, too.

There is a significant probability that an electron in the CB moving from n to N or from p to P will reflect because of the energy discontinuity at the junction.

The Double Heterojunctions



- if the active layer is sufficiently thin it can be assumed that the $\bar{E}$ concentration n = const within it.
- the recombin. rate per unit volume

$$-\frac{dn}{dt} = \frac{n}{\tau_{rr}} + \frac{n}{\tau_{nr}}$$

- an additional term should be added to include surface-recombination effects that occur at the heterojunction boundaries of the layer.

$$-\frac{dn}{dt} = n v_s$$

for GaAs/GaAlAs interface.

v_s – recombination velocity. $\approx 10^4$ m/s

- for an active layer of thickness "d" the overall recombination time τ satisfies

$$\frac{1}{\tau} = \frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}} + \frac{2v_s}{d}$$

- the internal quantum efficiency within the active layer is

$$\eta_{int} = \frac{\tau}{\tau_{rr}}$$

- the radiative recombination time τ_{rr} depends on doping level, injected current
  1 – 100 ns
- nonradiative lifetime τ_{nr} is decreased by the presence of impurities and lattice defects
  ~ 100 ns typical.
- for an active layer of thickness 0.05 μm and for $\tau = 1$ ns

$\eta_{int} \approx 0.7$.

The Spectrum of Recombination Radiation

- the photons generated by band-to-band recombination contain a range of energies rising from a min. value $\lambda_g = \frac{ch}{E_g}$
- to calc. the shape of the recomb. rad. → consider e & h population densities as a function of energy.
- For the electrons

$$n(E_2) = \rho_c(E_2) f(E_2)$$

E_2 - energies in CB
$\rho_c(E_2)$ - density of states within the band

- for the holes

$$p(E_1) = \rho_v(E_1)[1 - f(E_1)]$$

- $\rho_c(E_2) = 4\pi \left(\frac{2m_c^*}{h^2}\right)^{3/2} \sqrt{E_2 - E_c}$
- $\rho_v(E_1) = 4\pi \left(\frac{2m_h^*}{h^2}\right)^{3/2} \sqrt{E_v - E_1}$
- the rate of spontaneous emission resulting from recombination depend on several factors:
  1) # of electr. states within each small range of energy dE_2 in CB.
  2) the probability that these states are occupied $f(E_2)$
  3) # of electr. states within each small range of energies dE_1 in the V.B.
  4) the probability that these states are empty $1 - f(E_1)$.

- the total rate/unit volume at which recombinations occur

$$R_{spont} = \int\int A\, \rho_c(E_2) f_2(E_2)\, \rho_v(E_1)[1-f_1(E_1)]\, dE_1 dE_2$$

A - Einstein coeff. of spontan. emission
integrals run over the CB & VB.

- assume A doesn't depend on the E_2 & E_1

$$R_{spont} = A \int \rho_c(E_2) f_2(E_2)\, dE_2 \int \rho_v(E_1)[1-f_1(E_1)]\, dE_1 = Anp$$

where n and p - are total e and h densities.
typical $A_{GaAs} \sim 10^{-16}\ m^3 \cdot s^{-1}$

- simplifying assumptions - charge particle density decreases exponentially from the band edge

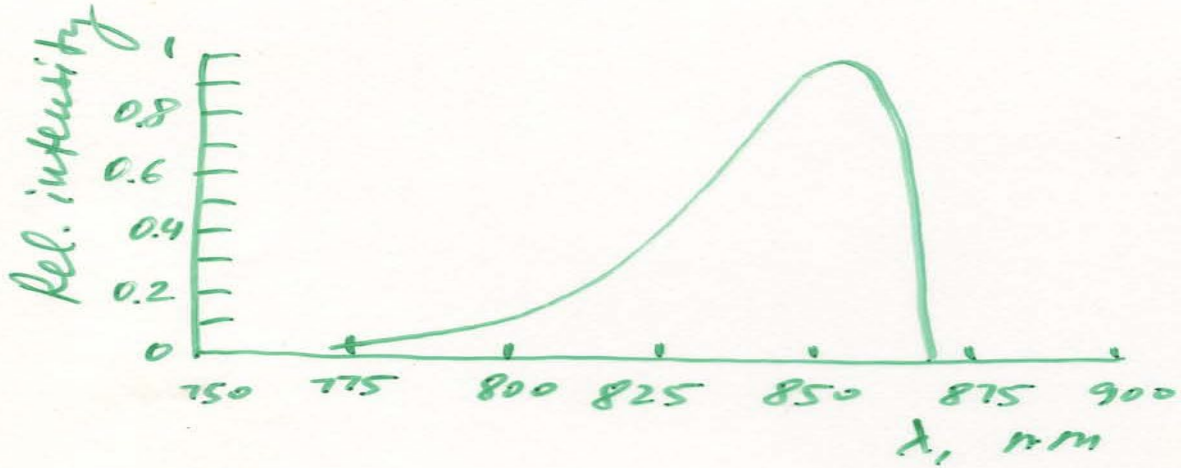
$$n(E_2) = n_c\, e^{-(E_2 - E_c)/kT}$$

$$p(E_1) = p_v\, e^{-(E_v - E_1)/kT}$$

- The luminescence spectrum will have a lineshape

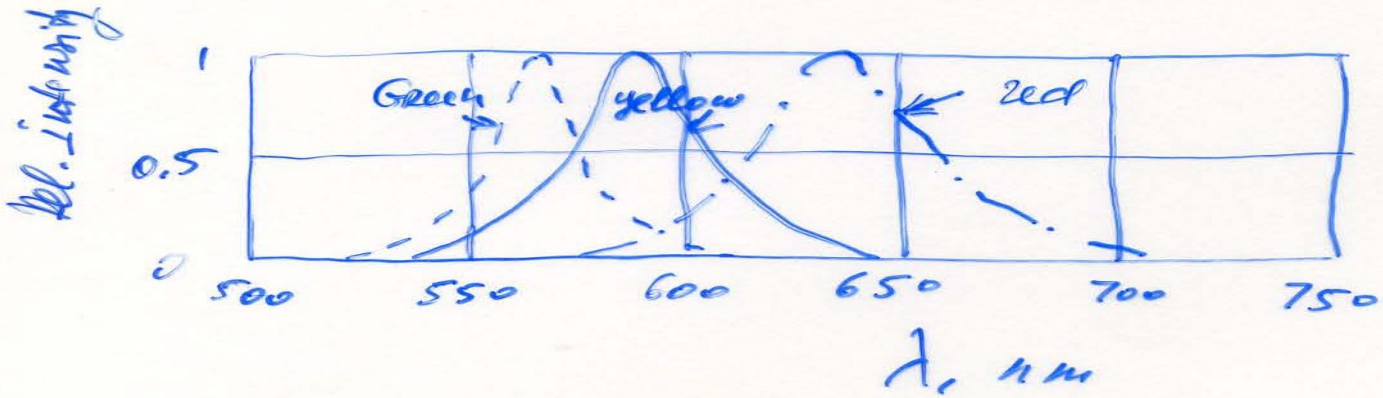
$$g(\nu) \sim \int_{E_c}^{E_v + h\nu} n(E_2)\, p(E_1)\, dE_2$$

$$g(\nu) \sim (h\nu - E_g)\, e^{-(h\nu - E_g)/kT}$$



Recombination spectrum of GaAs calculated from the simple model

$$g(V) \sim (hV - E_g) e^{-(hV - E_g)/kT}$$



$T_A = 25°C$

Spectra of HP red, yellow, and green LEDs.

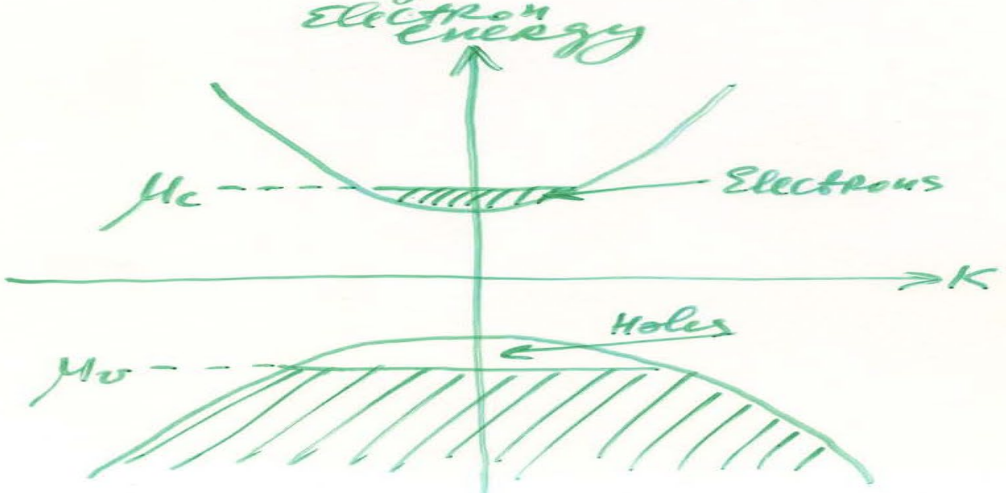
Actual recombin. spectra are more symmetric than we calculated, because of energy band smearing that takes place near the band edge of real semiconductors.

Semiconductor lasers

- What is required to convert the edge emitting LED into an edge emitting semiconductor laser.
  - condition of population inversion within the active region that provides sufficient gain to overcome losses.
- gain results if the rate of stimulated emission exceeds that of absorption
- the probability of a band-to-band stimulated emission depends on 4 factors
  1) B_{21} for the transition
  2) the energy density of the stimulating radiation, $\rho(\nu_{21})$
  3) the probability that the upper energy level is occupied by an electron, f_2
  4) probability $1-f_1$ that the lower energy level is not occupied by an e; namely it is occupied by a hole.
- gain $\propto [B_{21} f_2 (1-f_1) - B_{12} f_1 (1-f_2)] \rho(\nu_{21})$

  if $B_{12} = B_{21}$

  gain $\propto f_2 - f_1$



The existence of gain is a non equilibrium situation and can be represented schematically as the existance of a region of filled electron states above empty electron states.

One can represent the energies that bound the filled electron states in the conduction and valence bands as μ_c, μ_v, respectively, referred to as quasi-Fermi levels.

$$f_2(E_2) = \frac{1}{e^{(E_2-\mu_c)/KT}+1}$$

$$f_1(E_1) = \frac{1}{e^{(E_1-\mu_v)/KT}+1}$$

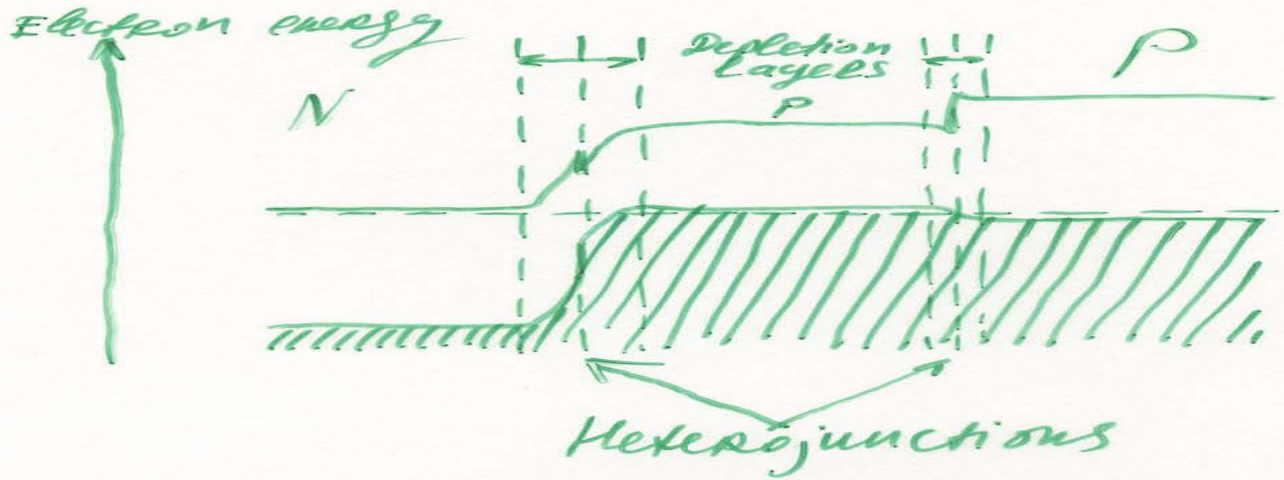
the condition $f_2 > f_1$ results if

- $(\mu_c - \mu_v) > h\nu_{21}$
- $h\nu_{21} > E_g$

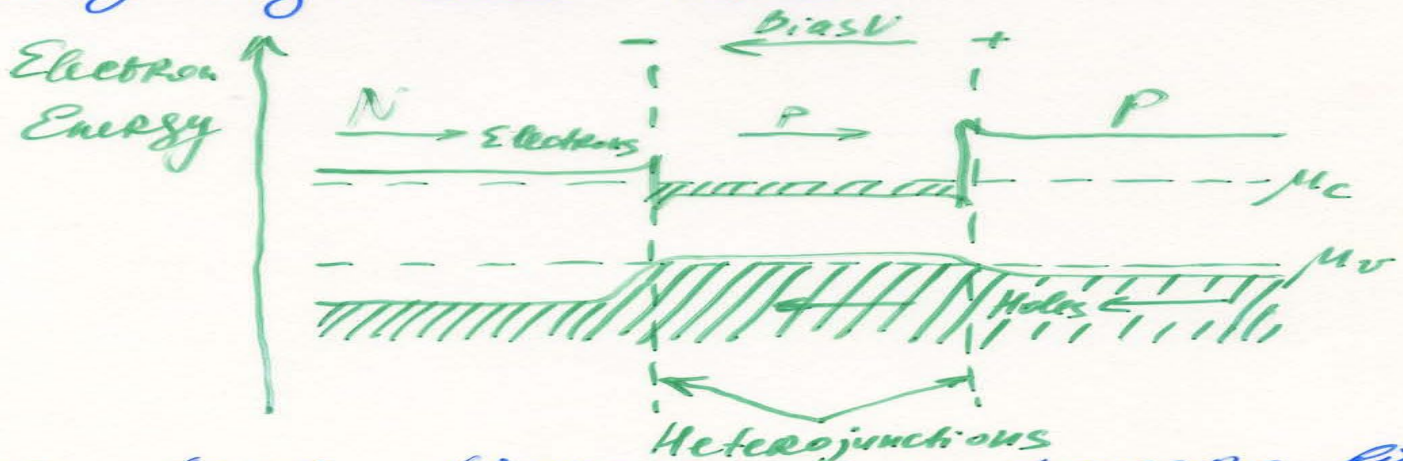
In a heavily doped p-type semiconductor the Fermi level can be forced down into the valence band.

If such a material is used as the active region in an N-p-P double heterostructure, then the energy levels in thermal equilibrium appear in the following manner:

Schematic band diagram in thermal equilibrium of an N–p–P double heterostructure in which the p region is heavily doped.

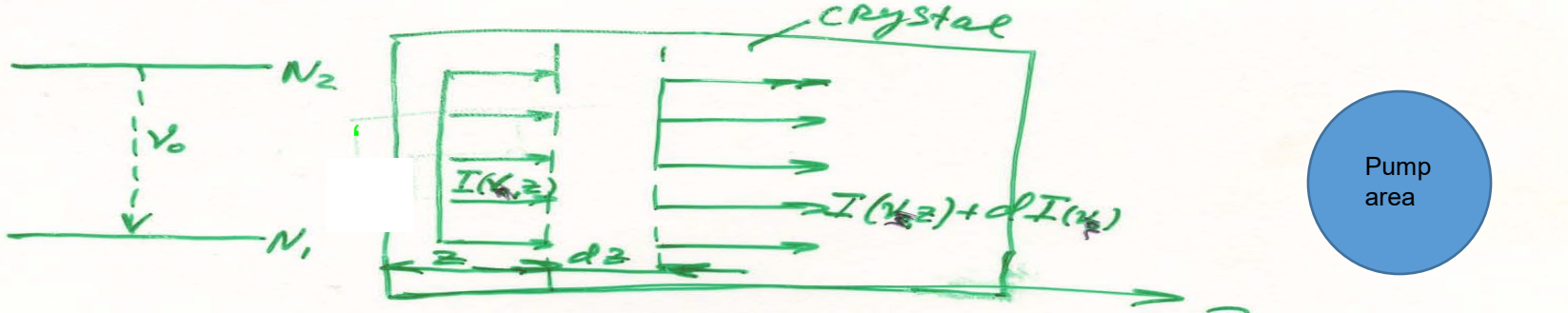


Under sufficient forward bias the situation shown in the following Fig. will result.



Schematic band diagram of a forward biased N–p–P double heterostructure in which the p region is heavily doped. Electrons flowing from the N region and holes flowing from the P region are trapped in the p region potential well creating a population inversion.

Gain coefficient
Recollection of PH581



- As the wave passes through the medium it grows in intensity if the # of stimulated emissions exceeds the # of absorptions.
- the change in intensity of the wave in travelling a small distance dz through the medium is

$$dI_\nu = \frac{\#\text{ stim. emiss} - \#\text{ of abs}}{\text{volume}} \times h\nu \times dz =$$

$$= \left[N_2 B_{21} g(\nu',\nu)\frac{I_\nu}{c} - N_1 B_{12} g(\nu',\nu)\frac{I_\nu}{c}\right] \times h\nu \cdot dz$$

- using Einstein relations

$$dI_\nu = \frac{I_\nu}{c}\left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^3 A_{21}}{8\pi h\nu^3} \cdot h\nu \cdot g(\nu',\nu)\,dz$$

$$\frac{dI_\nu}{dz} = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2 A_{21}}{8\pi\nu^2} g(\nu',\nu) I_\nu$$

solution $\boxed{I_\nu = I_\nu(0)\, e^{\gamma(\nu) z}}$ where $I_\nu(0)$ – int. at $z = 0$

$\boxed{\gamma = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2 A_{21}}{8\pi\nu^2} g(\nu',\nu)}$ gain coefficient

- if $N_2 > \frac{g_2}{g_1} N_1$ then $\gamma(\nu) > 0$ we have an optical frequency amplifier.

Cross section of emission $\sigma = \frac{c^2 A_{21}}{8\pi\nu^2} g(\nu',\nu)$

$\gamma = \left(N_2 - \frac{g_2}{g_1}N_1\right)\sigma$

The Gain Coefficient of a Semiconductor Laser

- the increase in intensity, dI_ν (W/m^2) of a small band of frequencies near ν on passing a distance dz through a medium could be written as

$$dI_\nu = (\#\text{ of stimulated emission/vol} - \#\text{ of absorpt/vol.}) \times h\nu\, dz =$$
$$= \left[B_{21} N_2 g(\nu';\nu) \frac{I_\nu}{c} - B_{12} N_1 g(\nu';\nu) \frac{I_\nu}{c} \right] h\nu\, dz$$

$N_2 g(\nu';\nu)$ – density of filled upper level states per frequency interval

$N_1 g(\nu';\nu)$ – density of unfilled lower level states per freq int.

I_ν (W/m^2) – intensity of output monochromatic radiation.

- the probability of stimulated emission depends on several factors:
  1) Einstein coeff. B
  2) the density of states in the upper level, $\rho_c(E_2)$, and the probability that these states are occupied, $f_2(E_2)$
  3) the density of states in the lower level, $\rho_v(E_1)$ and the probability that these states are empty, $1 - f_1(E_1)$.

- $$dI_\nu = \{ \rho_c(E_2)\rho_v(E_1) f_2(E_2)[1 - f_1(E_1)] B h \frac{I_\nu}{c} -$$
  $$- \rho_c(E_2)\rho_v(E_1) f_1(E_1)[1 - f_2(E_2)] B h \frac{I_\nu}{c} \} h\nu\, dz$$

- $\frac{A}{B} = \frac{8\pi h\nu^3 n^2 n_g}{c_0^3}$ for a dispersive medium where the refractive index varies with wavelength. $n_g = n - \lambda \frac{dn}{d\lambda}$ n_g – group refractive index

$$dI_\nu = \frac{h c_0^2 A}{8\pi n n_g \nu^2} \rho_c(E_2)\,\rho_v(E_1)\,[f_2(E_2) - f_1(E_1)]\,I_\nu\,d\nu,$$

$dI_\nu - \left[\frac{W}{m^2 \cdot y}\right]$

B - coeff. for stimulated emission for states distributed over an energy range dE

$\left[\frac{m^6}{y \cdot s^2}\right]$

integrate

$$g(\nu) = \frac{h c_0^2}{8\pi n n_g \nu^2} \int_{E_c}^{E_2 = h\nu} A\,\rho_c(E_2)\,\rho_v(E - h\nu)\,[f_2(E_2) - f_1(E_2 - h\nu)]\,dE_2$$

- how does the gain $g(\nu)$ depend on the current flowing through the junction?
- for a current I crossing a p-n junction of width W and height h the injected current density is

$$J = \frac{I}{h W}$$

- In a double heterojunction laser, ideally, all the injected carriers recombine in the region between the two heterojunctions, of spacing "d."

$$\Rightarrow R_{spont} = \frac{\eta_{int} \cdot J}{e d} = A n p$$

→ to calculate the gain is not straightforward because the Fermi-Dirac functions $f_2(E_2)$, $f_1(E_1)$ depend on the location of the quasi-Fermi levels μ_c, μ_v in the conduction and valence bands, respectively, in the active region.

Estimation of Semiconductor Laser Gain

- Consider the case of an n-doped GaAs active region with a donor concentration of $n_D = 10^{24}\,m^{-3}$, an internal quantum eff. of 0.6, and injected current of 10 mA that enters a 10μm wide by 10μm high active region whose thickness between heterojunctions is 0.5μm.

- From equations $A n_p = \frac{\eta_{int}\, J}{ed}$; $J = \frac{I}{hw}$

  taking $A = 10^{-16}\,\frac{m^3}{s}$

$$n_p = \frac{0.6 \times 0.01}{1.6\times10^{-19}\times10\times10\times0.5\times10^{-12}\times10^{-16}} = 7.5\times10^{47}\,m^{-6}$$

- in the active region

$$n - p = n_D = 10^{24}\,m^{-3}$$
$$\Rightarrow n = 1.5\times10^{24}\,; \quad p = 0.5\times10^{24}\,m^{-3}$$

- The electron density satisfies

$$n = \int_{E_c}^{\infty} \rho_c(E_2)\, f_2(E_2)\, dE_2$$

if T is low $f_2(E_2) = 1$ for $E_2 \le \mu_c$; $f_2(E_2) = 0$ for $E_2 > \mu_c$ } with similar relationships for the holes.

using $\rho_c(E_2) = 4\pi \left(\frac{2m_c^*}{h^2}\right)^{3/2} \sqrt{E_2 - E_c}$

$$n = \int_{E_c}^{\mu_c} \rho_c(E_2)\, dE_2 = \frac{1}{2\pi^2}\left(\frac{2m_c^*}{\hbar^2}\right)^{3/2} \int_{E_c}^{\mu_c} \sqrt{E_2 - E_c}\, dE_2$$

$$p = \int_{\mu_v}^{E_v} \rho_v(E_1)\, dE_1 = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2} \int_{\mu_v}^{E_v} \sqrt{E_v - E_1}\, dE_1$$

- In GaAs $m_e^* = 0.067\, m_e$; $m_h^* = 0.48\, m_e$ so evaluating the constants and performing the integration gives

$$n = 1.23 \times 10^{54} (\mu_c - E_c)^{3/2}$$

$$p = 2.35 \times 10^{55} (E_v - \mu_v)^{3/2}$$

- With the e and hole concentration calculated previously

$$\mu_c - E_c = 1.14 \times 10^{-20}\, J = 0.071\, eV$$

$$E_v - \mu_v = 7.88 \times 10^{-22}\, J = 0.0048\, eV$$

- the band gap energy in GaAs is 1.42 eV, so gain will result for photon energies from $\mu_c - \mu_v$ to E_g ; λ from 832 to 873 nm
- Calculate gain at $\lambda = 838\, nm \equiv 1.48\, eV$

$$\gamma(\nu) = \frac{h c_0^2}{8\pi n n_g \nu^2} \int_{E_A}^{E_B} A \times 6.5 \times 10^{103} \times \sqrt{E_2 - E_c}\, \sqrt{E_v - E_2 + h\nu}\, dE_2$$

$$\gamma(\nu) = \frac{h c_0^2}{8\pi n n_g \nu^2} \int_{E_c}^{E_v + h\nu} A\, \rho_c(E_2)\, \rho_v(E_2 - h\nu) [f_2(E_2) - f_1(E_2 - h\nu)]\, dE_2$$

$$\rho_c(E_2) = 4\pi \left(\frac{2 m_e^*}{h^2}\right)^{3/2} \sqrt{E_2 - E_c}$$

$$\rho_v(E_1) = 4\pi \left(\frac{2 m_h^*}{h^2}\right)^{3/2} \sqrt{E_v - E_1}$$

$f_2(E_2) = 1$ for $E_2 \le \mu_c$; $f_2(E_2) = e^{\frac{1}{(E_2 - \mu_c)/kT + 1}}$

$f_2(E_2) = 0$ for $E_2 > \mu_c$

- The limits on the integral are set by $f_2(E_2) > f_1(E_1)$ and $E_2 - E_1 = h\nu$
- Clearly

$$E_A = E_c + 0.055\, (eV)$$

$$E_B = E_c + 0.06\, (eV)$$

- if we assume that "A" is independent of energy and has a value of $10^{-16}\ m^3/s$ and using the values

$n = 3.6$; $n_g = 4$

$$J(r) = 2.52 \times 10^{7} \int_{E_c+0.055}^{E_c+0.06} \sqrt{E_2 - E_c}\ \sqrt{E_v - E_2 + 1.48}\ dE_2$$

- Now $E_v - E_2 + 1.48 = 0.06 - (E_2 - E_c)$
- So writing $E_2 - E_c = 0.06 \cos^2\theta$ the integral is transformed to give

$$J(r) = 1.62 \times 10^{5} \int_{0}^{0.293} \sin^2\theta\ \cos^2\theta\ d\theta = 1.06 \times 10^{4}\ m^{-1} =$$
$$= 10.6\ mm^{-1}$$

- high gain is typical for semic. lasers. the effective gain is reduced by the distributed loss coefficient α, which results from scattering.
Scattering increases with doping and $\sim 10^{3}-10^{4}\ m^{-1}$
- for $\alpha = 10^{3}\ m^{-1}$ the effective $J = 9.6\ mm^{-1}$. in a laser 500μm long the single pass intensity increase would be $e^{4.8} = 122$

Threshold Current and Power-Voltage Characteristics

- the active region in a semiconductor laser is typically very thin (few μm — few nm)
- the laser cavity length - few hundred μm long.
- external cavity lasers - few cm.
- gain is very high => natural Fresnel reflectance at the cleaved facet ends provides sufficient feedback for lasing.
- the performance of a semiconductor laser is defined by several parameters:
  - threshold current density J_{th};
  - power versus voltage (P-I) characteristic,
  - effective current density J_{eff}

$$J_{eff} = \frac{\eta_{int} \cdot J}{d}$$

where J - injected current density
η_{int} - internal quantum eff.
d - spacing between two hetero-junction

- threshold current density J_{th} is the value at which the gain exceeds the cavity losses

$$J_{th} = \alpha - \frac{1}{l} \ln \sqrt{R_1 R_2}$$

! An additional factor must be included.

- there is a refractive index discontinuity on both sides of the active layer in a double heterostructure laser.

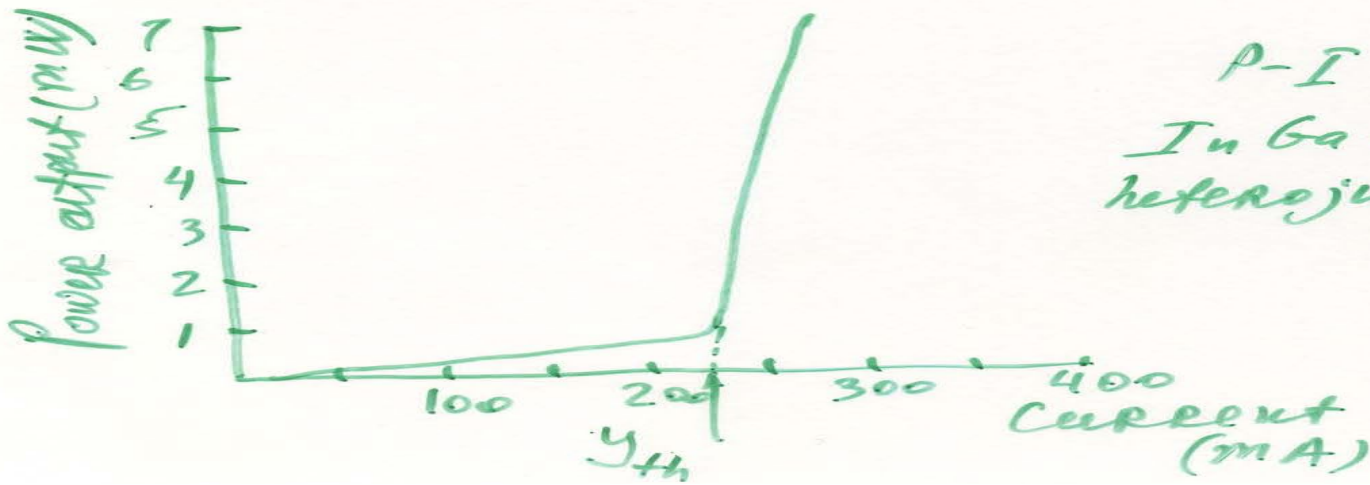


Schematic variation of refractive index across a N-p-P double heterojunction

- we have a wave guiding effect or confinement of the transverse mode of the laser to a region in, and on both sides of, the active region.
- the confinement factor, Γ, is a measure of the fraction of the oscillating field distribution that experiences gain within the active layer.

$$J_{th} = \frac{1}{\Gamma}\left(\alpha - \frac{1}{\ell}\ln\sqrt{R_1 R_2}\right)$$

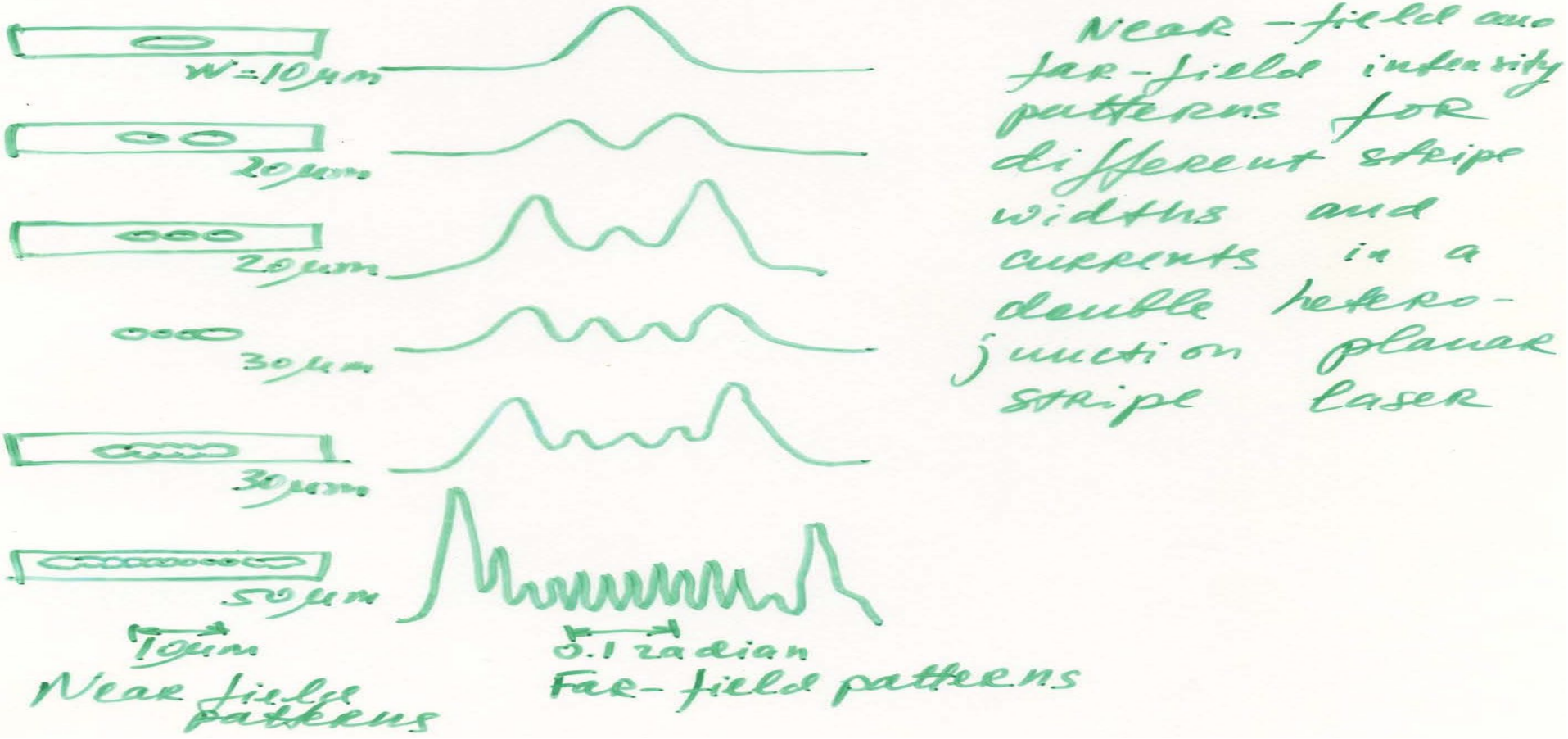
- in semic. lasers oscillating in a single transverse mode the gain is ~ to the excess current density above threshold $J = \beta[J_{eff} - (J_{eff})_{th}]$



P-I characteristic of a InGaAsP/InP double heterojunction laser

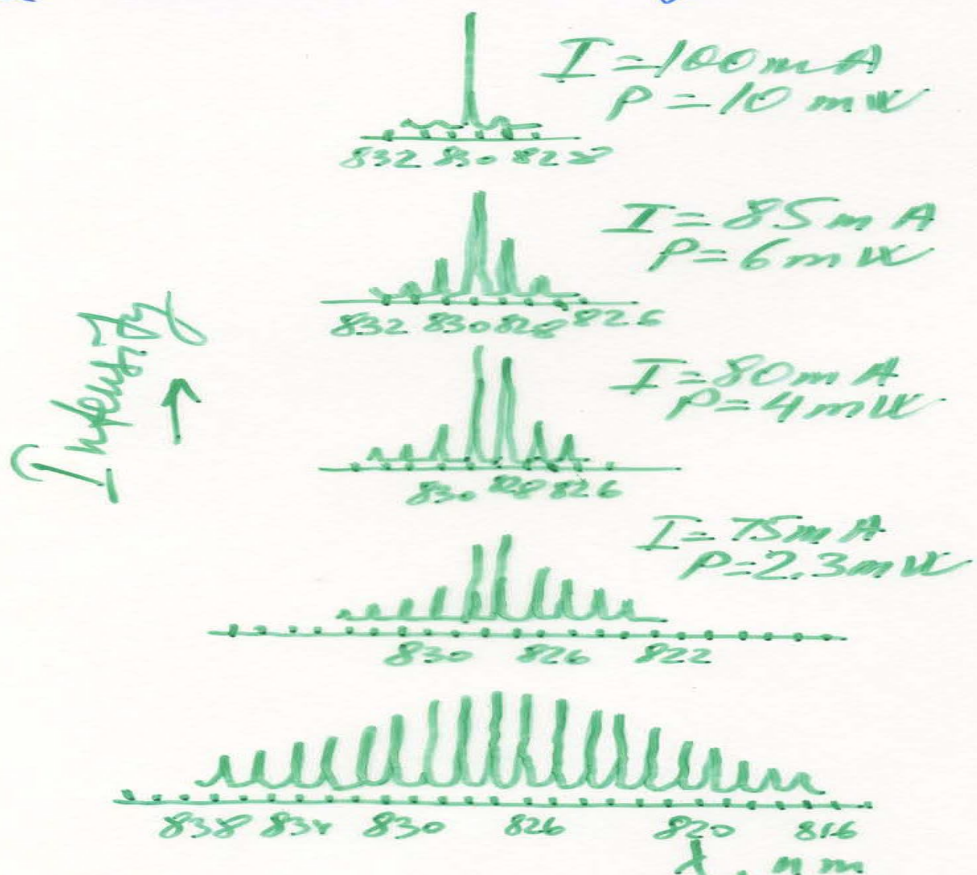
Longitudinal and Transvers Modes

- the active region is a rectangular parallelepiped with an end aperture of dimension $l \times d$
- waveguiding refractive index discontinuities in the direction $\perp$ to the junction, and sometimes in the direction $\parallel$ to the junction — index-guided stripe geometry exists
- the dimension $l \times d$ and y/y_m leads to the possibil. of different transverse emitted distributions as the current is increased.



Near-field and far-field intensity patterns for different stripe widths and currents in a double hetero-junction planar stripe laser

- Below threshold a semic. laser emits a broad, continuous spectrum like an LED.
- As the current density rises above threshold to distinct longitudinal modes will be observed to oscillate. One or two modes may begin to dominate as the current rises far above threshold.



Output frequency spectra of a CW laser diode at different currents

- the index of the active region, and the confining layer, varies with λ $\Rightarrow$ the longitudinal modes of a semic. laser are not equally spaced.

$$n_m \nu_m = \frac{mc}{2\ell}$$

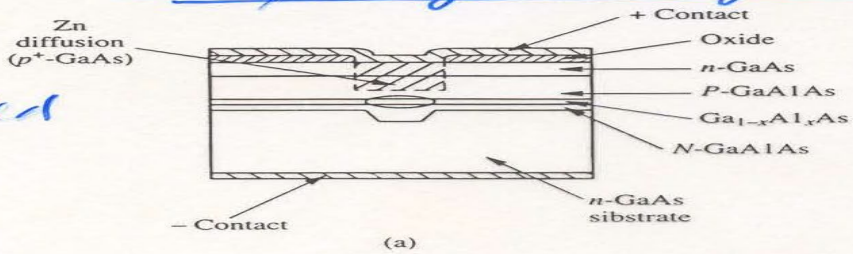
n_m – eff. index that applies at the frequency of the cavity reson. of index m, and $\Delta(n\nu) = n\delta\nu + \nu\frac{dn}{d\nu}\delta\nu$

Semiconductor Laser Structures

- double heterostructure configuration is a basic design of modern semiconductor lasers.
- difference - structure is fabricated so as to confine the current to a specific region within a device.

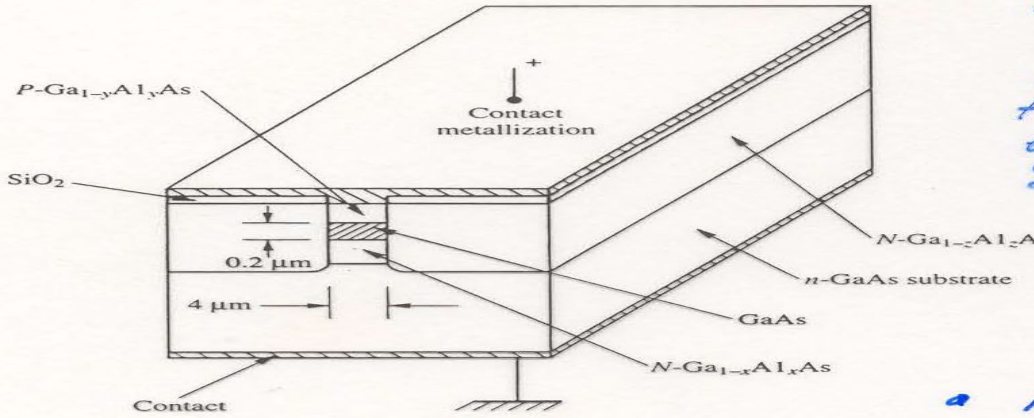
Stripe-geometry lasers

gain-guided



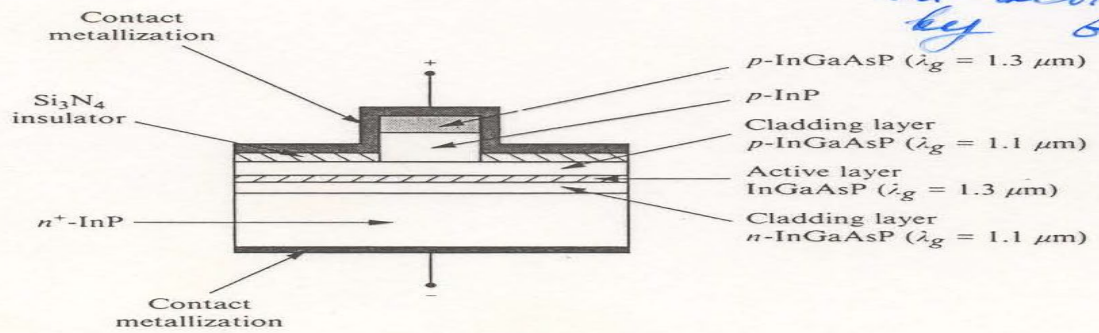
(a)

Fig. 13.30. Schematic construction of various types of stripe-geometry, double heterostructure lasers: (a) stripe confinement by oxide layer and deep zinc diffusion; (b) buried stripe; (c) cladded ridge[13.17].



(b)

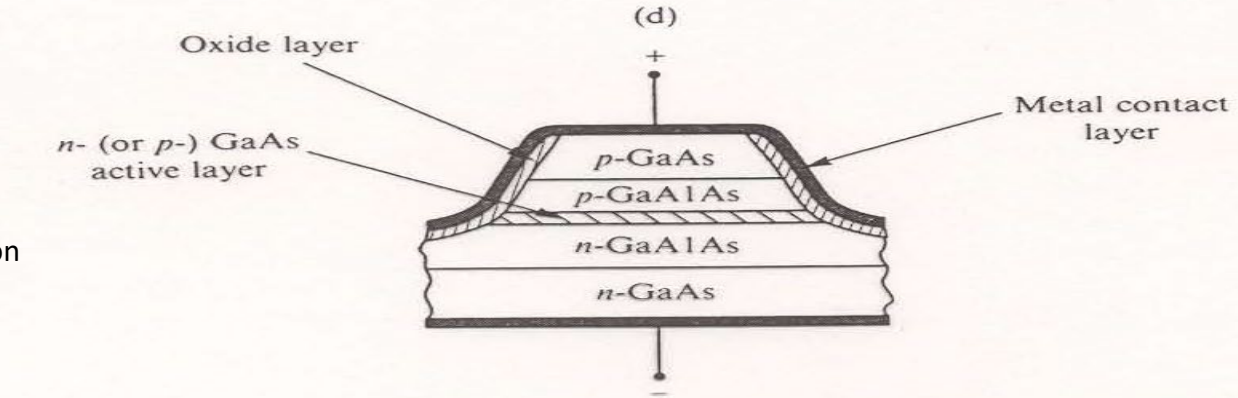
- If the region of current is defined by a conductive stripe on one of the electrodes or by making the confining layer resistive by proton bombardment or oxygen implantation then the active region will be laterally restricted to the region where current flows.

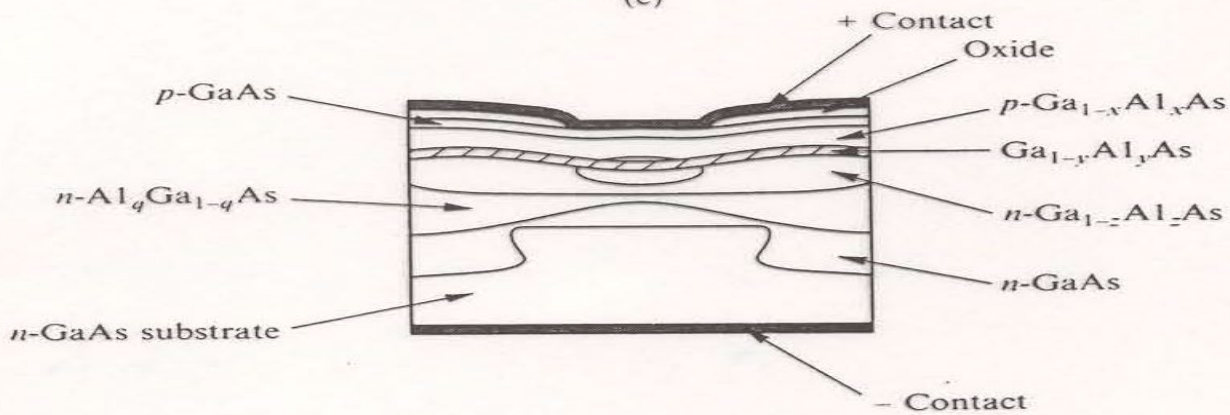


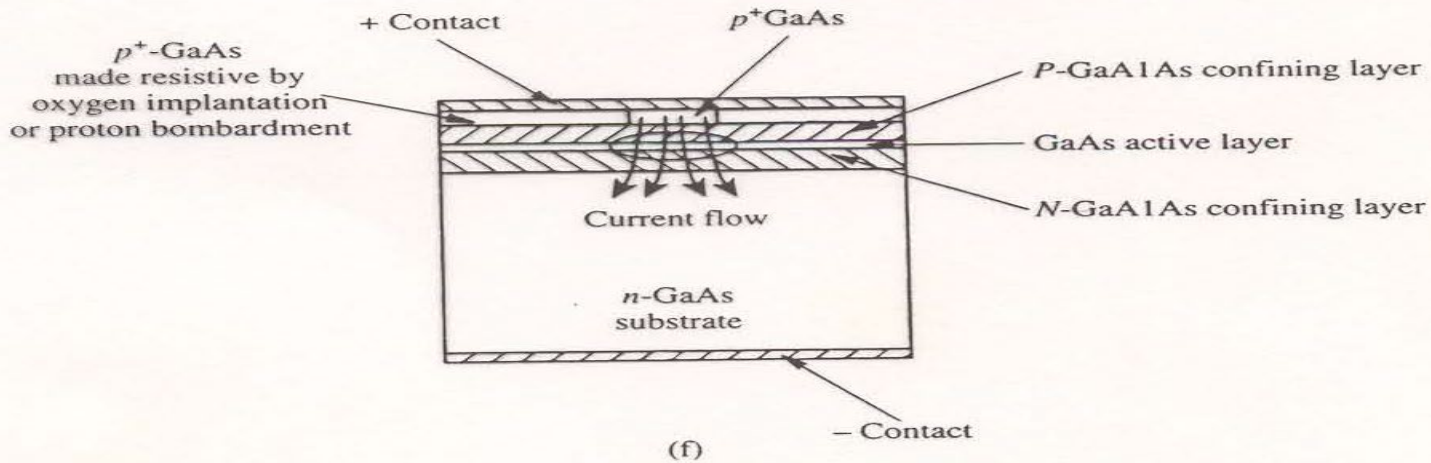
(c)

- if the fabrication process involves surrounding an active region of GaAs by GaAlAs on all four sides then the laser is described as index-guided.

Fig. 13.30. (*cont.*) (d) etched mesa; (e) constricted double heterostructure (CDH) (will give single transverse mode operation[13.18]; (f) stripe definition by oxygen implantation or proton bombardment.







gain guided

Graded-index separate confinement heterostructures

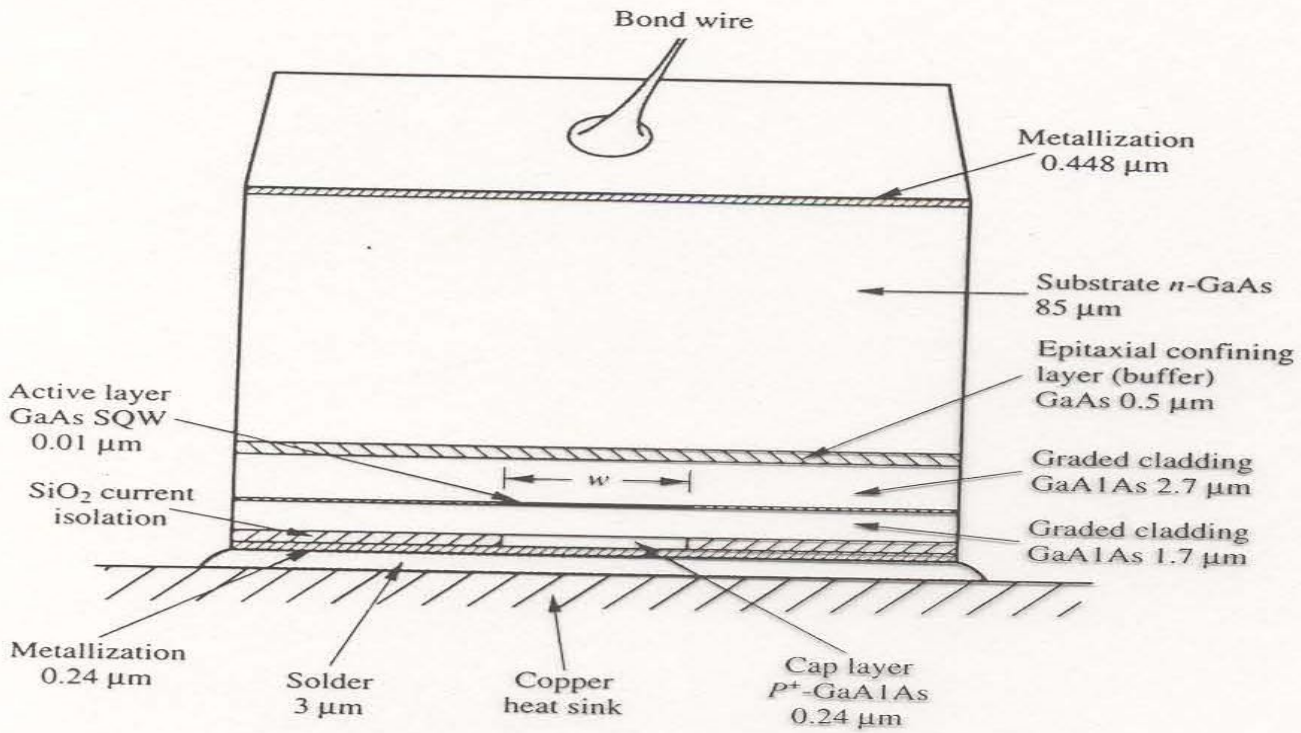
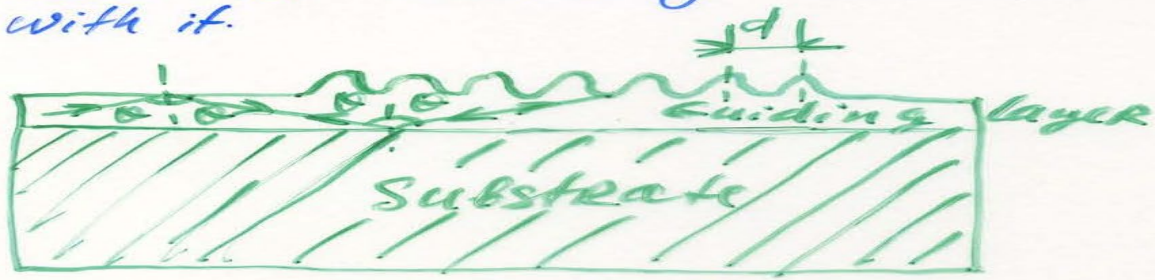


Fig. 13.31. GRINSCH SQW design used by McDonnell–Douglas Electronics Systems Co. Typical thicknesses of the various layers are shown. For a narrow stripe laser $w \sim 5$ μm, for a broad stripe laser $w \sim 60$ μm.

The active layer is very thin with graded layers of GaAlAs on each side to confine the mode optically.

Distributed Feedback (DFB) and Distributed Bragg Reflection (DBR) Lasers

- most edge emitting semic. lasers have a tendency to oscillate simultaneously on several cavity modes
- special features must be incorporated into the laser structure to encourage single mode operation.
- this can be done by including a periodic structure or grating in the device near the active layer where the oscillating transverse mode will interact with it.



Bragg reflection in a periodic waveguide structure that includes a surface corrugation of periodicity "d."

- Light that satisfies the Bragg condition will be reflected efficiently back. This condition is set by waves scattering off successive corrugations reflecting and constructively interferring.

$$2d \sin\theta = \frac{m\lambda_0}{n}$$

n - index of guiding layer.

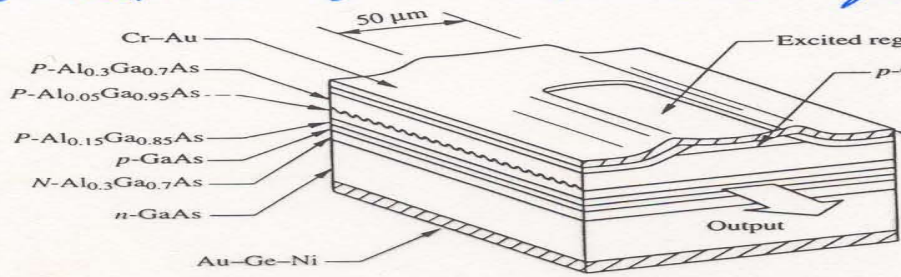
$\theta \simeq 90°$ its effect is included in an effective refr. index "n_{eff}" for the guided wave so that the grating period "d" required for operation at λ_0 is

$$\boxed{d = \frac{m\lambda_0}{2 n_{eff}}}$$

- A laser in which a periodic structure is placed in a close proximity to the active region over a substantial length of the gain region is called a distributed feedback DFB laser.

- the corrugations are separated from the active region and are formed between an $Al_{0.15}Ga_{0.85}As$ layer and an $Al_{0.05}Ga_{0.95}As$ layer. The thin p-type $Al_{0.15}Ga_{0.85}As$ layer and n-type $Al_{0.3}Ga_{0.7}As$ layer confine the optical field to the active region.

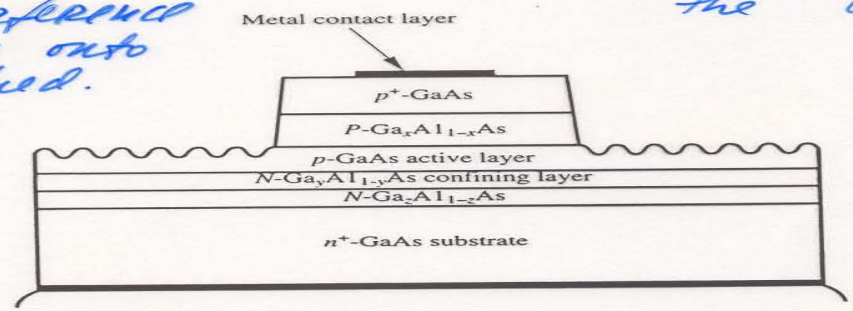
Fig. 13.33. (a) A GaAlAs double heterojunction DFB laser structure[13.20]. (b) DBR laser structure.



(a)

- the corrugations on the confining layer are produced by photolithography in which holographic techniques are used and a periodic interference pattern is projected onto the surface to be etched.

- Sufficient field amplitude leak through the p-type $Al_{0.15}Ga_{0.85}As$ to allow interaction with the corrugation.



(b)

- Selective cavity reflectance can be provided in a laser structure in which a grating is used at each end of the gain region to provide feedback at wavelength that satisfy the Bragg reflection condition. Distributed Bragg reflection lasers

- the gain and Bragg reflection regions are separate, which allow independent control of the carrier density in the grating region.
- Changes in carrier density in this region change the effective index of the cavity mode and tune the laser frequency.

Advantages
1) narrow linewidth
2) less dependent on temperature (temp. dep. of β and grating spacing).

Surface Emitting Lasers

- the laser beam is emitted ⊥ to the wafer.
- can be fabricated by conventional planar integrated circuit techniques
- in contrast to the edge emitting devices they can be scaled to very small size – smaller than the head of a pin.
- a periodic structure is etched onto the surface of the confining layer on one side of the junction.

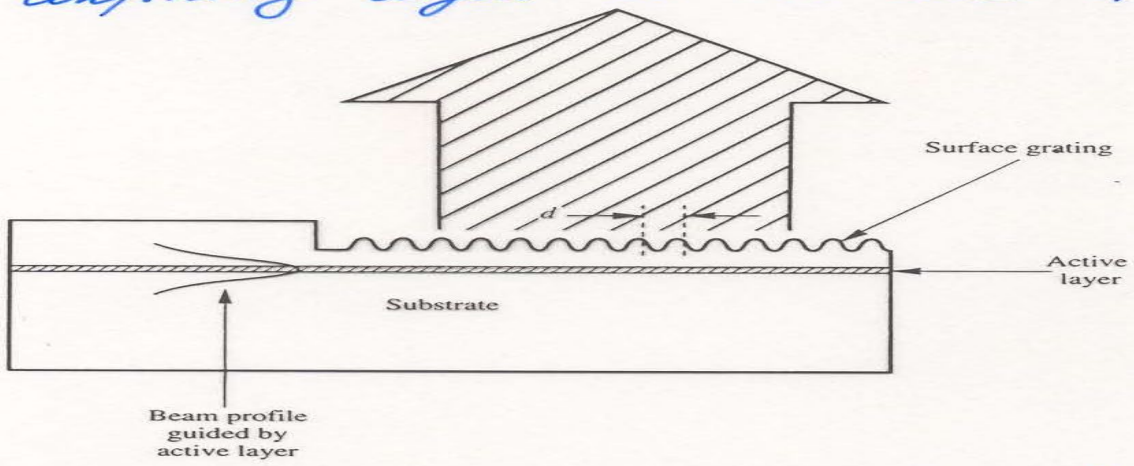
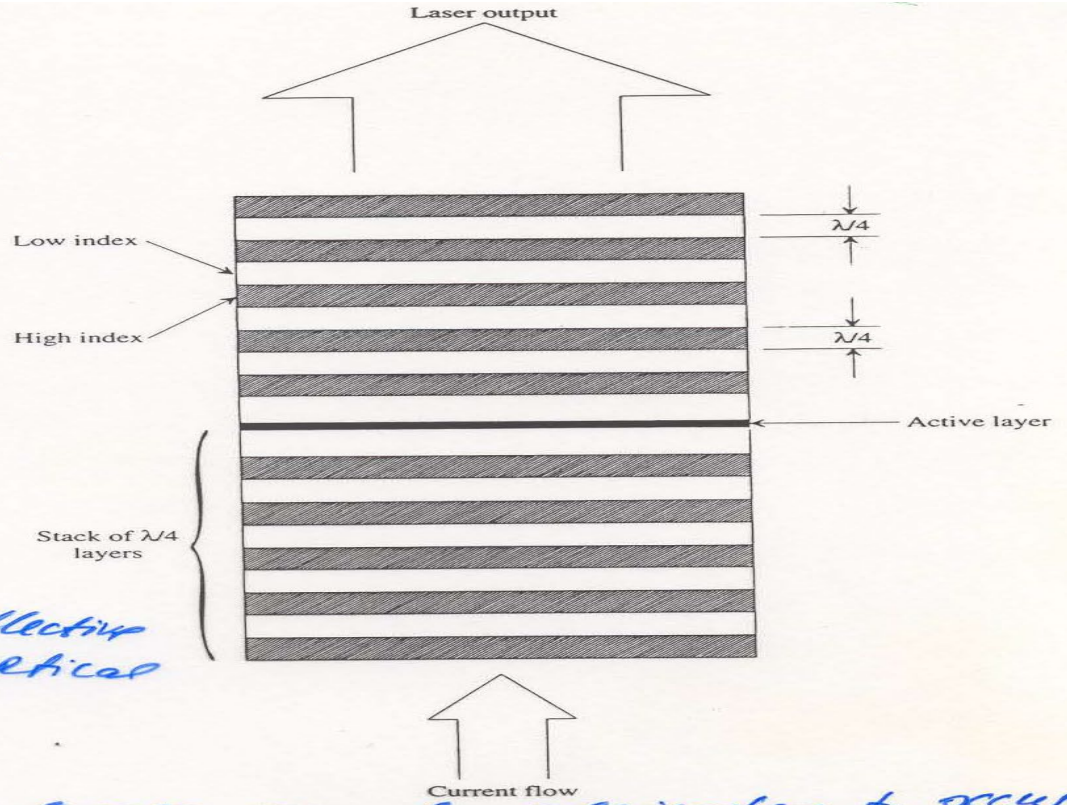


Fig. 13.34. Grating-coupled surface emitting laser.

- if the grating period, d, is chosen appropriately, then a diffraction max exists in the vertical direction for waves travelling in the active layer.
- this results if $m\lambda = d$
- a laser of this kind can emit over a large surface area, and provide a beam of high power and low beam divergence. (pulse powers > 4W, and high CW powers).

Vertical Cavity Surface Emitting Lasers (VCSEL)

Fig. 13.35. Schematic structure of a vertical cavity surface emitting laser.



- the active layer is bounded on each side in the vertical direction with alternate high and low index layers each of which is designed to be a quarter wavelength thick at the laser wavelength.
- Each of these stacks of $\frac{\lambda}{4}$ layers acts as a narrow band, high reflective mirror forming a vertical cavity.

Advantages

(a) a large # of small lasers can be fabricated to occupy a given area. density - 20,000/mm². Ø ~ 1-5μm. Hight ~ 5÷6μm. active layer thickness ~ 24 nm. $\gamma_{th} = 1 m A$

(b) Lasers have short cavity - cavity mode spacing is very large ~ 180nm for a 5.5μm laser. => single mode operation

(c) circular output beam - due to small Ø & cylindrical shape.

(d) the matrix of optical emitters that results has potential in many optical computing and optical interconnect applications.

Laser Diode Arrays and Broad Area Lasers

- there is a limit to the max. optical power that can be extracted from an edge emitting, stripe geometry laser.
- ↑power density in the cavity will damage laser facets. 4μm x 500nm high oscillating mode with 100mW output power is close to opt. damage. (power density ~ 1 MW/cm²)

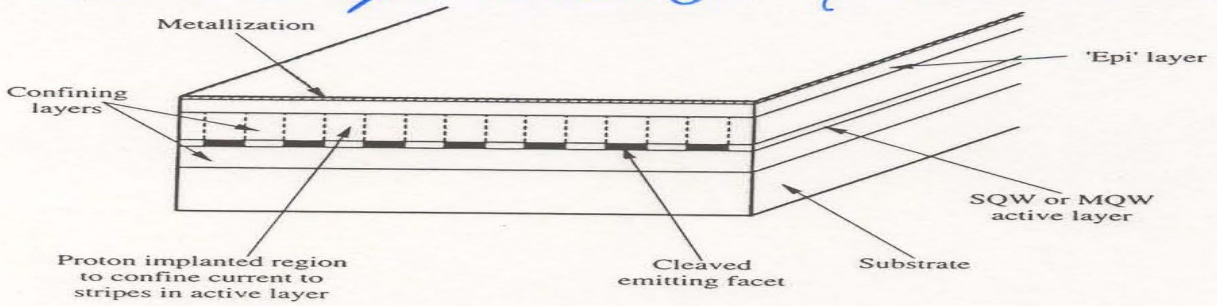


Fig. 13.36. Edge emitting laser diode array.

To produce higher output power — use laser diode arrays.

- many stripe lasers are fabricated in close proximity to each other.
- the current is confined to a series of stripe regions by producing regions of semiinsulating semicond. through proton implantation, which produces defects in the lattice.
- typically 10 stripe lasers would be fabricated with 4μm wide active regions and 10μm separation.
- a phase-locking of the fields in the separate stripes occurs leading to a combined, coherent wavefront emitted from the whole facet.
- total power ~ 1W.
- a broad area laser (BAL) is an alternative approach to generating a high power output. Active region is large and is not devided into stripes.

100μm wide SQW structure → CW power 3.8W. & 5A current.

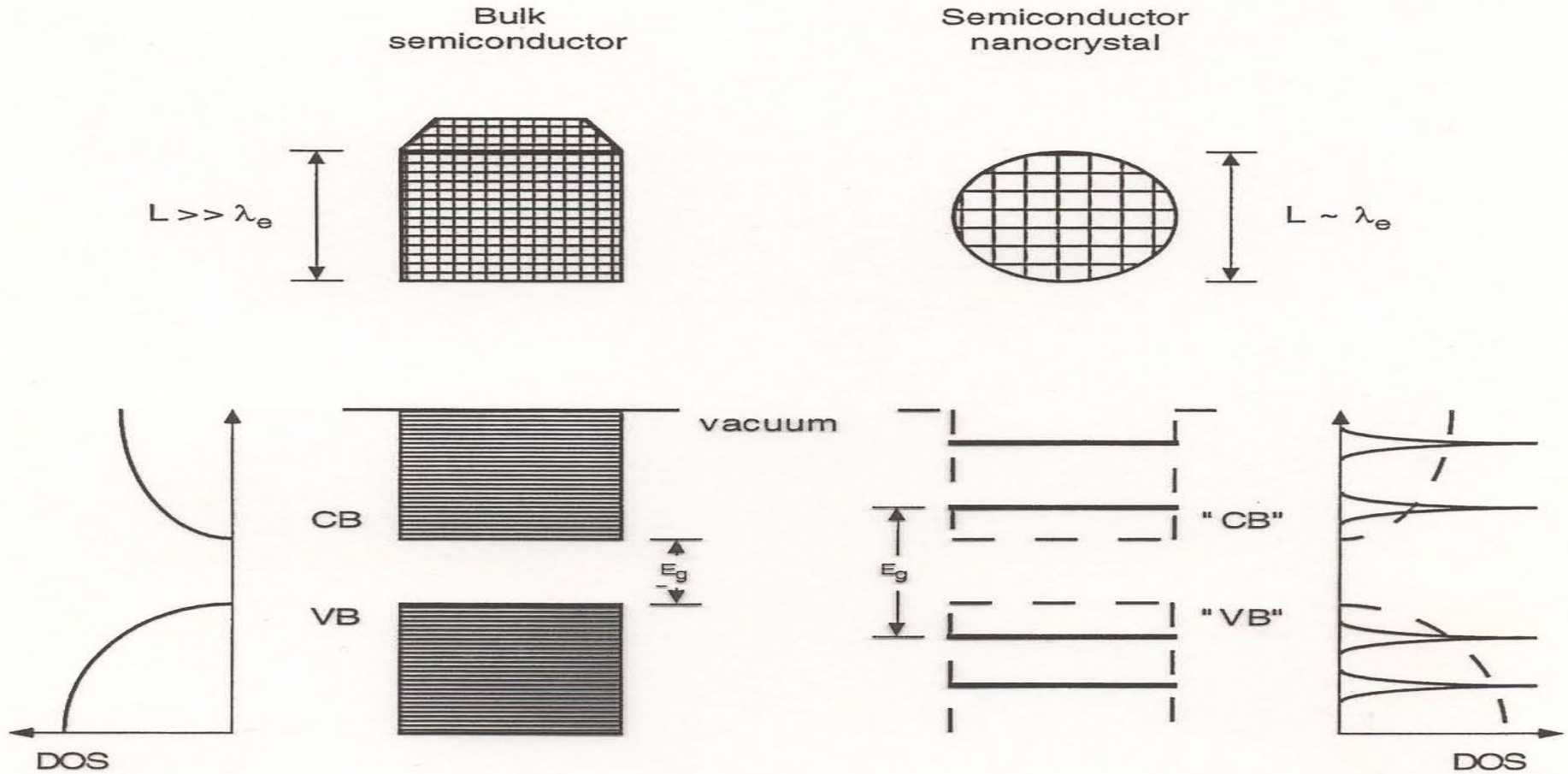


Figure 2.2: Comparison between a bulk semiconductor and a semiconductor nanocrystal showing the splitting of the bulk conduction and valence bands into discrete energy levels in the nanocrystal, accompanied by the concentration of the density of states into sharp features.

Quantum Well Lasers

- use a very thin active region (20 nm thick layer of GaAs bounded on each side by GaAlAs)
- if there is one such thin layer — single quantum well SQW laser
- if it is a multilayer sandwich of thin layers of alternating GaAs and GaAlAs — multiple quantum well MQW.

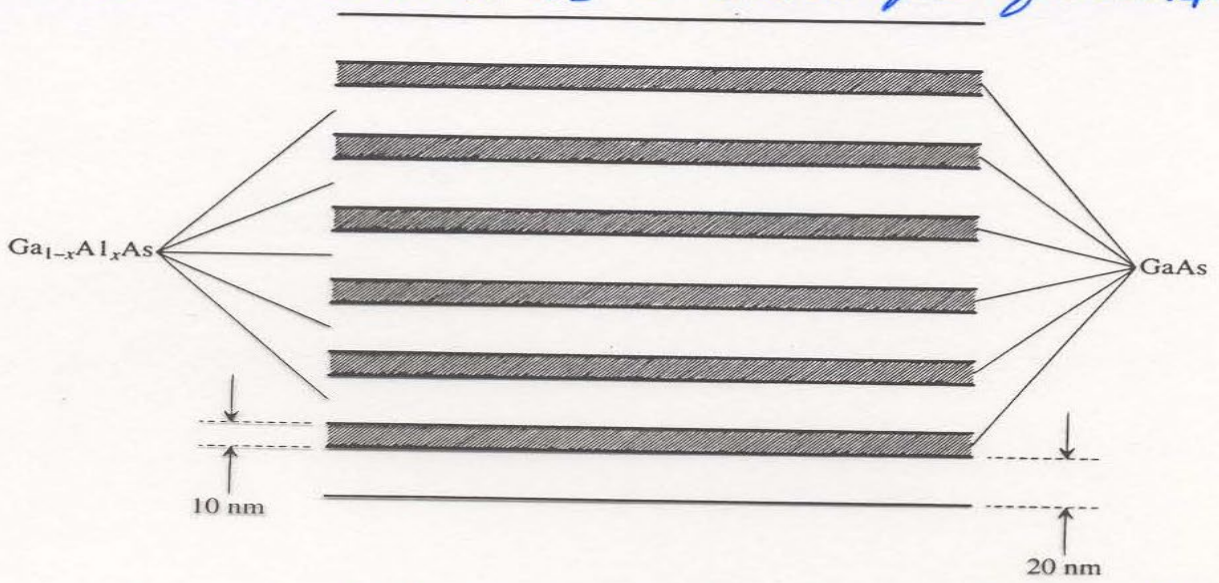


Fig. 13.37. Layered structure of the active region of a multiple quantum well semiconductor láser.

+ the total # of injected carriers necessary to achieve inversion is << to conventional heterostructure devices.

Why they have attractive properties?

- the quantum well is a region of min potential energy for electrons in conduct. band and holes in the V.B.
- the reduced threshold injection results because the very thin QW modifies the density of states available for e & h.

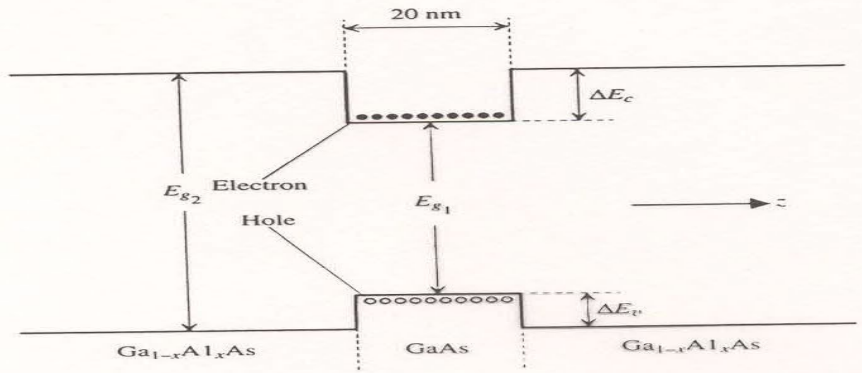


Fig. 13.38. Energy band diagram of a SQW laser.

- the # of modes that exist in the 2-dimensional region // to the layers

$$N_{2D}(K_{//}) = \frac{K_{//}^2 L_x L_y}{2\pi} = \frac{K_{//}^2 A}{2\pi}, \quad A = L_x L_y - \text{area of the junction}$$

- 2-dimensional density of states is

$$\rho(K_{//})\,dK_{//} = \frac{d N_{2D}(K_{//})}{dK_{//}} dK_{//} = \frac{A K_{//}}{\pi} dK_{//}$$

or in terms of energy per unit area

$$\rho_{QW}(E) = \frac{1}{A}\frac{dN_{2D}}{dE} = \frac{K_{//}}{\pi}\frac{dK_{//}}{dE}$$

- the # of modes that exist in the $\hat{z}$ ($\perp$) direction is reduced because of the small size of the direction.
- there ∃ only one mode, corresponding to a half-wavelength of the $\bar{e}$ wave fitting in the energy min in the C.B.
- the total energy of $\bar{e}$ in QW $E = \frac{\hbar^2 K^2}{2m_e} + E_Q$
- E_Q is associated with restricted motion of $\bar{e}$ $\perp$ junction.

$$K_{//} = \sqrt{\frac{2m_e}{\hbar^2}(E - E_Q)} \Rightarrow \frac{dK_{//}}{dE} = \frac{m_e/\hbar^2}{K_{//}} \quad \text{and} \quad \rho_{QW}(E) = \frac{m_e}{\pi\hbar^2}$$

at critical electron energies the $\bar{e}$ wavelength becomes short enough that additional modes $\perp$ to the layer become possible.

$$\rho_c^{3D}(E_2) = 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} \sqrt{E_2 - E_c}$$

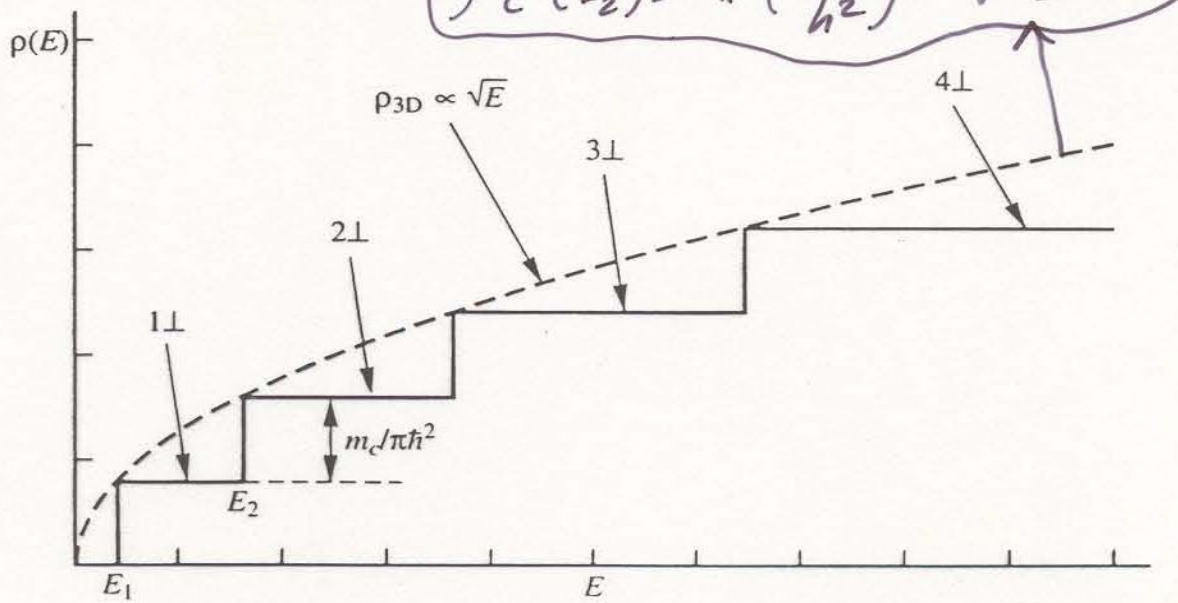


Fig. 13.39. Density of states in a SQW compared to the full three-dimensional density of the states function, E_1 is the minimum electron energy for one mode perpendicular to the layers, E_2 for two modes, and so on. The number 2⊥ on the curve, for example, indicates that there are two permitted modes in the perpendicular direction.

- QW lasers have lower thresholds, higher gains, and generate narrower linewidth laser radiation than more conventional heterostructure lasers.

The region on each side of the QW is frequently a region of graded index.

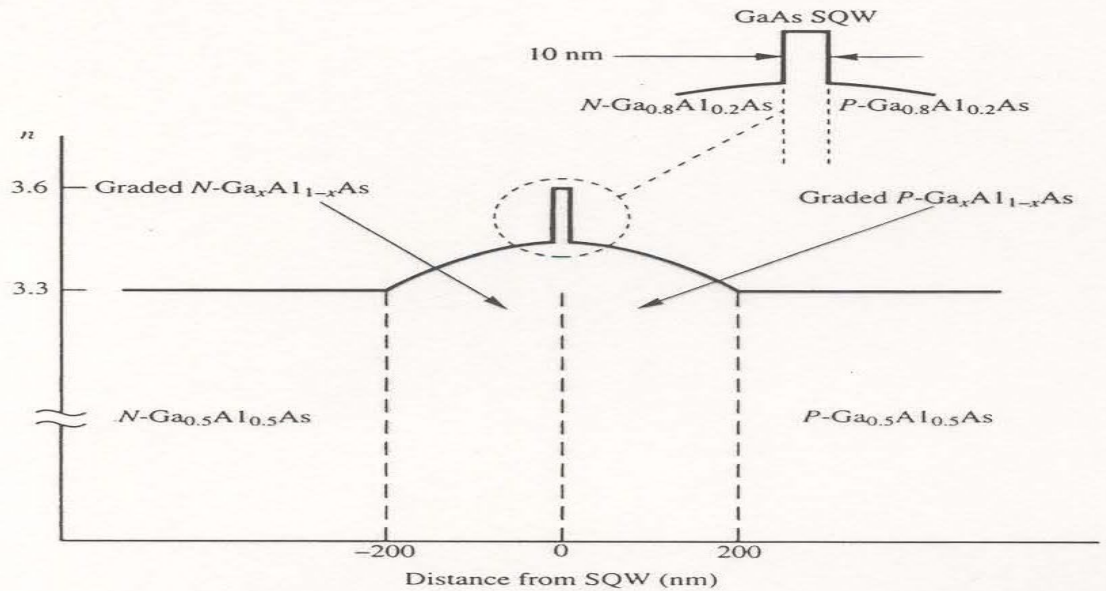
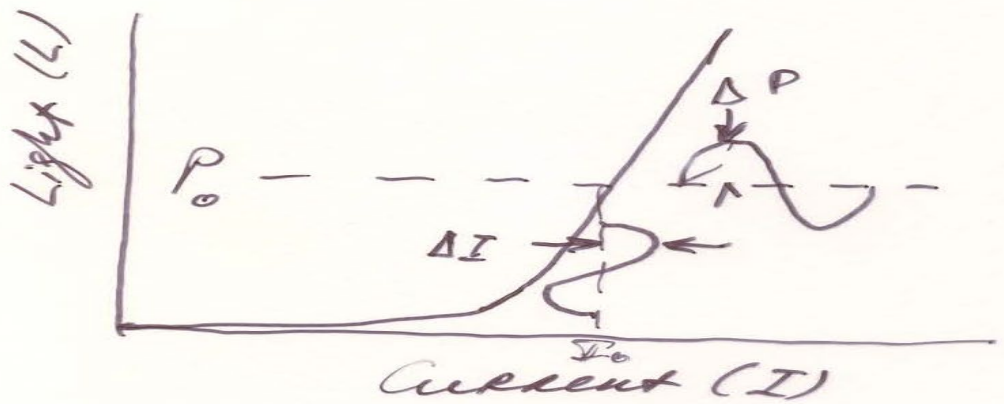


Fig. 13.40. Schematic variation of the refractive index change across a QW laser with a graded layer of $Ga_{0.5}Al_{0.5}As \rightarrow Ga_{0.8}Al_{0.2}As$ on each side of the QW (GaAs) region. The Ga concentration increases towards the QW region in the graded layers on either side.[13.24]

- Linewidths are wider than low pressure gas lasers.
- May operate in a single transverse mode
- Usually — several longitudinal modes of spacing 0.1 nm
- Gain profile is Lorentzian
- linewidths ~ few MHz — 10 MHz
- external cavity lasers — 10 kHz linewidths.

Rate Equations and Modulation of Semiconductor Lasers

- The fact that the diode lasers are so conveniently pumped by simple current injection (mA and at few volts drop) makes them the clear-cut candidate for communications.



if the current is varied by ΔI around an operating level I_0 the light output will also vary in a synchronous fashion

Typical light out current-in curve.

Rate equations

- we treat the system in a manner that ignore the spatial variations of carrier density and photons with Z.
- cross sectional overlap between the electromagn. mode and the inverted pop. as a constant

$$\Gamma = \frac{\int_{-d/2}^{d/2} E^2(x)\,dx}{\int_{-\infty}^{\infty} E^2(x)\,dx}$$

Γ is that fraction of the photon flux of the cavity mode that overlaps the inverted population (assumed to be in the width "d")

- density of the inverted population N is equal to the injected current

production term $\frac{dN}{dt} = \frac{J}{ed}$

- carriers are lost spontaneously by both radiative and nonradiative process.

$\frac{dN}{dt}$ (spontaneous recomb) $= -\frac{N}{\tau_s}$

where τ_s - carrier lifetime.

- stimulated emission depletes the inversion in direct proportion to the intensity of the stimulating wave. It is never very important until a laser is near or above threshold, and then it is all important.

$\frac{dN}{dt}$ (stimul. emission) $= -A(N - N_{tr})P$

N_{tr} is the value of the inversion that makes the material transparent (gain equal to losses)

A - host of other quantities:
1) gain coefficient per unit of inversion
2) group velocity of the wave composing the cavity mode.
3) volume of the cavity
4) Γ

$$\Rightarrow \boxed{\frac{dN}{dt} = \frac{J}{ed} - \frac{N}{\tau_s} - A(N - N_{tr})P}$$

this equation is a system of bookkeeping of the supply and use of the carriers and how they interact with photons.

- The photons in the cavity mode have a separate equation of motion that is coupled to the previous equation.

$$\frac{dP}{dt}(\text{stimul. emission}) = A(N - N_{thR})P$$

- $\frac{dP}{dt}(\text{spont}) = \beta \frac{N}{\tau_s}$

expln. β only fraction of the recombination $-\frac{N}{\tau_s}$ results in a photon, and only a fraction of that emission enters the cavity mode whose photon # is P.*

- Photons are lost from the cavity at a rate dictated by the resistive losses inside the cavity as well as by coupling to the external world.

$$\frac{dP}{dt}(\text{coupling, internal losses}) = -\frac{P}{\tau_p}$$

τ_p – photon lifetime in the cavity.

$$= 2) \quad \boxed{\frac{dP}{dt} = +A(N - N_{thR})P + \beta \frac{N}{\tau_s} - \frac{P}{\tau_p}}$$

$$1) \quad \boxed{\frac{dN}{dt} = \frac{\eta}{ed} - \frac{N}{\tau_s} - A(N - N_{thR})P}$$

Static Characteristics (DC excitation)

- all time derivatives are zero

assume $\gamma = \gamma_0$; $N = N_0 > N_{th}$, thus $P = P_0$ (to be determined) so that ② can be solved for the inversion density.

$$A\,[N_0 - N_{tr}]P_0 + \beta\frac{N_0}{\tau_s} = \frac{P_0}{\tau_p} \quad \text{or} \quad ④$$

$$A\,[N_0 - N_{tr}] = \frac{1}{\tau_p} - \beta\frac{N_0}{P_0\tau_s} \quad ⑤$$

$$0 = \frac{\gamma}{ed} - \frac{N}{\tau_s} - A(N_0 - N_{tr})P_0$$ where subscript $_0$ refers to the DC.

- 3a states that the sum of stimulated and spontaneous rates = photon loss rate as is true for any laser
- 3b - spont. emission becomes less and less important as the optical power becomes bigger.
- Now take 3a into ① and solve for the photon density

$$0 = \frac{\gamma_0}{ed} - \frac{N_0}{\tau_s} - \frac{P_0}{\tau_p} + \beta\frac{N_0}{\tau_s}$$

$$\Rightarrow P_0 = \frac{\gamma_0\tau_p}{ed} - (1-\beta)\frac{N_0\tau_p}{\tau_s} \quad \text{rewrite}$$

$$P_0 = \frac{\tau_p}{ed}\cdot\left[\gamma_0 - \frac{N_0 ed}{\tau_s}\right] + \beta\frac{N_0\tau_p}{\tau_s}$$

for N_0 is clamped more or less at threshold losing $\Rightarrow$

$$\boxed{P_0 = K(\gamma - \gamma_{th}) + P_{spont}}$$

where $\gamma_{th} = \frac{N_{th}\,ed}{\tau_s}$; consistent with sharp break in the L-I curves.

There is another way of explaining this sharp threshold behavior.

- below threshold, spont. emission goes into any of the electromagn. modes coupling to the active region $\left[\frac{8\pi\nu^2 n^3}{c^3}\cdot\Delta\nu\right]\cdot$ volume is number.

This is a huge # for even a small semiconductor.

GaAs; volume $= 1\,mm^3$; $n = 3.6$; $\lambda_0 = 840\,nm$;

$\Delta\lambda = 5\,nm$.

# of modes $\approx 1.17\times10^{10}$.

Most of those modes represent waves propagating in the wrong direction.

We see only those photons emitted in our direction and in the bandwidth of our receiving system, which is very small fraction of total (say 1 part in 10^3).

When the laser is above threshold, the excess pumping goes into one or at most a few (10-20 modes) which represent waves propagating toward us.

$\Rightarrow$ dramatic increase in intensity.

Frequency Response of Diode Lasers

- the above is sufficient when the injected current is a slowly varying function of time, but under high-freq. excitation (GHz rate) we have to be concerned about the ability of stim. emission to keep up with the rate of carrier injection.
- assume diode is biased above threshold by DC current $I_0 > I_{th}$ and an AC current $\Delta I(t)$ is added:

$$I = I_0 + \Delta I(t)$$

- $\Rightarrow$ carrier density N and photon density P will have a DC & time-varying comp.

$$N = N_0 + \Delta N(t)$$
$$P = P_0 + \Delta P(t)$$

- substitute N & P into (1) & (2)

$$\frac{d\Delta N(t)}{dt} = \frac{I_0}{eV} + \frac{\Delta I(t)}{eV} - \frac{N_0 + \Delta N(t)}{\tau_s} - A[N_0 + \Delta N(t) - N_{tr}] \cdot [P_0 + \Delta P(t)] \quad (4)$$

$$\frac{d\Delta P}{dt} = A[N_0 + \Delta N(t) - N_{tr}] \cdot [P_0 + \Delta P(t)] + \beta \frac{N_0 + \Delta N(t)}{\tau_s} - \frac{P_0 + \Delta P(t)}{\tau_p} \quad (5)$$

neglect products of time varying quantities (i.e. $\Delta N \cdot \Delta P$), and equate separately, the DC variables and time-varying components.

- the equations involving DC terms reproduce precisely already covered DC material
- the equations for the AC variables becomes

$$\frac{d\,\Delta N}{dt} = \frac{\Delta\gamma}{ed} - \left(\frac{1}{\tau_s} + A P_0\right)\cdot \Delta N - A\,[N_0 - N_{thR}]\cdot \Delta P \quad (6)$$

$$\frac{d\,\Delta P}{dt} = A\,[N_0 - N_{thR}]\,\Delta P + A P_0\,\Delta N - \frac{\Delta P}{\tau_p} + B\,\frac{\Delta N}{\tau_s} \quad (7)$$

- simplify by neglecting the spontaneous term as being small when laser is biased well above threshold. $\Rightarrow$ (6) & (7) become

$$\begin{cases} \dfrac{d\,\Delta N}{dt} = \dfrac{\Delta\gamma(t)}{ed} - \left(\dfrac{1}{\tau_s} + A P_0\right)\cdot \Delta N - \dfrac{\Delta P}{\tau_r} & (8) \\ \dfrac{d\,\Delta P}{dt} = \left[A P_0 + \dfrac{B}{\tau_s}\right]\Delta N & (9) \end{cases}$$

used $A\,[N_0 - N_{th}] = \frac{1}{\tau_r} - \frac{B N_0}{P_0 \tau_s}$

- we can eliminate the coupling by differentiating one and substituting ~~into~~ the other.

$$\frac{d^2\Delta N}{dt^2} + \left[\frac{1}{\tau_s} + A P_0\right]\frac{d\,\Delta N}{dt} + \frac{1}{\tau_p}\left[A P_0 + \frac{B}{\tau_s}\right]\Delta N = \frac{1}{ed}\cdot\frac{d\,\Delta\gamma}{dt} \quad (10)$$

$$\frac{d^2\Delta P}{dt^2} + \left[\frac{1}{\tau_s} + A P_0\right]\frac{d\,\Delta P}{dt} + \frac{1}{\tau_r}\left[A P_0 + \frac{B}{\tau_s}\right]\Delta P = \frac{\Delta\gamma}{ed}\left[A P_0 + \frac{B}{\tau_s}\right] \quad (11)$$

- it is most informative if we form the "transfer" characteristics by letting $\Delta\gamma(t) = \Delta\gamma_m \exp(j\omega t)$ in (11) and solving for $\Delta P(t) = \Delta P_m \exp(j\omega t)$.

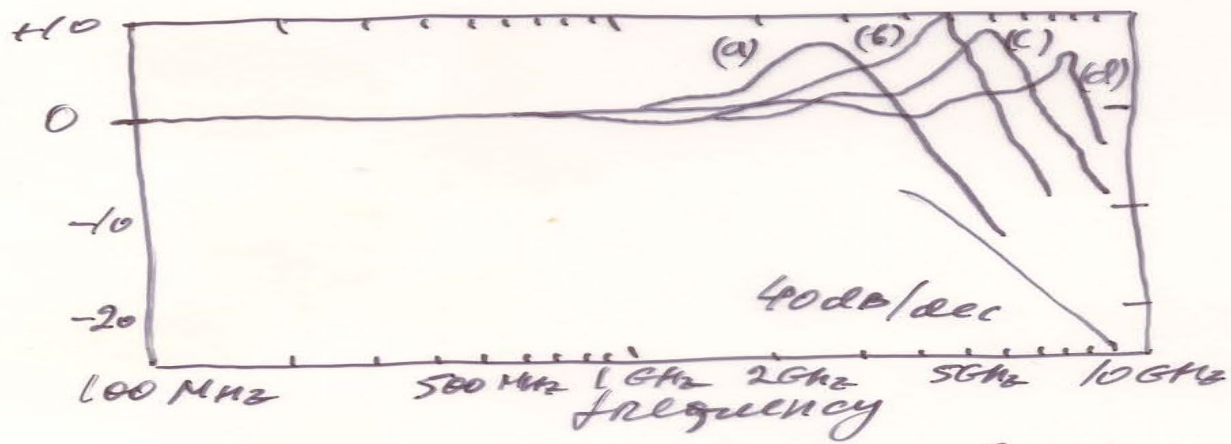
- the light - current transfer function becomes

$$\frac{\Delta P_m}{\Delta \gamma_m} = \frac{\left(\frac{1}{e d}\right) A \left(P_0 + \frac{B}{\tau_s}\right)}{\left[\left(\frac{1}{\tau_p}\right)\left(A P_0 + \frac{B}{\tau_s}\right) - \omega^2\right] + j\omega \cdot \left[\frac{1}{\tau_s} + A P_0\right]} \quad (12)$$

- The resonance in the transfer characteristics occurs when

$$\omega^2 = \frac{1}{\tau_p} \cdot \left[A P_0 + \frac{B}{\tau_s}\right] \simeq \frac{A P_0}{\tau_p} \quad (13)$$

that implies that the resonant frequency increases as photon lifetime decreases and increases as CW power increases. At higher frequencies the response drops as ω^{-2} or 40dB per decade of frequency.



modulation characteristics of a short cavity (20 μm) laser as a function of bias level.
a) 1 mW
b) 2 mW
c) 2.7 mW
d) 5 mW

=> High average power leads to a high modulation capability. There is a practical upper limit — optical damage of the surface.

⚠ Important point — multi-GHz modulation is possible.

1) Electron-hole pairs are created in an intrinsic GaAs semiconductor ($m_e^*=0.067m_e$, $m_h=0.48m_e$) at T=0°K by the absorption of the optical wave with $h\nu-E_g=0.05$ eV ($E_g=1.43$ eV), (since T=0, all the carriers are created by the optical pump).

(a) Identify the energy levels in the conduction and valence bands that can participate in the absorption or gain (i.e., find what is the spacing μ_c-E_c and $E_v-\mu_v$ in eV) Assume probability $f_2(E_2)=1$ for $E_2<\mu_c$ and $f_2(E_2)=0$ for $E_2>\mu_c$ with similar relations for the holes.

(b) What is the minimum number of electron-hole pairs necessary to achieve optical gain at the wavelength $\mu_c-\mu_v$?

a) The electron density satisfies

$$n=\int_{E_c}^{\infty} \rho_c(E_2) f_2(E_2)\, dE_2$$

since $f_2(E_2)=1$, for $E_2 \le \mu_c$ and $f_2(E_2)=0$ for $E_2>\mu_c$ with similar relations for the holes using

$$\rho_c(E_2)=4\pi\left(\frac{2m_e^*}{h^2}\right)^{3/2}\sqrt{E_2-E_c}\;;\quad \rho_v(E_1)=4\pi\left(\frac{2m_h^*}{h^2}\right)^{3/2}\sqrt{E_v-E_1}$$

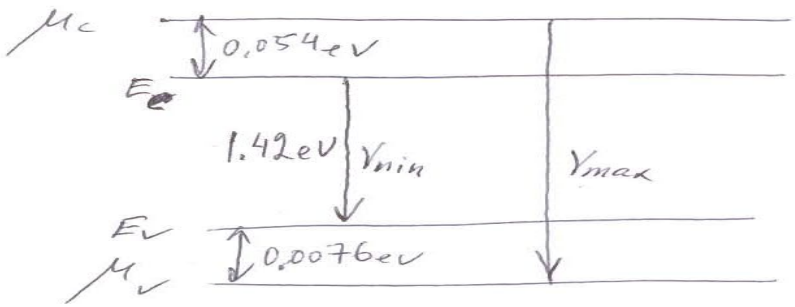
$$n=\int_{E_c}^{\mu_c}\rho_c(E_2)\,dE_2=\frac{1}{2\pi^2}\left(\frac{2m_e^*}{h^2}\right)^{3/2}\int_{E_c}^{\mu_c}\sqrt{E_2-E_c}\,dE_2$$

$$p=\int_{\mu_v}^{E_v}\rho_v(E_1)\,dE_1=\frac{1}{2\pi^2}\left(\frac{2m_h^*}{h^2}\right)^{3/2}\int_{\mu_v}^{E_v}\sqrt{E_v-E_1}\,dE_1$$

$m_e^*=0.067\,m_e$; $m_h^*=0.48\,m_e$, evaluating the constant, and performing integration gives:

$$n = 1.23\times10^{54}(\mu_c - E_c)^{3/2} = \sqrt{np} = 10^{24} \Rightarrow \mu_c - E_c = 8.71\times10^{-21}\,J = 0.054\,eV$$

$$P = 2.35\times10^{55}(E_v - \mu_v)^{3/2} = 10^{24} \Rightarrow E_v - \mu_v = 1.21\times10^{-21}\,J = 0.0076\,eV$$



$$\gamma_{min} = \frac{1.42\times1.6\times10^{-19}}{6.62\times10^{-34}} = 3.43\times10^{14}\,Hz$$

$$\gamma_{max} = \frac{(1.42+0.054+0.0076)\times1.6\times10^{-19}}{6.62\times10^{-34}} = 3.58\times10^{14}\,Hz$$

$$\lambda_{max} = \frac{c}{\gamma_{min}} = \frac{3\times10^{8}}{3.43\times10^{14}} = 875\,nm$$

$$\lambda_{min} = \frac{c}{\gamma_{max}} = \frac{3\times10^{8}}{3.58\times10^{14}} = 838\,nm$$

6) What is the min # of e-h pairs necessary to achieve optical gain at the wavelength $\mu_c - \mu_v \equiv \lambda_{min} = 838\,nm$

Since the laser field is space-dependent, the threshold condition must be derived by setting the condition that spatially averaged gain must equal spatially averaged losses.

$$\Rightarrow \langle g\rangle L = \langle\alpha_a\rangle L + \langle\alpha_n\rangle L + \langle\alpha_p\rangle L + \gamma_m \quad (1)$$

L – length of the AM.
g – gain coefficient
α_a – scattering loss of the active layer,
α_n & α_p losses in the n and p sides of the cladding.
γ_m – mirror loss.

The average values are taken over the field distribution so that, for example, average gain is given:

$$\langle g \rangle = a \frac{\int_c g |u|^2 dV}{\int_c |u|^2 dV} \qquad (2)$$

where $u(x,y,z)$ is the field distribution within the laser cavity. the integral of the numerator is taken over the volume of the active medium, integral of the denominator — over the whole volume of the cavity.

- The quantities $\langle g \rangle$ and $\langle g \rangle L$ are usually referred to as the <u>modal gain coefficient</u> and the <u>modal gain</u>.
- for simplicity we assume that $\alpha_n = \alpha_p = \alpha$, and we neglect the spatial variation of the cavity field along the longitudinal z coordinate (as produced by e.g. a standing wave pattern) and along the coordinate // to the j'unction.

Then with a help of (2) and corresponding expressions for $\langle \alpha_n \rangle$ and $\langle \alpha_p \rangle$ eq (1) gives

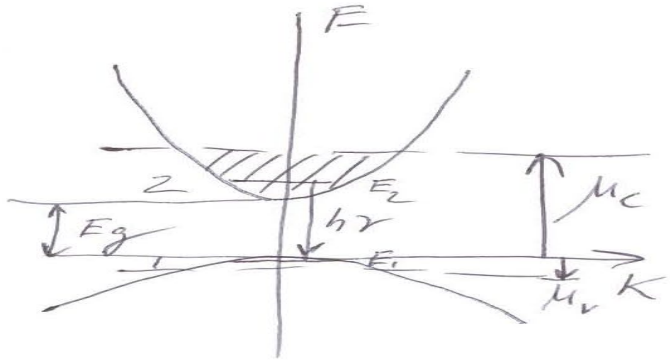
$$g\Gamma = \alpha_a \Gamma + \alpha(1-\Gamma) + [\ln(\tfrac{1}{R})/L] \qquad (3)$$

where $\Gamma = \dfrac{\int_{-d/2}^{d/2} |u|^2 dx}{\int_{-\infty}^{+\infty} |u|^2 dx}$ (4)

confinement factor.

x - coordinate ⊥ junction

R - reflectivity of the 2 end mirrors (assumed to be equal),



$$E_2 - E_1 = h\nu$$
$$h\nu > E_g$$
$$E_2 - E_1 \leq \mu_c - \mu_v$$
$$\Rightarrow \quad E_g \leq h\nu \leq \mu_c - \mu_v$$

to have the gain at any frequency requires $\mu_c - \mu_v \geq E_g$; the limiting case is referred to as the transparency condition. In this case one has $g = 0$ at $\nu = \frac{E_g}{h}$. To achieve this condition, one must inject a density of electrons into the conduction band and holes into the valence band which is called the transparency density N_{TR}.

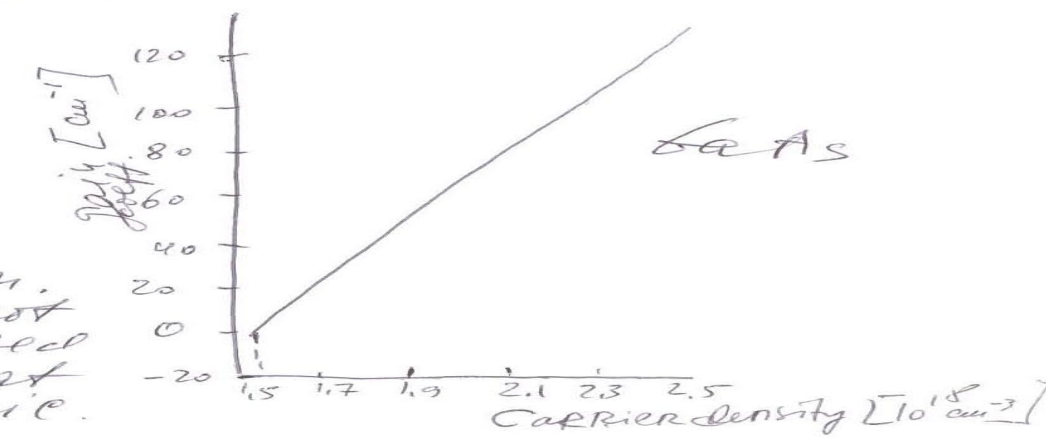
$N_{TR} = 1.2 \times 10^{18}\,cm^{-3}$ for GaAs. !

- $g = \sigma (N - N_{TR})$

$\sigma = 1.5 \times 10^{-16}\,cm^2$ for GaAs.

$\sigma = \frac{dg}{dN}$ - differential gain coefficient.

analogous to cross section. Cross-section concept is not appropriate for a delocalized wavefunction such as that of an electron in semic.



- Assume $\alpha_a = \alpha$ $\Rightarrow$ Eq ③ simplifies to

$$\Gamma = \alpha_a \Gamma + \alpha(1-\Gamma) + \left[\ln\left(\frac{1}{R}\right)/L\right]$$

$$\Gamma(N_{th} - N_{tR}) = \alpha + \left[\frac{\ln\left(\frac{1}{R}\right)}{L}\right] = \frac{\gamma}{L} \quad ⑤$$

N_{th} is the threshold carr. density

γ - total loss per pass

$$\gamma = \alpha L + \ln\left(\frac{1}{R}\right)$$

$\Rightarrow$ from ⑤

$$\boxed{N_{th} = \left(\frac{\gamma}{\sigma L \Gamma}\right) + N_{tR} \quad ⑥}$$

- Expression for Γ

$$\boxed{\Gamma \simeq \frac{D^2}{2+D^2} \quad ⑦}$$

where $\boxed{D = 2\pi (n_1^2 - n_2^2)^{1/2} \frac{d}{\lambda} \quad ⑧}$

is the normalized thickness of the active layer.

n_1 - refractive index of the active medium = 3.6

n_2 " " cladding = 3.4

$\lambda = 838$ nm.

assume $d = 0.1\,\mu m$

$\Rightarrow D \simeq 0.875$ $\Gamma \simeq 0.28$.

- Assume $R = 32\%$ (uncoated surfaces) $\alpha = 10\ cm^{-1}$

$L = 300\,\mu m$ $\Rightarrow \gamma = \ln\frac{1}{R} + \alpha L \simeq 1.44$.

if $\sigma = 3.6 \times 10^{-16}\ cm^2$; $N_{tR} = 2 \times 10^{18}$ carriers/cm^3

$$\boxed{N_{th} = (0.48 + 2) \times 10^{18} \text{ carriers}/cm^3}$$

$$J = \frac{I}{hw}$$

$$R_{spont} = \frac{\mu_{int} J}{ed}$$

$$R_{spont} = \frac{N}{\tau}$$

2. Consider a double-heterostructure (DH) laser consisting of GaAs active layer between two AlGaAs cladding layers, which emits at λ=840 nm. Assume a carrier density at transparency N_{tr}=1.2x10^{18}carriers/cm^3, a cavity length L=300μm, a differential gain σ=3.6x10^{-16}cm^2, a radiative time τ_r=4 ns, a thickness of the active layer d=100 nm, an internal quantum efficiency η_i=0.95 and a total loss per pass γ=1.43. Assume also that the refractive index of the active layer and of the cladding layers are n_1=3.6 and n_2=3.4, respectively. Calculate the current density at threshold required in this laser.

a) $J_{th} = \left(\frac{ed}{\eta_i \tau_r}\right) N_{th}$; d - thickness of the active layer
η_i - internal QEfficiency
τ_r - radiative recombin time.

b) From the ballance between gain and losses in the semiconductor we derived

$N_{th} = \left(\frac{\gamma}{\sigma L \Gamma}\right) + N_{tr}$; γ - total loss per pass
L - length of the active med.
Γ - beam confinement factor

$\Gamma = \frac{\mathcal{D}^2}{2+\mathcal{D}^2} = 0.2814$ where $\mathcal{D} = 2\pi(n_1^2 - n_2^2)^{1/2} \cdot \frac{d}{\lambda} = 0.885$

n_1 - active ; n_2 - cladding layers refr. indexes.

From a) & b)

$$J_{th} = \left(\frac{ed}{\eta_i \tau_r}\right)\left[\frac{\gamma}{\sigma L \Gamma} + N_{tr}\right] =$$

$$= \left(\frac{1.6\times10^{-19} \cdot 100\times10^{-9}}{0.95 \cdot 4\times10^{-9}}\right)\left[\frac{1.43}{3.6\times10^{-16} \cdot 300\times10^{-4} \cdot 0.2814} + 1.2\times10^{18}\right] =$$

$$= \boxed{701\ A/cm^2}$$