Front. Mech. Eng.
DOI 10.1007/s11465-011-0206-2

RESEARCH ARTICLE

Fabrizio SERGI, Dino ACCOTO, Nevio L. TAGLIAMONTE, Giorgio CARPINO, Eugenio GUGLIELMELLI

A systematic graph-based method for the kinematic synthesis of non-anthropomorphic wearable robots for the lower limbs

Abstract The choice of non-anthropomorphic kinematic solutions for wearable robots is motivated both by the necessity of improving the ergonomics of physical Human-Robot Interaction and by the chance of exploiting the intrinsic dynamical properties of the robotic structure so to improve its performances. Under these aspects, this new class of robotic solutions is potentially advantageous over the one of anthropomorphic robotic orthoses. However, the process of kinematic synthesis of non-anthropomorphic wearable robots can be too complex to be solved uniquely by relying on conventional synthesis methods, due to the large number of open design parameters. A systematic approach can be useful for this purpose, since it allows to obtain the complete list of independent kinematic solutions with desired properties. In this perspective, this paper presents a method, which allows to generalize the problem of kinematic synthesis of a non-anthropomorphic wearable robot for the assistance of a specified set of contiguous body segments. The methodology also includes two novel tests, specifically devised to solve the problem of enumeration of kinematic structures of wearable robots: the HR-isomorphism and the HR-degeneracy tests. This method has been implemented to derive the atlas of independent kinematic solutions suitable to be used for the kinematic design of a planar wearable robot for the lower limbs.

Keywords assistive robotics, non-anthropomorphic wearable robots, topology, kinematic synthesis, HR-isomorphism test, HR-degeneracy test

Received November 5, 2010; accepted November 15, 2010

Fabrizio SERGI (✉), Dino ACCOTO, Nevio L. TAGLIAMONTE, Giorgio CARPINO, Eugenio GUGLIELMELLI
Center for Integrated Research, Università Campus Bio-Medico di Roma, Rome, Italy
E-mail: fabrizio.sergi@unicampus.it

1 Introduction

Physical Human-Robot Interaction (PHRI) is an important factor in the design of robots operating in human environments. This factor becomes crucial in the case of wearable robots, which are person-oriented systems worn by human operators to extend, complement, substitute or enhance human function and capability [1]. In the design of robotic orthoses, both for human performance augmentation and for functional restoring, the most followed route has been that of designing the robot so to replicate as much as possible the kinematic structure of the human limbs [2–4].

Robots belonging to this class were thus named exoskeletons, according to the definition given in [5], ("an active mechanical device that is essentially anthropomorphic in nature, is "worn" by an operator and fits closely to his or her body, and works in concert with the operator's movements"). Robot kinematic chain is not a free design parameter for robotic exoskeletons, while wearable robots can be designed to have a possibly non-anthropomorphic kinematic structure (see sketch in Fig. 1), also according to the classification introduced in [6].

1.1 Possible advantages of non-anthropomorphic robots kinematic structures

Biological studies on terrestrial locomotion highlighted how morphology and control are intimately coupled [7]. Robotic researchers have afterwards demonstrated that these findings could be replicated to produce locomotion on hexapod and bipedal robots [8,9]. These studies globally demonstrated that the complexity of the control subsystem could be significantly reduced when the design of the robot structure is driven by the aim of providing a successful intrinsic dynamical interaction with the environment. This enabled to demonstrate the existence of a

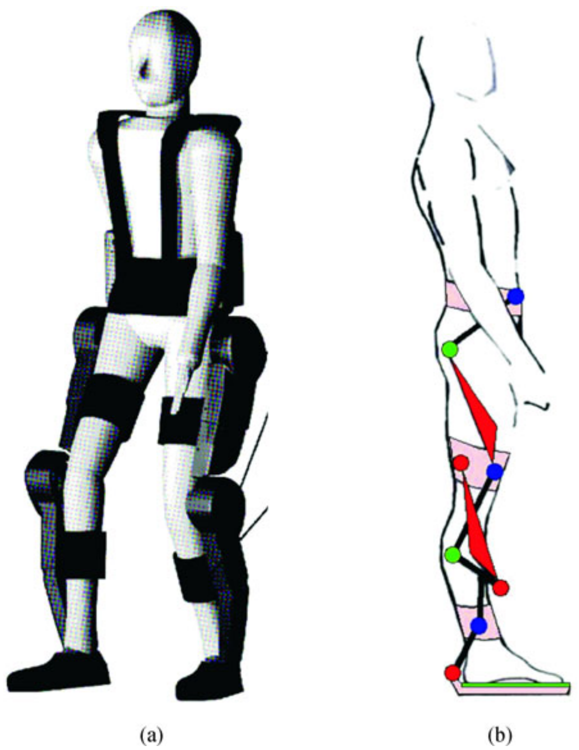


Fig. 1 (a) Example of an anthropomorphic wearable robot for the lower limbs [2]; (b) concept of a non-anthropomorphic wearable robot for the lower limbs

morphology and control trade-off [10] in the design of robots, which can be tuned to exploit the so-called extradimensional bypass [11] in the concurrent design of both robot morphology and control. These considerations suggest that wearable robots performances can benefit from a careful design of robot morphology, which is open in the case of non-anthropomorphic wearable robots, and can allow the achievement of a better dynamical interaction with the human body and with the environment.

Furthermore, the problem of kinematic compatibility is very relevant in the ergonomics of pHRI. In most cases, a kinematic model of a human limb is used for testing virtual concepts of wearable robots using 3D design software. Limbs models may not replicate accurately the biomechanical properties of the real human limbs for several reasons (e.g. inter-subject variability of parameters, or over-simplification of the kinematic model of human joints). Hence, kinematic incompatibilities between the real human limbs and the robot may occur, as described by [12]. These incompatibilities may be classified into: macro-misalignments, induced by a mismatch between the degrees of freedom of the human limb and those allowed by the exoskeleton; micro-misalignments, induced by the non-coincidence of joints axes (when the exoskeleton kinematic structure aims at replicating human kinematics) or by slippage of the exoskeleton attachments on the skin during motion. Both kinds of misalignments cause the exchange of unwanted interaction forces at the sites of contact between the human and the robot; experimental studies have also demonstrated that these interaction forces are source of discomfort and even pain for the user [13,14]. Macro-misalignments are unavoidable since the robot designer has the necessity of simplifying the structure of the robot thus restricting the number of degrees of freedom of the mechanism. Micro-misalignements can instead be avoided if there is no need to align the rotation axes of the robot with those of the human limbs, as it is the case of non-anthropomorphic wearable robots. In [13], a new paradigm for the design of kinematically compatible wearable robots for rehabilitation was proposed, postulating that a wearable robot must explicitly not copy the kinematic structure of the adjacent human limbs, and should provide a moving system acting in parallel with the human degrees of freedom.

1.2 Possible advantages of non-anthropomorphic robots kinematic structures

The problem of optimal kinematic synthesis of non-anthropomorphic wearable robots may be very difficult to be solved by human intuition and engineering insight alone, due to the large number of open parameters involved in the design. This task can be simplified by automatic tools in support of the designer.

In the last decade, evolutionary programming has been applied to solve the problem of co-designing from scratch both the mechanics and the control of mobile artificial machines, by just defining the basic building blocks of the structure and the rules to connect them [15]. This open-ended kind of design methodology has the advantage that it may lead to interesting and unexpected design solutions. However, such kind of design methods imply that the whole design process is completely demanded to the tool, which can autonomously decide to switch to a more complex structure during the optimization phase so to increase the fitness of the best individuals.

The authors are pursuing a systematic approach for the kinematic synthesis of wearable robots. In this approach the design process is divided into three stages. In the first stage a systematic search of all the plausible independent generalized kinematic solutions (i.e. topologies) is performed. In the second stage, an optimization algorithm acting on a fixed number of parameters (encoding both properties pertaining to the mechanical structure and to the control) is used to define the morphology providing the best performances in terms of some design objective. In the final stage, the best morphologies produced by the optimization on each topology are compared with each other, so to define the best solution. This approach appears more reliable since optimization algorithms acting on a fixed parameter space are simpler and with faster convergence properties. Furthermore, each optimization process is independent from the others and can run in parallel on different computers. Additionally, this approach assures that all interesting generalized solutions (i.e. topologies) are evaluated before producing the final design. However, this approach requires the a-priori knowledge of the list of independent topologies having the desired kinematic properties (i.e. maximum number of links and of degrees of freedom (DOFs)) and respecting some basic criteria of kinematic compatibility with the human body.

This paper describes a graph-based method for the exhaustive enumeration of the topologies of planar non-anthropomorphic wearable robotic orthoses, which represents the first stage of the design methodology described above. The method includes two special tests (i.e. the HR-isomorphism test and the HR-degeneracy test), which have purposively been devised to solve the problem of the enumeration of wearable robots kinematic structures. The paper provides the atlas listing all the possible independent kinematic structures of a planar wearable robotic orthosis with up to 7 robot links, to assist a human limb modeled as a 4-link/3-joint serial kinematic chain, satisfying the following kinematic requirements:

1) The number of DOFs is comprised between 3 and 5.

2) The robot structure must not impose unnatural kinematic constraints between one human joint and another one.

2 Kinematic structure encoding

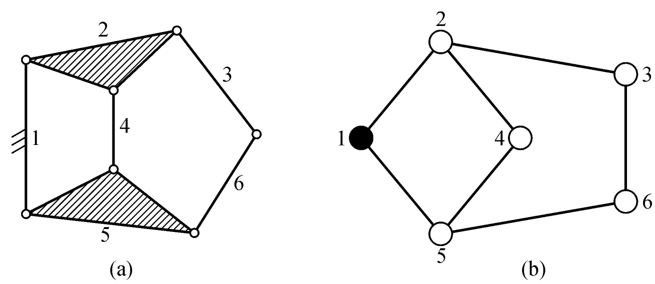
At first, an encoding needs to be defined, in order to represent the kinematic structure of robotic orthoses. Since the aim is to evaluate also the mobility of the human limbs connected to a robotic orthosis, the whole parallel kinematic chain consisting of both robot links and human limbs is considered. The description of this kinematic chain can be performed at two levels of abstraction: i) topology, which defines the number of links and the connections among them and ii) morphology, which instantiates a given topology, adding the geometrical properties of links and of joints.

2.1 Topology

Under some reasonable hypotheses [16], many properties of mechanisms kinematics, such as the number of degrees of freedom, are entirely determined only by the topology of the kinematic chain and unaltered by the geometric properties of its links. At this level of abstraction, the classical (graph)-(kinematic chain) analogy introduced in [17] can be employed, where graph vertexes correspond to the links of the chain and edges correspond to the joints. A graph can then be encoded through the Topology vertex-vertex Adjacency Matrix (TAM), which is a binary symmetric matrix of order n (where n corresponds to the number of links) where the element a_{ij} equals to 1 if link i and link j are connected through a joint, and to 0 otherwise (cfr. Fig. 2).

As a first assumption, we decide to focus on planar kinematic chains composed of only revolute joints. It is then unnecessary to discriminate on the type of joint connecting each link; hence the representation is complete in the description of kinematic chains topology allowing to convert the problem of kinematic synthesis into a problem of graphs enumeration. The mentioned assumption limits the relevance of the methodology for the design of assistive wearable robots for the lower limbs, since the hip and the ankle joints have spatial movements. However, it can be noticed that most of the power of the lower limbs is provided by actuation of movements in the sagittal plane, which is the dominant plane of motion during human locomotion.

The described representation is then complete in the description of kinematic chains topology and allows converting the problem of kinematic synthesis into a problem of graphs enumeration.



$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(c)

Fig. 2 Structural representation (a), graph representation (b) and TAM (c) of a six links kinematic chain (link 1 is filled in black since it is mechanical ground)

2.2 Morphology

The previously described representation for the topology of the kinematic chain can be adapted also for describing robot morphology, which gives details on the geometrical properties of links and is necessary to evaluate the possible kinematic configurations of the chain. We restrict our focus on planar kinematic chains containing only revolute joints. Then, the only necessary information to define robot morphology is the position of joints in a particular configuration.

In such a way a binary link is represented by a bar jointed at its extremities, a ternary link by a triangle jointed at its vertices and so on. Under these assumptions we only need to modify the adjacency matrix by adding the information concerning joints coordinates. This can be done by introducing the Morphology Adjacency Matrix (MAM), which is a square matrix of order n containing x-coordinates of joints (if any) in its strictly upper-triangular part and y-coordinates of joints in its strictly lower-triangular part. A "null" value is inserted in the indexes where the TAM contains zeroes, to avoid confusion between the absence of a joint and the superposition of one joint to the x or y axis of the reference frame.

3 Enumeration of kinematic chains

This section focuses on the description of the methodology for the enumeration of the whole set of independent kinematic solutions for the design of a robotic orthosis which can assist the movements of human limbs. The approach is presented for the case of a lower limb, which is schematized here as a serial kinematic chain composed of four links and three joints.

The problem is graphically represented in Fig. 3, which depicts the structural representation, the graph representation and the corresponding TAM of the kinematic chain comprising both human segments and robot links. The process of enumerating kinematic chains consists of three successive steps: (a) enumeration of graphs with the desired mobility, (b) degeneracy testing, (c) isomorphism detection. These steps will be described separately in the following paragraphs.

3.1 Enumeration of kinematic chains with the desired mobility

Without loss of generality, each kinematic solution is represented by a graph with $h + r$ edges, where h corresponds to the number of body segments (4 in our case) and r corresponds to the number of robot links. The following conditions are imposed:

1) links 1 (trunk), 2 (thigh), 3 (shank) and 4 (foot) compose the serial kinematic chain of the human limb supported by the wearable robot;

2) links 5, 6, 7 and 8 are by construction connected to the body segments. The necessity of providing assistance to each human joint implies that each body segment needs to be connected to a robot link. Additionally, each robot link connected to a body segment cannot be attached to another body segment not to reduce mobility of human joints (this would constitute a HR-degenerate sub-chain (see corresponding paragraph).

The complete list of independent kinematic solutions can be derived from the frame of the basic TAM shown in Fig. 3(c). Any topology can be encoded by a binary string of length l, where

$$l = h\cdot(r-h) + \frac{r(r-1)}{2}. \tag{1}$$

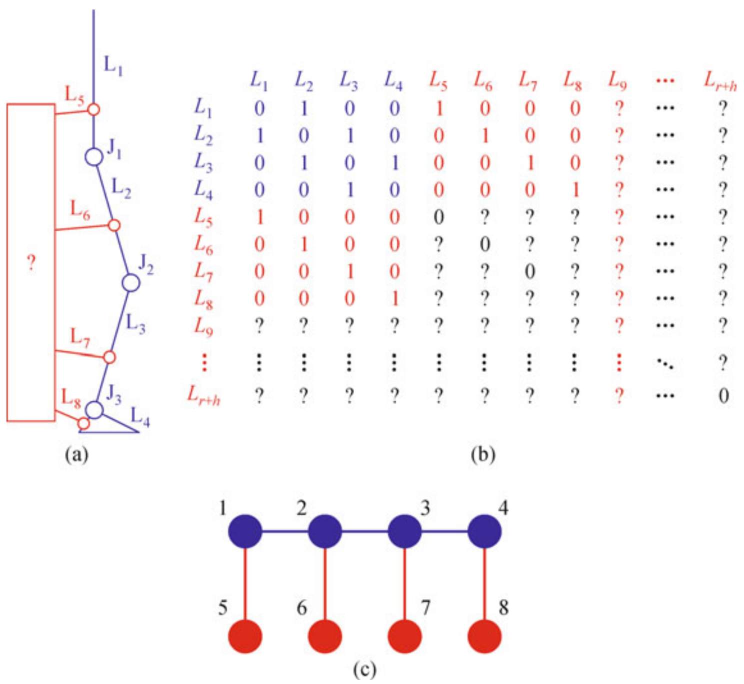


Fig. 3 Structural representation (a), generalized TAM (b) and graph representation (c) of the problem of structural synthesis of robotic orthoses for a planar wearable robot for the lower limbs. Human articulations and segments are in blue, while robot links and joints are in red. In the adjacency matrix, the blue color is used to represent entries which describe the connectivity of human limbs (condition (1) in paragraph IIIA), while the red color represents fixed entries provided by condition (2), paragraph IIIA

However, not any combination of parameters is adequate, since we are interested only in kinematic chains with a given number of DOFs, i.e. 3, 4 and 5. For a given planar kinematic chain with n links and f joints with 1 DOF, the total number of degrees of freedom (DOFs) is obtained by using the Kutzbach criterion [18]:

$$\text{DOFs} = 3(n-1)-2f. \tag{2}$$

Equation (2) implies that given any number of links, the kinematic chain can contain only a fixed number of joints. Since each joint between links i and j corresponds to a 1 in the (i, j) position of the corresponding TAM, the problem of enumeration of all topologies with a desired mobility is converted into the problem of exhaustively listing the binary strings of length l, with a fixed number of ones (e, as shown by Table 1).

This problem can be solved by considering all the possible $\binom{l}{e}$ combinations of the l integers from 1 to l taken e at a time and using those combinations as suitable locations where "one" values are to be inserted in the binary string encoding the TAM. This method implies a significant reduction (up to 99.99%) of the number of TAMs to be generated when comparing it to the "brute-force" approach where all 2^l combinations are enumerated. Table 1 shows that the number e of additional joints to be added in the kinematic structure takes into account that the basic structure implies the presence of a given set of joints (corresponding to the human articulations and to the connections with the basic set of robot links shown in Fig. 3).

After matrices enumeration, links connectivity is evaluated. The order of link i is defined as the number of joints in which link i participates. It can be easily calculated from the TAMs by summing elements in the i^{th} row:

$$order_i = \sum_{k=1}^{n} a_{ik}. \tag{3}$$

Since only closed kinematic chains are addressed by this enumeration, solutions including links with order lower than 2 must be discarded. Moreover, the upper bound for the order of any link must satisfy the inequality: max $(order_i) \leqslant L + 1$, where L represents the maximum number of independent loops.

Since the robotic structure is applied in parallel to three human joints the maximum order of robot links is set to 4.

3.2 Degeneracy and HR-degeneracy tests

A further selection over the list of enumerated topologies is performed, in order to filter out the kinematic chains which:

- contain rigid or over-constrained sub-chains;
- correspond to disconnected graphs (i.e., not all graphs vertices are connected by a path);
- impair the simultaneous motion of human joints.

A standard degenerate testing algorithm has been implemented to recognize and discard rigid sub-chains (such as 3 links-3 joints and 5 links-6 joints sub-chains). Degenerate kinematic chains are then eliminated. Kinematic chains containing at least one sub-chain with zero or negative DOFs according to Kutzbach formula (such as 3 links-3 joints and 5 links-6 joints sub-chains) are considered as degenerate solutions. Additionally, disconnected mechanisms (i.e. such that there is not a path connecting each couple of vertices of the corresponding graph) are eliminated with a purposively developed algorithm, which verifies the existence of a path between each couple of vertices.

Furthermore, an additional test was introduced so to exclude those solutions where a subset of p human joints is part of a subchain with less than p DOFs. In this case the robot would impair human movements by imposing unnatural kinematic constraints, violating the second kinematic requirement reported in section 1. This test is called HR-degeneracy test (Human-Robot degeneracy test) since it applies to kinematic chains including both human and robot structures. The test is performed by recognizing the presence of sub-chains where two adjacent human joints are constrained in a 1-DOF sub-chain (Fig. 4(a) and

Table 1 Parameters of the enumeration algorithm.

Robot links (r)	Total links (n)	DOFs	Indep. loops	Total joints (f)	Imposed joints	Additional joints (e)	Open Parameters (l)	Combinations considering desired DOFs $\binom{l}{e}$	Total combinations (2^l)	Reduction ratio/%
4	8	3	2	9	7	2	6	15	64	76.56
5	9	4	2	10	7	3	14	364	1.64×10^4	97.78
6	10	3	3	12	7	5	23	3.4×10^4	8.4×10^6	99.60
6	10	5	2	11	7	4	23	8.9×10^3	8.4×10^6	99.89
7	11	4	3	13	7	6	33	1.11×10^6	8.6×10^9	99.99

(c)) or where all three adjacent human joints are constrained in a 2-DOFs sub-chain (Fig. 4(b)). The exhaustive list of such HR-degenerate primitives (reported in Fig. 4) could be obtained by adapting results coming from standard atlases of kinematic chains [19] and was re-obtained in a previous work concerning the enumeration of orthoses for a 1-DOF human joint [20].

3.3 HR-isomorphism test

Since the chosen method is based on the enumeration of suitable matrices of adjacencies, an explicit isomorphism test is mandatory. Two kinematic chains K_1 and K_2 are said to be isomorphic if there exists a one-to-one correspondence between links of K_1 and K_2 such that any pair of links of K_1 are jointed if and only if the corresponding pair of links of K_2 are jointed. This means that from the graph corresponding to K_1 one can obtain the graph corresponding to K_2 by only relabeling link numbers. Many efforts have been dedicated in the past to the problem of finding an accurate and computationally efficient test for detecting isomorphisms. Most computationally efficient methods (as that described in [21]) move around the obstacle of explicit isomorphism detection between pairs of kinematic chains. Those methods are based on group theory applied to graphs and exhaustively generate isomorph-free classes of graphs. These methods would give the exhaustive list of all non-isomorphic graphs and produce as output only one graph for each homomorphic group of graphs. Unfortunately, they are not directly applicable to the described problem since they do not take into consideration the constraints indicated in section IIIA and would not assure that a valid topology (i.e., a topology which respects the constraints imposed in section IIIA) would be obtained, requiring a re-labeling of the obtained graphs and the application of the whole set of inversions of the four serially connected body segments.

For these reasons the method of progressive enumeration of kinematic chains has been adopted. This method is not computationally efficient, but it produces only valid kinematic chains. However, the chosen method implies the need for an isomorphism detection algorithm to avoid the generation of two kinematically identical solutions.

A function defined on a kinematic chain is called an index of isomorphism if any given pair of kinematic chains is isomorphic if and only if the corresponding values of the function are identical. The index of isomorphism used in the present work is the characteristic polynomial of the Extended Adjacency Matrix (EAM) $\boldsymbol{A}^{(d)}$ of order d, as also suggested in [22], which can be obtained from a TAM ($\boldsymbol{A}$, of elements a_{ij}), by employing the following formula:

$$\boldsymbol{A}^{(d)}=\begin{pmatrix} s_d(-\boldsymbol{a}_1) & a_{12} & \cdots & a_{1n} \\ a_{21} & s_d(-\boldsymbol{a}_2) & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & s_d(-\boldsymbol{a}_{\mathrm{n}}) \end{pmatrix} \tag{4}$$

where the vector $\boldsymbol{a}_{\mathrm{i}}=(a_{i1},a_{i2},...,a_{in})$ contains elements of the ith row and s_d is the elementary symmetric polynomial of order d in the n variables of vector $\boldsymbol{a}$, defined as

$$\begin{aligned} &s_0(\boldsymbol{a})=0,\\ &s_1(\boldsymbol{a})=\sum_i a_i,\\ &\ldots,\\ &s_k(\boldsymbol{a})=\sum\nolimits_{j_1<j_2<\cdots<j_k} a_{j_1}a_{j_2}\ldots a_{j_k}. \end{aligned} \tag{5}$$

Said $\boldsymbol{poly}_i^{(j)}$ the vector containing the normalized coefficients of the characteristic polynomial of matrix $\boldsymbol{A}_i^{(j)}$, our test of isomorphism is such that two matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are isomorphic if each of the three equations in Eq. (6) are verified.

$$\boldsymbol{poly}_1^{(j)}=\boldsymbol{poly}_2^{(j)}, \mathit{for}\ j=0,1,2. \tag{6}$$

In [22]. it is demonstrated that the simultaneous evaluation of the characteristic polynomial of both $\boldsymbol{A}^{(0)}$, $\boldsymbol{A}^{(1)}$ and $\boldsymbol{A}^{(2)}$ has a reliability of 100% for kinematic chains consisting of up to 11 links. This technique for isomorphism detection shows to be a very good compromise between reliability and computational efficiency, since it requires a polynomial time for assessing isomorphism and hence has been adopted.

However, when applying the isomorphism test to kinematic chains including both human segments and robot links, any kind of isomorphism test produces false-positives because robot and human links would be treated

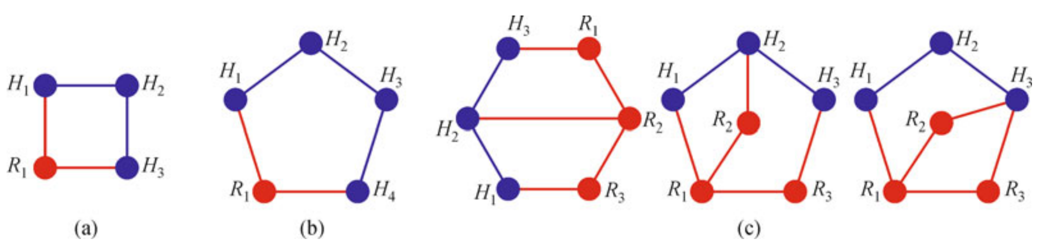


Fig. 4 HR-degenerate topologies. (a) Two adjacent human segments are constrained in a 1-DOF subchain; (b) three adjacent human segments are constrained in a 2-DOF subchain; (c) all possible independent assortments of two adjacent human joints involved in a 6 links, 1 DOF subchain are shown

the same way. This happens because isomorphism tests are "blind" with respect to the special condition regarding the scopes of this work, which involves considering both human segments and robot links as part of a unique kinematic chain. A false positive happens any time the permutation, which maps one graph into the other, affects any of the human joints. From the perspective of designing a wearable robot aimed at a certain kind of interaction with each of the human joints, such solutions correspond to actual different wearable robots topologies and must not be discarded. An example of two isomorphic but not HR-isomorphic solutions is shown in Fig. 5. To recognize such kind of solutions, a modified version of the isomorphism test has been introduced, the HR-isomorphism test (since it applies to kinematic chain including both human and robot structures). This test is described in the flow-chart shown in Fig. 6 and basically consists of assessing, after a classical characteristic polynomial-based isomorphism test, whether one of the permutations p_{adm} contained in a properly defined set P_{adm} is responsible for mapping one kinematic chain into another. Every permutation vector contained in the P_{adm} set is of the form $p_{adm}(i)$ = [1 2 3 4 $perms_i(5:n)$], where the function $perms_i$ provides the ith element of the set of permutations of the elements in the input array.

4 Results

The described algorithm has been implemented in MATLAB (The MathWorks, Inc.) and allowed to obtain the complete list of topologies describing the kinematic chains, which consist of both human segments (a serial 4-link, 3-joint kinematic chain) and robot links. We limited our search to the space of robot kinematic chains with up to 7 links, since we expect that a robot with more links would result in a too complex and overwhelming system. Since in the enumeration process also human kinematic chain is taken into account, the highest order of generated graphs for the case of lower limbs wearable robots is $n_{\mathrm{MAX}} = 11$.

4.1 Orthosis for a 1-DOF human joint

Preliminarily, the enumeration method has been applied to the simplest case, which consists of an orthosis for supporting a 1-DOF human joint. In this case, the distinction between human segments and robot links is meaningless and it is expected to re-obtain the same results obtained in the general case of mechanism enumeration. The results of the enumeration process are reported in Table 2 and are coherent with the results obtained for the general case of mechanism enumeration [16], thus confirming the completeness and technical soundness of the chosen approach.

4.2 Four robot links

The simplest solution respecting the constraints defined in Sect. 3.1 is composed of the basic set of 4 robot links, one for each body segment. To obtain a structure with 3 DOFs, two additional joints must be added to the basic configuration shown in Fig. 3. The two additional joints

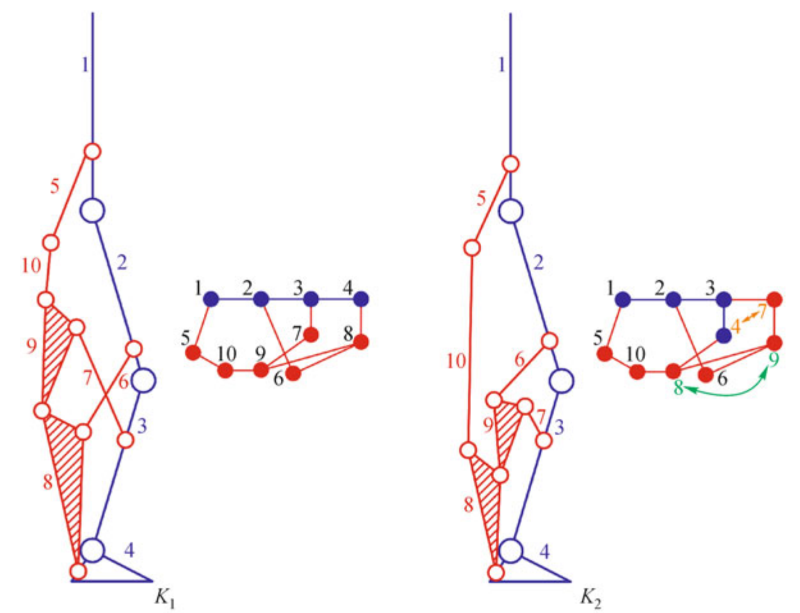


Fig. 5 Two isomorphic but not HR-isomorphic solutions. The permutation mapping K_1 into K_2 is given by the permutation vector [1 2 3 7 5 6 4 9 8]. This permutation maps link 4 (i.e., foot) into robot link 7. It can be noticed that local kinematic properties around each human joint (for example DOFs of the subchain including the hip, the knee and the ankle joints) are different

Table 2 Enumeration of kinematic structures for a planar orthosis assisting a 1-DOF human joint.

Number of links (human + robot)	DOFs	Independent topolgies
4	1	1
5	2	1
6	1	2
7	4	1
7	2	4
8	5	1
8	3	7
8	1	16

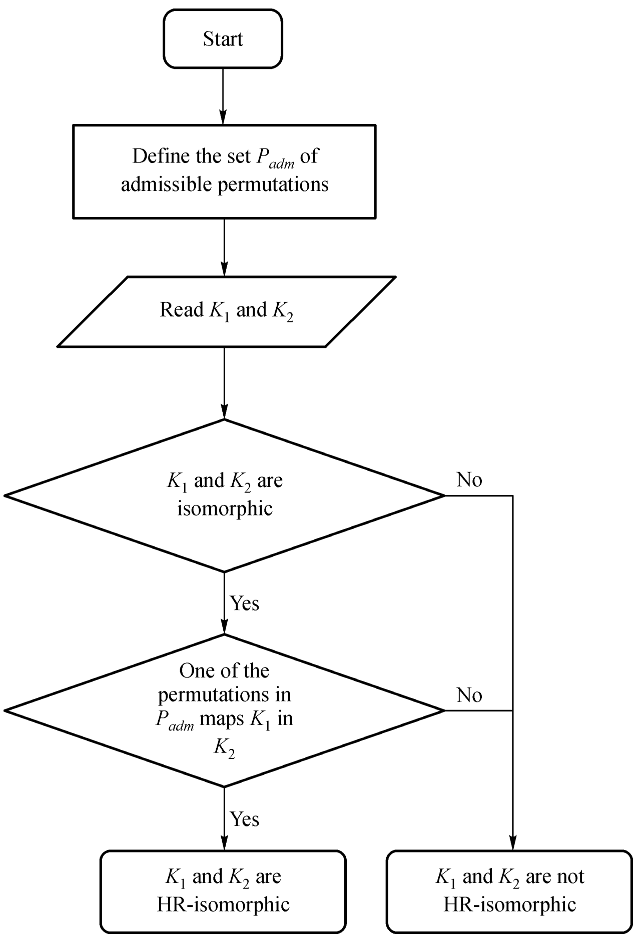


Fig. 6 Flow chart of the HR-isomorphism test algorithm. The set P_{adm} contains the $r!$ permutations which only act on subgraphs including robot links. They are all defined by a permutation vector of the form $p(i) = [1\ 2\ 3\ 4\ perms_i(5:n)]$, where the function $perms_i$ provides the ith element of the set of permutations of the elements in the input array. These permutations are needed so to verify if one of the permutations in P_{adm} is responsible for mapping K_1 into K_2 thus assessing the HR-isomorphism between K_1 and K_2

must involve each of the added robot links, so to avoid the presence of serial chains. Each of the added joint causes the generation of one independent loop. This implies a total number of 2 independent loops in the kinematic chain, which does not allow the robot to independently actuate each of the three human joints. Two independent solutions were found, as listed in Table 3, and shown in Fig. 7.

Table 3 Enumeration of independent topologies for wearable robots for the lower limbs

Number of links (human + robot)	DOFs	Independent topoliges
4	1	1
5	2	1
6	1	2
7	4	1
7	2	4
8	5	1
8	3	7
8	1	16

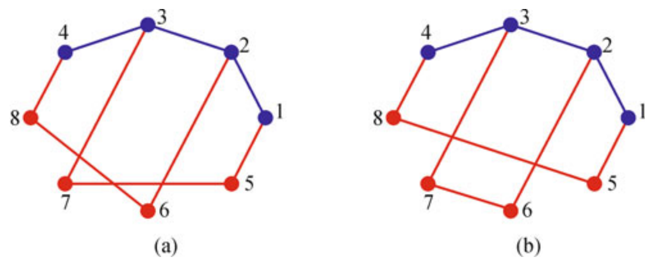


Fig. 7 Two possible topologies with four robot links

4.3 Five robot links

In this case the design implies the addition of 1 robot link and 3 joints to the basic kinematic structure shown in Fig. 3. The resulting kinematic chain has only 2 independent loops, and the total number of DOFs is 4. Four independent solutions exist, two of which are isomorphic but not HR-isomorphic (see Fig. 8(c) and (d)).

4.4 Six robot links

The addition of two robot links to the basic kinematic structure shown in Fig. 3 implies the possibility of having a structure with either 3 or 5 DOFs (adding five or four joints respectively). In the case of 3 DOFs, 293 independent solutions exist, while in the case of 5 DOFs, the number of solutions reduces to 6. In the case of 3 DOFs, there are 3 independent loops, while in the case of 5 DOFs only 2

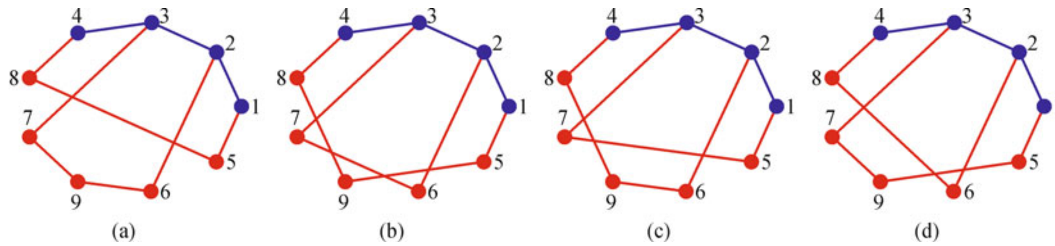


Fig. 8 Possible topologies including five robot links. In (c) and (d) two isomorphic but not HR-isomorphic solutions are shown. The corresponding permutation is defined by the permutation vector [4 3 2 1 8 7 6 5 9], which basically implies wearing the same robot structure bottom-up. However the two structures are independent from the wearer's standpoint since they impose different kinematic constraints on human joints

independent loops are present, which excludes the possibility of independently controlling the motion of each human joint.

4.5 Seven robot links

In this case 3 robot links and 6 joints are added, and the compound kinematic chain has a total of 4 DOFs, with 3 independent loops, making it possible to independently control motion of each human articulation. The number of independent enumerated kinematic solutions is 754, as summarized in Table 3.

5 Discussion and conclusions

Non-anthropomorphic kinematic solutions for robotic orthoses promise positive properties from both the standpoints of the dynamical interaction with the human body and of ergonomics. Despite of that, it is complex to synthesize an appropriate kinematic structure relying only on conventional kinematic synthesis methods. To facilitate this task, a systematic approach can be useful, since it allows to obtain the enumerate the whole list of independent solutions with desired kinematic properties, which generalize the set of solutions of the problem of kinematic synthesis of a non-anthropomorphic wearable robot with given requirements.

This paper presents a method for the systematic enumeration of all the topological solutions for a problem of kinematic synthesis of a robotic orthosis for the lower limbs. The design is bounded by a certain set of requirements, which were formulated on a kinematic basis, including the total number of DOFs and the local possibility of independent motion of adjacent human joints. The number of desired DOFs is comprised between 3 and 5. The lower boundary is motivated by having modeled the human limb as a planar 3 DOFs kinematic chain and by the specification of independently actuating each of the human joints, while the upper bound is provided by the assumption that structures with more than 5 DOFs would constitute a too complex and overwhelming mechanical system. The presence of redundant DOFs is supposed to be beneficial for providing a mean of changing the intrinsic dynamical properties of the system during gait assistance, but however this argument needs to be tested in simulation and then in a real prototype implementation.

The method described in the paper has been applied to the case of a planar wearable robot for the lower limbs, and has produced the atlas of independent topological solutions containing up to 7 robot links, which are summarized in Table 3. The number of independent solutions substantially differs from what reported by standard atlases of mechanisms for two main reasons: (i) also mechanism inversions (in which links corresponding to the human body are relabeled preserving the serial 4-link/3-joint structure) are considered as independent solutions (this is accomplished by using the HR-isomorphism test); (ii) solutions which impair the natural movements of human joints are filtered out (this is accomplished by the HR-degeneracy test).

The number of plausible topologies is around the thousand and can then be managed by optimization tools (e.g. evolutionary algorithms) for morphology optimization, acting separately on each family of solutions defined in terms of topology.

However, in the attempt to further reduce the dimensions of the search space and hence to limit the computational complexity of the optimization problem, additional design considerations can be employed. For example, by filtering out the solutions including a path of length 3 (two links) between the trunk and the foot (which would imply, for example, that links 5 and 8 are connected, making it necessary that links 5 and/or 8 are relatively long), the set of solutions reduces from 293 to 196 (10 links, 3 DOFs) and from 754 to 579 (11 links, 4 DOFs). Other similar filters, such as considering the maximum order of robot links, can be employed to reduce the number of solutions.

Acknowledgements This work was supported by the FP7 FET Proactive Initiative "Embodied Intelligence" of the European Commission, project No. ICT-2007.8.5-231451-EVRYON (evolving morphologies for human-robot symbiotic interaction).

References

1. Pons J L. Wearable Robots: Biomechatronic Exoskeletons. Wiley, 2008, 1–2
2. Kawamoto H, Sankai Y. Power assist system HAL-3 for gait disorder person, Lecture Notes on Computer Science, Berlin: Springer-Verlag, 2002, 2398: 196–203.
3. Walsh C J, Endo K, Herr H. Quasi-passive leg exoskeleton for load-carrying augmentation. International Journal of Humanoid Robotics, 2007, 4(3): 487–506
4. Veneman J F, Kruidhof R, Hekman E E G, Ekkelenkamp R, Van Asseldonk E H F, van der Kooij H. Design and evaluation of the LOPES exoskeleton robot for interactive gait rehabilitation. IEEE Transactions on Neural Systems and Rehabilitation Engineering, 2007, 15(3): 379–386
5. Dollar A M, Herr H. Lower extremity exoskeletons and active orthoses: Challenges and state-of-the-art. IEEE Transactions on Robotics, 2008, 24(1): 144–158
6. Guglielmelli E, Johson M J, Shibata T. Guest editorial, special issue on rehabilitation robotics. IEEE Transactions on Robotics, 2009, 25 (3): 477–480
7. Kubow T, Full R. The role of the mechanical system in control: A hypothesis of selfstabilization in hexapedal runners. Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, 1999, 354(1385): 849–861
8. Cham J G, Karpick J K, Cutkosky M R. Stride period adaptation for a biomimetic running hexapod. International Journal of Robotics Research, 2004, 23(2): 141–153
9. Collins S, Ruina A, Tedrake R, Wisse M. Efficient bipedal robots based on passive-dynamic walkers. Science, 2005, 307(5712): 1082–1085
10. Pfeifer R, Lungarella M, Iida F. Self-organization, embodiment, and biologically inspired robotics. Science, 2007, 318(5853): 1088–1093
11. Bongard J C, Paul C. Making evolution an offer It can't refuse: Morphology and the extradimensional bypass. Lecture Notes in Computer Science, 2001, 2159: 401–412
12. Schiele A, van der Helm F C T. Kinematic design to improve ergonomics in human machine interaction. IEEE Transactions on Neural Systems and Rehabilitation Engineering, 2006, 14(4): 456–469
13. Colombo G, Joerg M, Diez V. Driven gait orthosis to do locomotor training of paraplegic patients, 22nd Annual Conference of the Engineering in Medicine and Biology Society, Chicago, IL, 2000
14. Hidler J M, Wall A E. Alterations in muscle activation patterns during robotic-assisted walking. Clinical Biomechanics (Bristol, Avon), 2005, 20(2): 184–193
15. Lipson H, Pollack J B. Automatic design and manufacture of robotic lifeforms. Nature, 2000, 406(6799): 974–978
16. Mruthyunjaya T S. Kinematic structure of mechanisms revisited. Mechanism and Machine Theory, 2003, 38(4): 279–320
17. Dobrjanskyj L, Freudenstein F. Some applications of graph theory to structural analysis of mechanisms. Journal of Engineering for Industry-Transactions of the ASME, Series B, 1967, 89: 153–158
18. Kutzbach K. Mechanische Leitungsverzweigung. Maschinenbau, der Betrieb, 1929, 8, 710–716
19. Ding H F, Huang Z. A unique representation of the kinematic chain and the atlas database. Mechanism and Machine Theory, 2007, 42 (6): 637–651
20. Sergi F, Accoto D, Tagliamonte N L, Carpino G, Pathiyil L, Guglielmelli E. A systematic graph-based method for the kinematic synthesis of non-anthropomorphic wearable robots, IEEE Conference on Robotics Automation and Mechatronics (RAM), 2010, 100–105
21. McKay B D. Isomorph-free exhaustive generation. Journal of Algorithms, 1998, 26(2): 306–324
22. Sunkari R P. Structural synthesis and analysis of planar and spatial mechanisms satisfying Gruebler's degrees of freedom equation, Dissertation for the Doctoral Degree, Univ. of Maryland MD, 2006