Combined Refraction/Diffraction Model

REF/DIF 1
Version 2.4

Documentation and User's Manual

James T. Kirby and Robert A. Dalrymple

Center for Applied Coastal Research
Department of Civil Engineering
University of Delaware, Newark, DE 19716

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1 THEORETICAL BACKGROUND

The propagation of water waves over irregular bottom bathymetry and around islands involves many processes – shoaling, refraction, energy dissipation and diffraction. Until recently, only very approximate models existed to predict the wave behavior due to these effects. This manual describes the weakly nonlinear combined refraction and diffraction model initialy developed by Kirby and Dalrymple (1983a), which incorporates all of the effects mentioned above.

The practical analysis of the refraction of water waves has generally been carried out in the past using ray tracing techniques. This technique does not include wave diffraction, and therefore it is inaccurate whenever diffraction effects are important. Often due to complexities in the bottom topography, wave tracing diagrams have many intersecting wave rays which leads to difficulties in interpretation, as the theory predicts infinite wave heights at these locations. Recently, finite difference refraction models have been developed which have the advantage of providing wave heights and directions on a model grid rather than on irregularly spaced rays (see, for example, Dalrymple (1988, 1991)).

The diffraction of water waves around simple structures such as an offshore breakwater has been obtained analytically for a constant water depth, Sommerfeld (1896). Diagrams of the wave heights in the vicinity of such a structure have been presented by the Corps of Engineers (1973, also Wiegel, 1962). For a cylindrical structure, MacCamy and Fuchs (1954) presented the constant depth solution. These solutions give not only the wave height transmitted past the structure, but also the scattered, or reflected, wave radiating away from the structure. Generalized versions of these diffraction problems, using numerical techniques and the Green's function method, have yielded very powerful procedures for wave force calculation for cases where the drag force is much smaller than the inertia force.

In order to incorporate diffractive effects, the general practice has been to suspend refraction in areas where diffraction is dominant and only permit diffraction there, using Sommerfeld's analytic solution for a flat bottom. Far from the diffraction area, refraction is resumed. This ad-hoc technique clearly is inaccurate, but does permit the inclusion of diffraction in an approximate way.

Combined refraction/diffraction models include both effects explicitly, thus permitting the modelling of waves in regions where the bathymetry is irregular and where diffraction is important. Regions where wave rays cross due to local focussing or where caustics are caused by other means are treated correctly by the models and no infinite wave heights are predicted. The models, developed in parabolic form, do not predict the waves which are scattered upwave; that is, waves which are reflected directly back the way they came are

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not modelled and are neglected. This means, in general, wave reflection phenomena are not reproduced correctly.

Combined refraction/diffraction models are uniquely suited for the calculation of wave heights and wave direction in areas where one or both of these effects are present. Examples include the determination of wave heights in a bay given the offshore wave heights, periods and directions, and determination of the amount of wave energy penetrating an island chain, or calculation of the sheltering and hence the disturbance of the littoral processes by an island situated near a shoreline. They are not intended to replace diffraction theories currently in use for wave force calculations.

The weakly nonlinear combined refraction and diffraction model described here, denoted **REF/DIF 1**, is based on a Stokes expansion of the water wave problem and includes the third order correction to the wave phase speed. The wave height is known to second order (Liu and Tsay (1984)). It should be noted that it is not a complete third order theory, as all the third order terms are not retained. Known ambient currents, which effect the height and direction of wave propagation, are input for the model and enable it to predict waves where currents may be strong.

The application of the theoretical model to practical situations involves the use of a parabolic approximation, which restricts the model to cases where the wave propagation direction is within ±60° of the assumed wave direction, and the use of finite difference techniques for the wave amplitude, which results in tridiagonal matrices, which are computationally very fast to invert.

The **REF/DIF 1** model is described in detail in this manual, which also documents the application of the model to actual examples and provides explicit descriptions of the input and output.

1.1 Wave Models

1.1.1 Mild slope equation

The problem of water waves propagating over irregular bathymetry in arbitrary directions is a three-dimensional problem and involves complicated nonlinear boundary condition. Very few solutions to the three dimensional problem exist and those that do are only for flat bottoms. In one horizontal dimension, sophisticated models by Chu and Mei (1970) and Djordjevic and Redekopp (1978) predict the behavior of Stokes waves over slowly varying bathymetry. In order to simplify the problem in three dimensions, Berkhoff (1972), noted that the important properties of linear progressive water waves could be predicted by a weighted vertically integrated model. (The vertical integration reduces the problem to only the two horizontal dimensions, x and y.)

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Berkhoff's equation is known as the mild slope equation. It is written in terms of the surface displacement, $\eta(x, y)$. The equation, in terms of horizontal gradient operator, is

$$\nabla_h \cdot (CC_g \nabla_h \eta) + \sigma^2 \frac{C_g}{C} \eta = 0 \qquad (1)$$

Here,

$$C = \sqrt{(g/k) \tanh kh}, \text{ the wave celerity,}$$

$$C_g = C\{1 + 2kh/\sinh 2kh\}/2, \text{ the group velocity,}$$

where the local water depth is $h(x, y)$ and g is the acceleration of gravity. The local wave number, $k(x, y)$, is related to the angular frequency of the waves, σ, and the water depth h by the linear dispersion relationship,

$$\sigma^2 = gk \tanh kh \qquad (2)$$

The model equation (1) is an approximation; however, it is quite good even for moderately large local bottom slopes (see Booij, 1983). In both deep and shallow water, it is exact. Numerous authors have applied the mild slope model to various examples, primarily using finite element techniques. See, for example, Jonsson and Skovgaard (1979), Bettess and Zienkiewicz (1977), and Houston (1981).

For the linear mild slope equation, Radder (1979) developed a parabolic model, which had several advantages over the elliptic form presented by Berkhoff. First, the boundary condition at the downwave end of the model area is no longer necessary and, secondly, very efficient solution techniques are available for the finite difference form of the model. Radder used a splitting matrix approach, which involves separating the wave field into a forward propagating wave and a backward propagating wave, and then neglecting the backward scattered wave (which is justified in most applications as only the forward propagating wave is used for design). Radder's approximation for derivatives transverse to the wave direction results in a restriction on his parabolic model: the waves must propagate within 45° of the assumed wave direction. Booij (1981) also developed a splitting of the elliptic equation, but his procedure included more terms in the approximation to the lateral derivative and therefore his procedure enables the parabolic model to handle waves propagating within 60° of the assumed direction. Booij's procedure has been used in previous versions of the **REF/DIF 1** model (up through version 2.3).

More recently, Kirby (1986b) has developed an extension to the Booij approximation based on a Minimax principle, which further extends the range of validity of the model equations. The wave-current version of the resulting model is included here for the first time, and represents one of the chief enhancements in Version 2.4.

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1.1.2 Diffraction models

In contrast to the mild slope model which is valid for varying bathymetry, researchers in the area of wave diffraction were developing models for constant bottom applications. For example, Mei and Tuck (1980) developed a simple parabolic equation for wave diffraction and applied it to the diffraction of waves by a slender island. Their equation is

$$\frac{\partial A}{\partial x} = \frac{i}{2k} \frac{\partial^2 A}{\partial y^2} \tag{3}$$

where A is a *complex amplitude* related to the water surface displacement by

$$\eta = A e^{i\,(kx-\sigma t)} \tag{4}$$

Yue and Mei (1980), using a multiple scales approach, developed a nonlinear form of this equation, which accurately predicts the propagation of a third order Stokes wave. A striking result of their numerical experiments was the development of Mach stem reflection due to the reflection of obliquely incident waves from a breakwater. This phenomenon is uniquely a nonlinear effect and not predictable from a linear refraction theory.

The parabolic model described below combines the essential features of the two approaches described above. The variable depth features of the mild-slope equation (along with extensions to include effects of wave-current interaction) are retained, but the model is developed in parabolic form and in terms of a complex amplitude A.

1.1.3 Nonlinear combined refraction/diffraction models

Kirby (1983), using a Lagrangian approach, and Kirby and Dalrymple (1983a), with a multiple scales technique, developed the predecessor to the **REF/DIF 1** model, which bridged the gap between the nonlinear diffraction models and the linear mild slope equation. This model can be written in several forms depending on the application. The hyperbolic form, for time dependent applications, and the elliptic form, for steady state problems, require the use of boundary conditions on all sides of the model domain. This is a difficult requirement, as the reflected wave at a boundary is not generally known a priori. These models, however, have the advantage that there is no restriction on the wave direction.

A detailed comparison of results of the weakly-nonlinear model of Kirby and Dalrymple(1983a) to laboratory data was shown by Kirby and Dalrymple (1984). The laboratory test, conducted at the Delft Hydraulics Laboratory by Berkhoff, Booij and Radder (1982), consisted of determining the wave amplitude over a shoal on a sloping bottom. While results predicted by ray tracing techniques were shown by Berkhoff, Booij and Radder to be very poor, the agreement between the weakly-nonlinear model and the laboratory data was

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excellent. Comparisons between linear and nonlinear parabolic models clearly showed the importance of the nonlinear dispersion terms in the governing equations.

1.1.4 Wave-current interaction models

Booij (1981), using a Lagrangian approach, developed a version of the mild slope equation including the influence of current. This model is a weak current model in that the currents are assumed to be small and any products of currents are neglected as small. Kirby (1984) presented the corrected form of this mild slope model. A nonlinear correction and the ability to handle strong currents were added by Kirby and Dalrymple (1983b) and results for waves interacting with a current jet were obtained. Their equation is

$$(C_g + U)A_x + VA_y \; + \; i(\bar{k} - k)(C_g + U)A + \frac{\sigma}{2}\left\{\left(\frac{C_g + U}{\sigma}\right)_x + \left(\frac{V}{\sigma}\right)_y\right\}A$$
$$- \; \frac{i}{2\sigma}\left((p - V^2)A_y\right)_y - \sigma\frac{k^2}{2}D|A|^2A = 0 \tag{5}$$

where $p = CC_g$ and $\bar{k}$ = reference wave number, taken as the average wave number along the y axis, and U is the mean current velocity in the x coordinate direction and V is in the y direction. The nonlinear term includes D, which is

$$D = \frac{(\cosh 4kh + 8 - 2\tanh^2 kh)}{8\sinh^4 kh}$$

Kirby (1986a) rederived the above equation for a wide angle parabolic approximation, which allows the study of waves with larger angles of wave incidence with respect to the x axis. This more accurate equation was used as the basis for earlier versions of **REF/DIF 1**. The equation has been extended to include the more accurate minimax approximation (Kirby, 1986b) for the present version of **REF/DIF 1**. The revised governing equation is given by

$$(C_g + U)A_x - 2\Delta_1 VA_y + i(\bar{k} - a_0 k)(C_g + U)A + \left\{\frac{\sigma}{2}\left(\frac{C_g + U}{\sigma}\right)_x - \Delta_1\sigma\left(\frac{V}{\sigma}\right)_y\right\}A$$
$$+ \; i\Delta'\left[(p - V^2)\left(\frac{A}{\sigma}\right)_y\right]_y - i\Delta_1\left\{\left[UV\left(\frac{A}{\sigma}\right)_y\right]_x + \left[UV\left(\frac{A}{\sigma}\right)_x\right]_y\right\}$$
$$+ \; \frac{i\sigma k^2}{2}D|A|^2A + \frac{w}{2}A + \frac{-b_1}{k}\left\{\left[(p - V^2)\left(\frac{A}{\sigma}\right)_y\right]_{yx} + 2i\left(\sigma V\left(\frac{A}{\sigma}\right)_y\right)_x\right\}$$
$$+ \; b_1\beta\left\{2i\omega U\left(\frac{A}{\sigma}\right)_x + 2i\sigma V\left(\frac{A}{\sigma}\right)_y - 2UV\left(\frac{A}{\sigma}\right)_{xy} + \left[(p - V^2)\left(\frac{A}{\sigma}\right)_y\right]_y\right\}$$
$$- \; \frac{i}{k}b_1\left\{(\omega V)_y + 3(\omega U)_x\right\}\left(\frac{A}{\sigma}\right)_x - \Delta_2\left\{\omega U\left(\frac{A}{\sigma}\right)_x + \frac{1}{2}\omega U_x\left(\frac{A}{\sigma}\right)\right\}$$
$$+ \; ik\omega U(a_0 - 1)\left(\frac{A}{\sigma}\right) = 0 \tag{6}$$

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where

$$\beta = \frac{k_x}{k^2} + \frac{(k(p - U^2))_x}{2k^2(p - U^2)} \tag{7}$$

$$\Delta_1 = a_1 - b_1 \tag{8}$$

$$\Delta_2 = 1 + 2a_1 - 2b_1 \tag{9}$$

$$\Delta' = a_1 - b_1 \frac{\overline{k}}{k} \tag{10}$$

and w is a dissipation factor discussed in the next section. The coefficients a_0, a_1 and b_1 depend on the *aperture width* chosen to specify the minimax approximation; see Kirby (1986). The combination

$$
\begin{aligned}
a_0 &= 1 \\
a_1 &= -0.5 \\
b_1 &= 0
\end{aligned}
\tag{11}
$$

recovers Radder's approximation, while the choices

$$
\begin{aligned}
a_0 &= 1 \\
a_1 &= -0.75 \\
b_1 &= -0.25
\end{aligned}
\tag{12}
$$

recover the approximation of Booij (1981). The values of a_0, a_1 and b_1 used for the Minimax approximation depends on the range of angles to be considered; Kirby (1986b) found that the values for a maximum angular range of 70° gave reasonable results over the range of angles typically used. The corresponding coefficient values for this choice are

$$
\begin{aligned}
a_0 &= 0.994733 \\
a_1 &= -0.890065 \\
b_1 &= -0.451641
\end{aligned}
\tag{13}
$$

Equation (6) is the model equation used in **REF/DIF 1**. At present, the model still utilizes the Padé approximation form based on the coefficients in (12); testing is underway to extend the model to the minimax model with coefficients (13).

In the previous two equations, the dispersion relationship relating the angular frequency of the wave, the depth and the wave number is changed to reflect the Doppler shift due to currents. The new form of eq. (2) is

$$(\omega - kU)^2 = gk \tanh kh \tag{14}$$

where the absolute frequency, ω, is related to the intrinsic frequency, σ, by

$$\omega = \sigma + kU \tag{15}$$

where the assumption that the wave is primarily travelling in the x direction has been used.

1.2 Assumptions

The **REF/DIF 1** model, in parabolic form, has a number of assumptions inherent in it and it is necessary to discuss these directly. These assumptions are:

1. Mild bottom slope. The mathematical derivation of the model equations assumes that the variations in the bottom occur over distances which are long in comparison to a wave length. For the linear model, Booij (1983) performed a comparison between an accurate numerical model and the mild slope model for waves shoaling on a beach. He found that for bottom slopes up to 1:3 the mild slope model was accurate and for steeper slopes it still predicted the trends of wave height changes and reflection coefficients correctly.

2. Weak nonlinearity. Strictly, the model is based on a Stokes perturbation expansion and is therefore restricted to applications where Stokes waves are valid. A measure of the nonlinearity is the Ursell parameter which is given as

$$U = HL^2/h^3 \tag{16}$$

When this parameter exceeds 40, then the Stokes solution is no longer valid. In order to provide a model which is valid in much shallower water, a heuristic dispersion relationship developed by Hedges (1976) is provided as an option in the model. This relationship between the frequency and the water depth is

$$\sigma^2 = gk \tanh \left(kh(1 + |A|/h) \right) \tag{17}$$

In shallow water, this equation matches that of a solitary wave, while in deep water it asymptotically approaches the linear wave result, neglecting real amplitude dispersive effects. For this reason, a model with a dispersion relationship which is a smooth patch between the Hedges form (valid in shallow water) and the Stokes relationship (valid in deep water) is used. This hybrid model is described in Kirby and Dalrymple (1986b). There are, as a result of the different dispersion relationships possible, three options in **REF/DIF 1**: (1), a linear model, (2), a Stokes-to-Hedges nonlinear model,

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and (3), a Stokes model. Of these options, the second will cover a broader range of water depths and wave heights than the others.

3. The wave direction is confined to a sector $\pm 70°$ to the principal assumed wave direction, due to the use of the minimax wide angle parabolic approximation of Kirby (1986b).

1.3   Energy Dissipation

1.3.1   General form

Energy dissipation in the model occurs in a number of ways depending on the situation being modelled. An energy loss term, due to Booij (1981) and expanded by Dalrymple *et al.* (1984a), permits the model to treat bottom frictional losses due to rough, porous or viscous bottoms, surface films, and wave breaking. The linear form of the mild slope equation with dissipation is

$$\frac{\partial A}{\partial x} = \frac{i}{k}\frac{\partial^2 A}{\partial y^2} + wA \tag{18}$$

where the dissipation factor, w, is given by a number of different forms depending on the nature of the energy dissipation. The factor w is the energy dissipation divided by the energy and has the units of time^{-1}.

1.3.2   Laminar surface and bottom boundary layers

At the water surface and at the bottom, boundary layers occur due to the action of viscosity. For a contaminated surface, resulting from surface films (natural or otherwise), a significant amount of damping occurs, which is dependent on the value of the fluid viscosity. From Phillips (1966), the surface film damping is

$$w = \frac{\sigma k\sqrt{(\nu/2\sigma)}(1-i)}{\tanh kh} \tag{19}$$

where ν is the kinematic viscosity. The term under the square root sign is related to the thickness of the boundary layer, which is generally small. At the bottom, the boundary layer damping is

$$w = \frac{2\sigma k\sqrt{(\nu/2\sigma)}(1-i)}{\sinh 2kh} \tag{20}$$

By setting the input switch, *iff(3)=1* in the **REF/DIF** 1 model, surface and bottom damping are computed at all locations in the model.

1.3.3 Turbulent bottom boundary layer

In the field, the likely wave conditions are such that the bottom boundary layer is turbulent. In this case, an alternative specification of the energy dissipation must be provided. Utilizing a Darcy-Weisbach friction factor, f, the dissipation term can be shown to be

$$w = \frac{2\sigma k f |A|(1-i)}{3\pi \sinh 2kh \sinh kh} \tag{21}$$

See Dean and Dalrymple (1984). In order to implement this damping term, the value of $f = 0.01$ was assumed. In **REF/DIF 1**, if the input data switch $iff(1)=1$, then turbulent damping is computed at all locations.

1.3.4 Porous sand

Most sea bottoms are porous and the waves induce a flow into the bed. This results in wave damping due to the Darcy flow in the sand. For beds characterized by a given coefficient of permeability, Cp, the damping can be shown to be

$$w = \frac{gkC_p(1-i)}{\cosh^2 kh} \tag{22}$$

The coefficient of permeability, C_p, has the units of (m^2) and is of order 4.5 x 10^{-11} m^2. Liu and Dalrymple (1984) show, for very permeable sands, that the damping is inversely related to C_p and a different w term must be used. However, this case is not likely to occur in nature. Porous bottom damping is computed in **REF/DIF 1** when $iff(2) = 1$.

1.3.5 Wave breaking

For wave breaking, the model is more complicated. Dally *et al.* (1985) showed that the rate of loss of wave energy flux is dependent on the excess of energy flux over a stable value. This model has been tested for laboratory data for a number of different bottom slopes and predicts the wave height in the surf zone extremely well. Kirby and Dalrymple (1986a) show that the dissipation due to wave breaking is given as

$$w = \frac{KC_g \left(1 - (\gamma h/H)^2\right)}{h} \tag{23}$$

where K and γ are empirical constants, determined by Dally *et al.* to be equal to 0.017 and 0.4 respectively. Here, the wave height, $H = 2|A|$. By using this dissipation model and a breaking index relation ($H > 0.78h$) to determine the onset of breaking, the **REF/DIF 1** model is able to represent waves both outside and inside of a surf zone. The wave breaking algorithm is always active in the model.

Large surface piercing islands and causeways which would have surf zones are handled by the 'thin film' technique of Dalrymple, Kirby and Mann (1984b) and Kirby and Dalrymple (1986a). This procedure permits the easy computation of wave heights around arbitrarily shaped islands by replacing islands with shoals of extremely shallow depth (1 cm). The wave breaking routine reduces the wave heights over the shoal to less than one half centimeter, which results in a wave which carries negligible energy and therefore no longer affects any physical processes. Thus, the **REF/DIF 1** model does not distinguish between islands and deeper water computationally. However, the model output clearly shows the influence of the islands, as will be shown in section 3. Examples of wave breaking and the combined refraction/diffraction model appear in Kirby and Dalrymple (1986a) and Dalrymple *et al.* (1984b).

1.4 Wave Climate

1.4.1 Monochromatic waves

While the **REF/DIF 1** model is typically used with monochromatic wave trains propagating in one given direction, there is no intrinsic restriction to this case. As an example, for a given frequency, the wave direction is determined by the initial wave height distribution provided by the user on the offshore grid row, corresponding to $x = 0$. As this row is parallel to the y axis, the wave is generally prescribed as

$$A(0, y) = A_0 e^{i\ell y} \tag{24}$$

where A_0 is the given wave amplitude and ℓ is the wave number in the y direction. The ℓ is related to the wave number k by the relationship, $\ell = k \sin \theta$, where θ is the angle made by the wave to the x axis. This case is obtained by using the data switches, *iwave* and *nwavs* set to one.

1.4.2 Discrete directional waves

For several waves with different directions at a given frequency, the following relationship could be used for the initial wave condition:

$$A(0, y) = \sum_{n=1}^{nwavs} A_n e^{i\ell_n y} \tag{25}$$

The **REF/DIF 1** model is equipped to calculate the wave field produced by this boundary condition for a large number of user-supplied A_ns and θ_ns (up to 50). This mode is accessed by *iwave=1* and *nwavs* set to the number of discrete waves to be used.

1.4.3 Directional spectrum

Often, a $\cos^{2n} \theta$ directional spread is used with a given frequency component. This can be done with **REF/DIF 1** by specifying *iwave* equal to 2 and *nwavs* to the value of n. The total energy at frequency, σ, is

$$E(\sigma) = \int_0^{2\pi} E_n \, \cos^{2n}(\frac{\theta - \theta_o}{2}) \tag{26}$$

In order to avoid the problem of waves propagating at large angles to the propagation direction, θ_0, the directional distribution of energy is automatically truncated to include only those directions which contain more than 10% of the total energy.

To prescribe the initial conditions for the model, the directional distribution is discretized into 31 components, each with an amplitude characteristic of the waves in that particular directional band. These discrete waves are then assigned random phases and summed as in Eq. (21).

Note that this directional spectrum is for a given frequency, and not for a continuous distribution of frequencies as in a true directional spectrum. At the present time, the **REF/DIF 1** model can only calculate waves at a single frequency per calculation. The model will compute numerous frequencies per computer run (set *nfreqs* greater than one); however, they are not superimposable, as the wave-wave interactions between different frequencies are not included. Using the linear mode (*ntype*=0) and superimposing the results will provide a linear approximation to a directional spectrum.

The problem of computing the shoreward evolution of a directional spectrum of refracting, diffracting and breaking waves has recently been addressed in a new model called **REF/DIF S**. This model is essentially an enhancement to **REF/DIF 1** which allows for the simultaneous computation of many wave components on a large vectorizing computer. Since the amount of computer power required to run **REF/DIF S** in full spectral mode is enormous, the single component model **REF/DIF 1** will continue to be supported and developed as well.

1.5 Numerical Development

1.5.1 Crank-Nicolson Technique

The parabolic model is conveniently solved in finite difference form. In order to accomplish this, the study area bathymetry must be input as a grid with the (x, y) directions, divided into rectangles of Δx and Δy sizes. The complex amplitude $A(x, y)$ will then be sought at each grid and therefore we can keep track of A by denoting its location, not by (x, y), but by (i, j) where $x = (i - 1)\Delta x$ and $y = (j - 1)\Delta y$. Now we have to determine the

values of $A(i, j)$ which satisfy Eq.(6) for all i between 1 and m and for all j between 1 and n. The procedure involves expressing all the derivatives in the (x, y) directions in terms of the complex amplitude at various grid points. For example, the forward difference representation of

$$\frac{\partial A}{\partial x} = \frac{A_{i+1,j} - A_{i,j}}{\Delta x} \text{ at location } i, j \qquad (27)$$

If a forward difference is used for the x direction and a central difference representation centered at i is used for the second derivatives in the lateral direction for all the derivatives in Eq. (6), then an explicit finite difference equation results for $A_{i+1,j}$. This equation can be solved directly for all the $A_{i+1,j}, j = 1, 2, 3...n$, for a given i, provided appropriate lateral boundary conditions are prescribed. This explicit representation is not as accurate as an implicit scheme and therefore an implicit Crank-Nicolson procedure is used for the amplitude calculations. For a given i row, the Crank-Nicolson scheme can be written

$$aA_{i+1,k+1} + bA_{i+1,j} + cA_{i+1,j-1} = dA_{i,j+1} + eA_{i,j} + fAi, j-1 \qquad (28)$$

where the coefficients a, b, c, d, e, f involve variable, complex and nonlinear terms. The amplitudes on the left hand side of this equation are unknown, while the terms on the right hand side are known, from either the previous calculation or from the initial boundary condition on $j = 1$ and n. This equation is solved for all the $A_{i+1,j}, j = 2n - 1$ and i fixed, at once by a tridiagonal matrix solution procedure (Carnahan, Luther and Wilkes (1969)), adapted for complex-valued coefficients. Due to the nonlinearity of the finite difference equation, nonlinear terms are approximated on a first pass by using the $A_{i,j}$ values. Once the $A_{i+1,j}$ terms are computed, the equation is solved again for $A_{i+1,j}$ using now the just-calculated values in the nonlinear terms. This two-pass iterative method insures that the nonlinearities in the model are treated accurately (Kirby and Dalrymple, 1983a). The solution proceeds by moving one grid row in the x direction (incrementing i by one) and, using the two-pass implicit-implicit technique, determining the complex amplitude $A_{i+1,j}$ for all the values of j on this row. Marching in the wave direction these calculations are repeated until all the $A_{i,j}$ are known for all i and j. While it appears that the Crank-Nicolson procedure could be time consuming, given that there is a matrix inversion for each grid row, the coefficient matrix size is only 3 by n and the matrix inversion procedure is, in fact, very fast. The procedure is economical on storage as only the values for the rows i and $i + 1$ are necessary at each calculation.

1.5.2   Initial and Lateral Boundary Conditions

The initial condition is vital for the parabolic model. The furthest seaward grid row, corresponding to $i = 1$, is taken as constant depth and the incident wave(s) is prescribed

here. This wave is then propagated over the bathymetry by the model. The various initial conditions were discussed in the section, Wave Climate.

As in the solution of any differential equation in a domain, the lateral boundary conditions are important. There are several ways to treat the boundaries; however, none of the presently existing boundary conditions result in the total transmission of scattered waves. Therefore, for the **REF/DIF 1** model, a totally reflecting condition is generally used for each side ($j = 1$ and n). This requires that the specification of the model grid be done with care, as the reflection of the incident wave from the lateral boundaries can propagate into the region of interest rapidly and result in erroneous results.

In general, the width of the model should be such that no reflection occurs until far downwave of the region of interest. As a precaution, a graphical representation of the computed wave field should be examined to determine where the reflection from the boundaries is important. By using the switch, *ibc*, partially transmitting boundaries can be used (Kirby, 1986c). In general, this boundary condition will result in less reflection in the model domain; however, since some reflection will occur, it is recommended that runs be carried out with the reflecting boundary conditions in order to assess the regions potentially affected by reflection from the model boundaries.

1.5.3 Subgrids

In order to reduce the amount of data input and yet provide the user the ability to prescribe the fine scale bathymetry in areas of interest, **REF/DIF 1** utilizes a coarse scale user-specified reference grid and a fine scale subgrid, which can have many times the resolution of the reference grid. The principal purpose of the subgrid is to provide enough computational points to the numerical model to preserve accuracy. The user specifies the number of subgrid divisions in the y direction with the parameter *nd*. If *nd*=1, then the subgrid spacing in the y direction is the same as the reference grid. If *nd*=2, then the model uses twice as many computational points in the y direction as there are in the reference grid. In the propagation direction, x, the model will automatically determine the subgrid spacing if *ispace* has been set to unity. Otherwise, the user provides the subgrid spacing using the input, *mr*, which permits variable spacing in the x direction. For subgrids, the input flag, *isp*, must be set to one and an array, *isd*, must be specified.

1.5.4 Damping

When the large angle parabolic approximation is used as a basis for the computation of wave fields around islands, the presence of wave breaking, and resulting sharp lateral variations in wave height, leads to the generation of high-wavenumber spectral components in the

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computed complex amplitude A in the lateral (y) direction. Kirby (1986a) has shown that these components have propagation velocities which can become large in an unbounded fashion; as a result, they can propagate across the grid, filling the computational domain with high-wavenumber noise.

Previous versions of **REF/DIF 1** have attempted to control the high frequency noise generated by the breking process using smoothing filters, which are applied to the computed complex amplitude in between each computational row. In the present version of **REF/DIF 1**, this procedure is being replaced by a new algorithm which has recently been developed by Kirby (1993). Reference should be made to that document for a description of the algorithm. The damping is built into the computational algorithm and is turned on automatically if breaking has started anywhere in the computational domain.

2 USER'S MANUAL

This chapter provides the operating manual for the program **REF/DIF 1**. Chapter 3 provides sample documentation and calculations for example problems. A separate Fortran program *datgen.f* is provided which generates the input data files for these as well as a number of additional examples.

2.1 REF/DIF 1 Revision History.

2.1.1 Changes Appearing in Version 2.0

Several changes have been made to the program released as Version 1.0 of **REF/DIF 1**. These changes are outlined here for convenience.

Directional spectra application

The routine used to specify a directional spectrum as the initial condition for the wave calculations has been extensively revised and tested. The present model is based on a Mitsuyasu-type spreading factor and apportions wave directions and energy density in randomly-sized directional bins. In addition, the original random number generator supplied with Version 1.0 was found to not be sufficiently random, and a new version is supplied with Version 2.0.

ibc **- Open Boundary Condition Parameter**

Version 2.0 contains an option to use open lateral boundary conditions, which are designed to be reasonably transparent both to entering and exiting waves, if the topography near the boundary is reasonably uniform. The theory for these conditions are contained in the reference Kirby(1986c). It is recommended that initial runs for a particular site be done with the default reflective conditions in order to see the magnitude of the boundary effects and then to use the open conditions for final runs. The open boundary condition is invoked by choosing a value of *ibc*=1 in the input data.

icur **- No Currents Parameter**

A new parameter *icur* is included in the input data, and determines whether or not the program is to read in current values from the input data files. This change provides the option of not having to include zero current values on input whenno currents are being considered.

2.1.2 Changes Appearing in Version 2.1

Version 2.1 represents a minor modification. The change consists of a revision of the input file format structures for the file *indat.dat*. The formats for this file have been replaced by free format **read** statements, so that the user can enter data separated by comments

without regard to the column structure. Note that data entered in the previously defined formats will still be read properly, so existing *indat.dat* files will still work properly.

2.1.3   Changes Appearing in Version 2.2

Version 2.2 includes a revised version of a dissipation filter which is used to damp out noise after the onset of breaking in the numerical computations. This filter has been found to be much more suitable in applications to field situations than was the original filter.

In addition, Version 2.2 also provides the capability to input the first row of complex amplitude A from a new external data file *wave.dat*. The procedures for specifying *wave.dat* are described in section 2.4 of the manual.

2.1.4   Changes Appearing in Version 2.3

Version 2.3 provides a provision to save the last subdivided row of complex valued amplitudes computed on the last model grid row in file *owave.dat*. This option is potentially useful if the user wants to perform a run in several segments. *owave.dat* could then be used to store the intermediate calculation required to initialize a subsequent model run. The provision for using this option is described in section 2.5. Use of this option does not affect any internal model calculations.

2.1.5   Changes Appearing in Version 2.4

Version 2.4 incorporates two revisions to the basic model scheme. The first revision is an extension to the model equations to handle the Minimax approximation of Kirby (1986b) as well as the Padé large angle approximation (Booij, 1981; Kirby, 1986a). This algorithm is not used yet in the released version of the program.

The second revision is the replacement of the add-on noise filtering algorithm with an algorithm that functions as an imbedded part of the model equation and finite-difference approximation, as described by Kirby (1993). Additionally, several inconsistencies appearing in input error checking have been corrected. A more robust algorithm for computing wave angles (due to Medina (1991)) has been added.

2.2 Overview of Operating Manual

This section provides a description of the program structure (section 2.3), followed by some notes on problems which are likely to be encountered during the installation and use of the program on different computer systems (section 2.4). Section 2.5 presents the two levels of grid information used by the program. Section 2.6 describes the option of inputing the first row of complex amplitude values A from an additional external data file *wave.dat*. The input data file structure is then discussed. The program reads data in two essentially separate groups. The first group of data establishes the size of the model grid and gives the wave conditions to be studied; this group is discussed in section 2.7. The second group of data gives the reference grid data values and defines any user-specified subgrids; this is discussed in section 2.8. The structure of the program output is discussed in section 2.9. A listing of the program is included in Appendix A.

2.3 Program Outline and Flow Chart

The model **REF/DIF 1** is organized in one main program *refdif1* and thirteen subroutines. The program does not contain calls to any external package, and should be free standing on any system. The program is written in a subset of FORTRAN 77 and should be a legal code for any compiler which accepts the full FORTRAN 77 standard language.

REF/DIF 1 is structured in two levels; a main level, which reads in and checks input data and then starts the operation of the wave model level, and the model level itself, which performs the actual finite difference calculations. The flow charts for the two levels are given in Figures 1 and 2. A short description of each routine in the model follows.

1. *refdif1*: Main program controls the calls to *inref* and *inwave* to read in data, and to *model*, which performs the actual calculations. No calculations are performed by this routine.

2. *inref* : Called by *refdif1*. *inref* reads in data controlling reference grid dimensions and the grid interpolation scheme from logical device number *iun*(5), and reads in the reference grid values of depth dr, x-direction velocity ur and y-direction velocity vr from logical device number *iun*(1). Some data checking is performed. If data is read in in English units, *inref* converts it to MKS units using the *dconv* factor. Output files are initialized. A description of the data files may be found in sections 2.6-2.8 of this manual. At the end of the subroutine, control is returned to *refdif1*.

3. *inwave* : Called by *refdif1*. *inwave* reads in data specifying the initial wave field along the first row of reference grid points. Data is read from logical device number *iun*(5).

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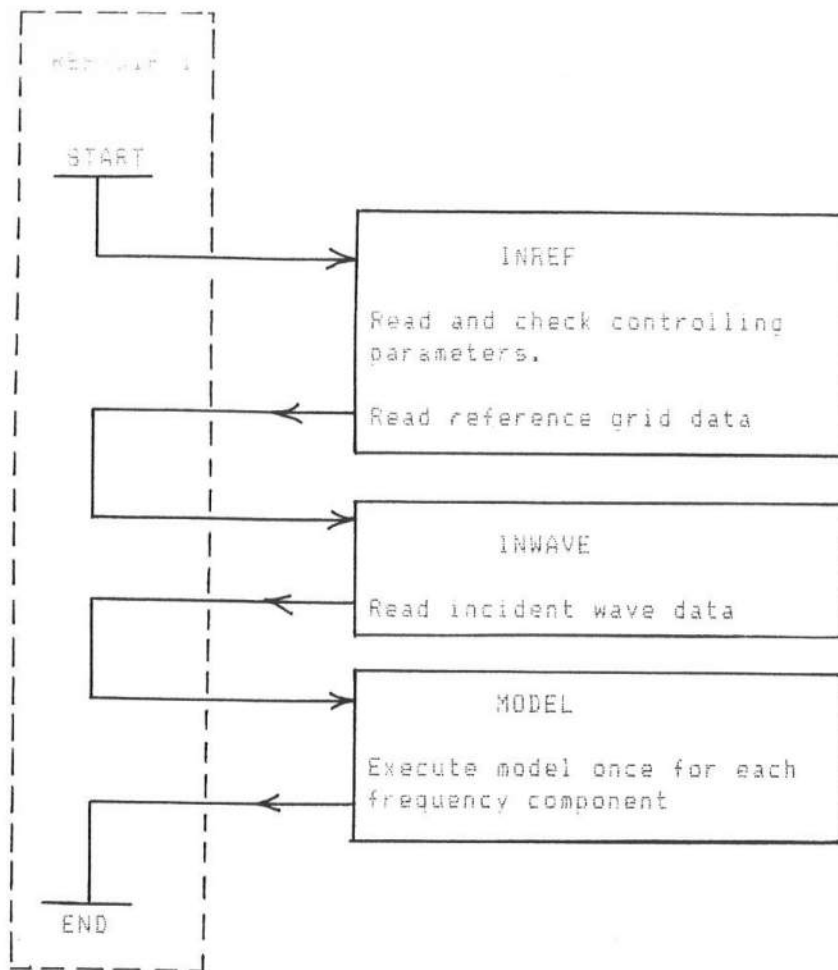
Figure 1: REF/DIF 1: *refdif1* program level
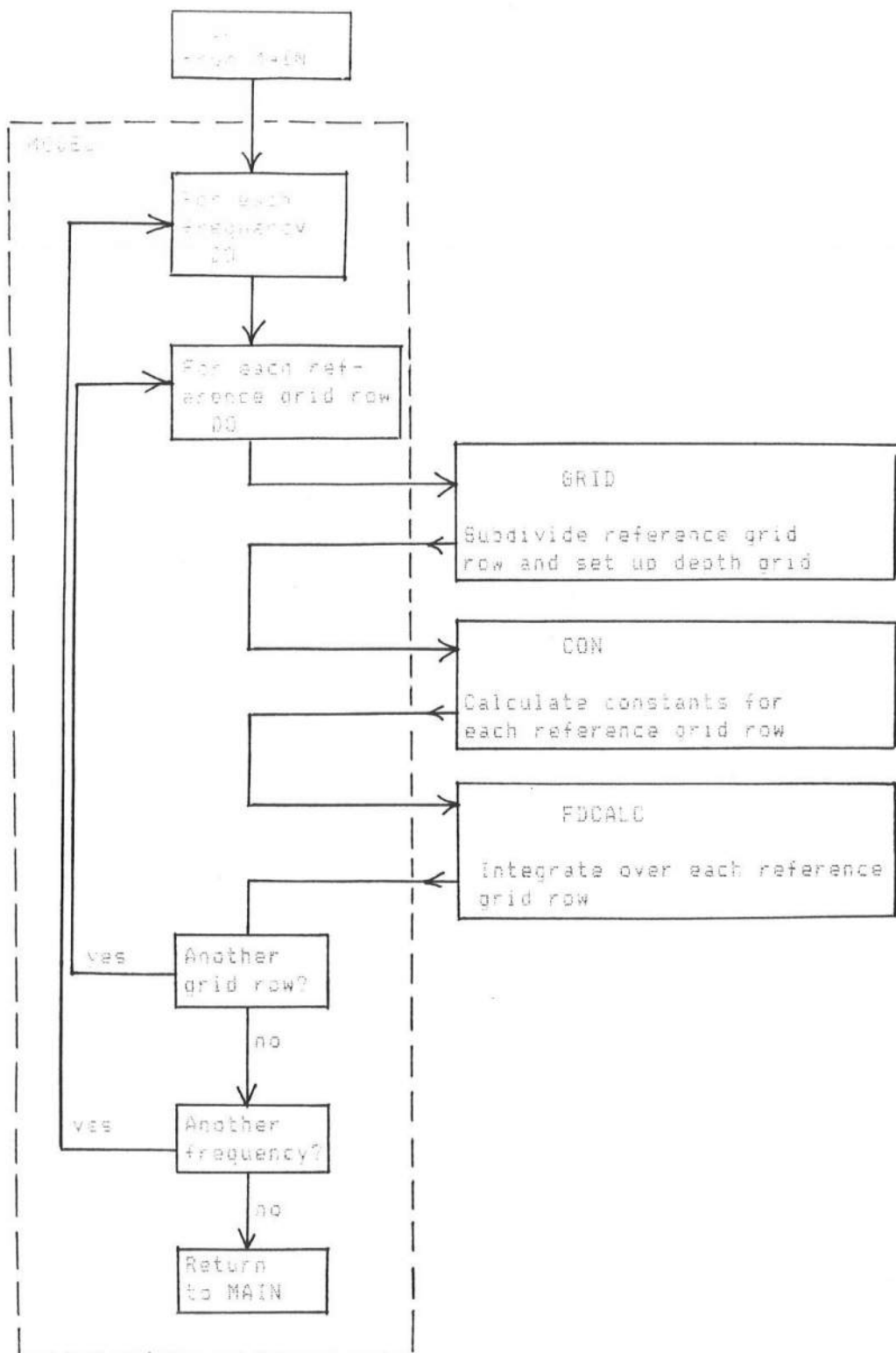
Figure 2: REF/DIF 1: *model* subroutine level

Conversion to MKS units is performed for data read in in English units. Control is returned to *refdif1*.

4. *model* : Called from *refdif1*. *model* controls execution of the computational part of the program. For each frequency component specified in the input, *model* performs the following series of operations:

(a) Initialize program by calculating the incident wave field on the first grid row.

(b) Then, for each grid block in the reference grid:

- Call *grid* to perform the grid interpolation specified in the input data.
- Call *con* to calculate constants on the interpolated grid.
- Call *fdcalc* to perform the numerical integration of the parabolic equation over the interpolated subgrid.

The model execution is then complete. Control is returned to *refdif1*.

5. *grid* : Called by *model*. *grid* performs the required interpolation over a single grid block of the reference grid as specified in the input data. The interpolation is performed as described in section 2.3. *grid* checks to see whether a user-specified subgrid feature should be read in, and reads it in from logical unit number *iun*(2) if needed. The interpolated depth grid is then corrected for tidal offset, and checked for surface-piercing features. These features are modified using the "thin-film" approach; see Kirby and Dalrymple (1986a). Control is returned to *model*.

6. *con* : Called by *model*. *con* calculates various constants for the reference grid created by *grid*. Control is returned to *model*.

7. *fdcalc* : Called from *model*. *fdcalc* performs the integration of the governing parabolic equation over the grid defined in *grid*. The coefficients of the finite difference form of the parabolic equation are developed according to the Crank-Nicolson method. A complete description of the equations and the treatment of nonlinearities may be found in Kirby (1986a) and Kirby and Dalrymple (1986b). The sequence of steps in *fdcalc* is as follows:

(a) An implicit step is performed to update complex amplitude A along an entire grid row.

(b) The model checks for the start or stop of breaking on the updated row.

(c) If the status of breaking changes, the model recomputes the breaking wave dissipation coefficient.

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(d) Then, if nonlinearity is being used or breaking status at any point along the row has changed, the model computes a new estimate of A on the updated row based on values obtained during the previous iteration.

This series of operations is performed for each row in the subdivided ir grid block, until the end of the grid defined in *grid* is reached. Control is then returned to *model*, which passes on to the next (ir+1) reference grid block.

8. *ctrida*: Utility routine which is called by *fdcalc* to perform the double sweep elimination to solve the implicit set of equations.

9. *diss*: Called by *con*. *diss* calculates frictional dissipation coefficients based on values of the switches read in by *inref*.

10. *wvnum* : Called by *model, grid* and *con*. *wvnum* performs a Newton-Raphson solution of the linear wave-current dispersion relation to obtain values of the wavenumber k.

11. *rand*: Called by *model*. This function is a simple random number generator used to initialize the random wave phases if the directional spreading model is being used.

12. *acalc*: Called by *model*. *acalc* normalizes the directional spectrum energy density over a 90° sector.

13. *bnum*: Called by *acalc*. *bnum* computes the Bernoulli number $n!/k!(n - k)!$

14. *fact*: Called by *bnum*. *fact* computes the factorial $n!$ of an integer n.

2.4   Special Installation Instructions

Several features of the program **REF/DIF 1** may require some modification or customizing during program installation on various systems. **REF/DIF 1** is written using the features of FORTRAN 77. No use is made of vectorized solution techniques, so the program should be useable on a wide range of systems with little or no modification.

If the program is to be used on machines with no upward limit placed on the size of the compiled, executable code, only two variables have to be checked during the initial installation of the program. These are the logical device numbers for the controlling input date file and the device number for line printer output. These numbers may vary from system to system. They appear in the program as:

$iun(4)$: logical device number for line printer output. (initialized in the main program *refdif1*).

$iun(5)$: logical device number for controlling input data file. (initialized near the top of subroutine *inref*).

The supplied version of **REF/DIF 1** has those values set to $iun(4)=$"*" and $iun(5)=5$. If a device number is to be given for —iun(4)—, the choice of 6 for output should be fairly standard (one exception is the DEC-10, which uses the device number 5 for both input from file or screen and output to the line printer). The version of the program currently used on SUN workstations has the default device indicator "*" substituted for $iun(4)$ everywhere. Output can be directed to a data file using the directed output switch in either UNIX or DOS.

All other device numbers for input and output disk files are read into the program as data.

Many systems require that access to disk files be initialized and terminated by the use of **open** and **close** statements in the program code. Since the parameter list of the **open** statement varies from system to system, the user should take care that the **open** statements are compatible with the system software being used. The **open** statements appear near the top of *refdif1* and *inref*. The corresponding **close** statements appear near the end of *refdif1*. The present version 2.4 is internally configured to run on a SUN system.

When writing **open** statements, a choice of a fixed record length of 132 (line printer width) will safely bracket all the **format** specifications in the program.

2.5 Computational Grids and Grid Interpolation

The reference grid terminology is defined in Figure 3. The grid consists of a mesh of points with dimensions mr x nr in x and y. The values of mr and nr must be less than or equal to the parameters ixr and iyr whose values are set in the **parameter** statements. Reference grid data of depth dr and ambient current components ur and vr are defined at the grid points. The program assumes that the x, y coordinate system is established with the origin at grid point $(ir,jr)=(1,1)$. In this manual, we make the distinction between the terms "grid row " ir, which is the row of points $jr = 1$, nr at location ir, and "grid block" ir, which is the physical space between grid rows ir and $ir+1$.

The reference points are separated by spacings dxr and dyr which are uniform in the x and y directions. The spacings dxr and dyr may have arbitrary, independent values.

Values of dr, ur and vr constitute the "reference grid data". Section 2.8 describes the required input data file for these quantities.

The computational grid for any particular application should be chosen with care. Since **REF/DIF 1** tries to use at least 5 points per wavelength, the length of the computational domain in the propagation direction is restricted (with the given **parameter** statements)
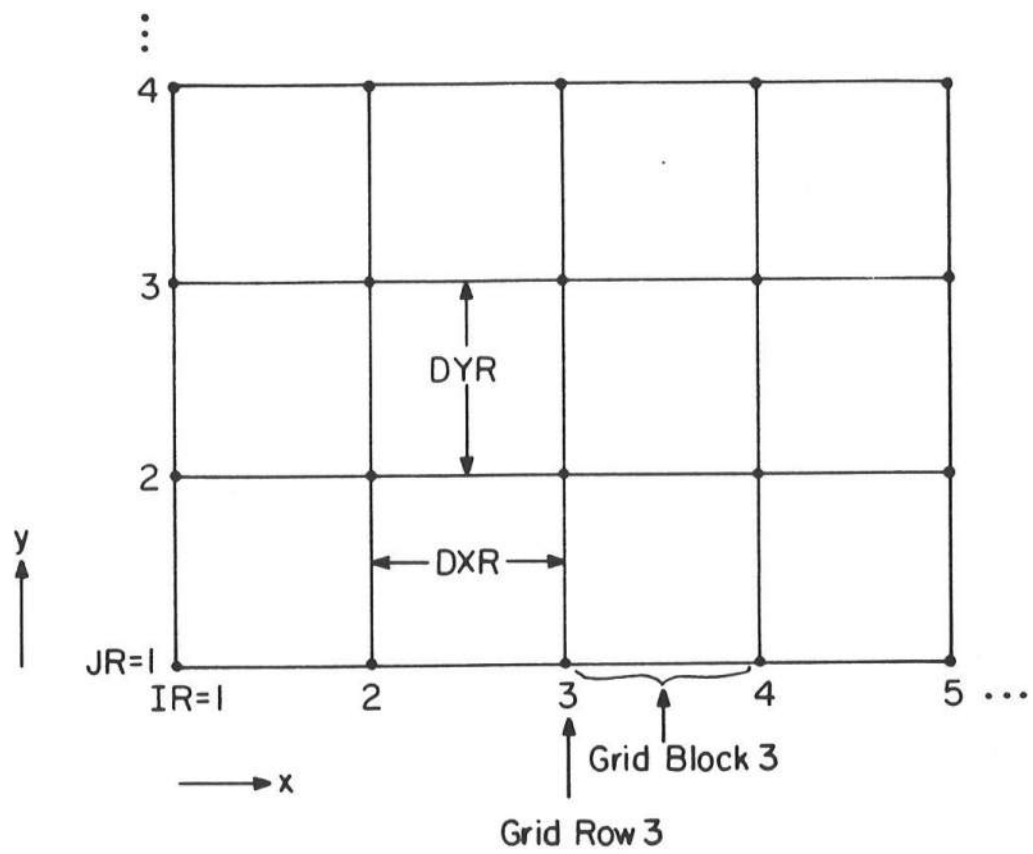
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Figure 3: Reference grid notation

to just over 200 wavelengths. This range can be extended by increasing ixr and/or ix in the **parameter** statements. The width of the model domain should be chosen such that interference from the boundaries does not affect the study area. If the reflective boundary conditions are used, the extent of boundary influence is usually obvious, particularly when viewing 2-dimensional plots of the amplitude envelope $|A(x, y)|$.

2.5.1 Grid Subdivision

The only major feature of **REF/DIF 1** which is not described in Chapter 1 is the ability to subdivide the given reference grid into a more finely subdivided computational grid. This would usually be done in cases where the reference grid spacing is too large to be used directly for calculations. In this case, the user may specify how the reference grid is to be subdivided, or the user may tell the program to attempt its own subdivisions.

The maximum reference grid dimensions have been set fairly arbitrarily and may be changed by modifying the parameter statements at the beginning of each routine in the program. The grid is large enough to comply with typical grids for various tidal computational models, which may be used to specify the ambient currents, and get small enough so that data values do not occupy too much internal storage. It is anticipated that the spacings dxr and dyr, if based on such a model, may be too large for an accurate integration of the parabolic model to be performed. Consequently, the model has been arranged so that the reference grid block between any two adjacent reference grid rows may be subdivided down to a finer mesh in order to provide sufficient resolution in the computational scheme. This subdivision process is performed internally in the program (with an exceptional feature to be described below), and may be controlled by the user or the program.

Remember that the computational scheme proceeds by marching in the x-direction, and, therefore, the only reference grid information required at any particular step is the data on row ir, where computation starts, and on row $ir+1$, where computation ends. The fine, subdivided mesh is set up on the intervening grid block. An example of a particular subdivision of a grid block is shown in Figure 4. Here, the choices of x-direction subdivision $md(ir)=5$ and $nd=2$ are illustrated, with md and nd being the number of spaces each reference grid cell is divided into, rather than the number of extra points being inserted.

Several restrictions are placed on the choice of nd and md's. First, the maximum dimensions of the subdivided grid cell is given by the parameters ix and iy. This implies that any md can be at most $(ix\text{-}1)$ and nd can be at most $(iy\text{-}1)/(nr\text{-}1)$. The maximum number of added spaces may be increased by increasing ix and iy in the **parameter** statements. Further, the y-subdivision specified by nd is applied uniformly along each grid row for the full extent to the reference grid. No provision is made for variable grid spacing in the

Reference Grid
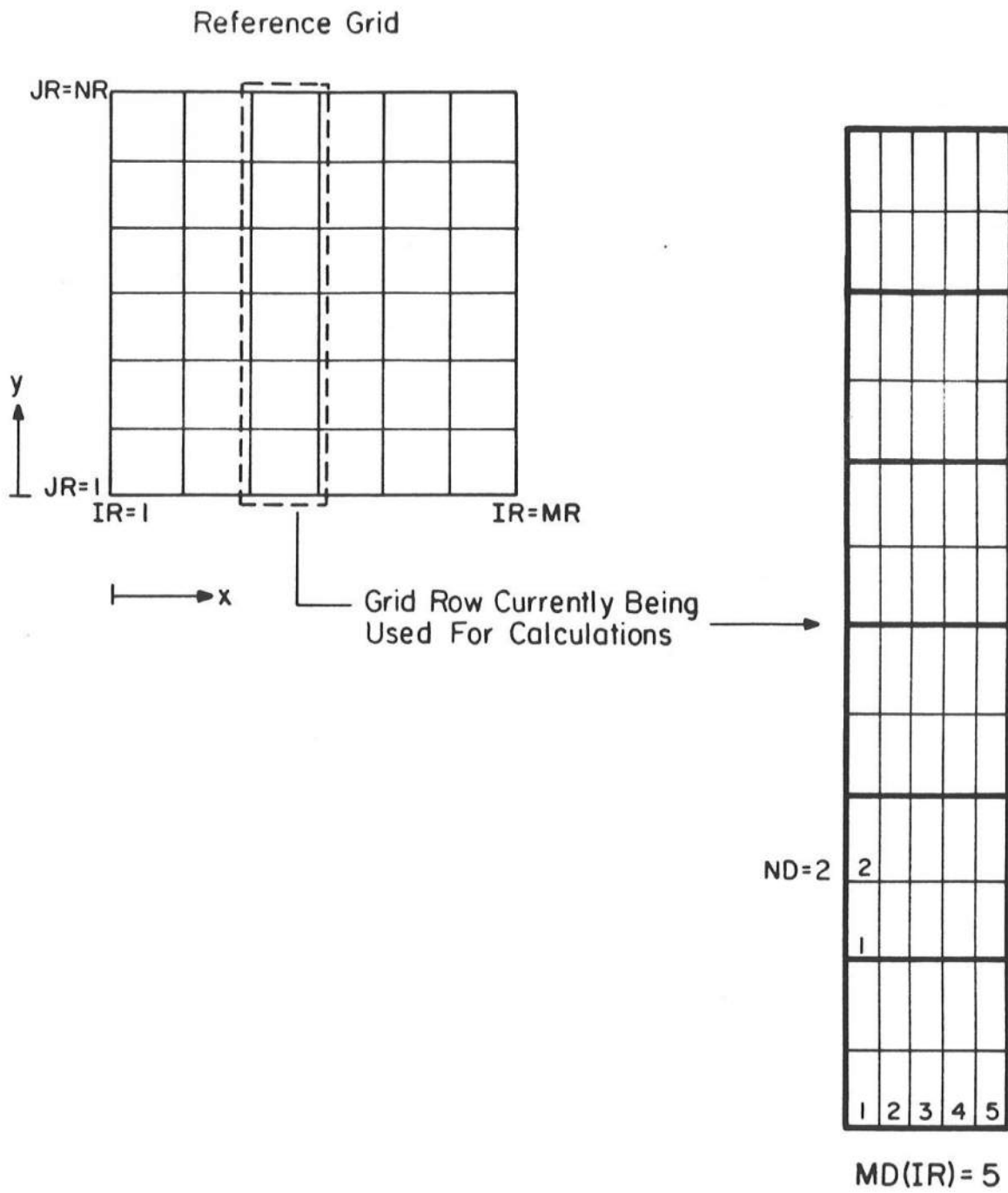
Figure 4: Sample grid subdivision

y-direction. Grid spacing in the x-direction is arbitrary, so md's may differ arbitrarily for each grid block.

The user must specify the single value of nd in the input data. Two choices may be made regarding md's, however.

1. The user may let the program calculate md. The program proceeds by calculating an average wavenumber along the reference grid row, and uses this to estimate the wavelength in the grid block. The program then chooses a subdivision so that at least 5 grid points per wavelength will occur in the grid block. The program checks to see that this desired number of subdivisions does not exceed the maximum. If it does, the program reduces the number to the maximum and prints a warning message indicating that the grid block can not be subdivided finely enough. Computed results in this case must be regarded as being suspect. The number of subdivisions used is indicated on the output. The user chooses this option by setting the switch $ispace=0$ on input.

2. The user may specify each value of $md(ir)$ from $ir=1$ to $(mr\text{-}1)$. This is done by setting the switch $ispace=1$ on input, and the values of md are then read in from the input data file. Note that this choice is necessary if the user-defined subgrids discussed below are to be used, since the user will be inputting subgrids with pre-specified spacings. As it is presently written, the program will only print a warning if it encounters subgrids with $ispace=0$ chosen; extensive garbling of input data may result.

After the subdivided grid block is established, the program uses this grid as the actual computation grid. New values of depth d and ambient current u and v are calculated at the extra grid points by fitting a twisted surface to the reference grid using linear interpolation. An example of the resulting bottom topography for a single grid cell is shown in Figure 5.

2.5.2 User-specified Subgrids

In some applications, an important topographic feature may be present at a subgrid scale within the reference grid. Examples include artificial islands, shoals, borrow pits, etc., which are superimposed on an otherwise slowly-varying topography which is represented by the sort of grid resolution appropriate to tidal models. An illustration of such a feature is shown in Figure 6, where a poorly resolved feature occupies portions of four reference grid cells. For cases such as this, the program includes the option for the user to input user-defined, subdivided grid cells in order to specify these features at the level of the computational grid.
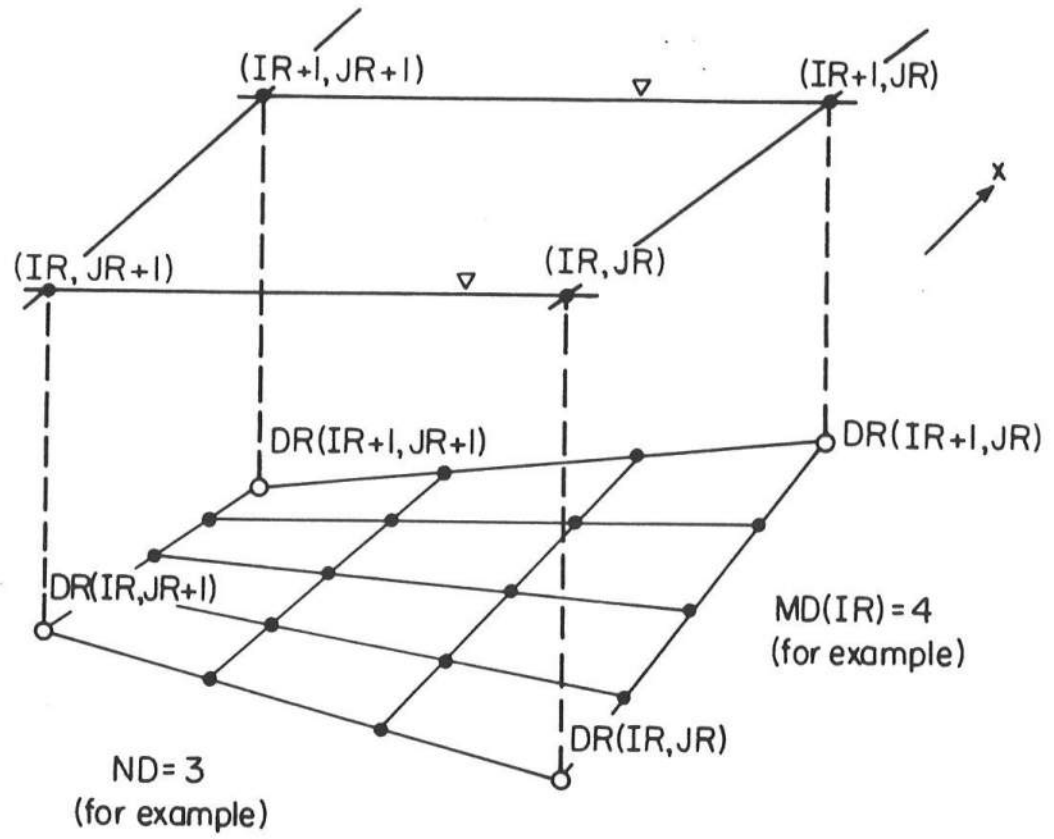
Figure 5: Interpolation of depth data

Reference Grid

JR

IR

Sub grid Feature

User – Specified Group
of Subgrid Blocks

Grid Block
(IR, JR)

(IR,JR+1)   (IR+1,JR+1)

ND = 5

Sub grid Feature
Depth Contours

(IR,JR)   (IR+1,JR)

Reference Grid Points

MD (IR) = 5
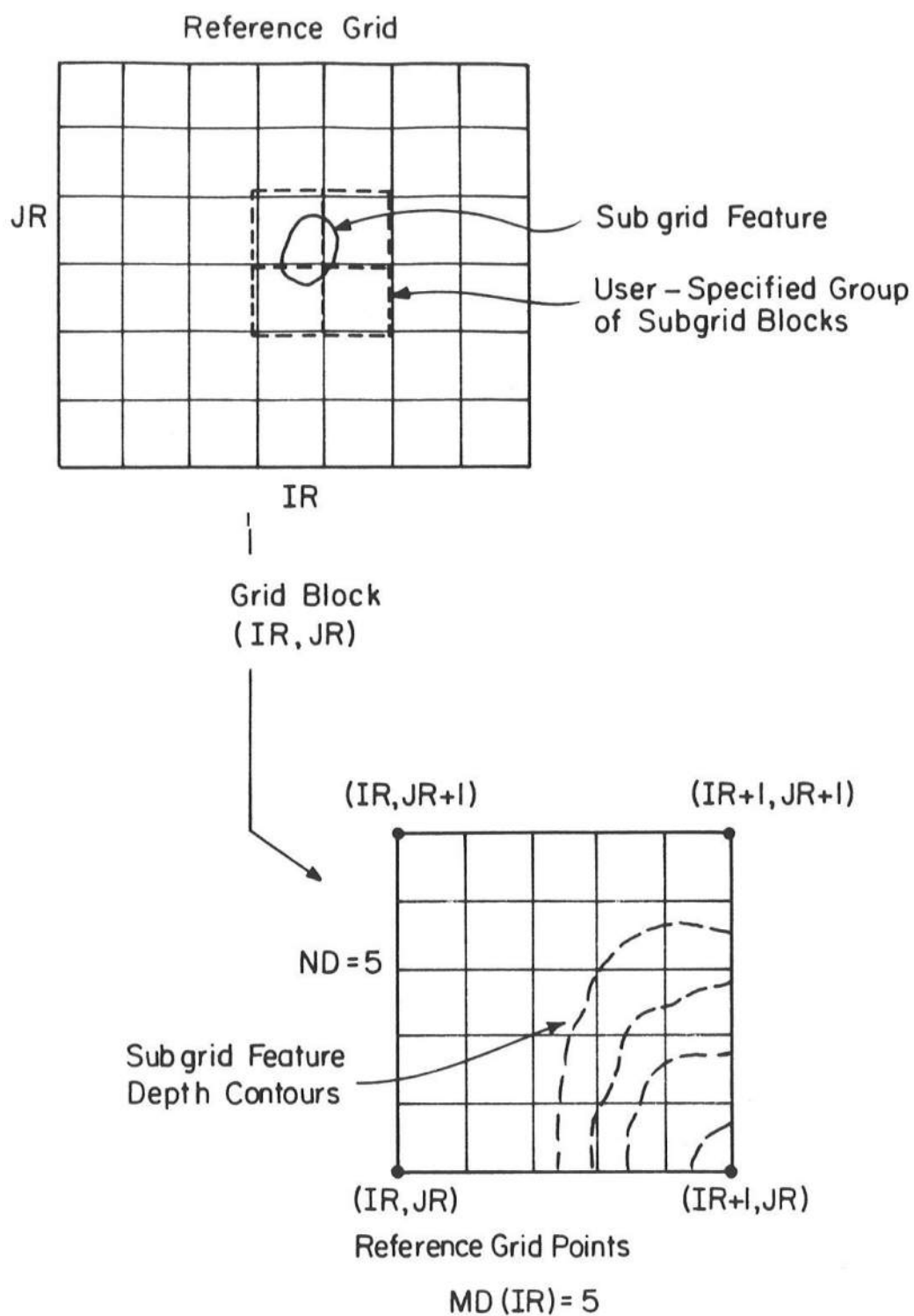
Figure 6: User-defined subgrids

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The use of user-defined subgrids implies that the user will be choosing the grid spacing of the subgrids. Since grid spacings must be uniform across a grid block, the user should choose the option, *ispace*=1 and specify the values of *md* in the input file, *indat.dat*. Then, the program will look for a data array of the correct, pre-specified dimension when it reads the subgrid data from the file, *subdat.dat* (unit number *iun(2)*).

Several aspects of the subgrid data should be noted. Data for depth *d* and current velocities *u* and *v* need to be specified at each of the subgrid points. These data are input in the same units, and with the same datum for depth as the data for the reference grid. Also, note that the subgrid includes the points on its boundaries; for example, the single subgrid shown in Figure 6 has a dimension of 6 by 6.

The data values on the outer borders of the subgrids should be setup to match with the linearly interpolated values in the region external to the user-defined subgrid. If the data do not vary linearly along the subgrid boundaries, there may be some mismatch between the subgrid boundaries and the external region. An exception to this rule occurs when two subgrids adjoin each other. Then, care should be taken that the data along the common boundary match. These common boundary point data are duplicated in the input data, since each subgrid data set includes the boundaries.

Instructions and formats for creating the subgrid data file *subdat.dat* are included in section 2.8.

2.6 User Specification of Complex Amplitude on First Grid Row

This section discusses the option of inputing the values of complex amplitude *A* for the first grid row from an external data file. This option is invoked by setting the data value *iinput*=2 in line 8 of *indat.dat*. (See the following section). In the event that the option is chosen, the data should be stored in a file *wave.dat*. The format for writing data to the file should be similar to:

```
complex a( iy)

write(*,*)(a(j),j=1,n)
```

The data will be read in by the *model* subroutine. The data is read in <u>after</u> grid subdivision in the *y* direction, and hence *n* rather than *nr* data values need to appear in the file *wave.dat*. An insuffient number of data points will simply trigger an "end of file" type of error during the **read** process.

The user may have any number of reasons for wanting to compute *A* externally to the program. Several possibilities are:

- Running tests with complex initial conditions (as in waves through a breakwater gap, etc).

- Testing a directional distribution model which is different from the model chosen here.

- Specifying a nearly planar wave field which has height and angle variations along the y direction.

To serve as an example, consider the case where you wish to specify a wave field having varying height $2a(y)$ and direction $\theta(y)$ along an offshore boundary having nonuniform depth $h(y)$. Assuming that the local wavenumber $k(y)$ has been determined from the dispersion relation

$$\frac{2\pi}{T} = (gk \tanh kh)^{1/2} \tag{29}$$

where T is the wave period, then the complex amplitude A can be specified by

$$A(y) = a(y)e^{i\psi(y)} \tag{30}$$

where the phase function $\psi(y)$ is given by

$$\psi(y) = \int_{y=0}^{y} k(y) \sin \theta(y) \mathrm{d}y \tag{31}$$

The discrete values $A(j)$ are then simply given by

$$A(j) = A(y_j); j = 1, \cdots, n \tag{32}$$

2.7   Program Input: Model Control and Wave Data

This section discusses the structure of the input data file *indat.dat* read in through logical device number *iun*(5). This file contains all the information needed to control the operations of the program, and all of the input wave data. Reference grid values of depth *dr* and currents *ur* and *vr* are treated in the following section.

Data is read from *iun*(5) in both the *inref* and *inwave* subroutines. Each line of input data is in free format. The arrangement of input data values in the data file varies a little depending on which parameters are chosen. The input file is set up as follows:

1. *iun(1), iun(2), iun(3)*:

   Logical device numbers for reference grid data, user-specified subdivision data, and reference grid output.

2. *mr, nr* :

   Reference grid dimensions. Maximum values are *ixr, iyr* respectively.

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3. *iu, ntype, icur, ibc*:

   iu is switch for physical units; $iu=1$, MKS; $iu=2$, English. (program defaults to $iu=1$ if an error is made on input).

   ntype is switch for nonlinearity; $ntype=0$, linear model; $ntype=1$, composite model; $ntype=2$, Stokes wave model.

   icur is switch for input current data. $icur=0$, no currents input; $icur=1$, currents input.

   ibc is the boundary condition switch. $ibc=0$, closed boundaries; $ibc=1$, open boundaries.

   Both *icur* and *ibc* default to zero on input error.

4. *dxr, dyr, dt* :

   Reference grid x-spacing and y-spacing (uniform over entire reference grid) and depth tolerance value.

5. *ispace, nd*:

   ispace is switch controlling subdivisions; $ispace=0$, program attempts its own x subdivisions. $ispace=1$, user specifies x subdivisions.

   nd is the number of y-direction subdivisions (needed in either case).

   If $ispace=1$, the x-direction subdivisions are inserted here (one for each reference grid block):

   $(md(ir), ir=1, mr\text{-}1)$

6. *iff(1), iff(2), iff(3)*:

   Dissipation switches; iff(1)=1;turn on turbulent boundary layer; iff(2)=1; turn on porous bottom damping; iff(3)=1; turn on laminar boundary layers. No damping if all values are zero.

7. *isp*:

   Switch for user-specified sub-grid specifications. $isp=0$, no subgrids to be read; $isp=1$, subgrids will be read.

8. *iinput, ioutput*:

   iinput specifies whether the program or the user will generate the first row of complex amplitude A values. If $iinput=1$, the program constructs A based on the input conditions specified as follows. If $iinput=2$, the user must specify A in an external data file *wave.dat*.

   ioutput specifies whether the last computed row of complex amplitudes A are to be

stored in file *owave.dat*. A value of *ioutput=1* skips this option. *ioutput=2* turns the option on.

If *iinput=1*, the rest of *indat.dat* is as follows:

9. *iwave, nfreqs*:

   iwave is switch for wave field type. *iwave=1*, discrete wave components. *iwave=2*, directional spreading model.

   nfreqs is the number of frequency components to be run.

   The remainder of the file depends on the choice of *iwave*.

   iwave=1

10. *freqs(ifreq), tide(ifreq)*:

    Wave period and tidal offset for each frequency component.

11. *nwavs(ifreq)*:

    Number of wave components for each frequency.

12. *amp(ifreq, iwaves), dir(ifreq, iwaves)*:

    Amplitude (not height) and direction (in degrees) for each component wave.

    repeat *nwaves* times

    repeat entire block *nfreqs* times

    iwave=2

10. *thet0*

    The central direction for the model spectrum.

11. *freqs(ifreq), tide(ifreq)*:

    As above.

12. *edens(ifreq)*:

    Variance density (m^2 or ft^2) for each frequency component.

13. *nwavs(ifreq), nseed*:

    Directional spreading factor (the factor n in $\cos^{2n}(\theta/2)$, and the seed value for the random number generator (between 0 and 9999).

    repeat for each frequency component

For *iinput=2*, the remainder of the data file is:

9. *freqs, tide*:

Wave period and tidal offset for the single frequency component.

Examples of data files are given in the example problem section.

2.8 Program Input: Reference Grid and Subgrid Data

This section describes two files which provide arrays of data on the various grids.

Reference Grid Data File

This file (named *refdat.dat*) is accessed as logical device number *iun*(1). Its contents consist of the arrays of depth dr, x-velocity ur, and y-velocity vr at the reference grid points. This file is accessed only once per model run, and its entire contents are read in by subroutine *inref*. If —icur=0—, only the depth data —dr— need to be specified.

Data for this file should be written in the following format:

```
        do 1 ir=1, mr
        write(unit,#) (dr(ir, jr), jr=1, nr)
1       continue

        (then, if icur=1)

        do 2 ir=1, mr
        write(unit,#) (ur(ir, jr), jr=1, nr)
2       continue

        do 3 ir=1, mr
        write(unit,#) (vr(ir, jr), jr=1, nr)
3       continue
#       format(16f8.4)
```

The data may be in either MKS or English units; set the units switch *iu* in the *iun*(5) data file *indat.dat* accordingly.

Subgrid Data File

This file (named *subdat.dat*) is accessed as logical device number *iun*(2). If no user-defined subgrids are to be read in, this file may be omitted. The file consists of two parts; an integer array of 1's and 0's indicating which reference grid cells are to be defined by the user, and then a sequence of groups of arrays of d, u and v, one group for each subgrid.

The integer array *isd* is dimensioned (mr-1) by (nr-1), with one point for each spacing in the reference grid. The array contains a 0 if that cell is not to be user-defined, and a

one if it is. For example, the $(mr,nr)=(7,6)$ reference grid shown in Figure 6 has four cells which are to be read in as user-defined subgrids.

The array *isd* should be written in the data file first, using the format:

```
      do 1 ir=1,mr-1
      write(unit,#)(isd(ir,jr),jr=1,nr-1)
 1    continue
 #    format(15I4)
```

Now, a group of arrays of d, u and v must be entered into the data file, with one group corresponding to each value of 1 in *isd*. The program accesses the data file by x-row (ir) and then by y-column (jr). For the above example, the subgrid array groups should thus be stored in the order of the coordinate pairs (4,3), (4,4), (5,3), (5,4). The dimensions of each subgrid are given by $m=md(ir)+1$ and $ns=nd+1$. The borders of adjacent subgrids share the same common boundary points.

Each group of subgrid data should be written using a format similar to:

```
      write(unit,#)((d(i,j),j=1,ns),i=1,m)

      (then, if icur=1)

      write(unit,#)((u(i,j),j=1,ns),i=1,m)
      write(unit,#)((v(i,j),j=1,ns),i=1,m)

 #    format(16f8.4)
```

using the appropriate value of m for the grid block in question. Data may be in MKS or English units, depending on the value of *iu* in the input data file. The integer array *isd* is read by *inref*, and the individual subgrids are read by *grid*.

2.9 Program Output

This section discusses output of two forms: output sent to the screen (or the chosen output device with logical unit number $iun(4)$), and output stored in disk file (logical unit number $iun(3)$) with a resolution of the reference grid.

2.9.1 Screen Output

Screen output consists of three types: title and header information which is printed in order to identify the run and the operating perameters, runtime error messages (either warnings or terminal error messages), and the results of calculations.

Header Information

At the start of each run, a set of two title pages are printed. Page 1 identifies the program and then prints out messages identifying the parameters which set up the model run, as read in by subroutine *inref*. A sample title page 1 is shown in Figure 7.

Title page 2 gives the input wave conditions which were read in by *inwave*. A sample title page 2 is given in Figure 8.

Dimensional quantities are printed on the title page in the units used for input. Title page 1 gives an indication whether quantities are in MKS or English units.

Run-time Error Messages

REF/DIF 1 performs some data checking and checking of calculations during a run. This checking may result in warnings or terminal errors which are beyond calculation errors which would lead to standard FORTRAN error messages. A list of possible errors and the resulting messages follow.

1. Reference grid dimensions were specified as being too large on input. $mr > ixr$ and/or $nr > iyr$.

   Message: dimensions for reference grid too large; stopping.

   Action: Program stops.

   Error occurs in: *inref*

2. User specifies a y-direction subdivision nd which will cause the number of y grid points n to exceed the maximum iy.

   Message: y-direction subdivision too fine. maximum number of y grid points will be exceeded. execution terminating.

   Action: Program stops.

   Error occurs in: *inref*

3. User specifies an x-direction subdivision on one of the grid blocks ir which exceeds the maximum amount (ix-1). As a result, the dimension of the subdivided grid will be too large.

   Message: x-direction subdivision too fine on grid block "ir", execution terminating.

   Action: program stops

   Error occurs in: *inref*

4. A depth value occurs in the reference grid which differs from the average of its neighbors by more than the tolerance value dt specified on input. This is basically a data checking feature. Printed values are in meters.

Refraction-Diffraction Model for
Weakly Nonlinear Surface Water Waves

REF/DIF 1,   Version 2.4

Center for Applied Coastal Research
Department of Civil Engineering
University of Delaware
Newark, Delaware 19716

James T. Kirby and Robert A. Dalrymple, July 1992

input section, reference grid values

reference grid dimensions   mr=100
                            nr=100

reference grid spacings   dxr= 20.0000
                           dyr= 20.0000

physical unit switch iu=2,   input in english units

icur=0, no current values read from input files

ibc=0, closed (reflective) lateral boundaries

ispace =0 chosen, program will attempt its own reference grid subdivisions

y-direction subdivision according to nd= 1

ntype = 1, stokes model matched to hedges model

  switches for dissipation terms

0   turbulent boundary layer
0   porous bottom
0   laminar boundary layer

isp=0, no user defined subgrids
iinput = 1, program specifies initial row of a

Figure 7: Sample title page 1

```
iwave=1, discrete wave amps and directions


the model is to be run for  1 separate frequency components

frequency component   1

wave period= 10.0000sec., tidal offset=  0.0000

wave component  1, amplitude = 14.0000, direction=  0.0000


             model execution, frequency component   1
```

Figure 8: Sample title page 2

Message: Depth "*dr*" (m) at reference grid location "*ir,jr*" differs from the average of its neighbors by more than "dt" (m). Execution continuing.

Action: None by program. Data in file *refdat.dat* should be corrected if wrong.

Error occurs in: *inref*

5. An ambient current value occurs which implies that the flow would be supercritical at the given location. This serves as both a check for anomalously large current values, and an indicator of possible subsequent computational problems.

Message: ambient current at reference grid location "*ir,jr*" is supercritical with froude number = "froude number", execution continuing

Action: None by program. Data in file *refdat.dat* should be corrected if wrong.

Error occurs in: *inref*

6. If the user specifies that predetermined subgrids are to be read in, while at the same time telling the program to performs its own subdivisions, the computed dimensions of the subgrid may be different than those of the subgrid included in the input. Runs requiring user-specified subgrids should choose the *ispace*=1 option. If an incompatible

set of dimensions occurs, the program will either garble the input array or run out of data.

Message: Warning: input specifies that user will be supplying specified subgrids (isp=1), while program has been told to generate its own subgrid spacings (ispace=0), possible incompatibility in any or all subgrid blocks.

Action: None by program. Should restart unit with correct *ispace.isp* values.

Error occurs in: *inref*

7. While calculating its own subdivision spacings, the model may try to put more division in a reference grid block than is allowed by by dimension *ix*. If this occurs, the program uses the maximum number of subdivisions allowed (*ix*-1), but prints a message indicating that the reference grid spacing is too large with respect to the waves being calculated. This problem may be circumvented by increasing the size of *ix* in *parameter* statements.

Message: *model* tried to put more spaces than allowed in grid block "*ir*"

Action: Program performs fixup and continues. Model resolution and accuracy may be poor, and a finer reference grid or increased value of *ix* in the *parameter* statements should be used.

Error occurs in: *grid*

8. While using the Stokes wave form of the model, *ntype*=2 the model may encounter large values of the Ursell number, indicating that the water is too shallow for that model to be appropriate. The cutoff point recognized by the program is $(A/h)/(kh)^2 = 0.5$.

Message: Warning: Ursell number = "u" encountered at grid location "i,j" should be using Stokes-Hedges model (ntype = 1) due to shallow water

Action: The program should be re-run with the composite nonlinear model.

Error occurs in: *fdcalc*

9. The Newton-Raphson iteration for wavenumber k may not converge in the specified number of steps. This may occur for waves on strong opposing currents.

Message: WAVENUMBER FAILED TO CONVERGE ON ROW "I", COLUMN "J"

K = last iterated value of wavenumber

D = depth

T = period calculated from last iterated value of k

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U = x-direction velocity

F = value of objective function (should be =0 for convergence)

Action: Program continues with last iterated value of k. Computed results are of questionable accuracy.

Error occurs in: *wvnum*

Results of Calculations

For each frequency component, the program starts a new page of output and indicates the number of the component in the input stack. The model then prints the x position and value of the reference phase function for each reference grid row. Also, for each grid row, the values of wave amplitude and wave direction are printed for each reference grid location. It should be noted that the wave directions given on output are meaningless if multiple direction components are being used, and, for single component runs, become meaningless if the waves become short crested or the crests become significantly curved. Amplitudes and directions are printed in 12(f10.4) format. Dimensions and amplitudes are printed out using the physical units (MKS or English) used for input.

2.9.2 Stored Output

The program stores a set of output with a resolution of the reference grid points, using logical device number *iun*(3). This data file (named *outdat.dat*) has been used to generate the plots given in Chapter 3. All data is stored using free format. The program first stores the values of *nr* and *mr*, and the values of the reference grid *y*-coordinates.

1. *nr, mr*

2. *yr(jr), jr=1,* nr

   Then, for each *x*-position *ir* in the reference grid, the program stores the following information.

3. *xr(ir), psibar* (value of the reference phase function)

4. *d(ir,jr), jr=1,* nr

5. *a(ir,jr), jr=1,* nr

The set of output (3-5) is repeated for each *x*-position *ir*, and the entire set of *x*-positions is repeated for each frequency component. The data for depth *d* include the effect of the tidal offset (input as *tide*) and also will show the thin film where exposed, dry land has been replaced by a small water depth. The data are stored using the units (MKS or English) used for input.

An example of a plotting program using data stored in *outdat.dat* is included in Section 3.2.2 to provide guidance on reading the stored output file.

An anotated listing of the program code is given in Appendix A. The final program code used for compilation has all documentation stripped off.

3 EXAMPLE CALCULATIONS

This chapter presents calculations performed using the combined refraction-diffraction model **REF/DIF** 1. The problems studied here were supplied by members of the joint industry program supporting the initial development of the computer program, or were chosen as representative tests of various features of the model. Further examples illustrating the use of the computational schemes upon which the program is based may be found in the technical report by Kirby (1983).

Each section of this chapter describes in full the model's application to a specific problem. Following a description of the problem and an indication of the type of results desired, the input data files for the program are displayed and explained. These data files are then used to run the program **REF/DIF** 1 with no job-specific modifications to the program involved. Program output is then presented in such a way as to adequately indicate the results. although, in application. individual users may wish to alter the nature of the presentation of output data.

The output for the various examples has been presented using some plotting programs which are external to the main body of the supplied program **REF/DIF** 1. These specialized programs have been included in order to provide some guidance in reading the data files generated by the main program. However, plotting routines are likely to vary from one computer system to another. The extra programs are therefore likely to be extensively machine-specific to the systems on which the computations were performed.

Section 3.1 presents calculations of waves around an artificial, surface piercing island. This example makes particular use of the breaking wave, thin film, and shallow water dispersion relation capabilities of the model.

Section 3.2 provides calculations for waves propagating over a submerged, elliptic shoal resting on a plane beach. This example has been studied experimentally and provides a means for checking the accuracy of the model calculations. It also provides an example of the type of results provided by a combined refraction-diffraction model in a situation where ray tracing predicts a strong convergence of wave rays, with resulting singularities in the prediction of wave height.

Section 3.3 provides example calculations for the case of waves shoaling on a plane beach and interacting with a rip current. This example illustrates the wave-current interaction feature of the model.

3.1 Waves Around an Artificial Island

The first example involves the calculation of the wave field around an artificial, surface piercing island of the type used in offshore operations. The island is circular with a base radius of $400ft$ and a crest elevation of $80ft$ above the flat seabed. The island radius at the crest is $160ft$, leading to a side slope of $1:3$. The water depth around the island is taken to be $60ft$. The island geometry is shown in Figure 9.

The wave conditions to be studied are given by:

Wave height: $H = 28ft$

Wave period: $T = 10sec.$

No currents

The model was run with a depth of $60ft$ away from the island and a tidal offset of $0.0ft$.

The required set of wave predictions consisted of wave height at 12 locations as indicated in Figure 10. The spacing between the points are in units of the base radius $r_b = 400ft$. Note that, since the model does not calculate reflected waves, the predicted wave heights at points 1 and 2 will be identical. Therefore, computations may be started arbitrarily close to the leading edge of the island base, in the absence of any current field distorted by the island's presence.

3.1.1 Setting up the Model

First, the island topography was established. Note that the region of the island above the water line is not treated explicitly in the computations. We therefore represented the island in the input data as a right circular cone with a peak height of $153.33ft$ and a base radius of $400ft$. The model will truncate the island in order to create a "thin film" over the exposed portions (see Figure 11).

Next, the reference grid spacing was chosen. Since the physical region to be modelled is small, we picked a reference grid with fine enough resolution so that no subdivision of the reference grid will be required. Using a wave period of 10 seconds and a depth of $60ft$, we use the dispersion relationship

$$\frac{4\pi^2}{T^2} = gk \tanh kh \tag{33}$$

to calculate $kh = .97$, giving a wave length of $L = 389ft$. L is just slightly less than $r_b = 400ft$. The grid spacings dxr and $dyr = 20ft$ were chosen for the reference grid, giving approximately 20 points per wavelength away from the island. We use 100 x 100 storage locations for the reference grid, indicating a model of approximately $5r_b$ by $5r_b$ in x and y. We sited the island center at $x = 460ft$ and $y = 10ft$, where x and y are measured from
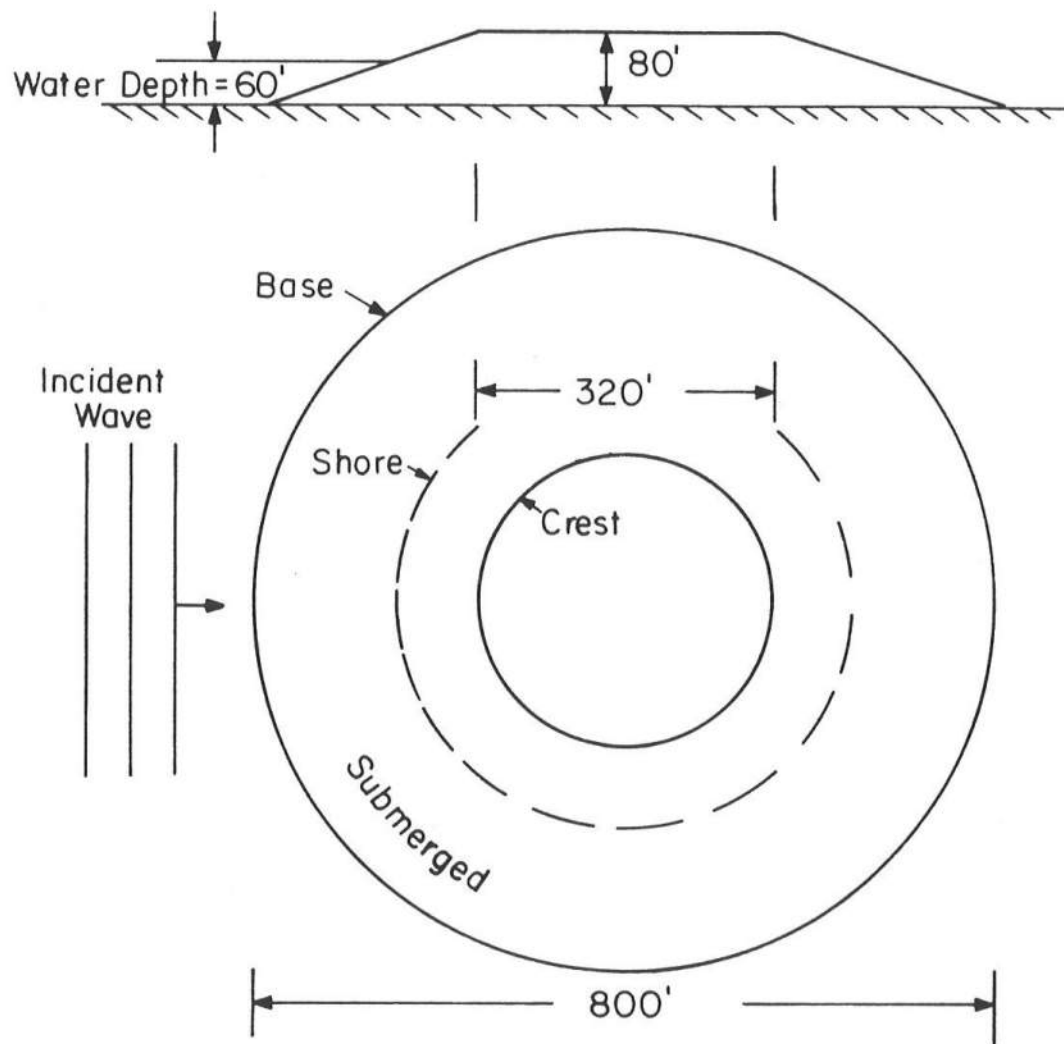
Figure 9: Artificial island geometry

Figure 10: Locations for wave height measurements
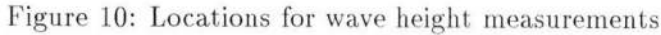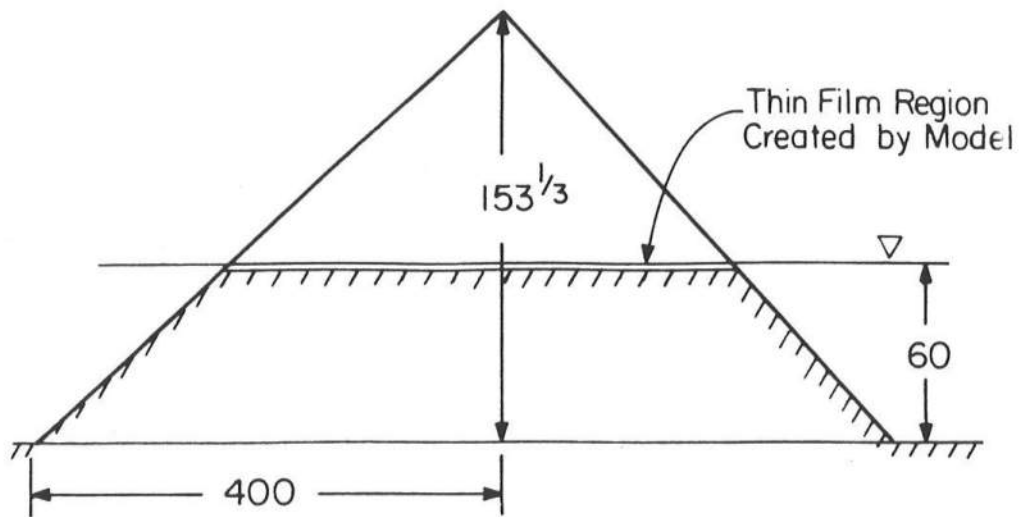
Figure 11: Representation of the island geometry in the program

the computational grid corner. The island and measurement points are shown in relation to the grid in Figure 12.
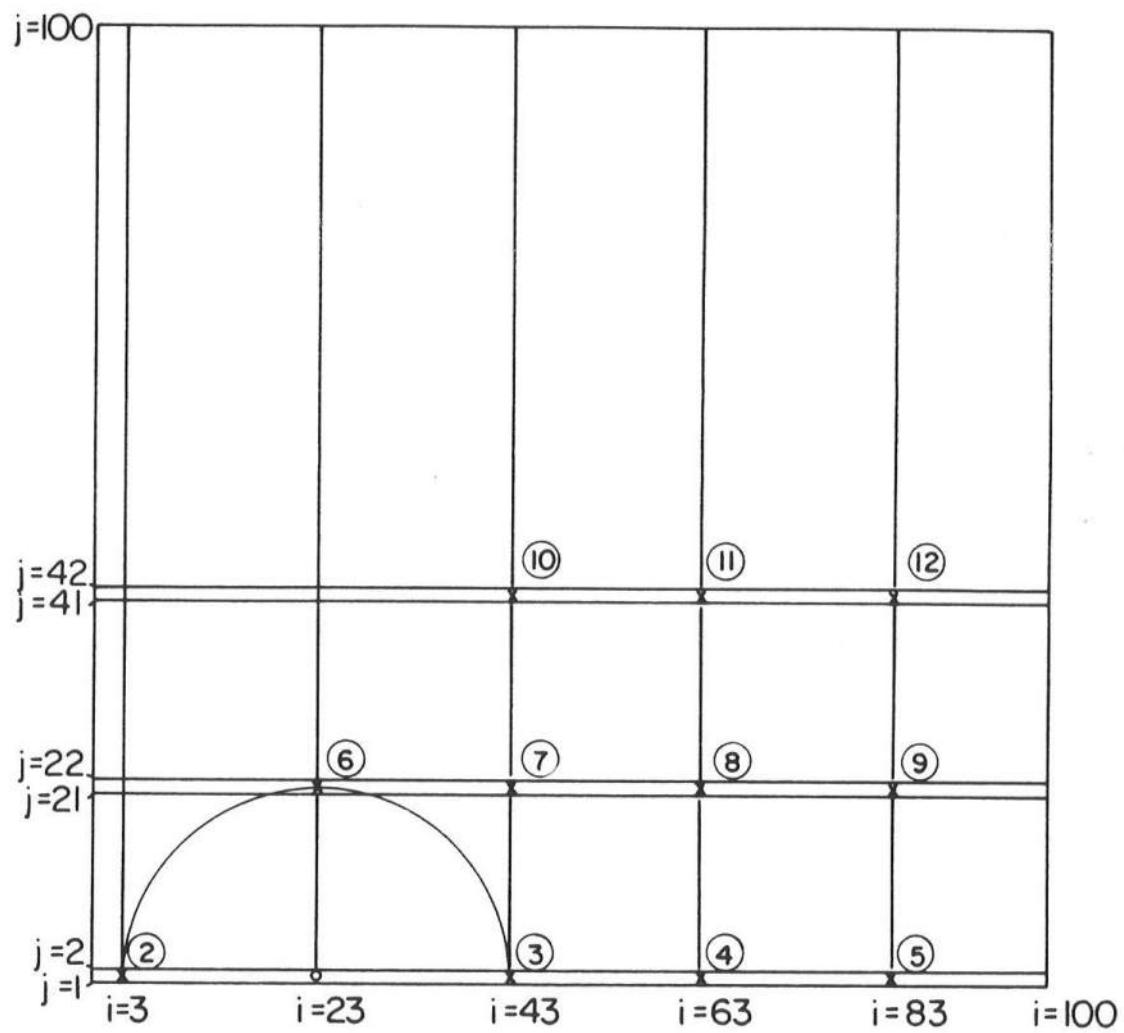
Figure 12: Measurement points in relation to reference grid

Location	Grid Points	Wave Height (ft)
2	(3,1)	28.00
3	(43,1)	17.28
4	(63,1)	14.36
5	(83,1)	16.94
6	(23,21)	23.52
7	(43,21)	19.06
8	(63,21)	20.88
9	(83,21)	18.60
10	(43,41)	32.05
11	(63,41)	29.74
12	(83,41)	23.46

Figure 13: Calculated wave heights at measurement locations

3.1.2 The Input Data Files

One run of the model was performed using the specified input conditions. The input data file *indat.dat* for the run follows. The reference grid data was stored in file *refdat.dat*.

```
1  2  3  4
100   100
2  1  0  0
   20.0000     20.0000     10.00000
0  1
0  0  0
0
1  1
1  1
  10.00000  0.
1
   14.0000  0.
```

3.1.3 Model Results

The output for the artificial island run is presented in two forms. First, Figure 13 provides values of wave heights at the measurement locations indicated in Figure 12. The values were obtained directly from the screen output.

In addition to this, we have constructed contour plots of instantaneous surface elevation

and wave amplitude $|A|$; these are shown in Figures 14 and 15, respectively. Contour elevations for surface elevation normalized by incident wave amplitude are in increments of 0.25, while the contour elevations for normalized amplitude are in increments of 0.2.
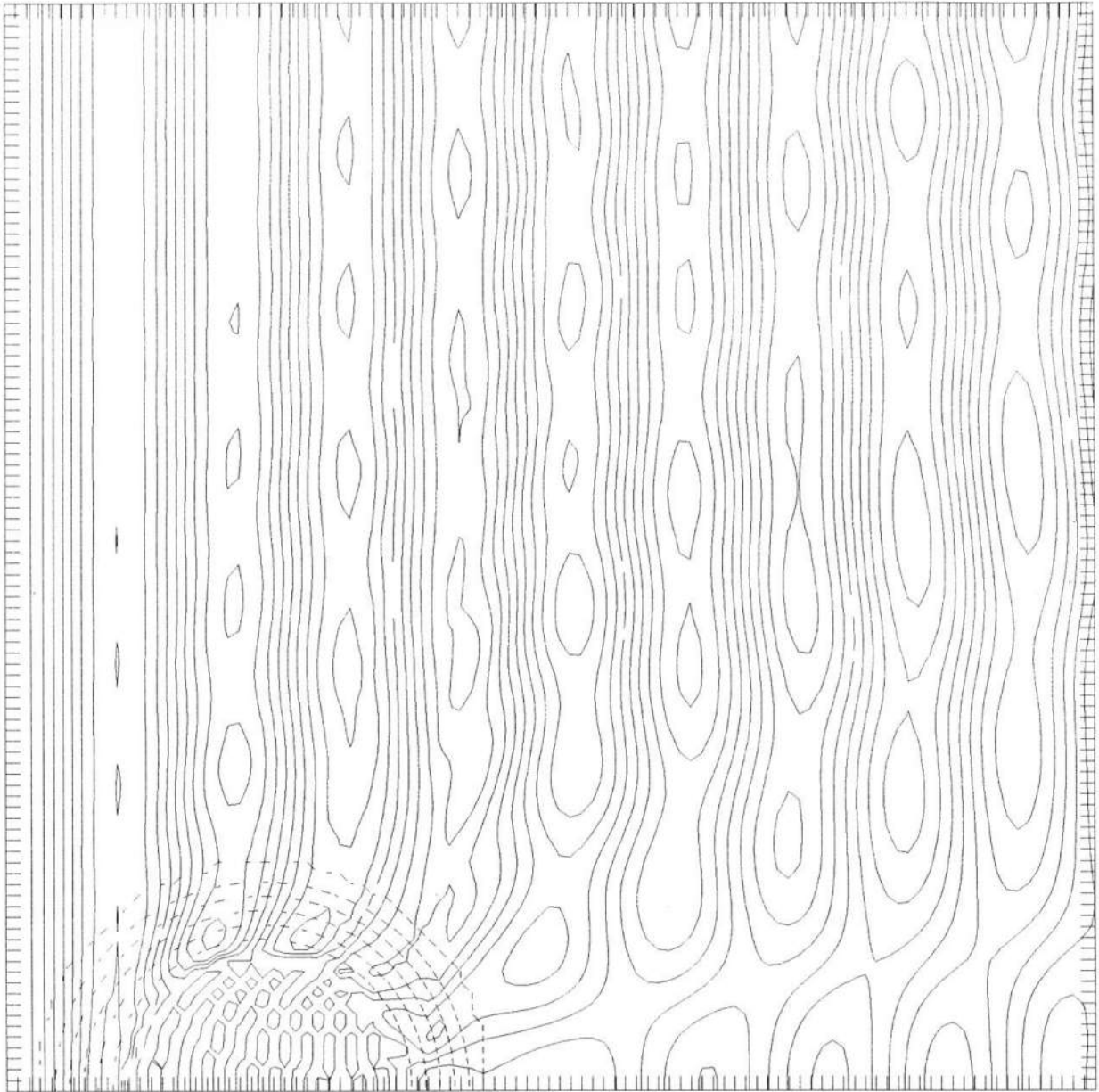
Figure 14: Artificial island example: contours of instantaneous surface elevation
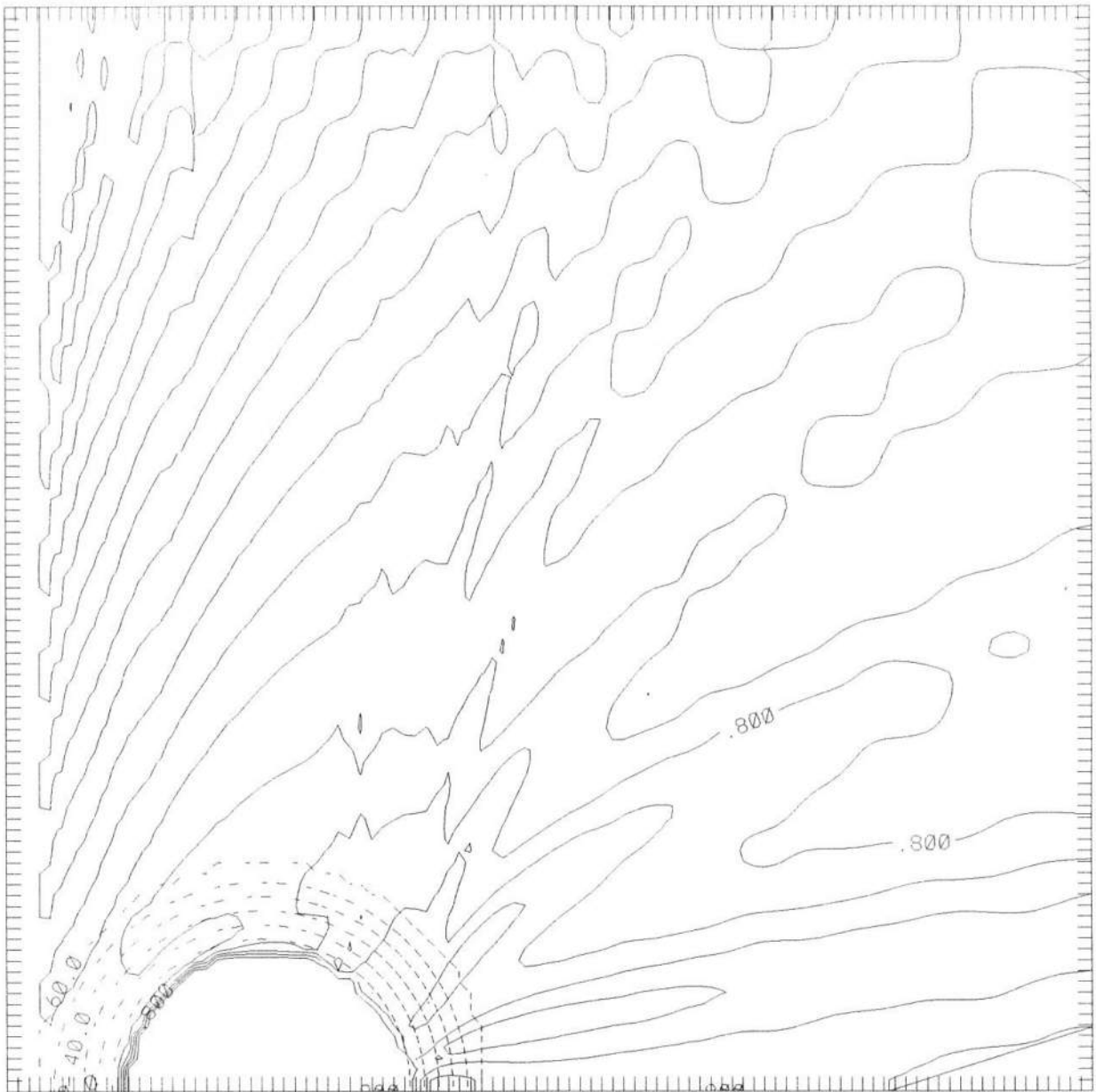
Figure 15: Artificial island example: contours of wave amplitude

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3.2   Wave Focussing by a Submerged Shoal

In this example, we study the propagation of an initially plane wave over a submerged shoal resting on a plane beach. This example has been chosen for several reasons:

1. A carefully controlled set of waves measurements has been made in the laboratory for this case (see Berkhoff et al (1982); Kirby (1986a)).

2. The wave pattern represents the case of ray crossing in the refraction method, and thus the computed results indicate present method's utility in situations where ray tracing breaks down.

3. The example gives a thorough test of the accuracy of the large angle and composite nonlinearity formulations.

The topography to be studied is shown in Figure 16. Details of the calculation of the topography may be found in Kirby (1986a).

For the case of an incident plane wave, we have performed a run of the model using input data corresponding to the experiment of Berkhoff et al (1982). The run was done using the full model with the Stokes-Hedges composite nonlinearity.

3.2.1   The Input Data Files

The input data file *indat.dat* for the present case follows.

```
    3     4    10
 100   100
    1     1    0    0
  .25   .25   15.
    0     1
    0     0    0
    0
    1     1
    1     1
    1.    0.
    1
0.0232  0.
```

The formulas to calculate the depth grid are given in Kirby (1986a).
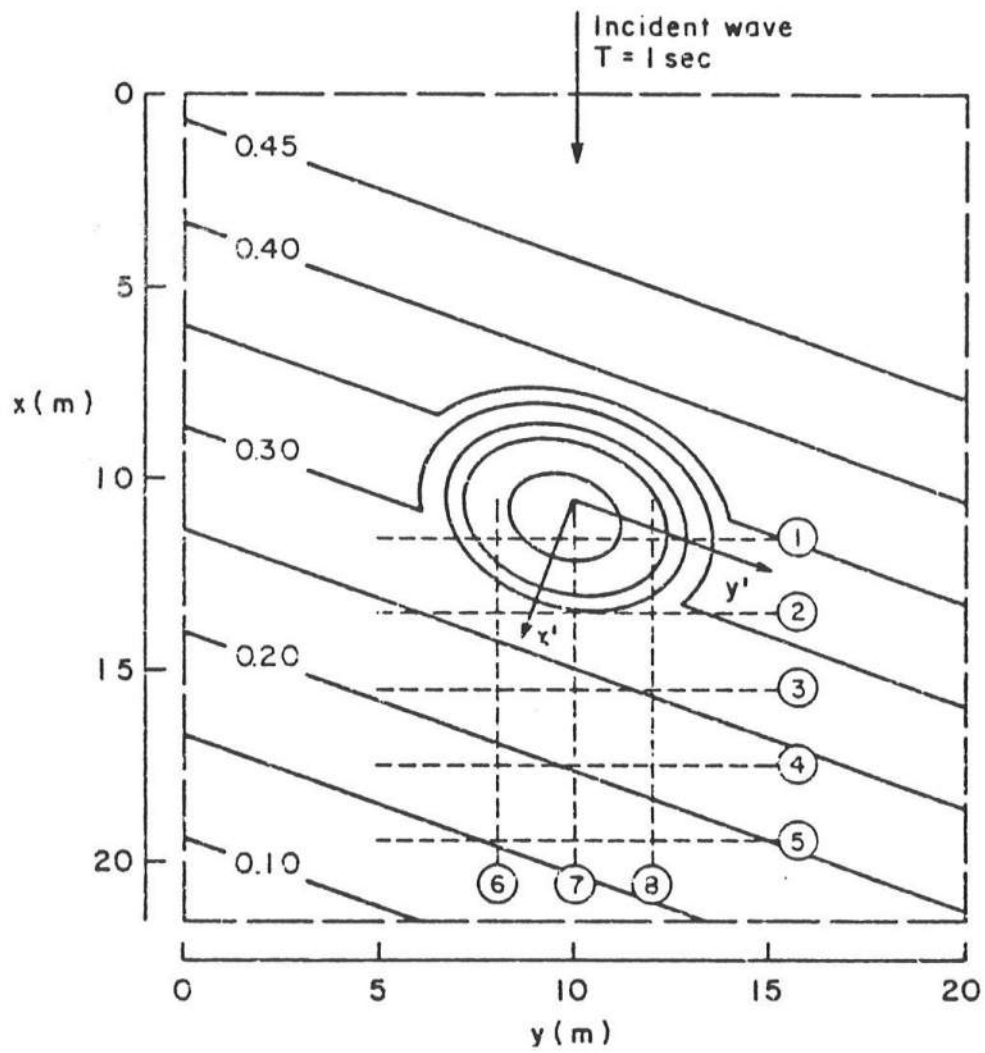
Figure 16: Bottom contours and computational domain for the experiment of Berkhoff et al (1982). Experimental data on transects 1-8.

3.2.2 Model Results

The output file *outdat.dat* for this example has been used to construct plots of wave amplitude. In this case, wave amplitudes have been non-dimensionalized using the incident wave amplitude. The resulting plot is shown in Figure 17. The plots show the effect of wave focussing over the shoal area, and show that the prediction of the wave field beyond the shoal does not involve the problems of refraction calculations used in similar situations.

The program used to plot contours of wave height based on the information stored in the output file *outdat.dat* is included here to provide some guidance in reading this file. The plotting is based on the NCAR package which may not be available on most systems.

```
c*------------------------------------------------------------------
c*      plotamp.f
c*
c*      plot amplitude field constructed from output in {\em outdat.dat}
c*      from a REFDIF1 model.
c*------------------------------------------------------------------
        dimension wave(100,100),h(100,100),x(100),y(100),psi(100)
        complex a(100,100)
        open(10,file='{\em outdat.dat}')
        read(10,*)n,m
        write(*,*)' grid dimensions are ',m,' by ',n
        read(10,*)(y(j),j=1,n)
        do 1 i=1,m
        read(10,*)x(i),psi(i)
        read(10,*)(h(i,j),j=1,n)
        read(10,*)(a(i,j),j=1,n)
1       continue
        close(10)
        a0=cabs(a(1,1))
c*------------------------------------------------------------------
c*      construct normalized wave height field
c*------------------------------------------------------------------
        do 3 i=1,m
        do 2 j=1,n
        wave(i,j)=cabs(a(i,j))/a0
2       continue
3       continue
c*------------------------------------------------------------------
c*      enter corner points for plotted grid
c*------------------------------------------------------------------
        write(*,*)' enter imin,imax,jmin,jmax'
        read(*,*)imin,imax,jmin,jmax
        mnew=(imax-imin)+1
        nnew=(jmax-jmin)+1
        do 5 i=1,mnew
        ii=(i+imin)-1
        do 4 j=1,nnew
        jj=(j+jmin)-1
        wave(i,j)=wave(ii,jj)
        h(i,j)=h(ii,jj)
```
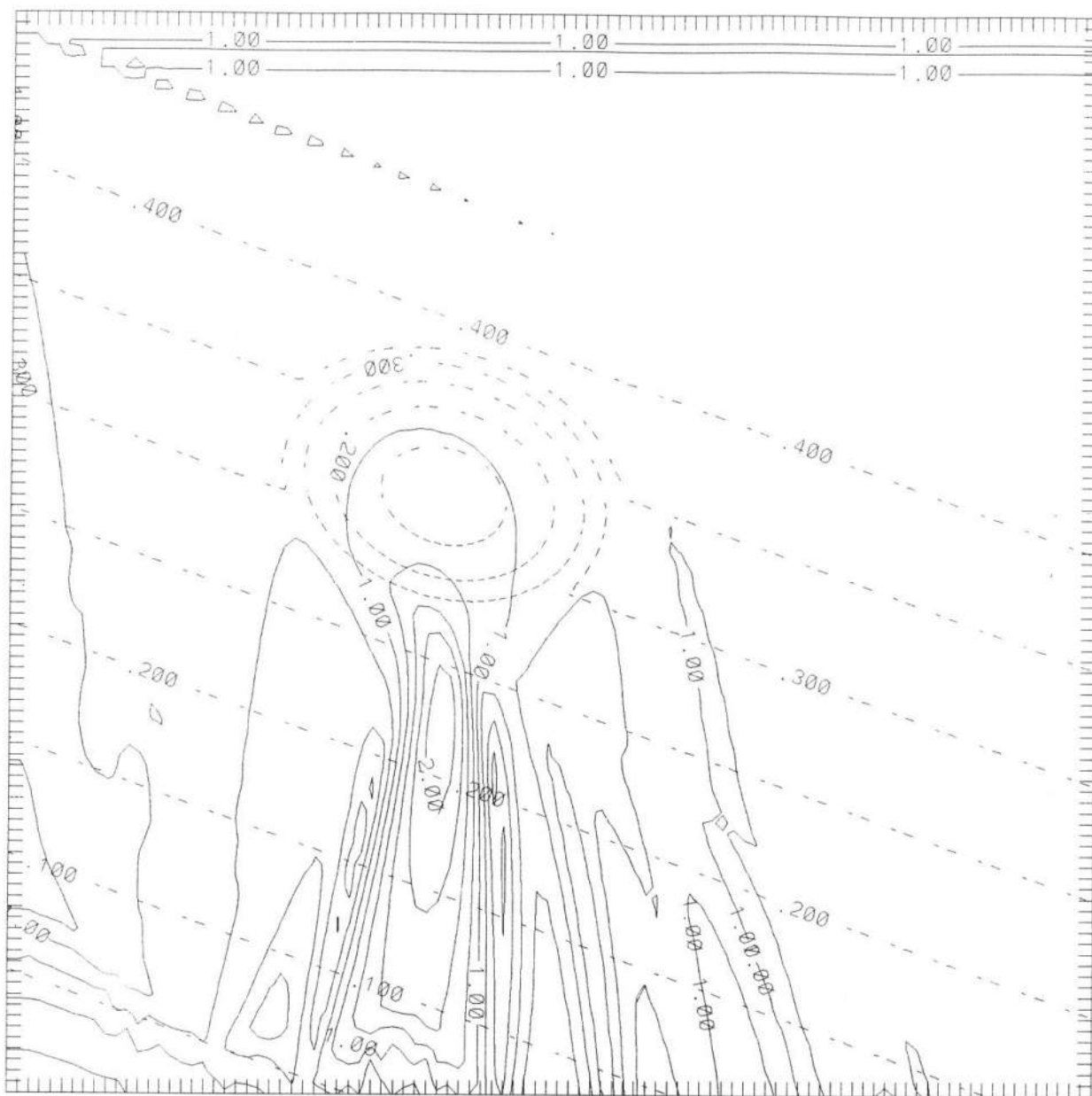
Figure 17: Results for waves propagating over a submerged shoal: wave amplitude contours

```
4       continue
5       continue
c*-----------------------------------------------------------------
c*      contour the normalized wave height field
c*-----------------------------------------------------------------
        call gopks(6,idum)
        call gopwk(1,2,1)
        call gacwk(1)
        call conrec(h,100,mnew,nnew,0.,0.,-10.,0,-1,200)
        call conrec(wave,100,mnew,nnew,-3.,3.,.25,0,-1,0)
        call frame
        call gdawk(1)
        call gclwk(1)
        call gclks
        stop
        end
```

Comparisons of the predictions of this model using both the Stokes wave nonlinearity and the composite model of Kirby and Dalrymple (1986b) are given in Kirby and Dalrymple (1986b); a comparison to laboratory data is also included and shows that the model is quite accurate in predicting the wave field.

3.3 Waves Interacting with a Rip-Current

An example of waves which are normally incident on a planar beach and interact with a steady rip current flowing offshore from the beach has been included here in order to show the effects of wave-current interaction in the model. This example was first used by Arthur (1950) to illustrate the effects of currents and depth changes acting together on results of ray tracing schemes. However, it also provides an important example of the usefulness of the combined refraction-diffraction scheme, since it represents a case where ray tracing breaks down due to the crossing of wave rays.

The velocity field studied by Arthur is shown together with computed wave orthogonals in Figure 18. Denoting a coordinate x' pointed offshore (the opposite of the x we will be using), the velocity distribution is given by

$$U = 0.02295x'e^{-(x'/76.2)^2/2}e^{-(y/7.62)^2/2} \tag{34}$$

$$V = -0.2188[2 - (x'/76.2)^2]e^{-(x'/76.2)^2/2}\mathrm{erf}(y/76.22)\mathrm{sign}(y) \tag{35}$$

where the velocities are in m/sec. In terms of x', the bottom topography is given by

$$h(x') = 0.02x' \tag{36}$$

Arthur ran his calculations for a wave period of $T = 8$ seconds.

A photograph of the wave field created in a laboratory study of waves interacting with an ebb tidal jet is shown in Figure 19. This photograph, taken from a paper by Hales and Herbich (1972), is provided for guidance in interpreting the contour plot of surface elevations provided below.

3.3.1 Setting Up the Model

We choose a grid spacing of $dxr = 5$m and $dyr = 5$m. We choose $mr = 100$ and $nr = 100$, giving an offshore and longshore extent of 495m. The most-shoreward grid row is established 5m from shore, giving a depth range of 10m to 0.1m. Arthur's wave period is retained, and we use an initial wave amplitude of 0.1m. The input conditions are a single, normally incident wave, no user specified grid subdivision, and one frequency component.
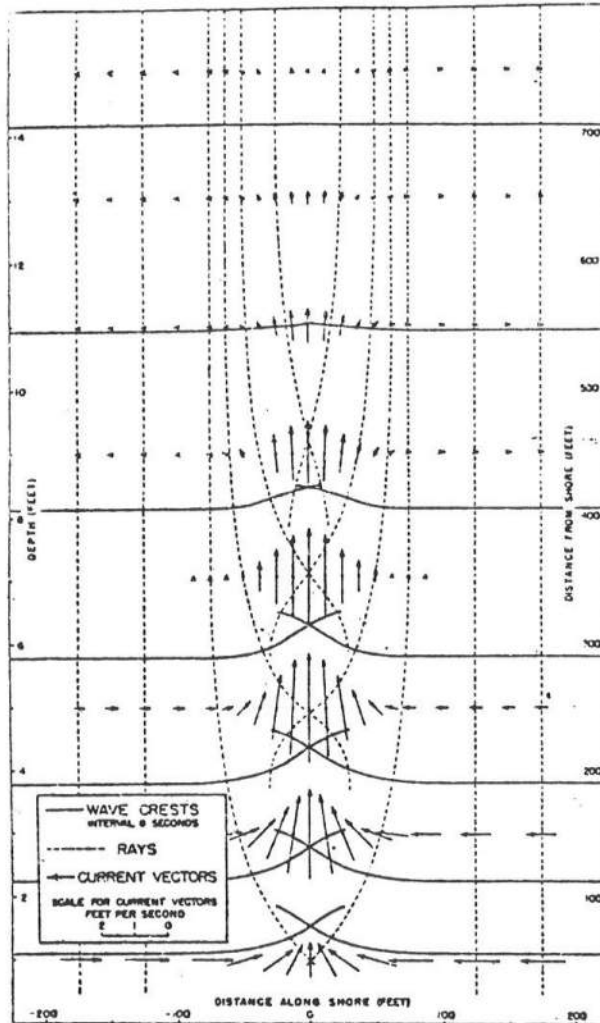
Figure 18: Pattern of orthogonals and wave crests for waves in presence of rip currents: refraction approximation. (from Arthur, 1950)
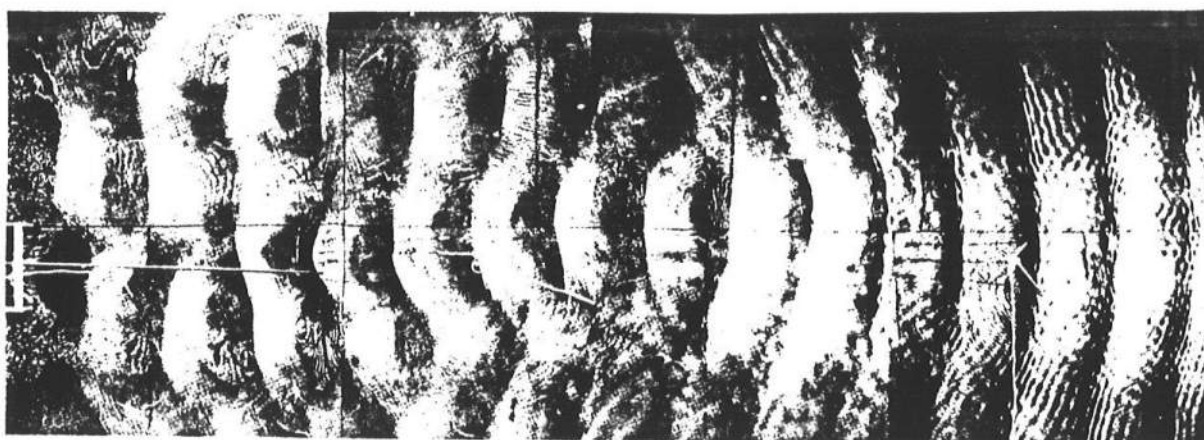
FIG. 12-WAVE TRAIN PROPAGATING ONTO AN EBB CURRENT. $U/C_o = 0.101$, $L_o/d = 4.58$, $H_o/L_o = 0.028$

Figure 19: Wave pattern on an ebb-tidal jet. (from Hales and Herbich, 1972)

The input data file for the present case follows.

```
1  2  3  4
100   100
1  1  1  0
   5.00000     5.00000    10.00000
0  1
0  0  0
0
1  1
1  1
   8.00000  0.
1
   1.00000E-01  0.
```

3.3.2   Model Results

Results for this case are limited to plots of surface contours and wave amplitude contours. These plots were constructed using the information stored in the data file *outdat.dat*. The plots are given in Figures 20 and 21. The plots show a shoaling, plane wave which approaches the beach with no distortion until the wave begins to interact with the rip current. The rip causes a focussing of waves and the formation of discontinuities in the wave crests, as in the photograph in Figure 19.
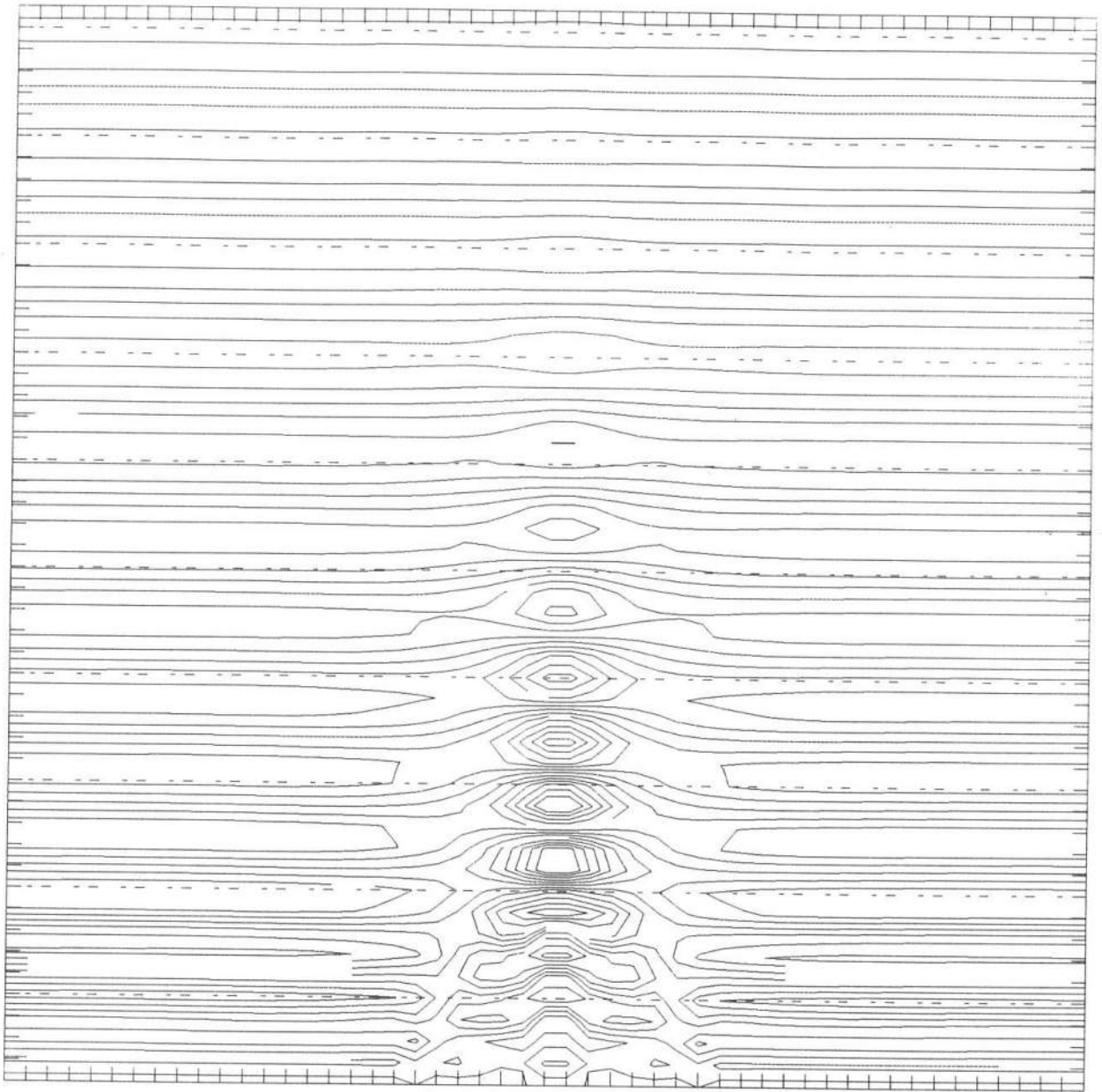
Figure 20: Waves interacting with a rip current. Shoreline at bottom. Surface displacement contours
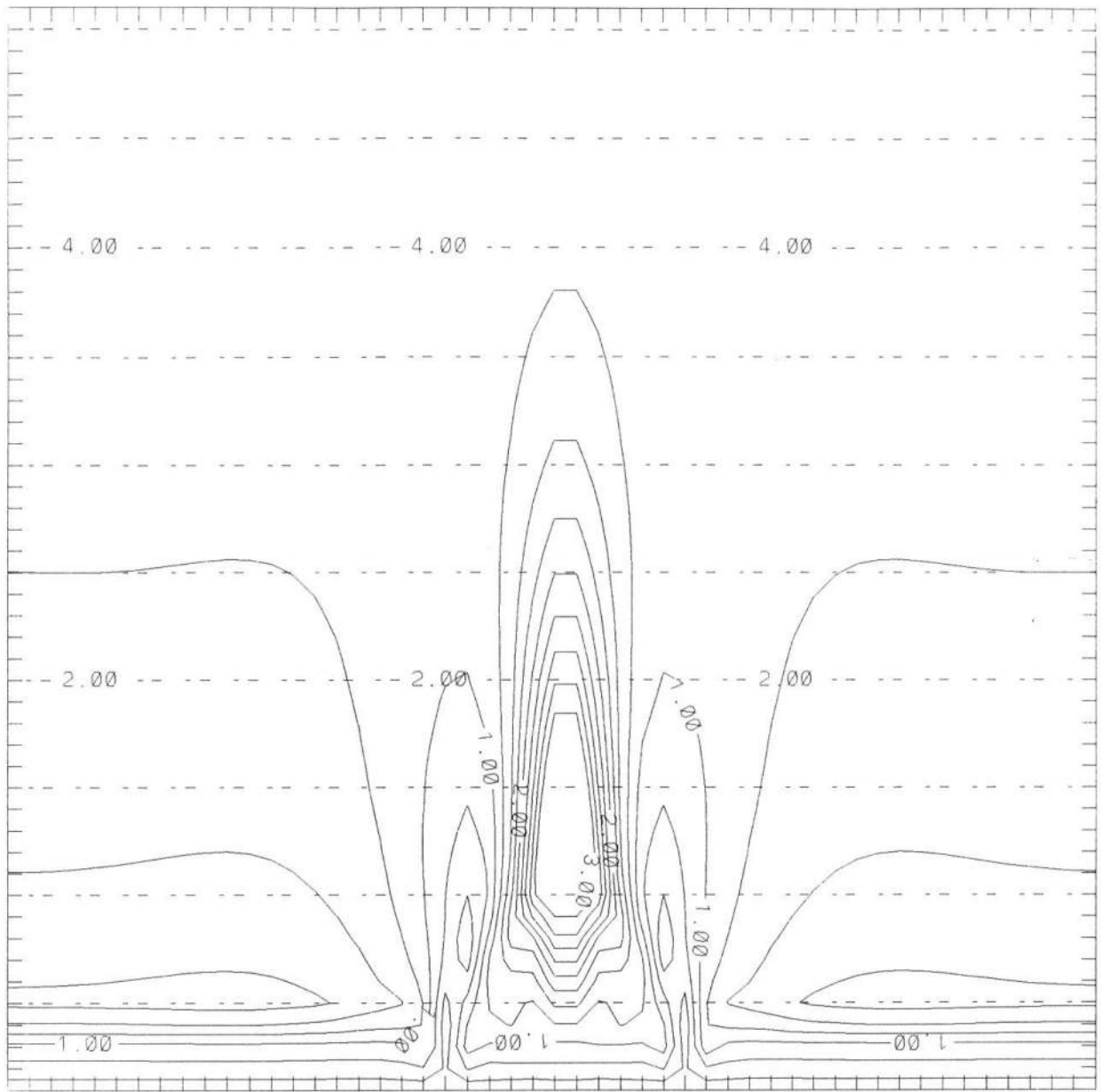
Figure 21: Waves interacting with a rip current. Shoreline at bottom. Wave amplitude contours

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5 Appendix A: *fweb* Documentation of the REF/DIF 1 Program Listing

The following section contains a document which provides a heavily anotated program listing for **REF/DIF 1**, which has been produced using the *fweb* documentation program (Krommes, 1992). The use of *fweb* here is somewhat experimental. In theory, *fweb* provides a programming environment which allows the programmer to specify the operation of a block of code in full, typeset detail, after which the actual *Fortran* or *C* code is spelled out. This procedure places a high premium on the use of a highly structured and modularized programming technique. In practice, many longer modelling codes are being essentially "retrofitted" for *fweb*, and thus the documentation does not go much beyond upgrading the comment statements appearing in standard *Fortran* style. As **REF/DIF 1** evolves, the *fweb* documentation will become a more interwoven portion of the documentation and user's manual

1.   Refraction-Diffraction Model REF/DIF 1, Version 2.4.

REF/DIF 1 calculates the forward scattered wave field in regions with slowly varying depth and current, including the effects of refraction and diffraction. The program is based on the parabolic equation method. Physical effects included in the present version include:

1. Parabolic approximation:

    (a) Minimax approximation given by Kirby (1986b).

2. Wave nonlinearity: choice of

    (a) Linear.

    (b) Composite nonlinear: approximate model of Kirby and Dalrymple (1986b).

    (c) Stokes nonlinear: model of Kirby and Dalrymple (1983a).

3. Wave breaking:

    (a) Model of Dally, Dean and Dalrymple(1985).

4. Absorbing structures and shorelines:

    (a) Thin film model surrounded by a natural surfzone (Kirby and Dalrymple, 1986a).

5. Energy dissipation: any of

    (a) Turbulent bottom friction damping.

    (b) Porous bottom damping.

    (c) Laminar boundary layer damping.

6. Lateral boundary conditions: either of

    (a) Reflective condition.

    (b) Open boundary condition (Kirby, 1986c).

7. Input wave field: either of

    (a) Model specification of monochromatic or directional wave field.

    (b) Input of initial row of data from disk file.

8. Output wave field:

    (a) Standard output.

    (b) Optional storage of last full calculated row of complex amplitudes.

The documentation of present program is contained in:

Combined Refraction/Diffraction Model REF/DIF 1, Version 2.4, Documentation and User's Manual, James T. Kirby and Robert A. Dalrymple, CACR Report No. 92‑04 , Center for Applied Coastal Research, Department of Civil Engineering, University of Delaware, June 1993.

Center for Applied Coastal Research
Department of Civil Engineering
University of Delaware
Newark, DE 19716

Program developed by James T. Kirby and Robert A. Dalrymple.

Version 2.4, July 1992.

program *refdif1*

    parameter($ixr = 120, iyr = 100, ix = 31, iy = 2500, ncomp = 50$)

    ⟨ common statements 16 ⟩

      // Constants which provide for conversion between MKS and english units on input and output.

    $dconv(1) = 1.$
    $dconv(2) = 0.30488$

      // Specify unit number for line printer output. Standard choice would be *; some systems may vary. this line must be set during installation. Line deleted in present version.

      // print headers on output

    write($*, 100$)

      // read control parameters and reference grid data

    call *inref*

      // read control parameters and initializing wave data

    call *inwave*

      // Pass program control to subroutine *model*.

      // For each frequency component specified in *inwave*, *model* executes the model throughout the entire grid and then reinitializes the model for the next frequency.

    call *model*

      // All done. Close output data files if **open** and **close** statements are being used.

    close($iun(1)$)
    close($iun(3)$)

    if($ioutput \equiv 2$) **close**(33)
    if($isp \equiv 1$) **close**($iun(2)$)

    stop

100: **format**($//////20x$, 'Refraction-Diffraction␣Model␣for'/$20x$, 'Weakly␣Nonlinear␣Surface␣\
    Water␣Waves'///$20x$, 'REF/DIF␣1,␣␣Version␣2.4'///$20x$,

```
'Center␣for␣Applied␣Coastal␣Research'/20x,
'Department␣of␣Civil␣Engineering'/20x, 'University␣of␣Delaware'/20x, 'Newark,\
␣Delaware␣19716'///10x, 'James␣T.␣Kirby␣and␣Robert␣A.␣Dalrymple,␣July␣1992')
;
```

end

2. INREF.

Subroutine reads in and checks dimensions and values for large scale reference grid. Wave parameters for the particular run are read in later by subroutine *inwave*.

The following unit (device) numbers are assumed:

1. *iun*(1): input reference grid values of *d*, *u*, and *v*

2. *iun*(2): input user specified subgrid divisions

3. *iun*(3): output results at reference grid locations to disk file

4. *: assumed unit for line printer output. This output used to be directed to a file *rundat.dat* in old versions. The output could be redirected during a model run if this feature were still desired.

5. *iun*(5): dimension information and run instructions (must be specified in program, machine specific).

Variable definitions.

mr, nr	reference grid dimensions (max ixr, iyr)
dxr, dyr	grid spacing for reference grid
iu	physical unit descriptor (1=mks, 2=english)
	default value is 1, mks units
dt	depth tolerence value (to check for anomalous depth values)
$ispace$	switch to control grid subdivision.
	=0, program attempts its own subdivisions
	=1, user specifies subdivisions
nd	y direction subdivision ($ispace = 0$ or 1)
	(must be $.lt.iy/nr - 1$)
$md(mr-1)$	x direction subdivisions (if $ispace = 1$)
	(must be $.le.ix - 1$)
$ntype$	nonlinearity control parameter
	=0, linear model
	=1, Stokes matched to Hedges in shallow water
	=2, Stokes throughout
$icur$	switch to tell program if current data is to be used and read on input
	=0, no input current data
	=1, input current data to be read
	program defaults to $icur = 0$
ibc	boundary condition switch
	=0, use closed lateral boundaries
	=1, use open lateral conditions
	program defaults to $ibc = 0$
dr	depths at reference grid points
	> 0, submerged areas
	< 0, elevation above surface datum
ur	x velocities at reference grid points
	(only entered if $icur = 1$)
vr	y velocities at reference grid points
	(only entered if $icur = 1$)

Subroutine is called from *refdif1* and returns control to calling location.

Center for Applied Coastal Research
Department of Civil Engineering
University of Delaware
Newark, DE 19716

Coded by James T. Kirby, October 1984.

subroutine *inref*

parameter $(ixr = 120, iyr = 100, ix = 31, iy = 2500, ncomp = 50)$

⟨ common statements 16 ⟩

// Constants.

$g = 9.80621$

// Define device number $iun(5)$ for reference grid input. This line must be set during program installation, prior to compilation. Also construct an **open** statement if needed.

$iun(5) = 5$
open $(unit = iun(5), file = \text{'indat.dat'}, status = \text{'old'})$

// Remaining unit numbers in $i4$ format.

read $(iun(5), *) (iun(i), i = 1, 3)$
open $(unit = iun(1), file = \text{'refdat.dat'}, status = \text{'old'})$
open $(unit = iun(3), file = \text{'outdat.dat'})$

// Headers on output file *.

write $(*, 106)$

// Read control data from unit $iun(5)$.

read $(iun(5), *)$ mr, nr
read $(iun(5), *)$ $iu, ntype, icur, ibc$
read $(iun(5), *)$ dxr, dyr, dt
read $(iun(5), *)$ $ispace, nd$

if $(ispace \equiv 1)$ **then**
    read $(iun(5), *) (md(i), i = 1, mr - 1)$
endif

write $(*, 107)$ mr, nr, dxr, dyr

if $(iu \equiv 1)$ **write** $(*, 114)$ iu
if $(iu \equiv 2)$ **write** $(*, 115)$ iu
if $(icur \equiv 0)$ **write** $(*, 200)$
if $(icur \equiv 1)$ **write** $(*, 201)$
if $(ibc \equiv 0)$ **write** $(*, 202)$
if $(ibc \equiv 1)$ **write** $(*, 203)$
if $(ispace \equiv 0)$ **write** $(*, 108)$
if $(ispace \equiv 1)$ **write** $(*, 109)$
write $(*, 119)$ nd
if $(ntype \equiv 0)$ **write** $(*, 110)$
if $(ntype \equiv 1)$ **write** $(*, 111)$
if $(ntype \equiv 2)$ **write** $(*, 112)$

// Check input from unit $iun(5)$.

```
      if ((mr > ixr) | (nr > iyr)) then
        stop 'dimensions␣for␣reference␣grid␣too␣large,␣stopping'
      end if

      if ((iu ≠ 1) ∧ (iu ≠ 2))  iu = 1

      dt = dt*dconv(iu)
      dxr = dxr*dconv(iu)
      dyr = dyr*dconv(iu)

      if (dt ≡ 0.)  dt = 2.

      if (nd > ifix(float(iy − 1)/float(nr − 1))) then
        write(∗, 102)
        stop
      endif

      if (ispace ≡ 1) then
        test = 0.
        do 1 i = 1, mr − 1
          if (md(i) > (ix − 1)) then
            write(∗, 103) i
            test = 1.
          endif
1:      continue
        if (test ≡ 1.) stop
      end if

      // read depth grid and velocities from unit iun(1)

      do 2 i = 1, mr
        read(iun(1), ∗) (dr(i, j), j = 1, nr)
2:    continue

      if (icur ≡ 1) then
        do 3 i = 1, mr
          read(iun(1), 101) (ur(i, j), j = 1, nr)
3:      continue
        do 4 i = 1, mr
          read(iun(1), 101) (vr(i, j), j = 1, nr)
4:      continue
      endif

      // convert depth and currents

      do 5 i = 1, mr
        do 5 j = 1, nr
          dr(i, j) = dr(i, j)*dconv(iu)
5:    continue

      if (icur ≡ 1) then
        do 55 i = 1, mr
          do 55 j = 1, nr
            ur(i, j) = ur(i, j)*dconv(iu)
            vr(i, j) = vr(i, j)*dconv(iu)
55:     continue
      endif

      // check for large depth changes and large currents in reference grid data
```

```
      do 6 i = 2, mr − 1
        do 6 j = 2, nr − 1
          dcheck = (dr(i + 1, j) + dr(i − 1, j) + dr(i, j − 1) + dr(i, j + 1))/4.
          if (abs(dcheck − dr(i, j)) > dt) write(∗, 104) dr(i,j), i, j, dt
6: continue

      if (icur ≡ 1) then
        do 7 i = 1, mr
          do 7 j = 1, nr
            if (dr(i, j) ≤ 0.0) go to 7
            fr = (ur(i, j)∗ur(i, j) + vr(i, j)∗vr(i, j))/(g∗dr(i, j))
            if (fr > 1.) write(∗, 105) i, j, fr
7: continue
      endif
```

// establish coordinates for reference grid

```
      do 8 ir = 1, mr
        xr(ir) = float(ir − 1)∗dxr
8: continue
      do 9 jr = 1, nr
        yr(jr) = float(jr − 1)∗dyr
9: continue
```

// establish y coordinates for interpolated grid

```
      n = nd∗(nr − 1) + 1
      dy = dyr/float(nd)
      do 10 j = 1, n
        y(j) = float(j − 1)∗dy
10: continue
```

// write grid information on output unit iun(3)

```
      write(iun(3), ∗) nr, mr
      write(iun(3), ∗) (yr(jr)/dconv(iu), jr = 1, nr)
```

// Read in changes for friction values.

// iff(1) = 1, turbulent boundary layer damping everywhere

// iff(2) = 1, porous bottom damping everywhere

// iff(3) = 1, laminar boundary layer damping everywhere

```
      read(iun(5), ∗) (iff(i), i = 1, 3)
      do 11 i = 1, 3
        if ((iff(i) ≠ 0) ∧ (iff(i) ≠ 1)) iff(i) = 0
11: continue
      write(∗, 116) (iff(i), i = 1, 3)
```

// Specify whether or not user specified subgrids are to be read in during model operation.

// isp = 0, no subgrids specified

// isp = 1, subgrids to be read in later from unit iun(2)

```
      read(iun(5), ∗) isp

      if (isp ≡ 0) write(∗, 117)

      if (isp ≡ 1) then
```

```
            write(*, 118)
            open(unit = iun(2), file = 'subdat.dat', status = 'old')
          endif

          if((isp ≡ 1) ∧ (ispace ≡ 0))  write(*, 113)

          if(isp ≡ 0) then
            do 14 ir = 1, mr
              do 14 jr = 1, nr
                isd(ir, jr) = 0
      14:  continue
          else
            do 15 ir = 1, mr − 1
              read(iun(2), 100) (isd(ir, jr), jr = 1, nr − 1)
      15:  continue
          end if
```

          // Input done, return to main program.

          return

```
100:  format(15i4) ;
101:  format(16f8.4) ;
102:  format('⎵y-direction⎵subdivision⎵too⎵fine.'/'⎵maximum⎵number⎵of⎵y⎵grid\
          ⎵points⎵will⎵be⎵exceeded.'/'⎵execution⎵terminating.') ;
103:  format('⎵x-direction⎵subdivision⎵too⎵fine⎵on⎵grid⎵block', 2x,
          i3/'⎵execution⎵terminating') ;
104:  format('⎵depth',
          2x, f7.2, '(m)⎵at⎵reference⎵grid⎵location', 2(2x, i3)/'⎵differs⎵from⎵the⎵average\
          ⎵of⎵its⎵neighbors⎵by', '⎵more⎵than', 2x, f7.2, '(m).'/'⎵execution⎵continuing')
          ;
105:  format('⎵ambient⎵current⎵at⎵reference⎵grid⎵location', 2(2x, i3),
          '⎵is⎵supercritical⎵with⎵froude⎵number⎵=', f7.4/'⎵execution⎵continuing') ;
106:  format('0'///20x, 'input⎵section,⎵reference⎵grid⎵values'///) ;
107:  format('⎵reference⎵grid⎵dimensions⎵⎵mr=', i3/'⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵nr=',
          i3///'⎵reference⎵grid⎵spacings⎵⎵⎵dxr=', f8.4/'⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵⎵dyr=', f8.4)
          ;
108:  format('⎵'/'⎵ispace⎵=0⎵chosen,⎵program⎵will⎵attempt⎵its⎵own⎵',
          'reference⎵grid⎵subdivisions') ;
109:  format('⎵'/'⎵ispace⎵=1⎵chosen,⎵subdivision⎵spacings⎵will⎵be', '⎵input⎵as⎵data') ;
110:  format('⎵'/'⎵ntype⎵=⎵0,⎵linear⎵model') ;
111:  format('⎵'/'⎵ntype⎵=⎵1,⎵stokes⎵model⎵matched⎵to⎵hedges⎵model') ;
112:  format('⎵'/'⎵ntype⎵=⎵2,⎵stokes⎵model') ;
113:  format('⎵warning:⎵input⎵specifies⎵that⎵user⎵will⎵be⎵supplying',
          '⎵specified⎵subgrids⎵(isp=1),'/'⎵while⎵program⎵has⎵been⎵t\
          old⎵to⎵generate⎵its⎵own⎵subgrid',
          '⎵spacings⎵(ispace=0).'/'⎵possible⎵incompatibility⎵in⎵any⎵or⎵all⎵subgrid⎵blocks')
          ;
114:  format('⎵'/'⎵physical⎵unit⎵switch⎵iu=', i1, ',⎵⎵input⎵in⎵mks⎵units') ;
115:  format('⎵'/'⎵physical⎵unit⎵switch⎵iu=', i1, ',⎵⎵input⎵in⎵english⎵units') ;
116:  format('⎵'///'⎵⎵⎵⎵switches⎵for⎵dissipation⎵terms'//'⎵', i1, '⎵⎵⎵turbulent⎵boundary⎵lay\
          er'/'⎵', i1, '⎵⎵⎵porous⎵bottom'/'⎵', i1, '⎵⎵⎵laminar⎵boundary⎵layer')
          ;
117:  format('⎵'/'⎵isp=0,⎵no⎵user⎵defined⎵subgrids') ;
118:  format('⎵'/'⎵isp=1,⎵user⎵defined⎵subgrids⎵to⎵be⎵read') ;
```

```
119: format('␣'/'␣y-direction␣subdivision␣according␣to␣nd=', i3) ;
200: format('␣'/'␣icur=0,␣no␣current␣values␣read␣from␣input␣files') ;
201: format('␣'/'␣icur=1,␣current␣values␣read␣from␣data␣files') ;
202: format('␣'/'␣ibc=0,␣closed␣(reflective)␣lateral␣boundaries') ;
203: format('␣'/'␣ibc=1,␣open␣lateral␣boundaries') ;

     end
```

3. INWAVE.

Read in wave parameters.

Variable definitions:

iinput	determine method of specifying the first row of computational values
	=1, input values from indat.dat according to value of *iwave*
	=2, input complex a values from file *wave.dat*
ioutput	determine whether last row of complex amplitudes are stored in separate file *owave.dat*
	=1, extra data not stored
	=2, extra data stored in file *owave.dat*

if *iinput* = 1:

iwave	input wave type
	=1, input discrete wave amplitudes and directions
	=2, read in dominant direction, total average energy density, and spreading factor
nfreqs	number of frequency components to be used (separate model run for each component)
freqs	wave frequency for each of *nfreqs* runs
tide	tidal offset for each of *nfreqs* runs

if *iwave* = 1

nwavs	number of discrete wave components at each of *nfreqs* runs
amp	initial amplitude of each component
dir	direction of each discrete component in + or - degrees from the x-direction

if *iwave* = 2

edens	total amplitude variance density over all directions at each frequency
nsp = *n*	spreading factor in $\cos^{2n}(\theta)$ directional distribution (stored in *nwavs*)
nseed	Seed value for random number generator. Integer value between 0 and 9999.

if *iinput* = 2:

freqs	Wave frequency for one run.
tide	Tidal offset for one run.

Center for Applied Coastal Research
Department of Civil Engineering
University of Delaware
Newark, DE 19716

Coded by James T. Kirby, Oct 1984, Sept 1989, Jan 1991

subroutine *inwave*

    parameter ($ixr = 120, iyr = 100, ix = 31, iy = 2500, ncomp = 50$)

    ⟨ common statements 16 ⟩

    $pi = 3.1415927$

      // Read value of *iinput*, *ioutput*.

    read ($iun(5), *$) *iinput*, *ioutput*

    if (($iinput \neq 1$) $\wedge$ ($iinput \neq 2$)) **then**

```
      write (*, *) '␣invalid␣value␣chosen␣for␣iinput,␣check␣indat.dat'
      stop
   endif

if ((ioutput ≠ 1) ∧ (ioutput ≠ 2)) then
      write (*, *) '␣invalid␣value␣chosen␣for␣ioutput,␣check␣indat.dat'
      stop
   endif

if (ioutput ≡ 2) then
   open (33, file = 'owave.dat')
   endif

if (iinput ≡ 1) then
      write (*, *) 'iinput␣=␣1,␣program␣specifies␣initial␣row␣of␣a'

      // Read iwave, nfreqs.

   write (*, 102)
   read (iun(5), *) iwave, nfreqs
   if (iwave ≡ 1) write (*, 103)
   if (iwave ≡ 2) write (*, 104)
   write (*, 105) nfreqs

   if (iwave ≡ 2) then
      read (iun(5), *) thet0
      thet0 = thet0 * pi / 180.
   endif

      // For each frequency, enter the wave period and tidal offset.

   do 3 ifreq = 1, nfreqs
      read (iun(5), *) freqs(ifreq), tide(ifreq)
      write (*, 107) ifreq, freqs(ifreq), tide(ifreq)
      freqs(ifreq) = 2. * pi / freqs(ifreq)
      tide(ifreq) = tide(ifreq) * dconv(iu)

        // If iwave = 1, read the number of discrete components.

      if (iwave ≡ 1) then
         read (iun(5), *) nwavs(ifreq)
         do 1 iwavs = 1, nwavs(ifreq)
            read (iun(5), *) amp(ifreq,iwavs), dir(ifreq,iwavs)
            write (*, 106) iwavs, amp(ifreq,iwavs), dir(ifreq,iwavs)
            dir(ifreq, iwavs) = dir(ifreq, iwavs) * pi / 180.
            amp(ifreq, iwavs) = amp(ifreq, iwavs) * dconv(iu)
      1: continue
      endif

        // If iwave = 2, read the parameters for each frequency.

      if (iwave ≡ 2) then
         read (iun(5), 101) edens(ifreq)
         read (iun(5), *) nwavs(ifreq), nseed
         seed = float(nseed) / 9999.
         write (*, 108) edens(ifreq), nwavs(ifreq), nseed
         dir(ifreq, 1) = thet0
         edens(ifreq) = edens(ifreq) * (dconv(iu)²·)
      endif
```

3: **continue**

endif

// If *iinput* = 2, read in wave period and tidal offset.

if (*iinput* ≡ 2) **then**
    write (∗, ∗) '␣iinput␣=␣2,␣user␣specifies␣a␣in␣wave.dat'
    nfreqs = 1
    read (*iun*(5), ∗) *freqs*(1), *tide*(1)
    write (∗, 102)
    write (∗, ∗) '␣wave␣period␣=', *freqs*(1), '␣sec.'
    write (∗, ∗) '␣tidal␣offset=', *tide*(1)
    freqs(1) = 2.∗*pi*/*freqs*(1)
    tide(1) = *tide*(1)∗*dconv*(*iu*)
endif

return

100: **format** (15*i4*) ;
101: **format** (16*f8.4*) ;
102: **format** ('1'///20*x*, '␣input␣section,␣wave␣data␣values'///) ;
103: **format** ('␣'///'␣iwave=1,␣discrete␣wave␣amps␣and␣directions') ;
104: **format** ('␣'///'␣iwave=2,␣directional␣spreading␣model␣chosen') ;
105: **format** ('␣'///'␣the␣model␣is␣to␣be␣run␣for', *i3*, '␣separate', '␣frequency␣components') ;
106: **format** ('␣'/'␣wave␣component␣', *i2*, ',␣amplitude␣=', *f8.4*, ',␣direction=', *f8.4*) ;
107: **format** ('␣'//'␣frequency␣component␣', *i2*//'␣wave␣period=', *f8.4*, 'sec.,␣tidal␣offset=',
    f8.4) ;
108: **format** ('␣'/'␣total␣variance␣density␣=', *f8.4*, ',␣spreading␣factor␣␣␣␣␣␣␣␣␣␣n=', *i2*,
    '␣seed␣number␣=', *i5*) ;

end

4. MODEL.

This subroutine is the control level for the actual wave model. Data read in during *inref* and *inwave* is conditioned and passed to the wave model. This routine is executed once for each frequency component specified in *inwave*.

The wave model is split in three parts which are run sequentially for each reference grid row.

grid	subroutine performs the interpolation of depth and current values.
con	calculate the constants needed by the finite difference scheme.
fdcalc	perform the finite difference calculations.

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Center for Applied Coastal Research
Department of Civil Engineering
University of Delaware
Newark, DE 19716

</div>

Coded by James T. Kirby, November 1984. Corrections made to specification of average k value on first row, April 5, 1993.

subroutine *model*

parameter $(ixr = 120, iyr = 100, ix = 31, iy = 2500, ncomp = 50)$

⟨ common statements 16 ⟩

dimension *dthi*(31), *thi*(31)

// Constants.

$g = 9.80621$
$rho = 1000.$
$pi = 3.1415927$
$eps = 1.0 \cdot 10^{-05}$

// Execute model once for each frequency.

// *ifreq* is the controlling index value.

do 200 *ifreq* $= 1, nfreqs$
  $psibar = 0.$
  write $(*, 203)$ *ifreq*

  // Specify initial nonlinear parameters for each run.

  if $(ntype \equiv 0)$ $an = 0.$
  if $(ntype \neq 0)$ $an = 1.$
  if $(ntype \neq 2)$ $anl = 0.$
  if $(ntype \equiv 2)$ $anl = 1.$

  // Calculate the mean *kb* on the first row, for use in specifying initial conditions.

  $npts = 0$
  $sumk = 0.$

  do 10 *jr* $= 1, nr$
    $d(1, jr) = dr(1, jr) + tide(ifreq)$

$\mathbf{call}\ wvnum(d(1,jr), ur(1,jr), freqs(ifreq), k(1,jr), eps, icdw, 1, 1)$

$\mathbf{if}\ (d(1,jr) > 0.05)\ \mathbf{then}$
$\qquad sumk = sumk + k(1,jr)$
$\qquad npts = npts + 1$
$\mathbf{endif}$

10: $\mathbf{continue}$

$kb(1) = sumk/\mathbf{float}(npts)$

// Establish initial wave conditions for the *ifreq* frequency

$\mathbf{if}\ (iinput \equiv 1)\ \mathbf{then}$

$\qquad$ // Compute wave from data given in *indat.dat*.

$\qquad \mathbf{if}\ (iwave \equiv 1)\ \mathbf{then}$

$\qquad\qquad$ // *iwave* $\equiv 1$, discrete components specified.

$\qquad \mathbf{do}\ 3\ j = 1, n$
$\qquad\qquad a(1,j) = \mathbf{cmplx}(0., 0.)$
$\qquad\qquad \mathbf{do}\ 2\ iwavs = 1, nwavs(ifreq)$
$\qquad\qquad\qquad a(1,j) = a(1,j) + amp(ifreq, iwavs) * \mathbf{cexp}(\mathbf{cmplx}(0., kb(1) * sin(dir(ifreq, iwavs)) * y(j)))$
$\qquad$ 2: $\mathbf{continue}$
$\qquad$ 3: $\mathbf{continue}$
$\qquad \mathbf{else}$

$\qquad\qquad$ // *iwave* $\equiv 2$, directional spreading model.

$\qquad sp = \mathbf{float}(nwavs(ifreq))$
$\qquad nsp = nwavs(ifreq)$
$\qquad thmax = pi/4.$
$\qquad \mathbf{call}\ acalc(thmax, nsp, a1)$
$\qquad edens(ifreq) = \mathbf{sqrt}(edens(ifreq)/a1)$
$\qquad nn = 31$
$\qquad ii = (nn - 1)/2 + 1$
$\qquad seed = rand(seed)$

$\qquad$ // Compute randomly distributed $\Delta\theta$'s.

$\qquad sum0 = 0.$
$\qquad \mathbf{do}\ 12\ i = 1, nn$
$\qquad\qquad seed = rand(seed)$
$\qquad\qquad dthi(i) = seed$
$\qquad\qquad sum0 = sum0 + seed$
$\qquad$ 12: $\mathbf{continue}$
$\qquad xnorm = 2.*thmax/sum0$
$\qquad \mathbf{do}\ 101\ i = 1, nn$
$\qquad\qquad dthi(i) = dthi(i)*xnorm$
$\qquad$ 101: $\mathbf{continue}$
$\qquad thi0 = -thmax$
$\qquad \mathbf{do}\ 4\ i = 1, nn$
$\qquad\qquad thi0 = thi0 + dthi(i)$
$\qquad\qquad thi(i) = thi0 - dthi(i)/2.$
$\qquad\qquad dth = dthi(i)$
$\qquad\qquad amp(ifreq,$
$\qquad\qquad\qquad i) = edens(ifreq)*\mathbf{sqrt}(dth)*\mathbf{sqrt}(\mathbf{cos}(thi(i) + dth/2.)^{2*nsp} + \mathbf{cos}(thi(i) - dth/2.)^{2*nsp})$

```
   4: continue
      do 5 i = 1, nn
         ip1 = i + 1
         seed = rand(seed)
         dir(ifreq, ip1) = 2.*pi*seed/100.
   5: continue
      do 7 j = 1, n
         a(1, j) = cmplx(0., 0.)
         do 6 i = 1, nn
            a(1, j) = a(1, j) + amp(ifreq, i)*cexp(cmplx(0., kb(1)*sin(thi(i) − thet0)*y(j) + dir(ifreq,
               i + 1)))*2.
   6: continue
   7: continue
      endif
   endif
```

   // if *iinput* = 2, read a from data file *wave.dat*.

```
if (iinput ≡ 2) then
   open(11, file = 'wave.dat')
   read(11, *) (a(1, j), j = 1, n)
   close(11)
endif
```

   // now execute model for the *ifreq* frequency over each of *mr* grid blocks. *ir* is the controlling
   index value.

```
do 100 ir = 1, (mr − 1)
```

   // Establish interpolated grid block for segment *ir*.

```
   call grid(ifreq, ir)
```

   // If *ir* = 1 write initial values on *iun*(3).

```
   if (ir ≡ 1) then
      write(*, 201) x(1)/dconv(iu), psibar
      write(*, 202) (cabs(a(1, j))/dconv(iu), j = 1, n, nd)
      write(iun(3), *) x(1)/dconv(iu), psibar
      write(iun(3), *) (d(1, j)/dconv(iu), j = 1, n, nd)
      write(iun(3), *) (a(1, j)/dconv(iu), j = 1, n, nd)
   endif
```

   // calculate constants for each grid block.

```
   call con(ifreq, ir)
```

   // perform finite difference calculations.

```
   call fdcalc(ifreq, ir)
```

   // grid block *ir* done, print output and go to next grid.

```
100: continue

   if (ioutput ≡ 2) then
      write(33, *) (a(m, j), j = 1, n)
   endif
```

   // Model complete for the *ifreq* frequency component, go to the next frequency component.

200: **continue**

  // Runs completed for all frequencies. Return to end of main program.

  return

201: **format** $(\,'_{\sqcup}\texttt{x=}', f10.2, '_{\sqcup\sqcup}\texttt{psibar=}', f20.4\,)$;
202: **format** $(\,'_{\sqcup}', 12(f10.4\,))$;
203: **format** $(\,'1', 20x, '\texttt{model}_{\sqcup}\texttt{execution},_{\sqcup}\texttt{frequency}', '_{\sqcup}\texttt{component}', i4//)$;

  end

5. GRID.

Interpolate the depth and current grids for reference grid block ir.

Center for Applied Coastal Research
Department of Civil Engineering
University of Delaware
Newark, DE 19716

Coded by James T. Kirby, October 1984.

```
subroutine grid(ifreq,ir)
   parameter(ixr = 120, iyr = 100, ix = 31, iy = 2500, ncomp = 50)
   ⟨ common statements 16 ⟩
      // Constants.
   pi = 3.1415927
   eps = 1.0 · 10⁻⁰⁵
      // Perform y-interpolation on reference grid.
      // Interpolate first row.
   do 10 j = 1, n, nd
      d(1, j) = dr(ir, ((j − 1)/nd + 1))
      u(1, j) = ur(ir, ((j − 1)/nd + 1))
      v(1, j) = vr(ir, ((j − 1)/nd + 1))
10: continue
   do 12 jj = 2, nr
      do 11 j = 1, (nd − 1)
         jjj = nd*(jj − 2) + (j + 1)
         d(1, jjj) = (dr(ir, jj) − dr(ir, jj − 1))*y(jjj)/dyr + (yr(jj)*dr(ir, jj − 1) − yr(jj − 1)*dr(ir, jj))/dyr
         u(1, jjj) = (ur(ir, jj) − ur(ir, jj − 1))*y(jjj)/dyr + (yr(jj)*ur(ir, jj − 1) − yr(jj − 1)*ur(ir, jj))/dyr
         v(1, jjj) = (vr(ir, jj) − vr(ir, jj − 1))*y(jjj)/dyr + (yr(jj)*vr(ir, jj − 1) − yr(jj − 1)*vr(ir, jj))/dyr
11: continue
12: continue
      // Set number of x points and define x values.
   if (ispace ≡ 0) then
      // ispace = 0, program sets subdivisions.
      do 13 j = 1, n
         dref = d(1, j) + tide(ifreq)
         if (dref < 0.001)  dref = 0.001
         call wvnum(dref, u(1, j), freqs(ifreq), k(1, j), eps, icdw, 1, j)
13: continue
      npts = 0
      sumk = 0.
      do 14 j = 1, n
```

```
          if (d(1, j) > 0.05) then
              sumk = sumk + k(1, j)
              npts = npts + 1
          endif
14:   continue

      kb(1) = sumk/float(npts)
      alw = 2.*pi/kb(1)
      anw = dxr/alw
      np = ifix(5.*anw)
      if (np < 1) np = 1
      md(ir) = min((ix − 1), np)
      if (np > (ix − 1)) write(∗, 100) ir

  endif

      // ispace = 1, user specified subdivision.

  m = md(ir) + 1
  dx = dxr/float(md(ir))
  do 15 i = 1, m
      x(i) = xr(ir) + float(i − 1)∗dx
15:   continue

      // interpolate values on m row.

  do 16 j = 1, n, nd
      d(m, j) = dr(ir + 1, ((j − 1)/nd + 1))
      u(m, j) = ur(ir + 1, ((j − 1)/nd + 1))
      v(m, j) = vr(ir + 1, ((j − 1)/nd + 1))
16:   continue

  do 18 jj = 2, nr
      do 17 j = 1, (nd − 1)
          jjj = nd∗(jj − 2) + (j + 1)
          d(m, jjj) = (dr(ir + 1, jj) − dr(ir + 1, jj − 1))∗y(jjj)/dyr + (yr(jj)∗dr(ir + 1,
              jj − 1) − yr(jj − 1)∗dr(ir + 1, jj))/dyr
          u(m, jjj) = (ur(ir + 1, jj) − ur(ir + 1, jj − 1))∗y(jjj)/dyr + (yr(jj)∗ur(ir + 1,
              jj − 1) − yr(jj − 1)∗ur(ir + 1, jj))/dyr
          v(m, jjj) = (vr(ir + 1, jj) − vr(ir + 1, jj − 1))∗y(jjj)/dyr + (yr(jj)∗vr(ir + 1,
              jj − 1) − yr(jj − 1)∗vr(ir + 1, jj))/dyr
17:   continue
18:   continue

      // interpolate values in x-direction

  do 19 i = 2, m − 1
      do 19 j = 1, n
          d(i, j) = (d(m, j) − d(1, j))∗x(i)/dxr + (x(m)∗d(1, j) − x(1)∗d(m, j))/dxr
          u(i, j) = (u(m, j) − u(1, j))∗x(i)/dxr + (x(m)∗u(1, j) − x(1)∗u(m, j))/dxr
          v(i, j) = (v(m, j) − v(1, j))∗x(i)/dxr + (x(m)∗v(1, j) − x(1)∗v(m, j))/dxr
19:   continue

      // Add in user specified grid subdivisions (read from unit iun(2)).

  do 30 jr = 1, nr − 1
      if (isd(ir, jr) ≡ 1) then
          js = nd∗jr + (1 − nd)
```

$jf = js + nd$

read $(iun(2), 101)$ $((d(i, j), j = js, jf), i = 1, m)$

if $(icur \equiv 1)$ **then**
    read $(iun(2), 101)$ $((u(i, j), j = js, jf), i = 1, m)$
    read $(iun(2), 101)$ $((v(i, j), j = js, jf), i = 1, m)$
endif

do 31 $i = 1, m$
    do 31 $j = js, jf$
        $d(i, j) = d(i, j) * dconv(iu)$
        $u(i, j) = u(i, j) * dconv(iu)$
        $v(i, j) = v(i, j) * dconv(iu)$
31: **continue**

    end if

30: **continue**

    // Add tidal offset to all rows and establish thin film.

do 20 $i = 1, m$
    do 20 $j = 1, n$
        $d(i, j) = d(i, j) + tide(ifreq)$
        if $(d(i, j) < 0.001)$ $d(i, j) = 0.001$
20: **continue**

    // Interpolation complete, return to *model*.

    return

100: **format** $('_\sqcup model_\sqcup tried_\sqcup to_\sqcup put_\sqcup more_\sqcup spaces_\sqcup than_\sqcup allowed_\sqcup in', '_\sqcup grid_\sqcup block_\sqcup', i3)$;
101: **format** $(16f8.4)$;

    end

6.   CON.

Subroutine calculates constants for reference grid block ir.

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Department of Civil Engineering
University of Delaware
Newark, DE 19716

</div>

Coded by James T. Kirby, October 1984.

```
subroutine con(ifreq,ir)

    parameter(ix = 31, iy = 2500, ixr = 120, iyr = 100, ncomp = 50)

    ⟨ common statements 16 ⟩

        // Constants.

    eps = 1.0 · 10⁻⁰⁵

        // Calculate constants.

    do 1 i = 1, m
        do 1 j = 1, n
            call wvnum(d(i,j),u(i,j),freqs(ifreq),k(i,j),eps,icdw,i,j)
            sig(i,j) = freqs(ifreq) − k(i,j)*u(i,j)
            akd = k(i,j)*d(i,j)
            q(i,j) = (1. + akd/(sinh(akd)*cosh(akd)))/2.
            p(i,j) = q(i,j)*9.80621*tanh(akd)/k(i,j)
            dd(i,j) = (cosh(4.*akd) + 8. − 2.*(tanh(akd)²))/(8.*(sinh(akd)⁴·))
1: continue

        // Calculate the dissipation term w.

    call diss

        // Calculate the mean kb on each row.

    do 11 i = 1, m
        npts = 0
        sumk = 0.
        do 10 j = 1, n
            if (d(i,j) > 0.05) then
                sumk = sumk + k(i,j)
                npts = npts + 1
            endif
10: continue
        if (npts ≡ 0) then
            kb(i) = k(i, 1)
        else
            kb(i) = sumk/float(npts)
        endif
11: continue

    return

end
```

7. FDCALC.

Perform the Crank-Nicolson finite-difference calculations on grid block *ir*. Method is the implicit-implicit iteration used by Kirby and Dalrymple(1983).

Parameters for use in determining the minimax approximation are defined here.

60 degree minimax coefficients.

$$a0 = 0.998214$$
$$a1 = -0.854229$$
$$b1 = -0.383283$$

70 degree minimax coefficients.

$$a0 = 0.994733$$
$$a1 = -0.890065$$
$$b1 = -0.451641$$

80 degree minimax coefficients.

$$a0 = 0.985273$$
$$a1 = -0.925464$$
$$b1 = -0.550974$$

Padé coefficients (refdif1-v2.3).

$$a0 = 1$$
$$a1 = -0.75$$
$$b1 = -0.25$$

Small angle coefficients (Radder's approximation).

$$a0 = 1.$$
$$a1 = -.5$$
$$b1 = 0.0$$

Coded by James T. Kirby, October 1984, January 1992, July 1992

Note to first users of Version 2.4: there is an unexplained and odd behavior in the minimax model when waves around islands are computed. For this reason, the program distributed here has the coefficients for the Padé model.

subroutine *fdcalc(ifreq,ir)*

parameter $(ixr = 120, iyr = 100, ix = 31, iy = 2500, ncomp = 50)$

⟨ common statements 16 ⟩

real *kap*, *ksth1*, *ksth2*
complex *c1*, *c2*, *c3*, *cp1*, *cp2*, *cp3*, *ci*, *damp*
complex *ac(iy)*, *bc(iy)*, *cc(iy)*, *rhs(iy)*, *sol(iy)*

dimension $thet(iy)$, $urs(iy)$

⟨ fdcalc statement functions 8 ⟩

    // Constants.

    // 70 degree minimax coefficients.

    // a0=0.994733 a1=-0.890065 b1=-0.451641

    // Padé coefficients.

$a0 = 1.0$
$a1 = -0.75$
$b1 = -0.25$

    // Additional constants.

$u2 = 1.0$
$kap = 0.78$
$gam = 0.4$
$omeg = freqs(ifreq)$
$pi = 3.1415927$
$ci = cmplx(0., 1.)$

$cdamp = 0.00025$

$alphn = 0.$
$delta1 = a1 - b1$
$delta2 = 1 + 2.*a1 - 2.*b1$

    // Initialize breaking index if $ir = 1$.

if $(ir \equiv 1)$ **then**
  $ifilt = 0$
  do 100 $j = 1, n$
    $ibr(j) = 0$
    $wb(1, j) = cmplx(0., 0.)$
100: **continue**
endif

    // Solution for m grid blocks in reference block ir.

do 200 $i = 1, (m - 1)$
  $ip1 = i + 1$
  $it = 1$
  $ih = 1$

    // r.h.s. of matrix equation.

$rhs(1) = cmplx(0., 0.)$

  do 1 $j = 2, (n - 1)$
    $rhs(j) = c1(i,j)*a(i,j) + c2(i,j)*a(i,j+1) + c3(i,j)*a(i,j-1) - dx*(w(i,j) + wb(1,j))*a(i,$
      $j)/2. - dx*cmplx(0.,1.)*an*anl*sig(i,j)*k(i,j)*k(i,j)*dd(i,j)*(1.-\textbf{float}(ibr(j)))*(cabs(a(i,$
      $j))^2)*a(i,j)/2. - dx*cmplx(0.,1.)*(1.-\textbf{float}(ibr(j)))*an*(1.-anl)*sig(i,j)*((1.+f1(i,$
      $j)*k(i,j)*k(i,j)*(cabs(a(i,j))^2)*dd(i,j))*\textbf{tanh}(k(i,j)*d(i,j) + f2(i,j)*k(i,j)*cabs(a(i,$
      $j)))/\textbf{tanh}(k(i,j)*d(i,j)) - 1.)*a(i,j)/2.$
1: **continue**

  $rhs(n) = cmplx(0., 0.)$

// Return here for iterations.

2: **if** $(it \equiv 1)$ $ii = i$
  if $(it \equiv 2)$ $ii = ip1$

  // Establish boundary conditions.

if $(ibc \equiv 1)$ **then** $ksth1 = $ **real** $((2.*(a(i,2) - a(i,1))/((a(i,2) + a(i,1))*dy))*$**cmplx** $(0., -1.))$ $ksth2$
      $= $ **real** $((2.*(a(i,n) - a(i, n-1))/((a(i,n) + a(i, n-1))*dy))*$**cmplx** $(0., -1.))$

  $bc(1) = $ **cmplx** $(1., ksth1*dy/2.)$
  $cc(1) = -$**cmplx** $(1., -ksth1*dy/2.)$
  $bc(n) = -$**cmplx** $(1., -ksth2*dy/2.)$
  $ac(n) = $ **cmplx** $(1., ksth2*dy/2.)$
else
  $bc(1) = $ **cmplx** $(1., 0.)$
  $cc(1) = -bc(1)$
  $bc(n) = $ **cmplx** $(1., 0.)$
  $ac(n) = -bc(n)$
endif

  // Calculate dissipation in rows where breaking occurs.

do 3 $j = 1, n$
  if $(ibr(j) \equiv 1)$
  $wb(2,j) = $ **cmplx** $(1., 0.)*0.15*cg(ip1, j)*(1. - (gam*d(ip1, j)/(2.*$**cabs** $(a(ii, j))))^{2\cdot})/d(ip1, j)$
  if $(ibr(j) \equiv 0)$ $wb(2,j) = $ **cmplx** $(0., 0.)$
3: **continue**

  // Coefficients for forward row.

do 4 $j = 2, (n-1)$
  $ac(j) = cp3(i, j)$
  $bc(j) = cp1(i, j) + (dx/2.)*(w(i+1, j) + wb(2, j)) + $ **cmplx** $(0., an*anl)*sig(i+1, j)*k(i+1, j)*k(i+1, j)*dd(i+1, j)*($ **cabs** $(a(ii, j))^{2\cdot})*(dx/2.) + $ **cmplx** $(0., an*(1.-anl))*sig(i+1, j)*(dx/2.)*((1.+f1(i+1, j)*k(i+1, j)*k(i+1, j)*($ **cabs** $(a(ii, j))^{2\cdot})*dd(i+1, j))*$**tanh** $(k(i+1, j)*d(i+1, j)) + f2(i+1, j)*k(i+1, j)*$**cabs** $(a(ii, j)))/$**tanh** $(k(i+1, j)*d(i+1, j)) - 1.)$
  $cc(j) = cp2(i, j)$
4: **continue**

  // Update solution one step.

call $ctrida(1, n, ac, bc, cc, rhs, sol)$
do 5 $j = 1, n$
  $a(ip1, j) = sol(j)$
  $sol(j) = $ **cmplx** $(0., 0.)$
5: **continue**
  if $((it \equiv 2) | (ih \equiv 2))$ **go to** 8

  // Check for start or stop of breaking in each row.

do 6 $j = 1, n$
  $urs(j) = 2.*$**cabs** $(a(ip1, j))$
6: **continue**

  $isave1 = 0$
  $isave2 = 0$
  do 7 $j = 1, n$
    $iset = 0$

$ireset = 0$

if $(((urs(j)/d(ip1,j)) > kap) \wedge (ibr(j) \equiv 0))$ $iset = 1$

if $(iset \equiv 1)$ **then**
   $ibr(j) = 1$
   $isave1 = 1$
end if

if $(((urs(j)/d(ip1,j)) < gam) \wedge (ibr(j) \equiv 1))$ $ireset = 1$

if $(ireset \equiv 1)$ **then**
   $ibr(j) = 0$
   $isave2 = 1$
end if

7: **continue**

  $ih = 2$

  // Redo initial calculation if breaking status changes.

if $((isave1 \equiv 1) | (isave2 \equiv 1))$ **go to** 2
8: **continue**
  if $(it \equiv 2)$ **go to** 9
  $it = 2$
  go to 2

9: **continue**

  // For Stokes model alone ($ntype \equiv 2$), test to see whether Ursell parameter is too large.

if $(ntype \equiv 2)$ **then**
   do 11 $j = 1, n$
     $urs(j) = (\boldsymbol{cabs}(a(ip1,j))/d(ip1,j))/((k(ip1,j)*d(ip1,j))^2)$
     if $(urs(j) > 0.5)$ **write**$(*, 204)$ $urs(j), i, j$
11: **continue**
end if

  // Roll back breaking dissipation coefficient at each row.

  do 12 $j = 1, n$
    $wb(1,j) = wb(2,j)$
12: **continue**

  // Calculate reference phase function for surface plotting.

  $psibar = psibar + (kb(ip1) + kb(i))*dx/2.$

  // Start filter if breaking is occuring.

  do 13 $j = 1, n$
    if $(ibr(j) \equiv 1)$ $ifilt = 1$
13: **continue**

200: **continue**

  // Calculate wave angles at reference grid rows. Note: angles are not well defined in a directional, multicomponent sea, or where waves become short crested. This routine was heavily modified by Raul Medina, University of Cantabria.

  do 15 $j = 1, n$
    if $(a(m,j) \equiv (0., 0.))$ **then**

```
        akx2 = 0.
     else
        akx2 = aimag(clog(a(m, j)))
     endif
     if (a(m − 1, j) ≡ (0., 0.)) then
        akx1 = 0.
     else
        akx1 = aimag(clog(a(m − 1, j)))
     endif
     if (abs(akx2 − akx1) > pi) then
        akx = sign((2.*pi − (abs(akx1) + abs(akx2)))/dx, akx1)
     else
        akx = (akx2 − akx1)/dx
     endif
     if (j ≠ n) then
        if (a(m, j + 1) ≡ (0., 0.)) then
           aky2 = 0.
        else
           aky2 = aimag(clog(a(m, j + 1)))
        endif
        if (a(m, j) ≡ (0., 0.)) then
           aky1 = 0.
        else
           aky1 = aimag(clog(a(m, j)))
        endif
     else
        if (a(m, j) ≡ (0., 0.)) then
           aky2 = 0.
        else
           aky2 = aimag(clog(a(m, j)))
        endif
        if (a(m, j − 1) ≡ (0., 0.)) then
           aky1 = 0.
        else
           aky1 = aimag(clog(a(m, j − 1)))
        endif
     endif
     if (abs(aky2 − aky1) > pi) then
        aky = sign((2.*pi − (abs(aky1) + abs(aky2)))/dy, aky1)
     else
        aky = (aky2 − aky1)/dy
     endif
     thet(j) = atan2(aky, (akx + kb(m)))
     thet(j) = 180.*thet(j)/pi
15: continue

   // Print out abs(a) at grid reference points on unit iun(4).

   mm1 = m − 1
   write(*, 205) (ir + 1), mm1
   write(*, 202) x(m)/dconv(iu), psibar
   write(*, 206)
   write(*, 203) (cabs(a(m, j))/dconv(iu), j = 1, n, nd)
```

    write $(*, 207)$
    write $(*, 203)$ $(thet(j), j = 1, n, nd)$

      // Write out reference grid data on disk file $iun(3)$.

    write $(iun(3), *)$ $x(m)/dconv(iu)$, *psibar*
    write $(iun(3), *)$ $(d(m, j)/dconv(iu), j = 1, n, nd)$
    write $(iun(3), *)$ $(a(m, j)/dconv(iu), j = 1, n, nd)$

      // Roll back solution to first grid level.

    do 201 $j = 1, n$
      $a(1, j) = a(m, j)$
201: **continue**

    return

202: **format** $('{}_\sqcup$x=$'$, *f10.2*, $'{}_{\sqcup\sqcup\sqcup\sqcup}$reference${}_\sqcup$phase${}_\sqcup$psibar=$'$, *f20.4*$)$;
203: **format** $('{}_\sqcup'$, $12(f10.4$)$)$;
204: **format** $('{}_\sqcup'//'{}_\sqcup$warning:${}_\sqcup$Ursell${}_\sqcup$number${}_\sqcup=$$'$, *f10.4*, $'{}_\sqcup$encountered${}_\sqcup$at$'$, $'$grid${}_\sqcup$location$'$, *i6*, $'$,$\backslash$
      $'$, *i6*$/'{}_\sqcup$should${}_\sqcup$be${}_\sqcup$using${}_\sqcup$Stokes$-$Hedges${}_\sqcup$model${}_\sqcup$(ntype=1)${}_\sqcup$due${}_\sqcup$to${}_\sqcup$shallow$'$, $'$water$'$)
      ;
205: **format** $('{}_\sqcup$grid${}_\sqcup$row${}_\sqcup$ir=$'$, *i3*, $'$,${}_{\sqcup\sqcup}'$, *i3*, $'$x-direction${}_\sqcup$subdivisions$'$, $'{}_\sqcup$used$'$) ;
206: **format** $('{}_\sqcup$wave${}_\sqcup$amplitudes$'$) ;
207: **format** $('{}_\sqcup$wave${}_\sqcup$directions$'$) ;

    end

8. FDCALC statement functions.

The following code provides the statement functions used in establishing the tridiagonal matrix structure used in *fdcalc*.

⟨ fdcalc statement functions 8 ⟩ ≡

$cg(i,j) = \textbf{\textit{sqrt}}(p(i,j)*q(i,j))$

$pv(i,j) = p(i,j) - v(i,j)*v(i,j)$

$bet(i,j) = -4.*(k(i+1,j) - k(i,j))/(dx*((k(i+1,j) + k(i,j))^2)) - 4.*(k(i+1,j)*(p(i+1,j) - u(i+1,j)^2) - k(i,j)*(p(i,j) - u(i,j)^2))/(dx*((k(i+1,j)+k(i,j))^2)*(p(i+1,j)+p(i,j) - (u(i+1,j)^2 + u(i,j)^2)))$

$dv(i,j) = (cg(i+1,j) + u(i+1,j))/sig(i+1,j) - (cg(i,j) + u(i,j))/sig(i,j) - delta1*dx*((v(i+1,j+1)/sig(i+1,j+1)) + (v(i,j+1)/sig(i,j+1)) - (v(i+1,j-1)/sig(i+1,j-1)) - (v(i,j-1)/sig(i,j-1)))/(2.*dy)$

$damp(i,j) = 2.*ci*cdamp*((cg(i+1,j) + u(i+1,j)) + (cg(i,j) + u(i,j)))/(dy*dy*(k(i+1,j)^2 + k(i,j)^2))$

$deltap(i,j) = a1 - b1*kb(i)/k(i,j)$

$cp1(i,j) = (cg(i+1,j) + u(i+1,j))*\textbf{\textit{cmplx}}(1., dx*(kb(i+1) - a0*k(i+1,j))) + \textbf{\textit{cmplx}}(1.,0.)*(cg(i,j) + u(i,j) + dv(i,j)*(sig(i+1,j) + sig(i,j))/4.) + 2.*omeg*\textbf{\textit{cmplx}}(0.,1.)*(-b1)*bet(i,j)*(u(i+1,j) + u(i,j))/sig(i+1,j) + 4.*omeg*(-b1)*\textbf{\textit{cmplx}}(0.,1.)*(3.*(u(i+1,j) - u(i,j))/dx + (v(i+1,j+1) + v(i,j+1) - v(i+1,j-1) - v(i,j-1))/(4.*dy))/(sig(i+1,j)*(k(i+1,j) + k(i,j))) + \textbf{\textit{cmplx}}(-2.*(-b1)/(dy*dy*(k(i+1,j) + k(i,j))) + b1*bet(i,j)*dx/(2.*dy*dy), -deltap(i,j)*dx/(2.*dy*dy))*(pv(i+1,j+1) + 2.*pv(i+1,j) + pv(i+1,j-1))/sig(i+1,j) - \textbf{\textit{cmplx}}(1.,0.)*omeg*delta2*(3.*u(i+1,j) + u(i,j))/(2.*sig(i+1,j)) + ci*omeg*(a0 - 1.)*k(i+1,j)*u(i+1,j)*dx/sig(i+1,j) + 2.*ifilt*damp(i,j) + \textbf{\textit{cmplx}}(1.,0.)*alphn*dx$

$cp2(i,j) = \textbf{\textit{cmplx}}((-delta1*dx)*(v(i+1,j) + v(i,j))/(2.*dy) + b1*u2*bet(i,j)*(u(i+1,j)*v(i+1,j) + u(i,j)*v(i,j))/(dy*sig(i+1,j+1)), (-delta1*u2)*(u(i+1,j+1)*v(i+1,j+1) + u(i,j+1)*v(i,j+1) + 2.*u(i+1,j)*v(i+1,j))/(2.*dy*sig(i+1,j+1)) + dx*(-b1)*bet(i,j)*(sig(i+1,j)*v(i+1,j) + sig(i,j)*v(i,j))/(2.*dy*sig(i+1,j+1))) + \textbf{\textit{cmplx}}(2.*(-b1)/(dy*dy*(k(i+1,j) + k(i,j))) + (-b1)*bet(i,j)*dx/(2.*dy*dy), +deltap(i,j)*dx/(2.*dy*dy))*(pv(i+1,j+1) + pv(i+1,j))/sig(i+1,j+1) + 4.*\textbf{\textit{cmplx}}(0.,1.)*(-b1)*sig(i+1,j)*v(i+1,j)/(dy*sig(i+1,j+1)*(k(i+1,j) + k(i,j))) - ifilt*damp(i,j)$

$cp3(i,j) = \textbf{\textit{cmplx}}(-(-delta1*dx)*(v(i+1,j) + v(i,j))/(2.*dy) + (-b1)*u2*bet(i,j)*(u(i+1,j)*v(i+1,j) + u(i,j)*v(i,j))/(dy*sig(i+1,j-1)), -(-delta1*u2)*(u(i+1,j-1)*v(i+1,j-1) + u(i,j-1)*v(i,j-1) + 2.*u(i+1,j)*v(i+1,j))/(2.*dy*sig(i+1,j-1)) - dx*(-b1)*bet(i,j)*(sig(i+1,j)*v(i+1,j) + sig(i,j)*v(i,j))/(2.*dy*sig(i+1,j-1))) + \textbf{\textit{cmplx}}(2.*(-b1)/(dy*dy*(k(i+1,j) + k(i,j))) + (-b1)*bet(i,j)*dx/(2.*dy*dy), -(-deltap(i,j)*dx)/(2.*dy*dy))*(pv(i+1,j) + pv(i+1,j-1))/sig(i+1,j-1) - 4.*\textbf{\textit{cmplx}}(0.,1.)*(-b1)*sig(i+1,j)*v(i+1,j)/(dy*sig(i+1,j-1)*(k(i+1,j) + k(i,j))) - ifilt*damp(i,j)$

$c1(i,j) = \textbf{\textit{cmplx}}(cg(i+1,j) + u(i+1,j) - dv(i,j)*(sig(i+1,j) + sig(i,j))/4.,0.) + \textbf{\textit{cmplx}}(1., -dx*(kb(i) - a0*k(i,j)))*(cg(i,j) + u(i,j)) + 2.*\textbf{\textit{cmplx}}(0.,1.)*omeg*(-b1)*bet(i,j)*(u(i+1,j) + u(i,j))/sig(i,j) + 4.*\textbf{\textit{cmplx}}(0.,1.)*omeg*(-b1)*(3.*(u(i+1,j) - u(i,j))/dx + (v(i+1,j+1) + v(i,j+1) - v(i+1,j-1) - v(i,j-1))/(4.*dy))/(sig(i,j)*(k(i+1,j) + k(i,j))) + \textbf{\textit{cmplx}}(2.*b1/(dy*dy*(k(i+1,j) + k(i,j))) - b1*bet(i,j)*dx/(2.*dy*dy), +deltap(i,j)*dx/(2.*dy*dy))*(pv(i,j+1) + 2.*pv(i,j) + pv(i,j-1))/sig(i,j) - \textbf{\textit{cmplx}}(1.,0.)*omeg*delta2*(3.*u(i+1,j) + u(i,j))/(2.*sig(i,j)) - ci*omeg*(a0 - 1.)*k(i,j)*u(i,j)*dx/sig(i,j) + 2.*ifilt*damp(i,j) - \textbf{\textit{cmplx}}(1.,0.)*alphn*dx$

$c2(i,j) = \textbf{\textit{cmplx}}(delta1*dx*(v(i+1,j) + v(i,j))/(2.*dy) + b1*u2*bet(i,j)*(u(i+1,j)*v(i+1,j) + u(i,j)*v(i,j))/(dy*sig(i,j+1)), (-delta1*u2)*(u(i+1,j+1)*v(i+1,j+1) + u(i,j+1)*v(i,j+1) + 2.*u(i,j)*v(i,j))/(2.*dy*sig(i,j+1)) + 4.*(-b1)*sig(i,j)*v(i,j)/(dy*(k(i+1,j) + k(i,$

$$j)) * sig(i, j + 1)) - dx * (-b1) * bet(i, j) * (sig(i + 1, j) * v(i + 1, j) + sig(i, j) * v(i, j))/(2. * dy * sig(i,$$
$$j + 1))) + \textbf{cmplx}\,(2. * (-b1)/(dy * dy * (k(i + 1, j) + k(i, j))) + b1 * bet(i, j) * dx/(2. * dy * dy),$$
$$(-deltap(i, j) * dx)/(2. * dy * dy)) * (pv(i, j + 1) + pv(i, j))/sig(i, j + 1) - ifilt * damp(i, j)$$

$$c3(i, j) = \textbf{cmplx}\,((-delta1 * dx) * (v(i + 1, j) + v(i, j))/(2. * dy) - b1 * u2 * bet(i, j) * (u(i + 1, j) * v(i + 1,$$
$$j) + u(i, j) * v(i, j))/(dy * sig(i, j - 1)), (delta1 * u2) * (u(i + 1, j - 1) * v(i + 1, j - 1) + u(i, j - 1) * v(i,$$
$$j - 1) + 2. * u(i, j) * v(i, j))/(2. * dy * sig(i, j - 1)) - 4. * (-b1) * sig(i, j) * v(i, j)/(dy * (k(i + 1, j) + k(i,$$
$$j)) * sig(i, j - 1)) + dx * (-b1) * bet(i, j) * (sig(i + 1, j) * v(i + 1, j) + sig(i, j) * v(i, j))/(2. * dy * sig(i,$$
$$j - 1))) + \textbf{cmplx}\,(-2. * b1/(dy * dy * (k(i + 1, j) + k(i, j))) + b1 * bet(i, j) * dx/(2. * dy * dy), (-deltap(i,$$
$$j) * dx)/(2. * dy * dy)) * (pv(i, j) + pv(i, j - 1))/sig(i, j - 1) - ifilt * damp(i, j)$$

$$f1(i, j) = \textbf{tanh}\,(k(i, j) * d(i, j))^{5.}$$

$$f2(i, j) = (k(i, j) * d(i, j)/\textbf{sinh}\,(k(i, j) * d(i, j)))^{4.}$$

This code is used in section 7.

9.   CTRIDA.

Tridiagonal matrix solution by double sweep algorithm. Present subroutine adopted from the subroutine described in:

Carnahan, Luther and Wilkes, *Applied Numerical Methods*, Wiley, 1969

The original subroutine has been modified to handle complex array coefficients and solution values. Input and output are

a, b, c	coefficients of row in tridiagonal matrix
d	right hand side vector of matrix equation
v	solution vector
ii, l	beginning and end indices of positions in the dimensioned range of the column vector.

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Coded by James T. Kirby, September 1984.

```
subroutine ctrida(ii,l,a,b,c,d,v)

   parameter(iy = 2500)

   complex a(iy), b(iy), c(iy), d(iy), v(iy), beta(iy), gamma(iy)

      // Compute intermediate vectors beta and gamma.

   beta(ii) = b(ii)
   gamma(ii) = d(ii)/beta(ii)
   iip1 = ii + 1

   do 1 i = iip1, l
      beta(i) = b(i) - a(i)*c(i-1)/beta(i-1)
      gamma(i) = (d(i) - a(i)*gamma(i-1))/beta(i)
1: continue

      // Compute solution vector v.

   v(l) = gamma(l)
   last = l - ii

   do 2 k = 1, last
      i = l - k
      v(i) = gamma(i) - c(i)*v(i+1)/beta(i)
2: continue

   return

end
```

10.   DISS.

Subroutine calculates the dissipation at a single grid point based on values of the switch iw at that point.

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```
subroutine diss
parameter(ixr = 120, iyr = 100, ix = 31, iy = 2500, ncomp = 50)
⟨ common statements 16 ⟩
real nu, cp, kd
   // Statement function.
sq(i, j) = sqrt(nu/(2.*sig(i, j)))
   // Constants.
nu = 1.3 · 10⁻⁰⁶
cp = 4.5 · 10⁻¹¹
g = 9.80621
pi = 3.1415927
```

// Value of f here is value assuming $\tau = (f/8)u^2$.

// $f = 4f_w$; f_w is the wave friction factor

$f = 0.01*4.0$

```
do 1 j = 1, n
   do 1 i = 1, m
      w(i, j) = cmplx(0., 0.)
      kd = k(i, j)*d(i, j)
```

// If $iff(1) = 1$, use turbulent boundary layer damping.

if $(iff(1) \equiv 1)$ $w(i, j) = 2.*f*cabs(a(1, j))*sig(i, j)*k(i, j)/(sinh(2.*kd)*sinh(kd)*3.*pi)$

// If $iff(2) = 1$, add porous bottom damping.

if $(iff(2) \equiv 1)$ $w(i, j) = w(i, j) + (g*k(i, j)*cp/(nu*(cosh(kd)^2)))*cmplx(1., 0.)$

// If $iff(3) = 1$, add boundary layer damping.

if $(iff(3) \equiv 1)$
$w(i, j) = w(i, j) + 2.*k(i, j)*sig(i, j)*sq(i, j)*(1. + (cosh(kd)^2))*cmplx(1., -1.)/sinh(2.*kd)$

```
1: continue
   return
   end
```

11. RAND.

Generate a floating point pseudo random number between 0 and 1 by the multiplicative congruential method. see knuth, d.e., 1969, p. 155.

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```
function rand(x)
    ix = ifix(32767.*x)
    irand = mod(4573*ix + 6923, 32767)
    rand = float(irand)/32767.
    return
end
```

12.   FACT.

compute the factorial of xi

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```
function fact(xi)

    prod = 1.
    if (xi > 1.) then
       do 17 ii = 2, int(xi)
          prod = prod * float(ii)
17: continue
    endif

    fact = prod

    return

end
```

13.   BNUM.

Compute the combination $in!/(n!(n - in)!)$.

<div style="text-align:center">

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```
subroutine bnum (in ,n ,bn )
    xin = in
    xt = fact (xin )
    xb = fact (float(n))*fact (float(in − n))
    bn = xt /xb

    return

end
```

14.   ACALC.

Calculate the normalization factor a for the directional spectrum such that $\int_{-\theta_m}^{\theta_m} \cos(\theta/2)^{2*nsp} d\theta = 1$, where, in code, $\theta_m = thmax$.

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```
subroutine acalc(thmax,nsp,a)

    itn = 2*nsp
    call bnum(itn,nsp,bn)
    a = thmax*bn/(2.^{itn-1})
    sum = 0.
    do 10 ik = 1, nsp
        ki = ik - 1
        call bnum(itn,ki,bn)
10:  sum = sum + bn*sin(float(nsp - ki)*thmax)/float(nsp - ki)
    a = a + sum/(2.^{itn-2})

    return

    end
```

15. WVNUM.

Calculate wavenumber k according to the form

$$(\sigma - ku)^2 = gk\tanh(kd)$$

where

d		local water depth
$s = \sigma$		absolute frequency
g		gravitational acceleration constant
u		x-component of ambient current
$eps = \epsilon$		tolerance for iteration convergence
i, j		indices in finite-difference grid
$icdw$		switch
		=0, no convergence failures encountered
		=1, at least one convergence failure

Solution is by Newton-Raphson iteration using Eckart's approximation as a seed value.

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Coded by James T. Kirby, September,1984

```
subroutine wvnum(d,u,s,k,eps,icdw,i,j)

    parameter(ixr = 120, iyr = 100)

    common /ref2/ dr(ixr,iyr), ur(ixr,iyr), vr(ixr,iyr), iun(8), iinput, ioutput

    real k, kn

        // Constants.

    g = 9.806
    pi = 3.1415927
    k = s*s/(g*sqrt(tanh(s*s*d/g)))

        // Newton-Raphson iteration.

    do 1 ii = 1,20
        f = s*s - 2.*s*k*u + k*k*u*u - g*k*tanh(k*d)
        fp = -2.*s*u + 2.*k*u*u - g*tanh(k*d) - g*k*d/(cosh(k*d)²·)
        kn = k - f/fp
        if((abs(kn - k)/kn) < eps) go to 2
        k = kn
1: continue
    t = 2.*pi/(sqrt(g*k*tanh(k*d)) + k*u)
    write(*,100) i, j, k, u, d, f, t
    icdw = 1
```

      return

2: $k = kn$

      return

100: **format** (' wavenumber iter. failed to converge on row', $i10$, ' column', $i10/$ ' k=',
$f15.8$, ' u=', $f15.8/$ ' d=', $f15.8$, ' f=', $f15.8/$ ' t=', $f15.8$) ;

   end

16.   COMMON STATEMENTS.

⟨ common statements 16 ⟩ ≡

```
common  /ref1/ mr, nr, ispace, nd, md(ixr), iu. dconv(2), iff(3), icur, ibc
common  /ref2/ dr(ixr,iyr), ur(ixr,iyr), vr(ixr,iyr), iun(8), iinput, ioutput
common  /ref3/ dxr, dyr, xr(ixr), yr(iyr), x(ix), y(iy)
common  /ref4/ isd(ixr,iyr)
common  /block1/ d(ix,iy), u(ix,iy), v(ix,iy), m. n, dx, dy, ibr(iy)
common  /con1/ q(ix,iy), p(ix,iy), sig(ix,iy)
common  /con2/ k(ix,iy), kb(ix), w(ix,iy), dd(ix,iy), wb(2, iy)
common  /nlin/ an, anl, ntype
common  /wav1/ iwave, nfreqs, freqs(ncomp), edens(ncomp), nwavs(ncomp)
common  /wav2/ amp(ncomp,ncomp), dir(ncomp,ncomp), tide(ncomp), seed, thet0
common  /comp/ a(ix,iy), psibar, ifilt

real k, kb
complex w, a, wb
```

This code is used in sections 1, 2, 3, 4, 5, 6, 7, and 10.

17. INDEX.