for large waves, 1956–1957 and 1962." *Tech. Report CERC-88-6*, U.S. Army Coast. Engrg. Res. Ctr., Vicksburg.

Kriebel, D. L., Dally, W. R., and Dean, R. G. (1986). "Undistorted Froude scale model for surf zone sediment transport." *Proc. 20th Intl. Coastal Engineering Conf.*, ASCE, 1296–1310.

Larson, M., and Kraus, N. C. (1989). "SBEACH: Numerical model for simulating storm-induced beach change." *Tech. Report CERC-89-9*, U.S. Army Corps of Engrs. Coastal Engrg. Res. Ctr., Vicksburg, Va.

Saville, T. (1957). "Scale effects in two dimensional beach studies." *Trans. 7th General Meeting*, International Association Hydraulic Research, 1, A3-1–A3-10.

Seymour, R. J. (1985). "Results of cross-shore transport experiments." *J. Waterway, Port, Coastal and Ocean Engrg.*, ASCE, 112(1), 168–173.

Sunamura, T. (1980). "Parameters for delimiting erosion and accretion of natural beaches." *Annual Resport*, Inst. of Geoscience, Univ. of Tsukuba, 6, 51–54.

CACR-92-05

PREDICTION OF STORM/NORMAL BEACH PROFILES

By Robert A. Dalrymple,[1] Fellow, ASCE

ABSTRACT: Using the results of large-scale wave tank tests of monochromatic waves breaking on sandy beaches, Larson and Kraus have shown that storm (barred) and normal (nonbarred) equilibrium beach profiles can be segregated in terms of two-dimensionless parameters, which involve wave and sediment characteristics. Here, by rearranging their results, a single dimensionless parameter is developed that predicts the occurrence of storm or normal profiles. The profile parameter ($P = gH_0^2/(w^3T)$) uses deep-water wave characteristics to distinguish the two profile types. Using shallow-water data the shallow-water Dean number, $H_b/(wT)$ can serve the same purpose. Further, a Froude number representation of the sediment fall velocity, $w^2/(gH_0)$, is shown to be an important parameter for equilibrium profiles.

INTRODUCTION

Waves impinging on sandy beaches shape them in two primary states, either by eroding and flattening the foreshore to build a breakpoint bar, resulting in a storm profile, or by steepening the profile to a nonbarred, or normal, profile.

One of the major advances in discriminating between the two profile shapes based on wave and sediment characteristics was made by Dean (1973), who used a heuristic argument and small-scale laboratory data to show that (1) The dimensionless parameter, $D = H/wT$ (the ratio of wave height to sand fall velocity and wave period, now referred to as the Dean number), plays a major role in determining nature of the beach profile; and (2) the following equation, relating the wave steepness to dimensionless fall velocity, could discriminate between the two types of profiles.

$$\frac{H_0}{L_0} = 1.7\left(\frac{\pi w}{gT}\right) \quad \text{(1)}$$

where H_0, L_0 = the deep-water wave height and length, respectively; and g = the gravitational acceleration. (The right-hand side of this expression can also be expressed as the ratio of the fall velocity to the deep-water wave celerity, C_0.) Starting with a small value of wave steepness and a given value of the dimensionless fall velocity, increasing the wave steepness [to exceed the value given by (1)], the profile changes from a normal profile to a storm profile, thus providing an explanation for the seasonal differences in profiles.

By substituting the linear wave theory representation for deep-water wave length, $L_0 = gT^2/2\pi$, this equation can be written concisely as a single parameter, distinguishing between normal and storm profiles

---

[1] Prof. and Dir., Ctr. for Appl. Res., Dept. of Civ. Engrg., Univ. of Delaware, Newark, DE 19716.

Note. Discussion open until August 1, 1992. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on February 19, 1991. This paper is part of the ***Journal of Waterway, Port, Coastal, and Ocean Engineering***, Vol. 118, No. 2, March/April, 1992. ©ASCE, ISSN 0733-950X/92/0002-0193/$1.00 + $.15 per page. Paper No. 1416.

$$D_0 = \frac{H_0}{wT} = 0.85$$

Kriebel et al. (1986) carried out a number of laboratory tests of beach profiles and re-evaluated Dean's (1973) criteria. They found, using more full-scale tests, that there is a difference between small-scale laboratory tests (used by Dean) and full-scale tests [those of the Corps of Engineers, see Saville (1957), and Kajima et al. (1982)], due to scale effects in the models. From their study, the following relationship was found

$$\frac{H_0}{L_0} = c_1 \frac{\pi w}{gT} \quad \cdots\cdots (2)$$

and $D_0 = c_2$, where the constants fall in the following ranges, $4 < c_1 < 5$, and $2 < c_2 < 2.5$. The low values were recommended for the small-scale laboratory tests and the large values for the full-scale tests (and for natural beaches).

Kraus and Larson (1988) and Larson and Kraus (1989) examined only large-scale wave tank data, using the data of Saville (CE data) and Kajima et al. (CRIEPI data), and found that $c_1 = 5.5$ in (2) and that the separation between barred and normal profiles was distinguished better by several other curves, depending on the variables chosen for analysis:

$$\frac{H_0}{L_0} = 115 \left(\frac{\pi w}{gT}\right)^{3/2} \quad \cdots\cdots (3)$$

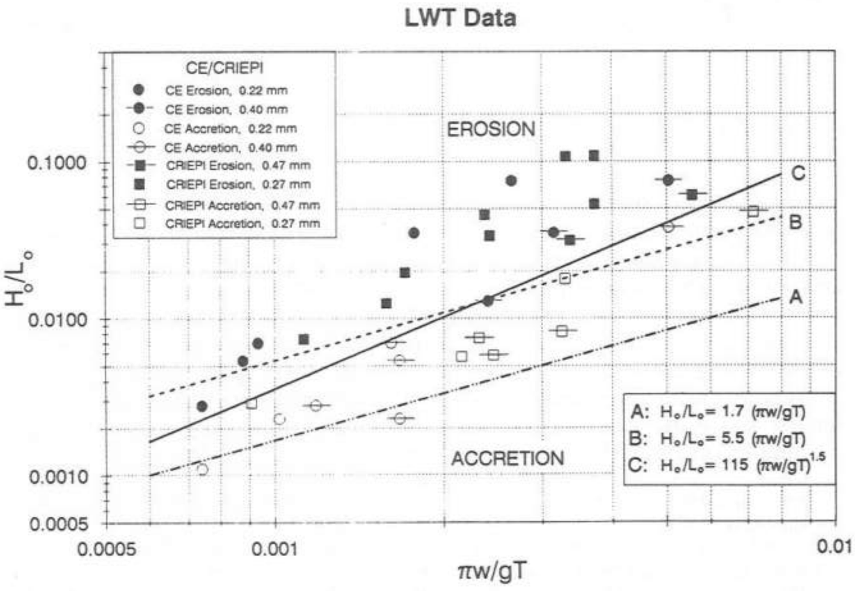

FIG. 1. Deep-Water Wave Steepness versus Dimensionless Fall Velocity, from Larson and Kraus (1989). Curves represent Dean, Kriebel et al., and Larson and Kraus fit to data

(case 201, $P = 894$) clearly shows a normal profile, with a large deposition of sand on the berm, while Fig. 4(*b*) (case 500, $P = 240{,}000$) is a strongly barred profile. However, Fig. 4(*c*) (case 801, $P = 8774$), which is right about at the border line between the two profile types, shows a slightly accretionary profile.

SHALLOW WATER PARAMETERS

The profile parameter is defined in terms of deep-water parameters and segregates all of the full-scale laboratory data correctly. If shallow-water data are available is the parameter still useful? For example, if we define $P_b = gH_b^2/w^3T$, does this work as well? What about D_b?

A value of $P_b \sim 22{,}200$ was able to discriminate between most of the data; however, two tests were not correctly predicted. For the shallow-water Dean number, the value of $D_b = 4$ worked for all the data, but for one test. It appears that the shallow-water Dean number may be more successful than the shallow-water profile parameter; but in any case, the deep-water profile parameter was more successful.

CONCLUSIONS

The consistency of the curve fitting of the large wave tank data [the two relationships of Larson and Kraus (1988) and Kraus and Larson (1989)] shows that a single parameter, based on deep-water wave characteristics, can determine the nature of the beach profile; this is the profile parameter, defined in (7). This means that it is no longer necessary to use a plot or an equation to determine the nature of the beach profile.

Using the shallow-water breaking wave heights, the Dean number is a more successful predictor of the beach profile than the equivalent shallow-water profile parameter. However, neither was more successful than the deep-water profile parameter.

More work needs to be done with additional large-scale laboratory results to verify the accuracy of the profile parameter and then apply it to the existing field data.

ACKNOWLEDGMENTS

Nicholas C. Kraus of the U.S. Army Coastal Engineering Research Center, Waterways Experimental Station, Vicksburg, Mississippi, kindly provided the CE/CRIEPI tank data and figures 1 and 2. David L. Kriebel provided the writer with helpful comments. This work was funded in part under the Cooperative Partnership with the Department of Natural Resources and Environmental Control, state of Delaware.

APPENDIX. REFERENCES

Dean, R. G. (1973). "Heuristic models of sand transport in the surf zone." *Proc. Conf. on Engrg. Dynamics in the Surf Zone*, Institution of Engineers, 208–214.
Dan, R. G. (1985). "Physical modelling of littoral processes." *Physical modeling in coastal engineering*, R. A. Dalrymple, ed., A. A. Bakema, Rotterdam.
Kajima, R., Shimizu, T., Maruyama, K., Hasegawa, H., and Sakakiyama, T. (1982). "Experiments of beach profile change with a large wave flume." *Proc. 18th Int. Coastal Engrg. Conf.*, ASCE, 1385–1404.
Kraus, N. C., and Larson, M. (1988). "Beach profile change measured in the tank

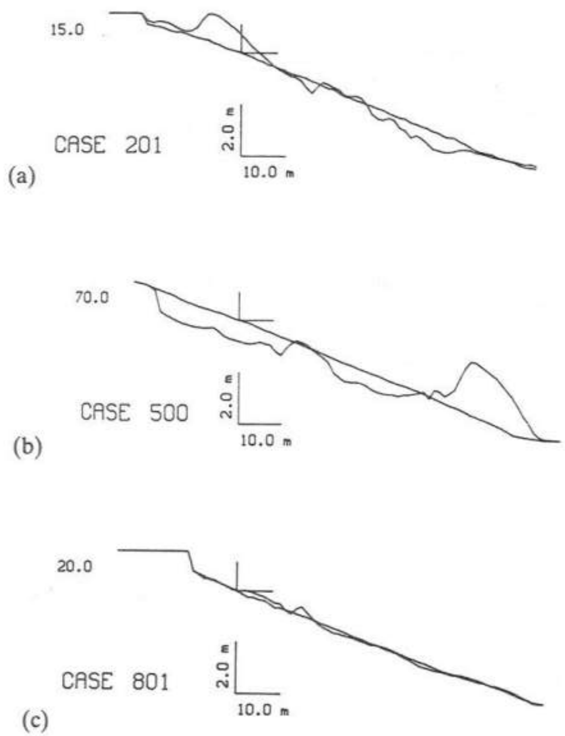

FIG. 4. (a). Normal Beach Profile, CE Data, P = 984, after 15.0 Hours of Testing; (b) Storm Profile, CE Data, P = 240,000, after 70.0 Hours of Testing; and (c) Slightly Accretionary Profile, CE Data, P = 8774, after 20.0 Hours of Testing. All Profiles from Kraus and Larson (1988)

$$\frac{gH_0^2}{w^3T} = P = \frac{2\pi}{0.0007} \sim 9{,}000 \quad \text{(11)}$$

From both empirical relationships, because of the consistency of the data and the fitting of the empirical relationships by Larson and Kraus, the same profile parameter results with approximately the same value (9,000–10,400) separating barred from normal profiles.

The large-scale wave tank data from Kraus and Larson are shown in Table 1, which includes the experimental case reference number, wave period, T, deep-water wave height, breaking wave height, depth in the tank in the horizontal section, h, fall velocity of the sand, median grain size, d_{50}, initial beach slope, tan β, Dean number, deep-water wave steepness, profile parameter, and whether the experiment was considered to have produced a storm (E) or a normal (A) profile, as determined by Kraus and Larson (1988). The correspondence between P and the type of profile is clear. The horizontal line divides the data: above the line the data is from the Corps of Engineers (Saville tests); below the line, the CRIEPI data.

As examples of the value of the profile parameter and the corresponding beach profile, the equilibrium profile for three tests are shown with their corresponding profile parameter value in Figs. 4(a), (b), and (c). Fig. 4(a)

$$\frac{H_0}{L_0} = 0.00070\, D_0^3 \quad \text{(4)}$$

The first formula, using the same variables as Dean (1973), but with the dimensionless fall velocity ratio raised to the 3/2 power, provided a better fit to the data than that of Dean or Kriebel et al. The second relationship relates the wave steepness to the Dean number. All of the data points were correctly segregated by these two relationships, which are shown in Figs. 1 and 2; further, a constant value of D_0 is not capable of segregating this data set. Additionally, using field data collected by Seymour (1985) and Sunamura (1980), Larson and Kraus also showed that the second formula [(4)], with the same value of the empirical coefficient, was a reasonable predictor for the field as well, provided the mean wave height was used, rather than, say, the significant wave height.

The second relationship [(4)] can be misleading if interpreted incorrectly. For a fixed value of the Dean number, increasing the wave steepness from a small value leads from storm profiles to normal profiles, whereas the first relationship, with fixed dimensionless fall velocity, correctly goes from normal to barred profiles as the wave steepness increases. The apparent contradiction arises through the relationship between the wave steepness and the Dean number.

$$\frac{H_0}{L_0} = \frac{2\pi H_0}{gT^2} = \left(\frac{2\pi w}{gT}\right) D_0 \quad \text{(5)}$$

Therefore, for fixed Dean number, increasing the wave steepness in (4) actually means increasing the dimensionless fall velocity. Conceptually then, the first of the two curves [(3)] is preferable for interpretation.

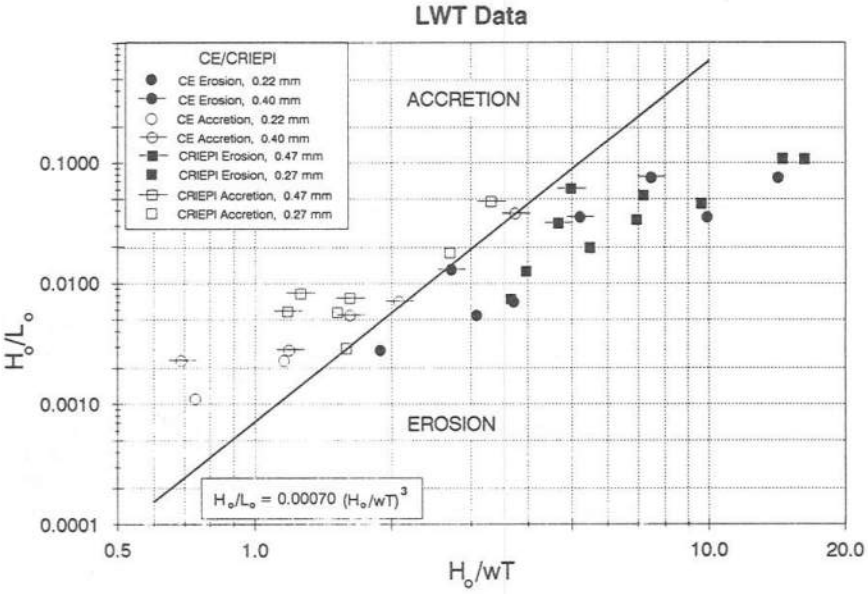

FIG. 2. Wave Steepness versus Dean Number, from Larson and Kraus (1989)

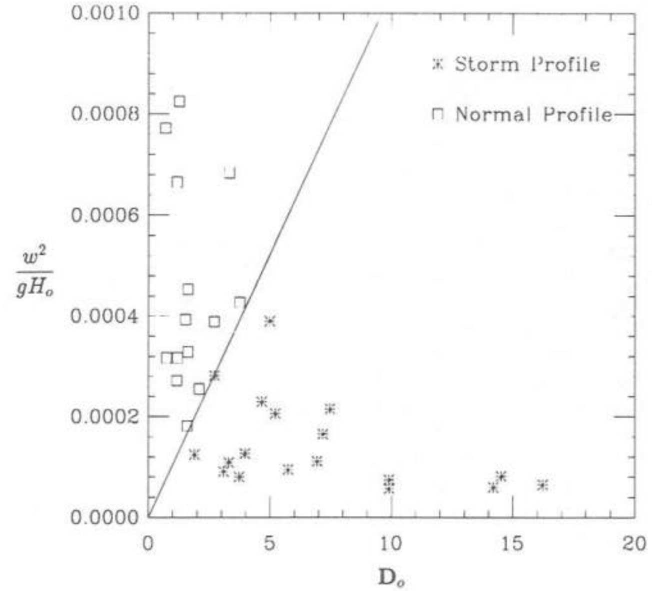

FIG. 3. Dimensionless Fall Velocity versus Dean Number. Open Squares Denote Normal Profiles

THE PROFILE PARAMETER

Using the Larson and Kraus results, it is interesting to rewrite the equations in terms of a single dimensionless variable. Beginning with (3) we can rewrite it as:

$$\frac{\dfrac{H_0}{L_0}}{\left(\dfrac{\pi w}{gT}\right)^{3/2}} = 115 \quad \text{.........} \quad (6)$$

or, squaring both sides and cancelling the common variables

$$P \equiv \frac{gH_0^2}{w^3T} = \frac{115^2\pi}{4} \sim 10{,}400 \quad \text{.........} \quad (7)$$

where P is defined as the profile parameter. This single parameter then is capable of discriminating between the two profile types and it is not necessary to examine a plot or an equation to determine if the wave and sediment conditions will lead to a barred or normal profile. If the profile parameter exceeds 10,400, then the beach is barred; for small values of P, the beach profile is normal. This profile number is the product of two other dimensionless numbers, the fall velocity nondimensionalized by gravity times the wave height (this is almost w/C where C is the shallow water wave speed) and the Dean number.

$$P = \left(\frac{gH_0}{w^2}\right) D_0 \sim 10{,}400 \quad \text{.........} \quad (8)$$

The gH_0/w^2 is another nondimensional fall velocity. [It has been used by Dean (1985) to examine the dependency of the sediment transport with sand size.] This implies that there is a linear relationship between the redimensionalized fall velocity and the Dean number. This is shown in Fig. 3, for the same data as used by Larson and Kraus. A linear fit was drawn by eye through the data given the relationship

$$\frac{w^2}{gH_0} = 0.000104\, D_0 \quad \text{.........} \quad (9)$$

When rearranged, this equation yields the fixed profile parameter value of 9,600 to separate the two profile types.

Alternatively, the second Larson and Kraus formula, (4) can be used to form the profile parameter in the same manner as before

$$0.0007 \left(\frac{D_0^3}{\dfrac{H_0}{L_0}}\right) = 1 \quad \text{.........} \quad (10)$$

or, cancelling

TABLE 1. Large Tank Cases (Larson and Kraus 1989; Kraus, Personal Communication, 1991)

Case (1)	T (2)	H_0 (3)	H_b (4)	h (5)	w (6)	d_{50} (7)	D_0 (8)	D_b (9)	H_0/L_0 (10)	P (11)	E/A (12)
100	11.33	1.080	1.680	4.570	0.031	0.220	3.08	4.80	0.00542	34,000	*E*
200	11.33	0.460	1.070	4.570	0.035	0.220	1.16	2.71	0.00231	4,285	*A*
300	11.33	1.390	2.000	4.270	0.033	0.220	3.73	5.36	0.00697	46,670	*E*
400	5.60	1.720	2.300	4.420	0.031	0.220	9.91	13.2	0.03513	174,000	*E*
500	3.75	1.650	1.900	4.570	0.031	0.220	14.2	16.3	0.07515	240,000	*E*
600	16.00	0.440	1.150	4.570	0.037	0.220	0.74	1.94	0.00110	2,343	*A*
700	16.00	1.120	2.100	3.810	0.037	0.220	1.89	3.54	0.00280	15,200	*E*
101	11.33	1.080	1.800	4.570	0.059	0.400	1.62	2.70	0.00542	4,930	*A*
201	11.33	0.460	1.900	4.570	0.059	0.400	0.69	2.85	0.00231	894	*A*
301	11.33	1.390	2.400	4.270	0.059	0.400	2.09	3.60	0.00697	8,167	*A*
401	5.60	1.720	2.400	4.420	0.059	0.400	5.21	7.26	0.03513	25,000	*E*
501	3.75	1.650	1.600	4.570	0.059	0.400	7.46	7.23	0.07515	34,700	*E*
701	16.00	1.120	1.950	3.810	0.059	0.400	1.19	2.07	0.00280	3,744	*A*
801	3.75	0.830	0.760	4.570	0.059	0.400	3.75	3.44	0.03780	8,774	*A*
901	7.87	1.260	2.000	3.960	0.059	0.400	2.71	4.30	0.01303	9,640	*E*
1-1	6.00	0.460	0.950	4.500	0.061	0.470	1.26	2.60	0.00818	1,524	*A*
1-3	9.00	0.950	1.400	4.500	0.065	0.470	1.62	2.39	0.00751	3,582	*A*
1-8	3.00	0.850	0.850	4.500	0.057	0.470	4.97	4.97	0.06049	12,760	*E*
2-1	6.00	1.760	1.940	3.500	0.063	0.470	4.66	5.13	0.03131	20,250	*E*
2-2	9.00	0.730	1.540	3.500	0.069	0.470	1.18	2.48	0.00577	1,768	*A*
2-3	3.10	0.710	0.800	3.500	0.069	0.470	3.32	3.74	0.04732	4,860	*A*
3-1	9.10	0.960	1.880	4.500	0.032	0.270	3.30	6.46	0.00743	30,320	*E*
3-2	6.00	1.100	1.580	4.500	0.032	0.270	5.73	8.23	0.01957	60,370	*E*
3-3	12.00	0.650	1.470	4.500	0.034	0.270	1.59	3.60	0.00289	8.790	*A*
3-4	3.10	1.620	1.500	4.500	0.036	0.270	14.5	13.4	0.10797	178,000	*E*
4-1	3.50	0.340	0.500	3.500	0.036	0.270	2.70	3.97	0.01778	6,944	*A*
4-2	4.50	1.060	1.270	4.000	0.034	0.270	6.93	8.30	0.03353	62,320	*E*
4-3	3.10	1.610	1.520	4.000	0.032	0.270	16.20	15.3	0.10730	250,000	*E*
5-1	5.80	0.300	0.630	3.500	0.034	0.270	1.52	3.19	0.00571	3,870	*A*
5-2	3.10	0.800	0.890	3.500	0.036	0.270	7.17	7.97	0.05332	43,400	*E*
6-1	5.00	1.780	1.910	4.000	0.036	0.270	9.89	10.6	0.04560	133,200	*E*
6-2	7.50	1.100	1.420	4.500	0.037	0.270	3.96	5.12	0.01253	31,200	*E*

Note: *A* = accretion; E = erosion.