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Surf-Zone Modeling

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Abstract

In the present paper we review the state-of-the-art of surf zone modelling. The last decade has seen a tremendous development in our understanding of wave breaking and of the mechanisms that are active in creating nearshore circulation and infragravity wave motion. At the present time, the models representing our combined knowledge can be divided into two categories: The depth and short-wave averaged circulation models which are capable of resolving the slow variations on a coast such as currents and infragravity waves, and the time domain models, which aim to describe the phase motion of the short-waves. Both types of models will be reviewed.

Introduction

When waves approach the shoreline they typically break at some location. The breaking process is highly complex (see Peregrine (1983) for a review). During the breaking process, the waves loose a significant amount of energy which is turned into turbulence. In addition, as the waves continue to break while propagating towards the shoreline they also experience a loss in their momentum flux. This loss of momentum flux is responsible for the generation of steady currents and low-frequency waves in the surf zone. The topic of "surf zone modeling" is therefore highly complex as it involves the modeling of the high-frequency turbulence generated by the breaking waves, modeling the propagation of breaking waves, and modeling the generation of steady currents and low-frequency by the breaking waves.

Significant progress has been made in the modeling of various aspects of surf zone hydrodynamics. In this paper we review some of this progress. Due to space limitations, we have been forced to keep our discussions very brief. We have, for the same reason, also been forced to completely leave out some topics (e.g., the modeling of the turbulence generated by the breaking waves, the modeling of swash motions). For more detailed overviews of the topic of surf zone hydrodynamics, the readers are referred to the review papers by Battjes (1988), Battjes *et al.* (1990), and Svendsen & Putrevu (1995).

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Models of surf zone hydrodynamics can be divided into two categories: short-wave-averaged models and short-wave resolving models. The first class of models (short-wave-averaged models) aim to model only the circulations generated by the breaking waves – these models assume that the short-wave motion is known. The second class of models (the short-wave resolving models) seek to model the propagation of breaking waves.

We first review short-wave-averaged models. This is followed by a review of short-wave resolving models.

Short-wave-averaged models

Basic Equations

Short-wave-averaged models model fluid motions that have time scales much larger than those of the incident wind waves (or short-waves).

In this class of models, it is implicitly assumed that the short-wave motion is known. In practice, this means that the short-wave motion is determined from another model which we call the "wave-driver." Significant progress has been made in the modeling of the short-wave-averaged circulation patterns using the short-wave-averaged equations since the concept of the radiation stress was introduced by Longuet-Higgins & Stewart (1962, 1964).

The derivation of the short-wave-averaged equations involves the following steps:

- The Navier-Stokes equations are integrated from the bottom ($z = -h_0$) to the surface ($z = \zeta$)
- Leibnitz rule is used to express

$$\int_{-h_0}^{\zeta} \frac{\partial}{\partial t}, \frac{\partial}{\partial x} \quad \text{as} \quad \frac{\partial}{\partial t}\int_{-h_0}^{\zeta}, \frac{\partial}{\partial x}\int_{-h_0}^{\zeta} \quad + \text{ Bdry. terms, e.g., } \frac{\partial \zeta}{\partial t}$$

  etc.
- The boundary conditions at the bottom and the free surface are used to eliminate most of the boundary terms
- The variation of the pressure is derived by integrating the vertical momentum equation from the free surface to an arbitrary location in the fluid z
- The expression for the pressure in the previous step is used to eliminate the pressure from the depth-integrated horizontal momentum equations
- The horizontal velocity, u_α, is divided into a current part, V_α, a wave-part, $u_{w\alpha}$, and a turbulent part, $u_\alpha^{'}$ as

$$u_\alpha = V_\alpha + u_{w\alpha} + u_\alpha^{'} \tag{1}$$

- The resulting equations are then averaged over a short-wave period. Quantities averaged over a short-wave period are denoted hereafter by an overbar

$$\bar{\cdot} = \frac{1}{T}\int_t^{t+T} .dt \tag{2}$$

where T represents the short-wave period.

The steps involved in these manipulations are explained in detail in standard textbooks, e.g., Phillips (1977) or Mei (1983). The two derivations differ at an essential point. Whereas Phillips defines $\overline{u_{w\alpha}} = 0$ below trough level, Mei defines $u_{w\alpha}$ so that $\overline{\int_{-h_o}^{\zeta} u_{w\alpha} dz} = 0$. In brief, this implies that the two definitions are related by

$$u_{w\alpha,\,\mathrm{Mei}} = u_{w\alpha,\,\mathrm{Phillips}} - \frac{Q_w}{h} \tag{3}$$

where Q_w is the volume flux of the waves and $h = h_0 + \overline{\zeta}$. The derivations given in Phillips and Mei consider the case of depth-uniform currents. Extension of these derivations to the case of depth-varying currents is straightforward and results in the following set of equations

Continuity

$$\frac{\partial \bar{\zeta}}{\partial t} + \frac{\partial \bar{Q}_\alpha}{\partial x_\alpha} = 0 \tag{4}$$

where $\bar{Q}_\alpha$ represents the net short-wave-averaged volume flux and is given by

$$\bar{Q}_\alpha = \overline{\int_{-h_o}^{\zeta} u_\alpha \, dz} \tag{5}$$

Momentum

$$\rho \frac{\partial}{\partial t} \bar{Q}_\beta + \rho \frac{\partial}{\partial x_\alpha} \int_{-h_0}^{\bar{\zeta}} V_\alpha V_\beta dz + \rho \frac{\partial}{\partial x_\alpha} \overline{\int_{\zeta_t}^{\zeta} u_{w\alpha} V_\beta + u_{w\beta} V_\alpha dz}$$

$$+ \rho g (\bar{\zeta} + h_0) \frac{\partial \bar{\zeta}}{\partial x_\beta} + \frac{\partial}{\partial x_\alpha} \left[S'_{\alpha\beta} - \overline{\int_{-h_0}^{\zeta} \tau_{\alpha\beta} dz} \right]$$

$$- \tau_\beta^S + \tau_\beta^B = 0 \tag{6}$$

where the quantity $S'_{\alpha\beta}$ is defined (exactly) as

$$S'_{\alpha\beta} = \rho \overline{\int_{-h_0}^{\zeta} \left[u_{w\alpha} u_{w\beta} - \delta_{\alpha\beta} w_w^2 \right] dz} + \frac{1}{2} \rho g \overline{\eta^2} \delta_{\alpha\beta} \tag{7}$$

where $\eta = \zeta - \overline{\zeta}$.

The momentum equation given above may benefit from some explanation. The first term in this equation represents the temporal acceleration, the next two terms represent convective acceleration, the $\partial \bar{\zeta} / \partial x_\beta$ term represents the pressure gradient from the short-wave-averaged surface elevation, $S'_{\alpha\beta}$ is the so-called radiation stress (which will be discussed below), $\tau_{\alpha\beta}$ represents the turbulent Reynolds' stresses, τ_β^S represents the surface stresses (from wind for example), and τ_β^B represents the bottom stress.

The quantity $S'_{\alpha\beta}$ that appears in the momentum equation is called the radiation stress and represents the excess momentum flux due to the presence of the wave motion. This is an extremely important quantity because it is the gradients of this quantity that drive short-wave-averaged motions in the nearshore. As can be seen from (7), the radiation stress is defined entirely in terms of short-wave quantities and can be calculated exactly if the short-wave motion were completely known. However, at the present time we do not have an accurate theory for breaking waves in the surf zone. Thus, we need to introduce approximations to calculate the radiation stresses. It is important to note that these approximations are one of the most significant causes of errors in the modeling of short-wave-averaged circulations. One assumption that is frequently made is the use of linear (sine) wave theory to calculate the radiation stresses. While linear theory may do a qualitatively accurate job of predicting the variations of the radiation stresses in the surf zone, it is quantitatively inaccurate because breaking waves in the surf zone grossly violate the assumptions of linear theory in two important ways: 1) the wave height to water depth ratio in surf zone waves is not small and 2) surf zone waves are not sinusoidal. Early doubts on the validity of linear theory in the surf zone were raised by Bowen *et al.* (1968), Thornton (1970), Miller & Barcillon (1978), and Svendsen (1984a). Analysis of laboratory data by Stive & Wind (1982) and Svendsen & Putrevu (1993) has demonstrated that linear theory predictions of the radiation stresses are grossly inaccurate through most of the surf zone. This is important because the gradients of the radiation stresses provide the forcing for short-wave-averaged circulations in the surf zone. Thus, an inaccuracy in the predictions of the radiation stresses translates into an equivalent inaccuracy in the prediction of the forcing for short-wave-averaged circulations. A semi-empirical approach introduced by Svendsen (1984a) is now frequently used to calculate the radiation stresses (and other wave-averaged quantities) in the surf zone. Empirical variations of the important parameters that go into Svendsen's model have been derived by Hansen (1990) by analyzing a number of laboratory experiments.

Models of depth-uniform, short-wave-averaged circulation

We start our discussion of the short-wave-averaged circulation by considering simplified models where the currents are assumed to be uniform over depth ($V_\alpha = V_{m\alpha}$). The continuity equation remains the same

$$\frac{\partial \bar{\zeta}}{\partial t} + \frac{\partial \bar{Q}_\alpha}{\partial x_\alpha} = 0 \tag{8}$$

but the total volume flux $\bar{Q}_\alpha$ can be expressed directly in terms of $V_{m\alpha}$ as

$$\bar{Q}_\alpha = \overline{\int_{-h_o}^{\zeta} u_\alpha \, dz} \;=\; \int_{-h_0}^{\bar{\zeta}} V_\alpha dz + Q_{w\alpha} \tag{9}$$

$$= V_{m\alpha} h + Q_{w\alpha} \tag{10}$$

In the above $Q_{w\alpha}$ represents the volume flux due to the wave motion.

In the momentum equation the terms that depend on the vertical structure of the currents reduce to

$$\int_{-h_0}^{\bar{\zeta}} V_\alpha V_\beta dz + \overline{\int_{\zeta_t}^{\zeta} u_{w\alpha} V_\beta + u_{w\beta} V_\alpha dz} \;=\; V_{m\alpha} V_{m\beta} h + V_{m\alpha} Q_{w\beta} + V_{m\beta} Q_{w\alpha} \tag{11}$$

$$= \frac{\bar{Q}_\alpha \bar{Q}_\beta}{h} - \frac{Q_{w\alpha} Q_{w\beta}}{h} \tag{12}$$

The last term in the above is either absorbed by the different definition of $u_{w\alpha}$ (Mei 1983) or included in the definition of the radiation stress (Phillips 1977). In either case, the momentum equation then reduces to

$$\rho \frac{\partial \bar{Q}_\beta}{\partial t} + \frac{\partial}{\partial x_\alpha}\left(\rho \frac{\bar{Q}_\alpha \bar{Q}_\beta}{h} + S_{\alpha\beta} - \overline{\int_{-h_0}^{\zeta} \tau_{\alpha\beta} dz}\right) = -\rho g h \frac{\partial \bar{\zeta}}{\partial x_\beta} + \tau_\beta^S - \tau_\beta^B \tag{13}$$

where the definition of the radiation stress depends on the way $u_{w\alpha}$ is defined. With Mei's definition $S_{\alpha\beta} = S'_{\alpha\beta}$ given by (7)[3], whereas the definition of $u_{w\alpha}$ used by Phillips leads to

$$S_{\alpha\beta} = S'_{\alpha\beta} - \frac{Q_{w\alpha} Q_{w\beta}}{h} \tag{14}$$

Steady State Models

In the simplest possible situation of steady state, no alongshore variation ($\partial/\partial y = 0$), and no wind stress the above equations reduce to the following:

Continuity

$$\frac{\partial \bar{Q}_x}{\partial x} = 0 \tag{15}$$

Cross-shore momentum

$$g(h_0 + \overline{\zeta})\frac{d\overline{\zeta}}{dx} = \frac{-1}{\rho}\frac{dS_{xx}}{dx} \tag{16}$$

Longshore momentum

$$\frac{dS_{xy}}{dx} - h\frac{d\tau_{xy}}{dx} + \tau_y^B = 0 \tag{17}$$

In addition to the assumptions stated above, the turbulent stresses and the bottom shear stresses have been neglected in the cross-shore momentum equation (these terms have been found to be small as far as the cross-shore momentum balance is concerned (see, e.g., Svendsen *et al.* 1987).

The cross-shore momentum equation (16) shows that changes in the cross-shore component of the radiation stress are balanced by corresponding changes in the mean water level (Longuet-Higgins & Stewart 1963, Bowen *et al.* 1968). This is an important effect, in particular near the shoreline where the wave-induced set-up could be as much as 15% of the water depth at the break point. The longshore momentum equation shows that changes in longshore component of the radiation stresses are balanced by a bottom shear stress (and the turbulent lateral stress). The presence of the bottom shear stress implies that a current is generated in the alongshore direction. Thus, longshore currents are generated by changes in the longshore component of the radiation stress (Bowen 1969, Thornton 1970, Longuet-Higgins 1970).

Simple models for infragravity and shear wave motion

[3] Mei (1983), in a footnote, seems to indicate that the last term in (12) is omitted. Careful scrutiny of the algebra shows that this is not the case.

Note that the assumption of steady state made above implies that it is also assumed that the incident short-waves do not vary with time (or that the short-waves are monochromatic). The next level of sophistication on a longshore uniform coast involves relaxing the assumption of steady state. Neglecting the nonlinear terms, the turbulent stresses, and the bottom stresses but retaining the unsteady terms, the short-wave-averaged equations of continuity and momentum may be combined to give (Symonds *et al.* 1982, Schäffer & Svendsen 1988)

$$\frac{\partial^2 \overline{\zeta}}{\partial t^2} - \frac{\partial}{\partial x_\alpha}\left(gh\left(\frac{\partial \overline{\zeta}}{\partial x_\alpha}\right)\right) = \frac{1}{\rho}\,\frac{\partial^2 S_{\alpha\beta}}{\partial x_\alpha\,\partial x_\beta} \tag{18}$$

Equation 18 is a wave equation with forcing. The solutions of the homogeneous version of this equation represent the so-called "free waves" which are the normal modes of oscillation of the beach. The free waves consist of both edge waves and leaky waves. Edge waves (Eckart 1951, Ursell 1952) are waves that propagate alongshore while being trapped close to the shore by refraction. For a given beach geometry there exist only a discrete set of such modes.

Leaky waves on the other hand are not trapped in the nearshore – they propagate back out to the deep ocean. The spectrum of the leaky waves is continuous. The term surf beat is also used for leaky waves with strictly shore normal motion (1D-horizontal motion).

Equation 18 shows that temporal variations of the radiation stresses in the nearshore (caused by temporal variations of the incoming short-wave field) can generate low frequency waves in the nearshore. This equation or some variation thereof (like including the nonlinear terms) has been used to study the generation of both leaky waves (Symonds *et al.* 1982, Symonds & Bowen 1984, Schäffer & Svendsen 1988, List 1992, Roelvink 1993) and edge waves (Schäffer 1994) in the surf zone. Some results of these studies are summarized below.

Symmonds *et al.* showed that temporal variations of the break point (caused by the temporal variations of the incoming short-waves) can generate infragravity waves. Schäffer & Svendsen showed that strong infragravity generation takes place inside the surf zone if the temporal variations of the short-wave field is not completely destroyed by the breaking process. Schäffer (1994) showed that if the short-waves were incident at an angle to the shore-normal then generation of edge waves takes place. It should be pointed out here that at present there are indications that a significant generation of infragravity energy takes place in the surf zone. However, there are no clear indications as to how important the mechanism outlined here is in comparison with other mechanisms of infragravity wave generation such as nonlinear wave-wave interaction.

The depth-uniform, short-wave-averaged equations have also been used to model the effects that longshore currents have on edge wave propagation (Howd *et al.* 1992, Falques & Iranzo 1992, Oltman-Shay & Howd 1993) as well as the instability of longshore currents (Bowen & Holman 1989, Dodd *et al.* 1992, Putrevu & Svendsen 1992, Falques & Iranzo 1994). The modeling of these phenomena start with (8) and (13) neglecting the forcing, the bottom shear stress, and lateral mixing[4]. Under these

[4] Dodd *et al.* (1992) included bottom friction and Falques & Iranzo included both the bottom

conditions the equations read

$$\frac{\partial \overline{\zeta}}{\partial t} + \frac{\partial}{\partial x}\left[U(h_0 + \overline{\zeta})\right] + \frac{\partial}{\partial y}\left[\tilde{V}(h_0 + \overline{\zeta})\right] = 0 \tag{19}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \tilde{V}\frac{\partial U}{\partial y} + g\frac{\partial \overline{\zeta}}{\partial x} = 0 \tag{20}$$

$$\frac{\partial \tilde{V}}{\partial t} + U\frac{\partial \tilde{V}}{\partial x} + \tilde{V}\frac{\partial \tilde{V}}{\partial y} + g\frac{\partial \overline{\zeta}}{\partial y} = 0 \tag{21}$$

where $\tilde{V}$ represents the longshore current. It is then assumed that the total flow consists of a mean longshore current ($V(x)$) and a perturbation of that current. In addition, one also assumes alongshore uniformity of the bottom topography. Hence we let

$$\overline{\zeta} = 0 + \eta'(x,y,t) \tag{22}$$

$$U = 0 + u'(x,y,t) \tag{23}$$

$$\tilde{V} = V(x) + v'(x,y,t) \tag{24}$$

If we substitute the above into (19)–(21), linearize the expressions with respect to the perturbation, and seek solutions that propagate alongshore by assuming, solutions of the form $\eta' = \eta(x)\exp[i(ky - \sigma t)]$ we get the following set of equations

$$-i\sigma\eta + \frac{d(uh_0)}{dx} + ik(vh_0) + ikV\eta = 0 \tag{25}$$

$$-i\sigma u + ikVu + g\frac{d\eta}{dx} = 0 \tag{26}$$

$$-i\sigma v + u\frac{dV}{dx} + ikVv + ikg\eta = 0 \tag{27}$$

which constitute an eigenvalue problem: for a given k non trivial solutions are possible only for certain values of σ.

If σ is real then the solutions represent alongshore propagating waves. These solutions are free edge waves modified by the presence of the longshore current. It turns out that both the cross-shore structure of the edge waves and their dispersion relationship are sensitive to the presence of the longshore current (Howd *et al.* 1992, Falques & Iranzo 1992). Another important consequence of the presence of the longshore current is that it introduces an asymmetry: at a given frequency the longshore wave number increases for a longshore current opposing the edge wave propagation and decreases for a longshore current in the direction of edge wave propagation. Such effects are evident in field observations of edge waves (Oltman-Shay & Guza 1987, Oltman-Shay & Howd 1993). Falques & Iranzo further showed that strong longshore currents can have subtle effects on edge waves – these are not discussed here.

If in the eigenvalue-problem (25)–(27) σ turns out to have a positive imaginary component then the solution indicates that the longshore current is unstable – the perturbations tend to grow in time. The stability problem was first formulated by

friction and lateral mixing in their modeling of the instabilities of longshore currents

Bowen & Holman (1989) who were motivated by field observations which showed the presence of alongshore propagating wave-like motions with wavelengths an order of magnitude too short to be gravity waves (Oltman-Shay *et al.* 1989). Solutions of the instability problem (Bowen & Holman 1989, Dodd *et al.* 1992, Putrevu & Svendsen 1992, Falques & Iranzo 1994) have shown that the predicted characteristics of the unstable modes closely match the characteristics of wave-like motions observed in the field thus suggesting that these motions are generated by a shear instability of the longshore current (for this reason these motions are referred to as shear waves). Additional support for the instability mechanism comes from a recent laboratory experiment reported by Reniers *et al.* (1994). Reniers *et al.* also found that the presence of a longshore bar is conducive to the generation of shear waves; this is important because this feature is one of the predictions of the instability theory. Thus, at present there are strong indications that shear waves may be successfully modeled as being generated by instabilities of a longshore current.

Comprehensive models of depth-uniform, short-wave-averaged currents

So far, we have discussed individual processes that have been modeled using the short-wave-averaged, depth-uniform current equations. Several numerical models have been developed which solve equations (8) and (13) in the general situation (e.g., Noda *et al.* 1974, Ebersole & Dalrymple 1979, Kirby & Dalrymple 1982, Watanabe 1982, Wu & Liu 1985, Wind & Vreugdenhil 1985). These models do have some limitations. For example, they are obviously restricted to the case of depth-uniform currents, they use empirical results for the lateral mixing, they often use simplified wave-drivers, and some of them are only steady state.

Nevertheless, such models are quite powerful in the sense that they can handle complex topographies and complicated current situations. For example, Wind & Vreugdenhil (1985) present a calculation which shows that their model predicts the development of an intense rip current. Given the strengths of the comprehensive models, it is somewhat surprising that these models do not seem to have been used to run numerical experiments from which physical insight may be gained.

Models with vertical resolution

In addition to the depth-integrated flow models discussed above models have been developed that provide information about the vertical structure of the flow. These models resolve the vertical structure locally using the flow parameters $\overline{\zeta}$ and $\overline{Q}$ determined through the solution of the depth integrated equations.

Early examples of such local models were the solutions for the undertow (Svendsen 1984b, Dally & Dean 1984, Stive & Wind 1986, Svendsen *et al.* 1987, Okayasu *et al.* 1988, and many others), and, later, solutions for the local vertical structure of the longshore current (Svendsen & Lorenz 1989), and for the vertical structure of the velocity under infragravity waves (Putrevu & Svendsen 1995).

Combination of the local vertical flow resolving models and the depth integrated models have then lead to the so-called quasi-3D models (deVriend & Stive 1987; Svendsen & Putrevu 1990; Sanchez-Arcilla *et al.* 1990, 1992; Van Dongeren *et al.* 1994, 1995;

Sancho *et al.* 1995; and many others). These models essentially represent a combination of the complete set of depth integrated equations (4) and (6) (or approximations thereof) which include the effects the vertical structure has on the horizontal flow, and local flow models that resolve the vertical structure locally.

We will first discuss the local models.

Undertow models

The mechanism that determines the vertical structure of the undertow (the steady cross-shore circulation) inside the surf zone may be illustrated by considering the local short-wave-averaged momentum balance on a fluid particle. Recall that on a longshore uniform coast the depth-averaged cross-shore momentum balance is given by (16) which states that the changes in the cross-shore component of the radiation stress is balance by a pressure gradient due to changes in the short-wave-averaged surface elevation. However, the forcing from the pressure gradient is uniformly distributed over the entire water column whereas the forcing from the radiation stress is not (most of the contribution to the radiation stress comes from the upper part of the water column). Therefore, at any given vertical location there is a net imbalance between the forcing from the pressure gradient and the forcing from the radiation stress – this imbalance is taken up by the turbulent shear stresses (see Svendsen (1984b) for a more detailed discussion). This leads to the following equation that determines the local momentum balance

$$\frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} = g\frac{d\bar{\zeta}}{dx} + \overline{\frac{\partial u_w^2}{\partial x}} + \overline{\frac{\partial u_w w_w}{\partial z}} \tag{28}$$

Typically, the Reynolds' stress is modeled using an eddy viscosity formulation

$$\tau_{zx} = \rho \nu_t \frac{\partial U}{\partial z} \tag{29}$$

Substitution of (29) into (28) leads to a second-order equation for the undertow (U) which can be solved subject to appropriate boundary conditions. The solution for the undertow has been discussed extensively in the literature (e.g., the references cited above) and numerous comparisons with data have been made (e.g., Stive & Wind 1986, Svendsen *et al.* 1987, Okayasu *et al.* 1988). Inside the surf zone the first term on the right hand side is the dominating term. In earlier solutions, the last term in (28) was neglected. Deigaard & Fredsoe (1989) discusses this term.

The arguments presented above can be readily extended to temporally varying situations to determine the velocity profiles in infragravity waves (Putrevu & Svendsen 1995, Smith & Svendsen 1995). These solutions are not discussed here.

Quasi 3D models

The term "quasi 3D models" is used to denote the class of models that resolve the three dimensional structure of the short-wave-averaged velocity without resorting to a fully three dimensional calculation. Many such models have been developed (see the references cited earlier), and while all these models attempt to do the same thing, there are some differences in approach – which will not be discussed here.

There are several incentives for including the vertical structure in the models. One is the realization that the currents (and velocities in infragravity waves as well) change over the depth. Hence the velocity at the bottom, which is of particular concern for the analysis of sediment transport, may have a magnitude and direction that is quite different from the magnitude and direction of the depth averaged current predicted by the simpler models. A second reason that points to the importance of pursuing the quasi 3D approach is the effect the 3D structure of the current has on the horizontal mixing. As an example, we consider uniform longshore currents on a straight coast.

Longshore currents have traditionally been modeled using the short-wave-averaged, depth-uniform current equation of longshore momentum (equation 17) where the Reynolds' stress is usually modelled using an eddy viscosity closure. Comparison of the solution of (17) with data indicates that a mixing coefficient of order $0.2-0.6h\sqrt{gh}$ is needed to make the predictions match the observations. On the other hand, a favorable comparison of the predictions of the undertow to measurements requires the use of a mixing coefficient of order $0.01h\sqrt{gh}$. Clearly, therefore, there is an order of magnitude difference between the mixing coefficients required to predict the horizontal and vertical structure of the currents. While it is not unreasonable to expect that the horizontal and lateral mixing coefficients could be different, existing turbulence measurements show that it is unreasonable to expect them to differ by an order of magnitude. Svendsen & Putrevu (1994) showed that accounting for the vertical structure of cross and longshore currents could explain the difference in the lateral and vertical mixing coefficients. This is briefly discussed below.

If the assumption of depth-uniform currents is abandoned, then the steady longshore momentum equation (subject to assumptions of longshore uniformity, steady state, etc.) can be derived from (6) and reads

$$\frac{d}{dx}\left(\int_{-h_0}^{\bar{\zeta}} UV\,dz + \overline{\int_{\zeta_t}^{\zeta} u_w V + v_w U\,dz}\right) + \frac{1}{\rho}\frac{d}{dx}\left(S_{xy} - \overline{\int_{-h_0}^{\zeta} \tau_{xy}dz}\right) + \frac{\tau_y^B}{\rho} = 0 \quad (30)$$

and the vertical structure of the longshore current is governed by (see Svendsen & Lorenz 1989)

$$\frac{\partial}{\partial z}\left(\nu_t \frac{\partial V}{\partial z}\right) = \overline{u_w \frac{\partial v_w}{\partial x}} + U\frac{\partial V}{\partial x} \quad (31)$$

Observational evidence (e.g., Visser 1984) shows that the vertical shear in the longshore current is weak. Therefore, it is reasonable to approximate the last term on the RHS of (31) by $U dV_m/dx$. With this approximation it is straightforward to integrate (31) twice to determine the vertical structure of the longshore current.

When this is done and the result is substituted into (30) the following equation results

$$\frac{d}{dx}\left[\left(D_c h + \int_{-h_0}^{\bar{\zeta}} \nu_{tx} dz\right)\frac{dV_m}{dx}\right] - \frac{\tau_y^B}{\rho} - \frac{d}{dx}\left(F_2 V_m\right) = \frac{1}{\rho}\frac{dS_{xy}}{dx} + \frac{dF_1}{dx} \quad (32)$$

where the "dispersion coefficient" D_c is given by

$$D_c = \frac{1}{h}\int_{-h_0}^{\bar{\zeta}} U \int_{z}^{\bar{\zeta}} \frac{1}{\nu_t}\int_{-h_0}^{z} U\;dz\;dz\;dz \quad (33)$$

and F_1 and F_2 are defined by similar integrals (see Svendsen & Putrevu 1994).

Equation 32 shows that the D_c term has an effect that is similar to lateral mixing. In addition, for typical undertow profiles it turns out that the dispersion coefficient is an order of magnitude larger than the vertical turbulent mixing coefficient. This result suggests that to model short-wave-averaged nearshore circulations it is important to resolve the vertical structure of the flow, because similar mechanisms are likely to be present also in more general nearshore circulation situations. This finding also points to the importance of pursuing the quasi 3D approach to modeling nearshore circulations.

As mentioned earlier, a number of quasi 3D models have been presented. All these models have three basic components: a short-wave driver that determines the forcing for the short-wave averaged circulations; a module that numerically solves the local short-wave-averaged equations to determine the vertical structure of the short-wave-averaged currents (this step is largely analytical); and a module that solves the depth-integrated, short-wave-averaged equations using the results from the previous step for the velocity profiles to calculate, e.g., the dispersive mixing described above. Due to space limitations the details are not discussed here – for the details reference is made to the original publications cited earlier.

Wave Drivers

As mentioned, the forcing of the short-wave-averaged motion is created by the variation of mass and momentum flux in the short wave motion.

A short wave model or wave driver is needed to determine this forcing. The simplest type is the one-dimensional model for the cross-shore wave height variation on a beach with no longshore variation determined from the wave averaged energy equation in combination with assumptions about the nature of the wave motion. Examples are Svendsen (1984a), Dally et al.(1984). This 1D method has been extended to cover wave spectra for irregular waves (Battjes & Janssen, 1978, Thornton & Guza, 1983, Dally, 1990).

On a straight coast with no longshore variation of a monochromatic wave motion, such models can be used in 2D-horizontal computations (see some of the earlier cited works), because the pattern of wave propagation is determined by Snell's law. For more general topographies numerical models such as REF/DIF version 2.5 (Kirby & Dalrymple, 1995) can be used if the waves are monochromatic. It includes a simulation of wave breaking.

It is observed, however, that for the present time the absence of wave drivers that can provide the forcing for the wave averaged models in situations with irregular short wave motion such as storm waves represents an important obstacle for the use of these models for analysis of infragravity waves and time varying currents in more general situations. See also the discussion in section 2.0 about the accuracy of existing wave theories.

Short-wave resolving models

As mentioned earlier, short-wave-resolving models aim to model the breaking waves directly. Basically all such models assume that the horizontal length scale of the wave motion is large in comparison to the local depth. Operating in the time domain these models are capable of analyzing regular as well as irregular wave motions to the extent the corresponding incident motion can be specified at the boundary of the computational domain.

The short wave resolving surf zone models can be classified into two categories: 1) models based on the nonlinear shallow water equations and 2) models based on the Boussinesq approximation.

To compare these two models, it is useful to introduce a dispersion parameter μ and a nonlinearity parameter δ defined as

$$\mu = kh_0 \tag{34}$$

$$\delta = a/h_0 \tag{35}$$

where a is the amplitude of the wave motion, h_0 is the water depth (the vertical length scale), and $k = 2\pi/L$ where L is the length of the wave (a horizontal length scale).

The nonlinear shallow water (NSW) equations are derived under the assumptions $\mu \ll 1$ and $\delta \sim 1$. These two assumptions lead to the conclusion that the horizontal velocity is uniform over depth. The Boussinesq equations, on the other hand, are derived under the assumption $\mu \sim \delta \ll 1$ while $\delta/\mu^2 = O(1)$. These assumptions lead to the conclusion that to the first approximation the horizontal velocity has a quadratic variation over depth. We discuss the two types of models separately.

Nonlinear shallow water equation models

The nonlinear shallow water equations read

$$\frac{\partial \zeta}{\partial t} + \nabla_h(\mathbf{u}(h_0 + \zeta)) = 0 \tag{36}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\nabla_h \mathbf{u} + g\nabla_h \zeta = 0 \tag{37}$$

where $\mathbf{u}$ represents the (depth uniform) horizontal velocity vector and ∇_h is the horizontal gradient operator. Often a bottom friction term is added to the momentum equations (37).

These equations are essentially the continuity and momentum equations and hence they conserve mass and momentum. Since they do not include any mechanisms that represent wave breaking an exact solution to (36) and (37) will also conserve energy (apart from a small loss if bottom friction is included). It appears that this can only happen if the waves keep changing form. Any initial wave, no matter how small, will consequently steepen at its front as it propagates even over a horizontal bottom and since there are no dispersive effects in the model that can counteract this process the front eventually becomes vertical.

The use of a Lax-Wendroff (or similar) dissipative scheme introduces (numerical) dissipation into the computation in such a way that the shape of the front freezes just before it becomes vertical. It turns out that the ensuing energy dissipation equals the dissipation in a bore of the same height.

This method for describing breaking waves has been developed extensively in the past fifteen years. Examples are Hibberd & Peregrine (1979), Packwood & Peregrine (1980), Packwood (1983) which focused on the general theory and the effect of porous beds on sandy beaches. Later contributions from the same group are Watson & Peregrine (1992), Watson *et al.* (1992), and Barnes *et al.* (1994).

It turns out that the NSW-equations are particularly efficient for analyzing the fully nonlinear runup in the swash zone where the bottom slope is normally steep. Operational computer models that focus on this application for breaking and nonbreaking waves have also been developed (Kobayashi *et al.* 1989, Kobayashi & Wurjanto 1992, Cox *et al.* 1992, Kobayashi & Karjadi 1994, Cox *et al.* 1994).

The advantage of the NSW-equation-method is its relative simplicity and its capability to simulate many of the features of breaking waves with plausible accuracy. As indicated above this capability includes the energy dissipation once the front has been frozen meaning the waves have started "breaking", but not where this will happen because that depends on the distance it takes the initial waves to steepen sufficiently; that process is not correctly represented by the method. Also the shape of the front once it is frozen depends on the spatial discretization used.

Boussinesq models

The second class of time domain models for surf zone waves that have been developed in recent years are based on the Boussinesq assumption that $\delta/\mu^2 = O(1)$. In these models the classical Boussinesq equations for conservation of mass and momentum have been modified in various ways to incorporate the effect of breaking.

One version adds on a heuristic basis a dissipation term $\nu_t u_{xx}$ to the momentum equation (Karambas *et al.* 1990, Zelt 1991, Karambas & Koutitas 1992, Wei & Kirby 1995).

A second approach consists of acknowledging that the breaking process modifies the horizontal velocity profiles, in particular under the turbulent front. This in turn modifies the nonlinear and dispersive terms in the momentum equation. The resulting equations are conveniently considered in the depth integrated form, which may be written as (for one horizontal dimension)

$$\zeta_t + Q_x = 0 \tag{38}$$

$$\begin{aligned} Q_t + g(h_0+\zeta)\zeta_x &+ (\frac{Q^2}{h_o})_x + \frac{h_0^3}{6}(\frac{Q}{h})_{xxt} - \frac{h_o^2}{2}Q_{xxt} \\ &+ (\Delta M)_x + (\Delta P)_{xxt} + \frac{\tau_b}{\rho} = 0 \end{aligned} \tag{39}$$

in which Q is the total discharge through a vertical, and the ΔM and ΔP terms represent the modifications of the nonlinear and dispersive terms, respectively, caused by the breaking.

Bronchini *et al.* (1992) and Schäffer *et al.* (1992, 1993) determined this effect by assuming that it was caused only by the roller which they included by using a simplified velocity profile with a discontinuity at the lower edge of the roller. They found that modifying the nonlinear terms in the momentum equation accordingly creates a wave height decay and also changes the surface profile similarly to the situation in a surf zone. Schäffer *et al.* only included the ΔM-term in (39) while Bronchini *et al.* focused on the ΔP term.

Yu & Svendsen (1995) determined ΔM and ΔP directly from the fundamental equations by assuming that the motion in the breaking waves is rotational. This leads to a formulation where the vorticity generated by the breaking is determined by solving a vorticity transport equation in addition to the equations of mass and momentum (37) and (38) that give Q and ζ. The resulting velocity profiles are continuous and hence holds the possibility of providing insight also into the flow field.

It is interesting to notice that, although the terms added to the Boussinesq equations to simulate the effect of breaking are determined in very different ways, a closer inspection of all these methods shows that numerically these terms are remarkably similar with respect to their numerical magnitude, temporal variation, and duration. One can say that in all cases the terms represent the signature of the breaking on the mass and momentum balance. It is the physical assumptions and ideas about how to determine these signatures that differ in the different methods.

Concluding Remarks

In all we find that in recent years the art of surf zone modelling has been turned into a science. Intensive research in the field, in the laboratories and on modelling, has lead to discovery and conceptualization of a large number of new mechanisms. The analysis, primarily through numerical modelling of the exact equations, and careful scrutiny of the assumptions used has strengthened the accuracy of and our confidence in the modelling capabilities. It is expected this development will continue and be supported by future direct model simulations of field conditions.

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