A Coastal Engineering Assessment of Fenwick Island, Delaware

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ABSTRACT

Fenwick Island, Delaware has experienced beach erosion for many decades; at the present time, the erosion is threatening to destroy many of the beach front homes. Hurricane Gloria accelerated the erosion process by stripping away some of the protective sand dune located between the houses and the sea.

This study focussed on the present erosion problem by examining historical shoreline positions, bathymetric surveys obtained during 1985, and hindcast wave data. Empirical eigenfunction analysis was applied to beach profiles at Fenwick Island, to show the variation of the beach over the year. Littoral transport volumes and littoral drift roses were calculated, using the Wave Information Study data set, which were used to determine the location of the littoral drift nodal point on the Delaware shoreline.

The above analyses and a simple numerical model study indicate the pervasive erosion can be reduced through the use of a groin field with beach fill.

INTRODUCTION

The Delaware Atlantic shoreline responds dynamically to the forces of nature. Storms as well as the daily onslaught of waves force the beach to change orientation and shape, as high wave energies move the beach sand either on- or offshore and along the shoreline. This shoreline reshaping presents no problems as long as there are no fixed structures on the beach; however, when communities are located on the shore, human intervention is needed to maintain both the location (and existence) of the beach and the community. Rising sea levels exacerbate the problem, as formerly safe communities, once high above storm water levels, are now within the reach of storm surges and large waves.

Most of the Delaware Atlantic shoreline is in a state of erosion. (Exceptions are Cape Henlopen, Rehoboth Beach, Bethany Beach and the area next to the south jetty at Indian River Inlet.) In 1972 the State Legislature recognized the seriousness of this erosion to the State of Delaware and enacted the Beach Preservation Act. This law gave the Department of Natural Resources and Environmental Control (DNREC), through its Beach Preservation Section, the responsibility for the enhancement, preservation and protection of the beaches by means of preventing and repairing beach erosion.

Various methods have been used by DNREC to stabilize eroding shorelines, such as groins and beach nourishment. They also cooperated with the Corps of Engineers in the Low Cost Shore Protection project, which experimented with floating offshore breakwaters and a perched beach scheme in Delaware Bay. Dune stabilization has also played a major role in beach maintenance.

The DNREC has contracted with the Ocean Engineering Group to examine the causes, effects and extent of the beach erosion at Fenwick Island and to make recommendation for erosion mitigation. This report is the culmination of this study.

The town of Fenwick Island is located on the border with the State of Maryland on a narrow barrier. See Figure 1 for the location of the town. Erosion has reduced the width of the barrier dune located on State property in front of the privately held community, making the dune highly susceptible to breaching in a major storm. Significant storm damage has not occurred in the last two decades due to the uncharacteristic absence of major hurricanes. Loss of the dune will expose the community to the high hazard of both storm surge flooding and storm waves, leading to the destruction of numerous homes in a large storm.

In this report, estimates of beach recession rates are presented based on historical records. Twenty years of U.S. Army hindcast wave data were used to estimate the amount of longshore sediment transport. An investigation was carried out for the location of the littoral transport nodal point, which is the dividing position for northward moving littoral drift along the majority of the Delaware beaches and the southward drift which prevails in Maryland. Beach profiles provided by DNREC during 1985 were used to determine the seasonal variability of the beach. Finally, recommendation are provided, which should result in reducing the beach erosion.

BEACH RECESSION RATES

Historical Estimates of Shoreline Erosion--Over the past twenty years a number of reports have been written describing various aspects of the Delaware shoreline. In these reports there have been estimates of the volumetric losses and gains along sections of the coast.

In a report by the Delaware State Highway Department (1956), estimates were given for annual erosion or accretion. These estimates can be seen in Table 1.

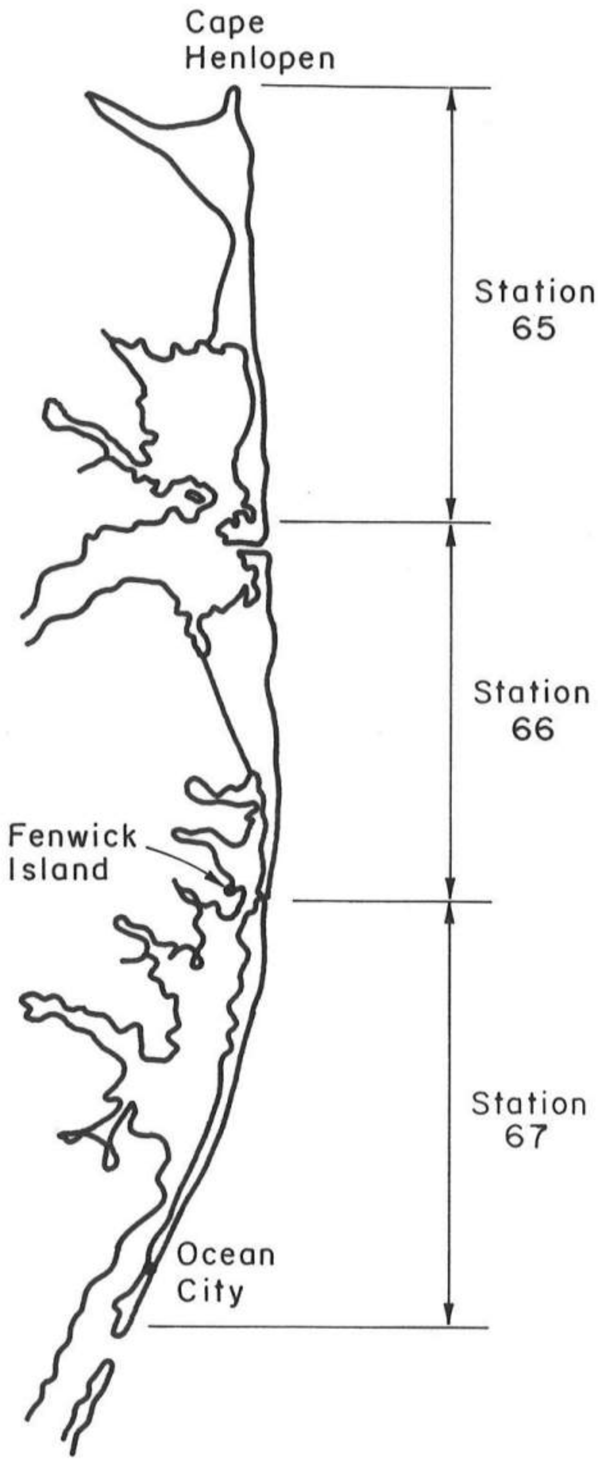


Figure 1. Schematic of Delaware Shoreline and Locations of Wave Information Study Stations.

TABLE 1

VOLUMETRIC ACCRETIONS AND EROSION FOR THE DELAWARE COAST FROM 1929-1954

Location	Annual Volume (yds^3)
Reach ½-1½ miles South of Cape Henlopen	-41,000
Reach 1½ mi. - Silver Lake	-68,000
Silver Lake - 2 mi. North of I.R. Inlet	0
2 mi. North of I.R.I. - I.R. Inlet	-130,000
I.R. Inlet - North of Bethany Beach	44,000
North of Bethany - South of Bethany	18,000
South of Bethany Beach to State Line	-54,000

The values in Table 1 are based on a twenty five year average and are calculated from plots of shoreline position. The surveys were performed by the U.S.C.& G.S. in 1929 and the Corps of Engineers in 1954. From these values, it can be seen that there is an area of high erosion just south of Bethany Beach. If a littoral drift nodal point receives no sediment, then nodal point should be an area of high erosion. Thus, the nodal point from 1929-1954 could have been located just south of Bethany Beach. The volumetric erosion data provided by the Corps of Engineers (1968) shown in Table 2 supports this conclusion. These data were calculated by determining the area changes in beach profiles from one survey period to the next and then multiplying the result by the distance between the profiles. The measurement profiles are shown in Figure 2.

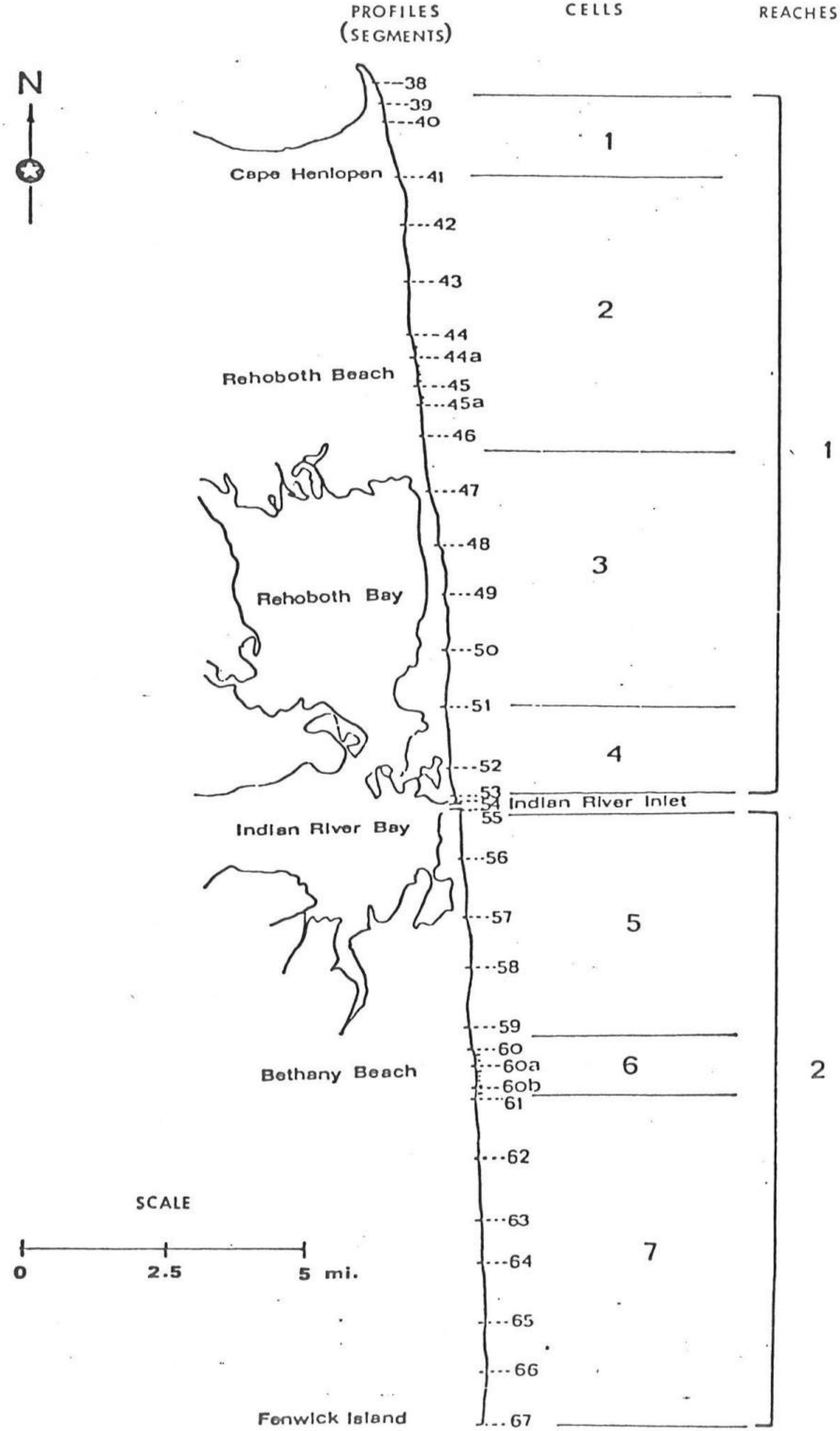


Figure 2 - Profile Locations and Definitions of Shoreline Elements.

TABLE 2

VOLUMETRIC EROSION AND ACCRETION FOR INDIAN RIVER INLET(I.R.I.) TO THE DELAWARE/MARYLAND STATE LINE

(yds^3 /mile/year)

Location	1843-1929	1929-1954	1954-1964	1843-1964
Profiles 55-60 (IRI to Bethany)	-25,890	10,450	-13,065	-17,346
Profiles 60-61 (Bethany Beach)	-34,880	20,930	2,325	-19,770
Profiles 61-67 (South Bethany to Fenwick Island)	-16,640	-9,360	-43,330	-17,330

The most recent data from Table 2 indicates the area between Profiles 61 and 67 is an area of high erosion. The earliest data indicates that Bethany Beach was an area of high erosion and the 121 year average (1843-1964, the rightmost column in the table) indicates that the area would be an average nodal point. However, the 121 year averaged is highly biased since large amount of erosion occurred from 1843-1929 when compared to the rest of the time period (1929-1964). It should be noted that during 1954-1964 the profiles north of the Bethany profiles showed accretion while the profiles to the south showed significant erosion. This implies that the nodal point has shifted to the south during this time. The accretion at Bethany may be in part due to the repair of nine groins in 1958 (Dalrymple et al.,1976).

In the Dalrymple et al. report, data from the Corps of Engineers 1956 report are presented, which indicates that the area of highest erosion occurs at profiles 21 and 22 (York Beach-Fenwick Island State Park) during 1929-1954.

Coastal and Offshore Engineering and Research Incorporated (COER, 1983) calculated the average rate of change in the volume of sand for various profiles along the Delaware coast between 1964-1982. Some of the results are presented in Table 3.

TABLE 3

SEDIMENT TRANSPORT RATES FOR THE DELAWARE COAST(1964-1982)

Profile	Change (yds^3/yr)
67	-17,893
66	-33,247
65	-36,482
64	-48,388
63	-28,732
62	-3,160
61	-4,899
60	-6,293

From Table 3 it can be seen that the profile with the largest erosion is profile 64. In the COER report, the estimates are based on bathymetric surveys, which unfortunately had no depth of closure. An attempt by COER to alleviate the depth of closure problem produced results that profile 65 had the greatest erosion. When looking at Table 3, the exact numbers are not as important as the relative magnitudes of the values since there is an error in the estimates due to the lack of closure. Profile 64 corresponds to York Beach and Profile 65 corresponds to approximately 0.75 miles to the south of York Beach (Fenwick Island State Park).

Procedure--The calculation of historical erosion rates provides reliable data for predicting future recession. One method of determining these rates relies on the use of aerial photographs, which preserve the shape of the shoreline. Photographs, taken since 1926, were provided by DNREC, for determination of the movement of the shoreline over a 41 year period.

The dates of the ten aerial photographs are as follows: 1926 (U.S. Army Air Corps); May 7, 1938 (U.S.D.A. Agricultural and Conservation Service, ASCS) ; February 1, 1954 (ASCS); July 20, 1954 (ASCS); November 12, 1960 (ASCS); March 8, 1962 (Corps of Engineers); May 2, 1968 (ASCS); May 17, 1977 (ASCS); June 5, 1979 (DNREC). The lack of suitable fixed controls prohibited the use of the 1926 photo.

A set of landmarks (buildings, roads, canals) was established in order to reference the shoreline. By using the same landmarks for photographs of different years, it is possible to see shoreline variations with respect to time.

The shoreline position for each photograph was overlayed with the use of a Bausch and Lomb Zoom Transfer Scope (Model ZT-4). This device permits the viewing of two photographs simultaneously; thus, by enlarging one of the photographs so that it is same scale as the other photograph, an accurate comparison can be made.

When overlaying the shoreline positions, the high water line (HWL) was used as the shoreline position. The HWL was used rather than the actual water line for the following reasons. The waterline is influenced by tidal variations which would have to be taken into account when calculating the actual shoreline position. Nonuniform wave runup could cause inaccuracies. The HWL is detected easily by a change in color between the moist sand seaward of the HWL and the dry sand landward of the HWL (Stafford and Langfelder, 1971). This change in color is detectable for both black and white and color photographs. The HWL responds quickly to erosion or accretion and is not greatly affected by tidal variations.

The photographically determined shorelines represent the position of the HWL at the time of the photograph. Seasonal variations, as well as storm-induced short-term shoreline changes, are interpreted by this method as part of the long-term erosional trend. However, the use of a long period of time and a sufficient number of photographs averages out the short-term changes and provides a reliable estimate of the shoreline recession.

The shoreline positions can be seen in Figures 3 and 4. The error in the overlaying is approximately the width of the drawn line which is approximately 9.5 feet. As shown in the two figures, there are gaps in the shoreline where it was impossible to accurately determine the shoreline. For example, in some of the March 1962 photographs, the shoreline was not shown since the photograph was probably taken to show the destruction caused by the storm. Also the 1926 photograph was not used since Route 1 was neither in its present position nor straight .

Shoreline Comparisons--From May 1938 to February 1954, the beach receded. The July 1954 shoreline was approximately in the same position as the May 1938 shoreline, so it appears that from 1938 to 1954 the beach was stable, and the comparison of the February and July 1954 shorelines gives a good estimate on the seasonal variations of the beach during 1954. (Shoreline recession calculations should be performed with photographs of the same season.)

The 1954, 1960, 1962 shorelines indicate that the beach was receding during these years. A maximum recession of 190 feet occurred at one point, located 600 feet north of Route 54, between the 1938 shoreline and the 1962 (post storm) shoreline. After the March, 1962 storm, the beach took approximately three and one half years to recover to the 1960 shoreline position. This includes the beach nourishment by the State of Delaware. The 1968 shoreline shows that the beach never did recover back to the 1954 position.

Comparing the 1968 shoreline with the 1977 shoreline, the latter shoreline shows that some areas of the beach were eroding while other areas were accreting. Between 1977 and 1979, the

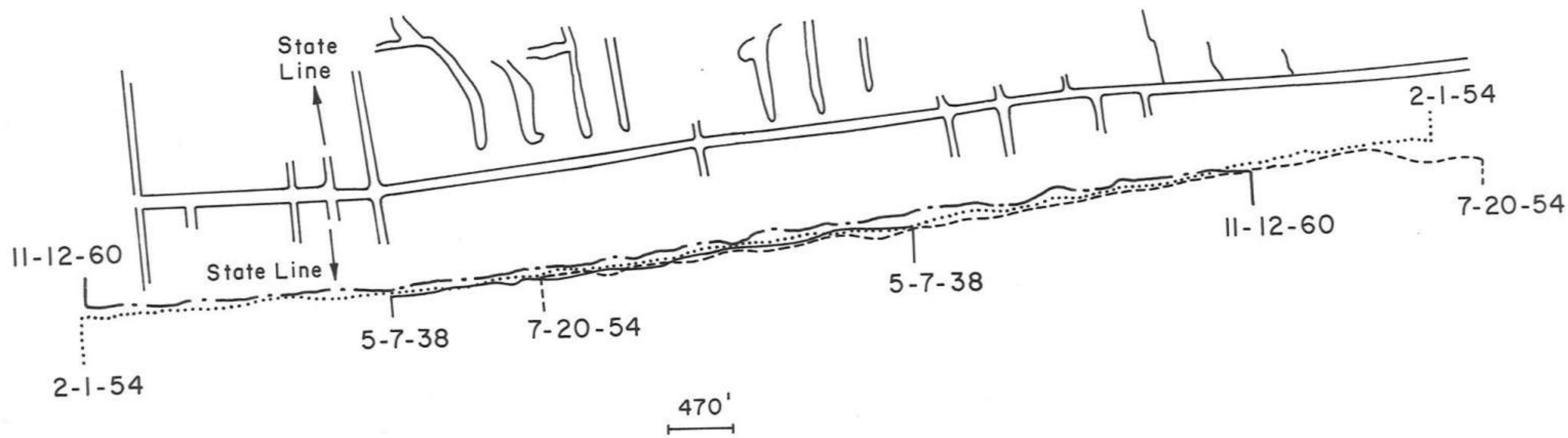


Figure 3. Historical Shoreline Positions
1938 - 1960

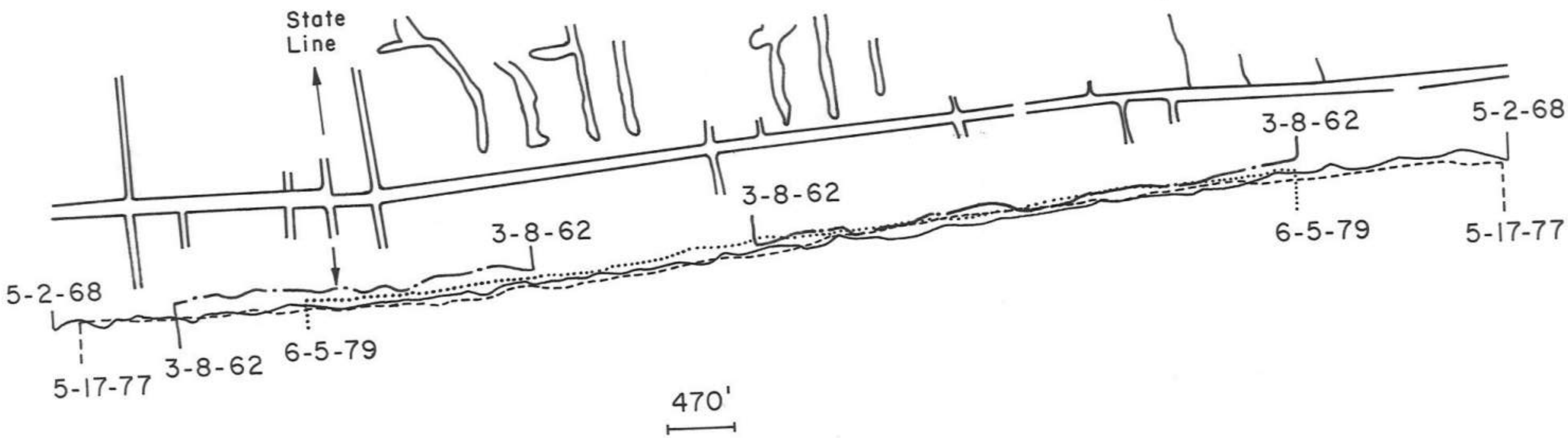


Figure 4. Historical Shoreline Positions
1962 - 1979

beach underwent severe erosion with some parts of the 1979 shoreline landward of the shoreline position just after the March 1962 storm! Between May 1977 and June 1979 the shoreline was eroding at an annual rate of 31.5 feet/year (standard deviation is 18.6 feet/year)!

Based on a forty-one year average (1938-1979), the beach is eroding at a rate of 1.7 feet/year (with a standard deviation of 0.8 feet/year). This agrees quite well with the value of 1.9 feet/year (standard deviation of 3.9 feet/year) that Hayden et al. (1979) found for the entire barrier.

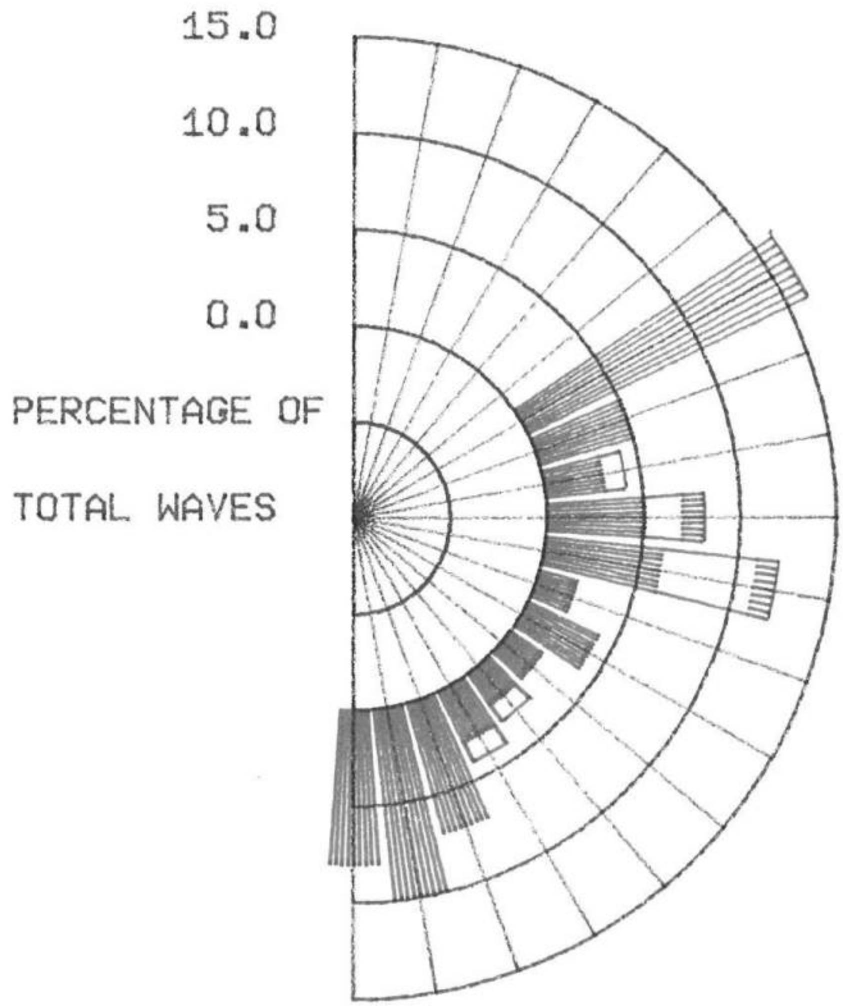
From the analysis of the aerial photographs, the beach at Fenwick Island has historically eroded at 1.7 feet/year and the evidence suggests that the current erosion could be much higher. A current aerial photograph would clarify whether the erosion between 1977 and 1979 was a short term trend or whether it is typical of the current erosion problem.

WAVE INFORMATION STUDY AND SEDIMENT TRANSPORT CALCULATIONS

Delaware's Wave Climate--In order to verify the recession rates calculated by the aerial photogrammetric method, the shoreline recession was also calculated with a sediment transport model. The sediment transport model requires that the wave climate for the specific area be known. In the past there have been wave data bases available for coastal studies, but they were usually averages of the total wave climate and were not site specific (eg, Goldsmith et al.,1974; U.S. Naval Oceanographic, 1963). Recently the Corps of Engineers have created a data base that accurately simulates nearshore wave conditions for sections of the Atlantic coast.

The Wave Information Study (WIS) provides the most accurate hindcast wave data for the Atlantic coast. The study is broken into three sections: a deepwater swell generating area, a transition region, and a nearshore region. Phase III of the study, the nearshore region, represents hindcast wave data (wave height, period, and direction) calculated for every three hours for 1956 to 1975. The data is provided for every ten miles of coastline at a depth of 30 feet. For this study, data from Stations 65, 66, and 67 were used (Figure 1). The depth of thirty feet was chosen by the Corps of Engineers because it accurately represents the nearshore wave conditions and it is only slightly effected by tides or storm surges. The WIS data set currently does not include the effects of tropical storms. The formulation of the WIS data base can be seen in Appendix 1.

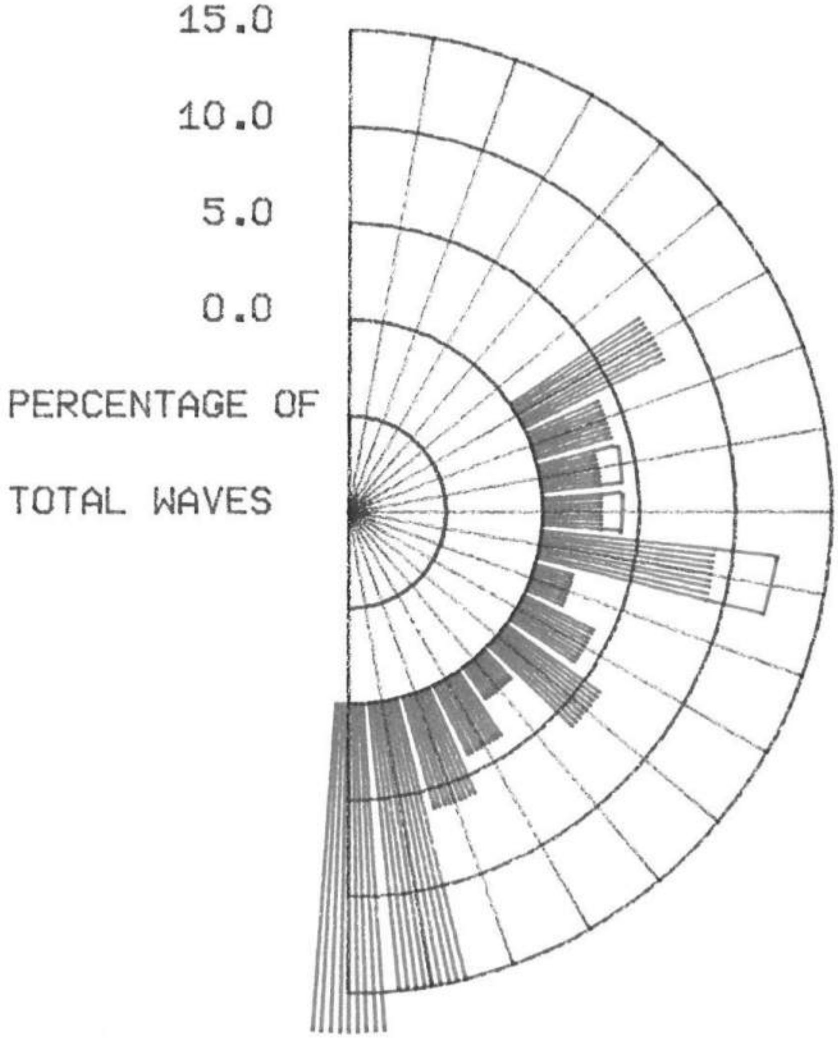
In order to gain a better insight into the wave climate off the Delaware coast, a statistical analysis was performed using the WIS data. Seasonal wave roses were constructed for Stations 65, 66, and 67. They can be seen in Figures 5 to 16. In these figures there is no plot for any percentage that was less



STATION 65 SEASONAL WAVE ROSE

SEASON: WINTER
BASED ON 20 YEARS OF DATA

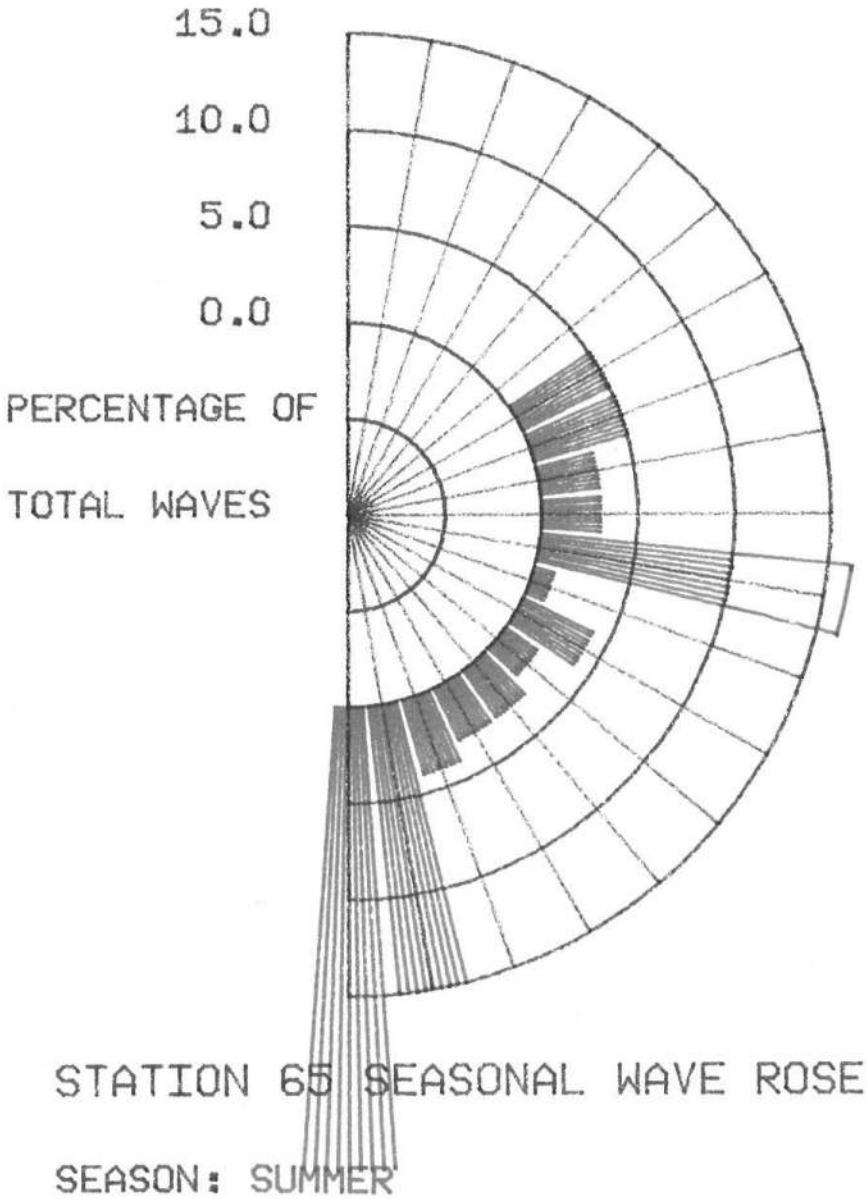
FIGURE 5



STATION 65 SEASONAL WAVE ROSE

SEASON: SPRING
BASED ON 20 YEARS OF DATA

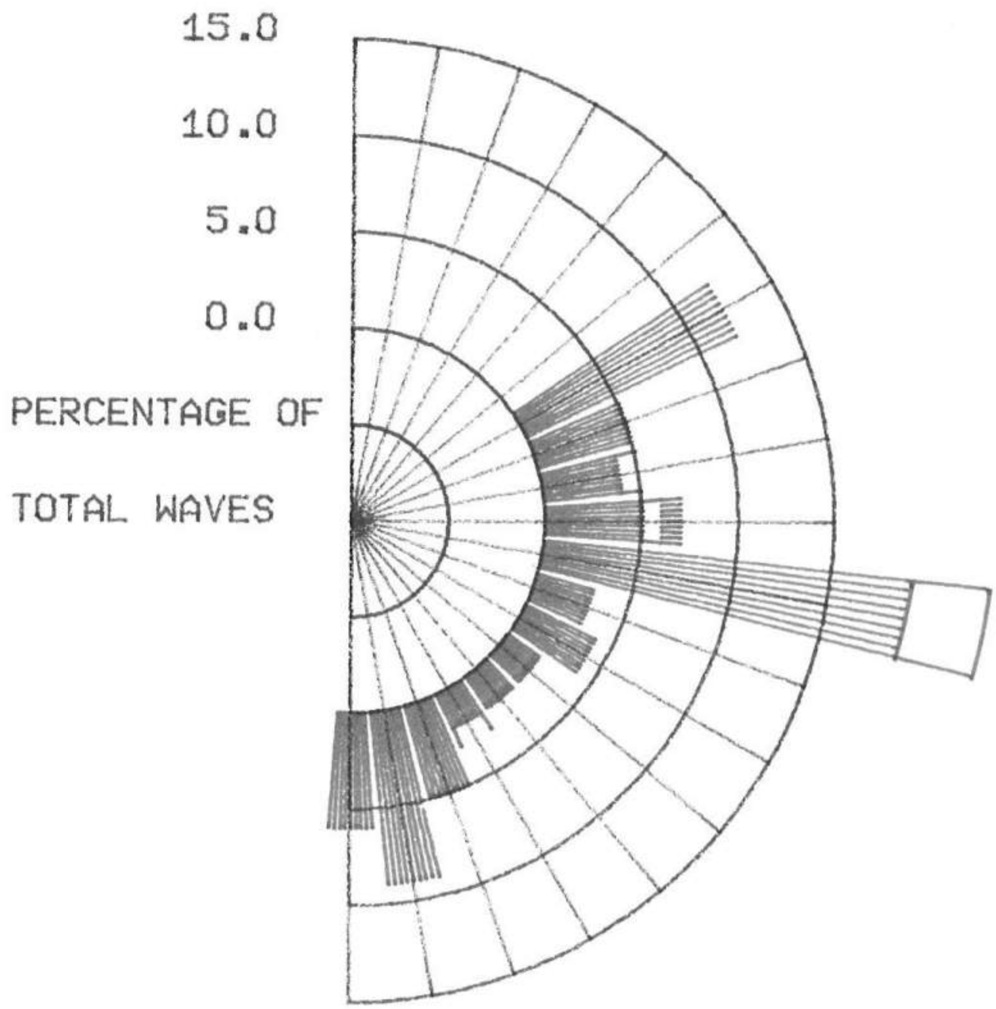
FIGURE 6



STATION 65 SEASONAL WAVE ROSE

SEASON: SUMMER
BASED ON 20 YEARS OF DATA

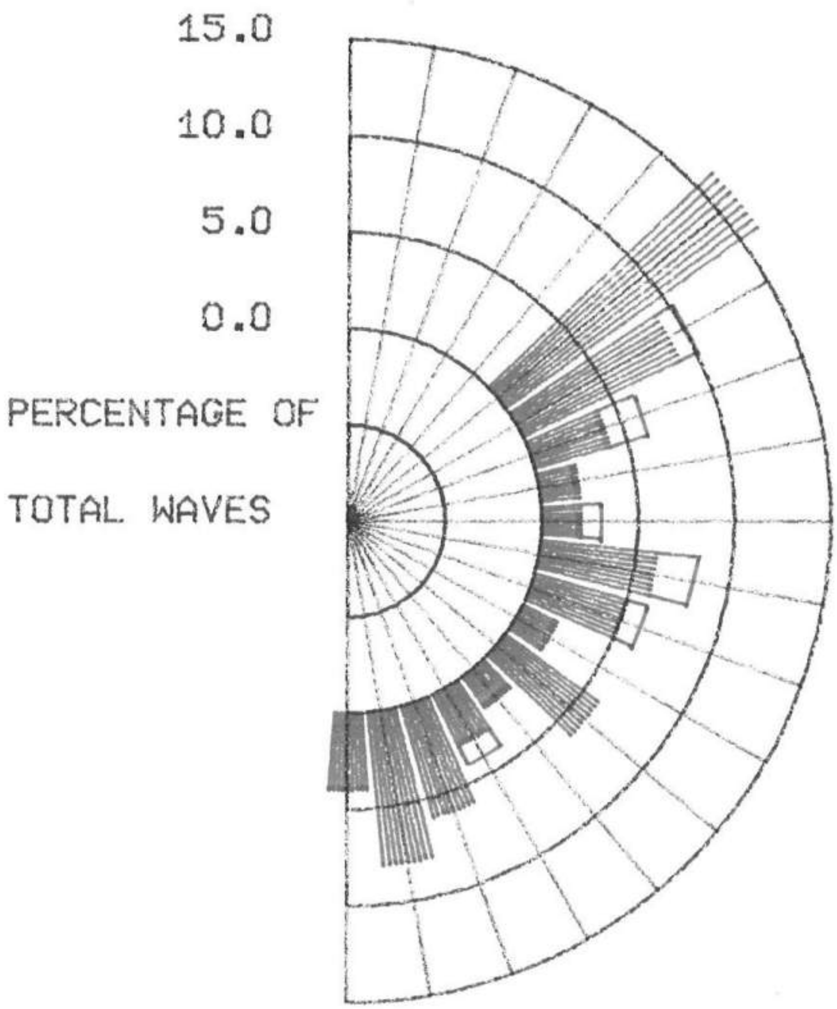
figure 7



STATION 65 SEASONAL WAVE ROSE

SEASON: FALL
BASED ON 20 YEARS OF DATA

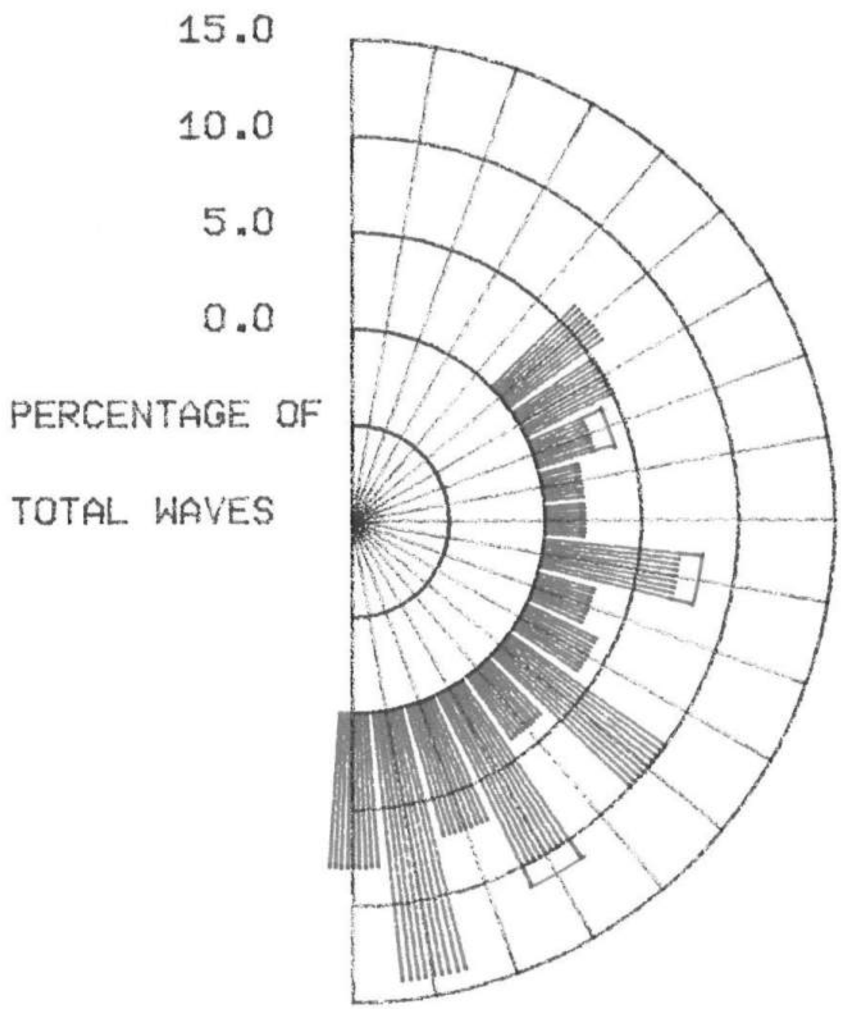
figure 8



STATION 66 SEASONAL WAVE ROSE

SEASON: WINTER
BASED ON 20 YEARS OF DATA

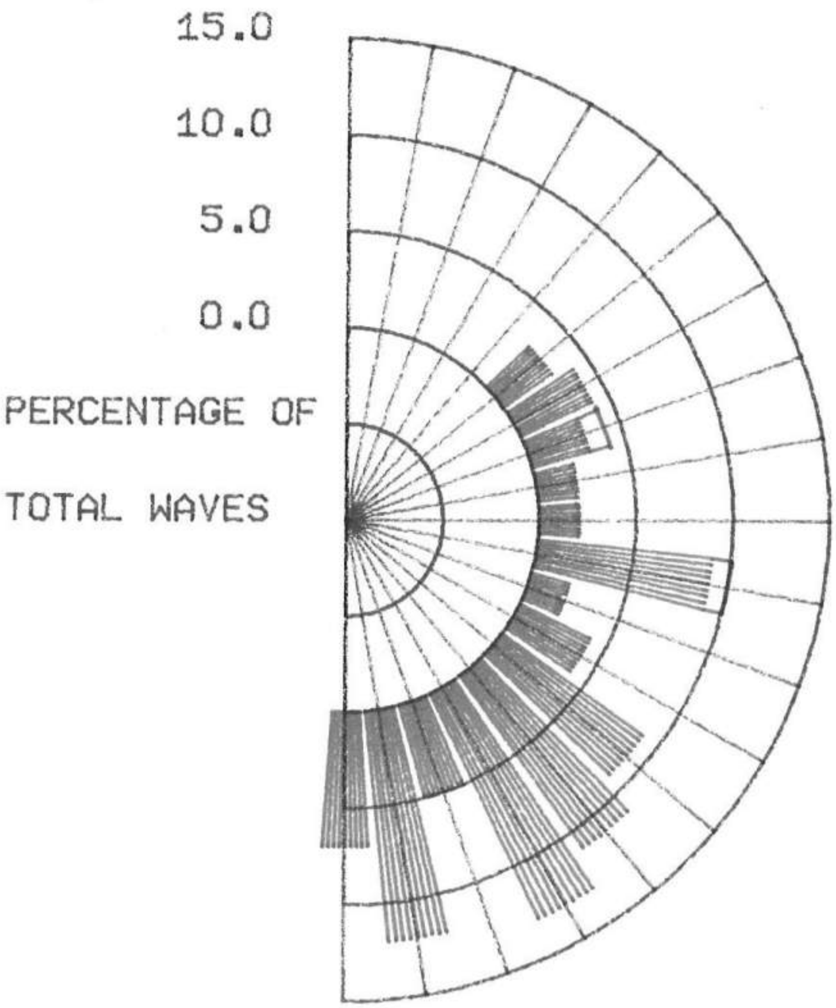
figure 9



STATION 66 SEASONAL WAVE ROSE

SEASON: SPRING
BASED ON 20 YEARS OF DATA

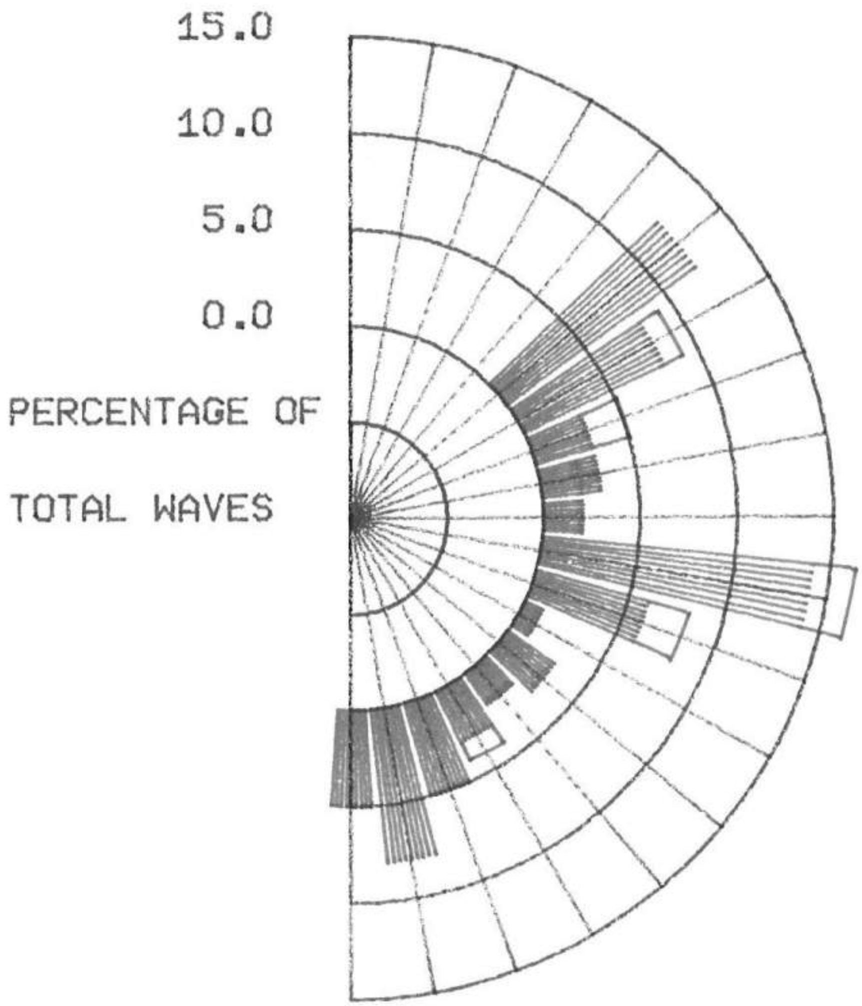
figure 10



STATION 66 SEASONAL WAVE ROSE

SEASON: SUMMER
BASED ON 20 YEARS OF DATA

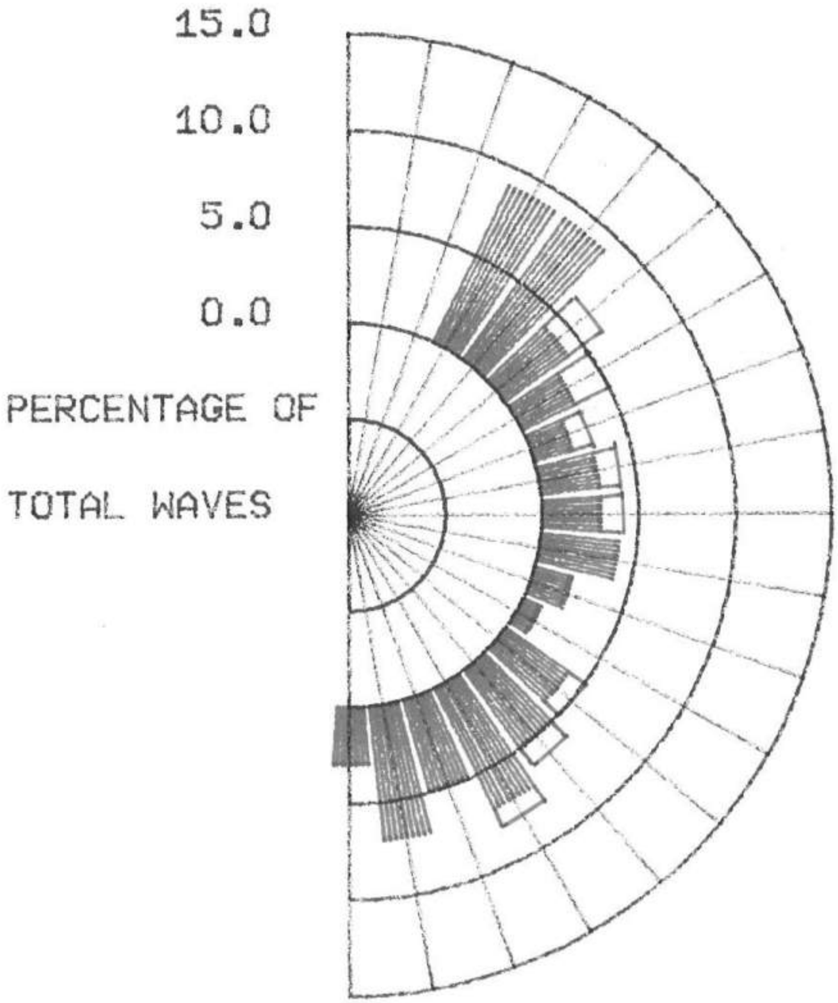
figure 11



STATION 66 SEASONAL WAVE ROSE

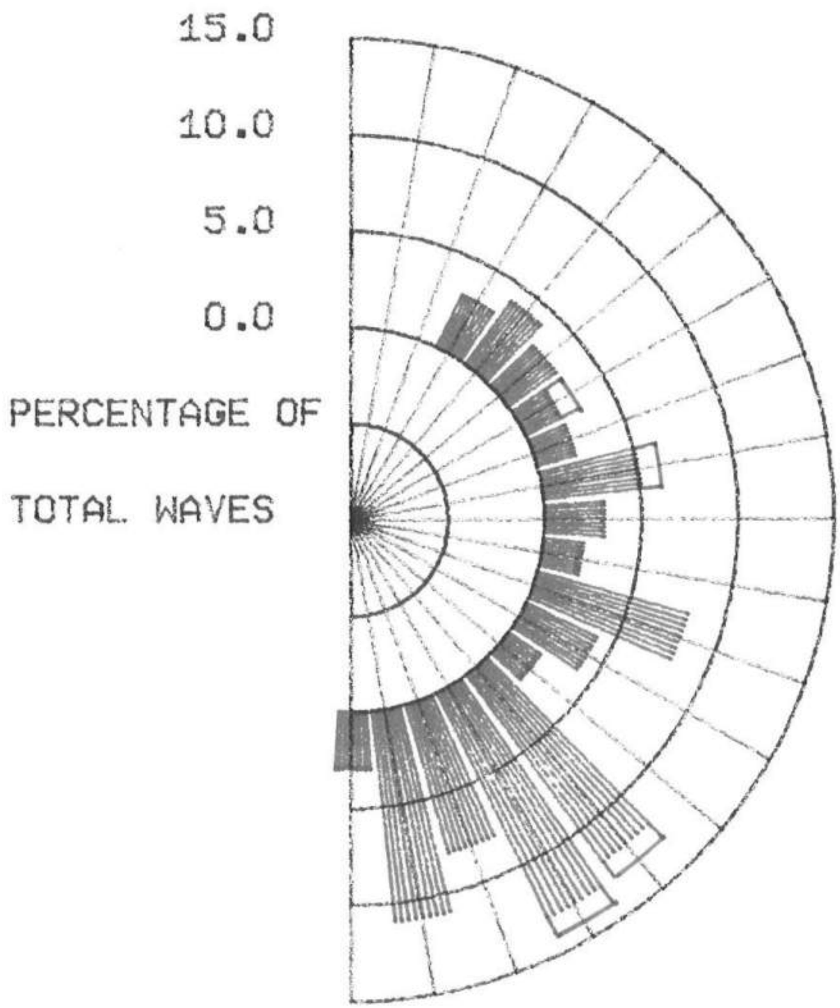
SEASON: FALL
BASED ON 20 YEARS OF DATA

figure 12



STATION 67 SEASONAL WAVE ROSE

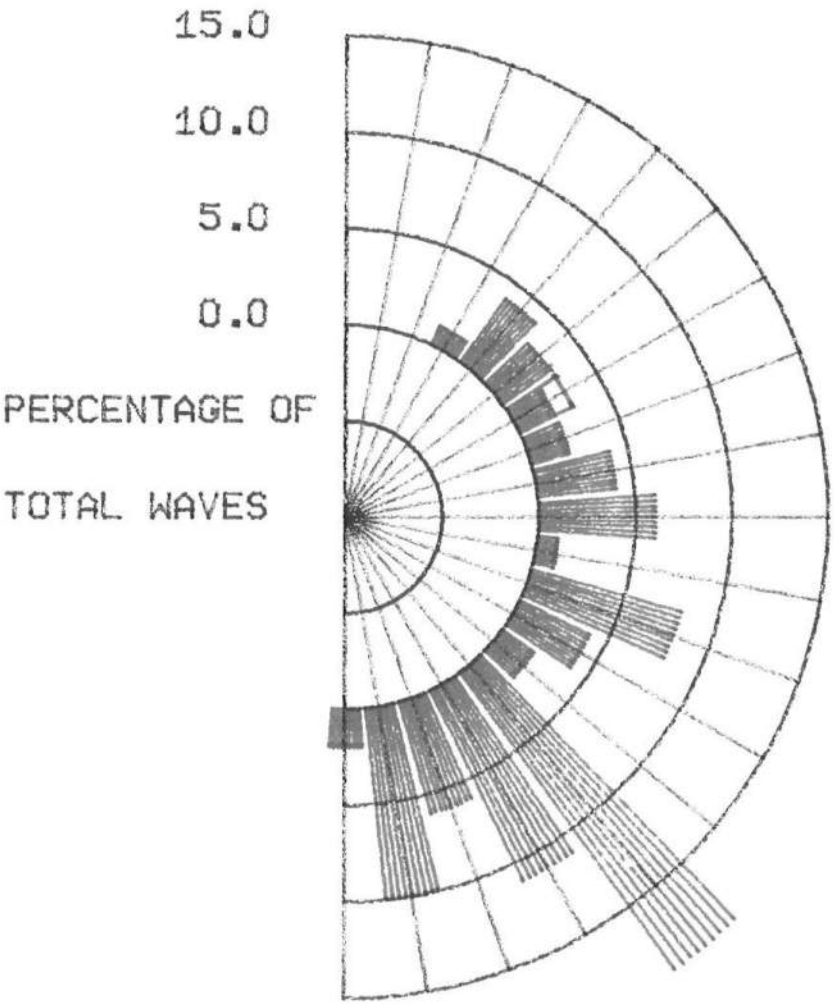
SEASON: WINTER
BASED ON 20 YEARS OF DATA
figure 13



STATION 67 SEASONAL WAVE ROSE

SEASON: SPRING
BASED ON 20 YEARS OF DATA

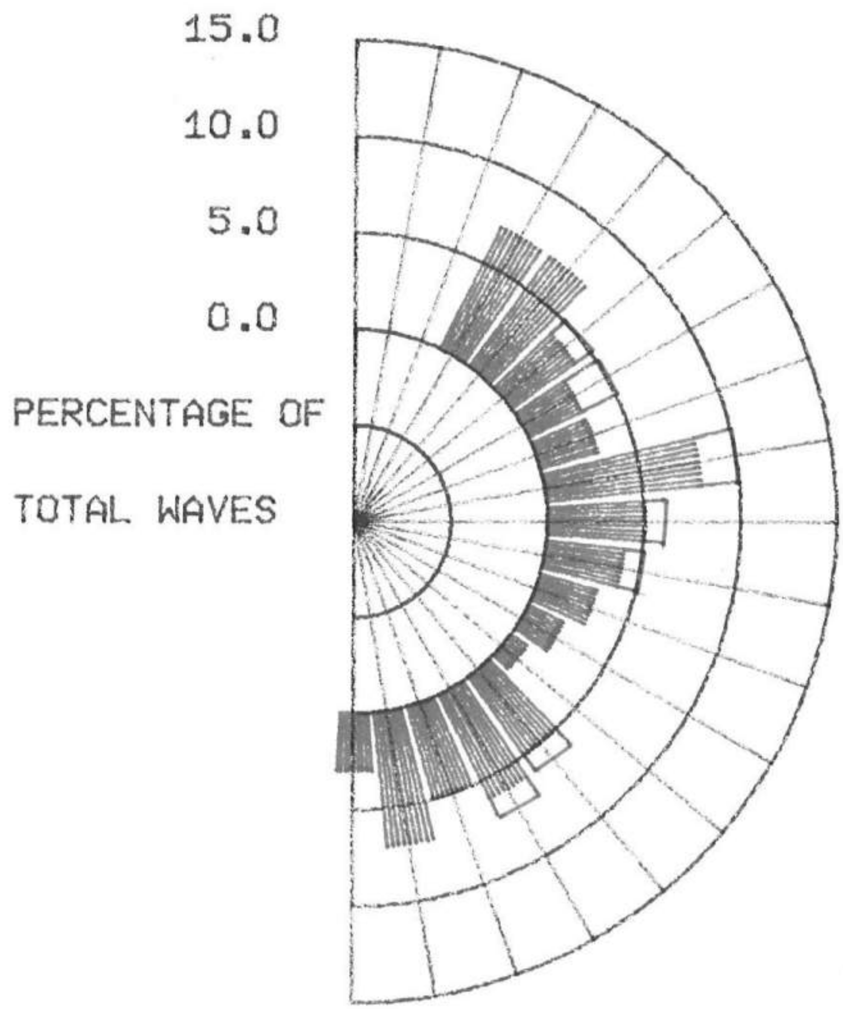
figure 14



STATION 67 SEASONAL WAVE ROSE

SEASON: SUMMER
BASED ON 20 YEARS OF DATA

figure 15



STATION 67 SEASONAL WAVE ROSE

SEASON: FALL
BASED ON 20 YEARS OF DATA

figure 16

than one percent. The seasons are as follows:

Winter: January to March
Spring: April to June
Summer: July to September
Fall: October to December

The seasons correspond to the littoral drift patterns that were found at Bethany Beach (Dick and Dalrymple, 1983). When comparing the wave roses for Station 65 to Stations 66 and 67, it can be seen that there is some sheltering effects due to the presence of the Delaware Bay and Cape May, New Jersey.

During the winter the predominant wave direction is from the east and northeast, while in the summer the waves come from the east and southeast. The littoral drift should be to the south during the winter and to the north in the summer. Delaware's sand supply is being depleted since the littoral drift towards Maryland in winter is greater than the return drift during the summer. The wave roses show a large percentage of waves come from due south. These waves probably don't exist, but are predicted by the Corps of Engineers' model.

Sediment Transport Model--In order to estimate the amount of shoreline recession, it is necessary to be able to calculate the amount of sediment transport along the shoreline based on a given wave climate. The sediment transport model outline below follows the wave energy flux method as described by Coastal and Offshore Engineering and Research, Inc. (1983).

The wave heights provided in the WIS data are the four sigma (4σ) wave heights. According to the Shore Protection Manual (1977), the correct wave height to use for sediment transport models of the type that is used in this study, is the root-mean-square wave height, H_{rms}. The significant wave height is approximately equal to the four sigma wave height

$$H_s = H_{4\sigma} \tag{1}$$

and that the root-mean square wave height is related to significant wave height by

$$H_{rms} = H_s / \sqrt{2} \tag{2}$$

Therefore, dividing the WIS wave height data by $\sqrt{2}$ is necessary for the sediment transport calculations. To simplify the analysis, the sea and swell components were converted into one monochromatic unidirectional wave, as done by Jensen, 1983.

$$H = \sqrt{(H_{sea}{}^2 + H_{swell}{}^2)} \tag{3}$$

The period and direction of the resultant wave are equal to the period and direction of the sea or swell component with the larger wave height. This data given at a depth, d must be transformed to the breaker depth, d_b. (Subscript b denotes condition at breaking.)

Using linear wave theory and the assumption of straight and parallel bottom contours, the breaking wave height, H_b is given by

$$H_b = H\ K_r\ K_s \tag{4}$$

where K_s is the shoaling coefficient, and K_r is the refraction coefficient.

$$K_s = \sqrt{(C_{g\,b} / C_g)} \tag{5}$$

where C_g is the wave group celerity.

$$K_r = \sqrt{(\cos(\Theta_b) / \cos(\Theta))} \tag{6}$$

For Equation (6),

$$C_{g\,b} \approx \sqrt{g\ d_b} = \sqrt{g\ H_b / \kappa} \tag{7}$$

where, κ is the wave breaking criterion, defined by

$$\kappa = 0.78 \tag{8}$$

We also have

$$\cos(\Theta_b) = \sqrt{(1 - \sin^2(\Theta_b))} = \sqrt{(1 - \sin^2(\Theta)\ C_b^{\,2}/C^2)} \tag{9}$$

by Snell's law. Equation 4 becomes:

$$H_b = H\sqrt{Cg/\sqrt{g\ Hb/\kappa}}\quad \sqrt{\cos\theta \Big/ \sqrt{1 - \sin^2\theta \left(\frac{Cb}{C}\right)^2}} \tag{10}$$

To determine H_b we define the function, G

$$G = H_b - \text{R.H.S. of equation 16} \tag{11}$$

Equation 10 is solved iteratively for H_b using the secant method and a 1% relative change convergence test for H_b was used. The breaker wave angle is given by:

$$\Theta_b = \sin^{-1}(\sin(\Theta)(C_b/C)) \tag{13}$$

The volumetric sediment transport rate is given by:

$$Q = C\ H_B^{\,2.5}\ \sin(2\ \Theta_b) \tag{14}$$

where $C = K\rho g^{\frac{1}{2}}/((\rho_s - \rho)\ (1 - P)\ 16\kappa^{\frac{1}{2}}) = 0.325\ \text{feet}^{.5}/\text{second}$ (15)

The following constants were used in equation 21: K=0.77 (Komar and Inman, 1977); g= gravity=32.17 ft/sec^2; ρ = density of water= 1.99 slugs/ft^3; ρ_s = density of sediment= 5.14 slugs/ ft^3;

P=porosity of the sediment=0.4; and κ = breaking wave criterion, taken as 0.78.

Sediment Transport Results--Potential sediment transport rates for the Delaware-Maryland coast were obtained by using twenty years of WIS data from Stations 65, 66 and 67 along with Equation 14. Every three hours, the sediment transport rate was calculated and then summed for the year. The results for net littoral transport rates are shown in Table 4. The littoral drift rates are referred to as potential because the empirical equation (14) implies an availability of transportable material and the WIS wave climate is only an approximation to the actual data.

TABLE 4

ANNUAL SEDIMENT TRANSPORT FOR DELAWARE-MARYLAND COAST
IN CUBIC YARDS PER YEAR.

YEAR	STATION 65	STATION 66	STATION 67
1956	-63720	272426	1131544
1957	-113292	-40295	110485
1958	-176954	-450115	175900
1959	-172608	-175944	-43191
1960	-113479	67417	338391
1961	-96657	100242	245857
1962	-67646	584679	869985
1963	-130146	35547	153723
1964	-75295	230978	229643
1965	-88862	83786	167369
1966	-191221	-20213	83561
1967	-47625	79274	143494
1968	-188871	44609	76025
1969	-119023	396041	798964
1970	-149771	-65215	187657
1971	-225432	-72666	134313
1972	-201196	126919	272020
1973	-420150	-160474	-210039
1974	-175128	-36368	-48388
1975	-243095	136825	666079
Mean	-153000	56900	274200
Standard Deviation	84530	217610	336178

Note: the negative sign(-) denotes littoral drift to the north.

As shown in Table 4 there are large annual fluctuations in sediment transport, with the standard deviations in Stations 66 and 67 greater than the mean! As expected, there is a change in direction of the littoral drift along the Delaware-Maryland coast. Calculating the sediment transport difference between Stations 66 and 67 yields a mean net annual transport of 217,300 $yds^3/yr.$ to the south at Fenwick Island (standard deviation = 226,800 $yds^3/yr.$). If there are no other sources of sand, this implies that the Fenwick Island area is losing 3.6 cubic yards of sand per foot of beach to the State of Maryland each year.

In order to better understand the implications of the varying sediment transport rates, a conversion from a volumetric transport rate to a beach recession rate, using the control volume in Figure 17, is calculated.

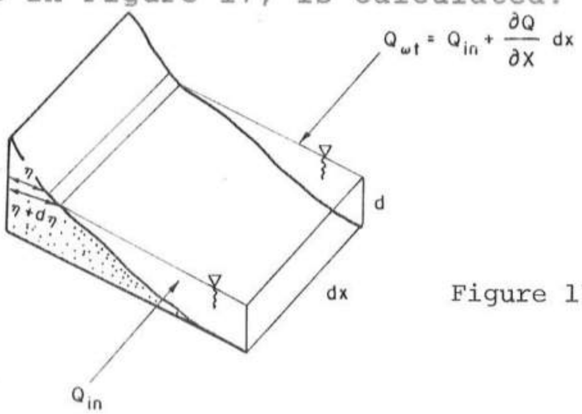


Figure 17. Control Volume for Conservation of Sand Argument.

Assuming a mild sloping beach , the change in volume of sand with time due to a change in shoreline position is equal to $\frac{\partial \eta}{\partial t}$ (d dx) where η is the distance from a datum to the shoreline, t is time, dx is an incremental shoreline distance, and d is the depth of closure. The depth of closure is a fictitious depth, which denotes the depth beyond which there is no sediment transport. As will be discussed later, this depth can only be surmised by theory and experience as it can (or has) not been measured. The volume of sand eroded or accreted per unit time is equal to the difference in the rate at which sediment is carried into the control volume minus the rate at which sediment is carried out, $\frac{\partial Q}{\partial x}$ dx.

Equating volumes,

$$\frac{\partial \eta}{\partial t} (d\ dx) = \frac{\partial Q}{\partial x} dx \tag{16}$$

Simplification of Equation 16 yields,

$$\frac{\partial \eta}{\partial t} = \frac{1}{d} \frac{\partial Q}{\partial x} = R \tag{17}$$

where R is defined as the recession rate. Equation 17 is the conservation of sand equation (eg, Dean and Maurmeyer, 1983).

To determine the recession rates at Fenwick Island, dx was taken as ten nautical miles (five nautical miles north of the state line and five miles south of the state line). Since the state line happens to lie on the division between Station 66 and 67, the sediment transport rates for these two stations were used as the influx and efflux of the Fenwick Island control volume. The net sediment transport was determined by subtracting the sediment transport rate at Station 66 from the sediment transport rate at Station 67.

Based on a depth of closure of thirty feet, the sediment transport values in Table 4 indicate a mean recession rate of 3.2 feet per year (standard deviation = 3.3 feet per year). If the depth of closure is taken as twenty five feet, the recession rate is 3.8 feet per year. Since the recession rate of 3.2 feet per year is approximately the same as the recession rate of 1.7 feet per year found by aerial photogrammetry (allowing for errors due to potential versus the actual littoral drift), there would appear to be negligible net onshore offshore sediment transport. Thus, the primary mode of sediment transport at Fenwick Island is taken as alongshore.

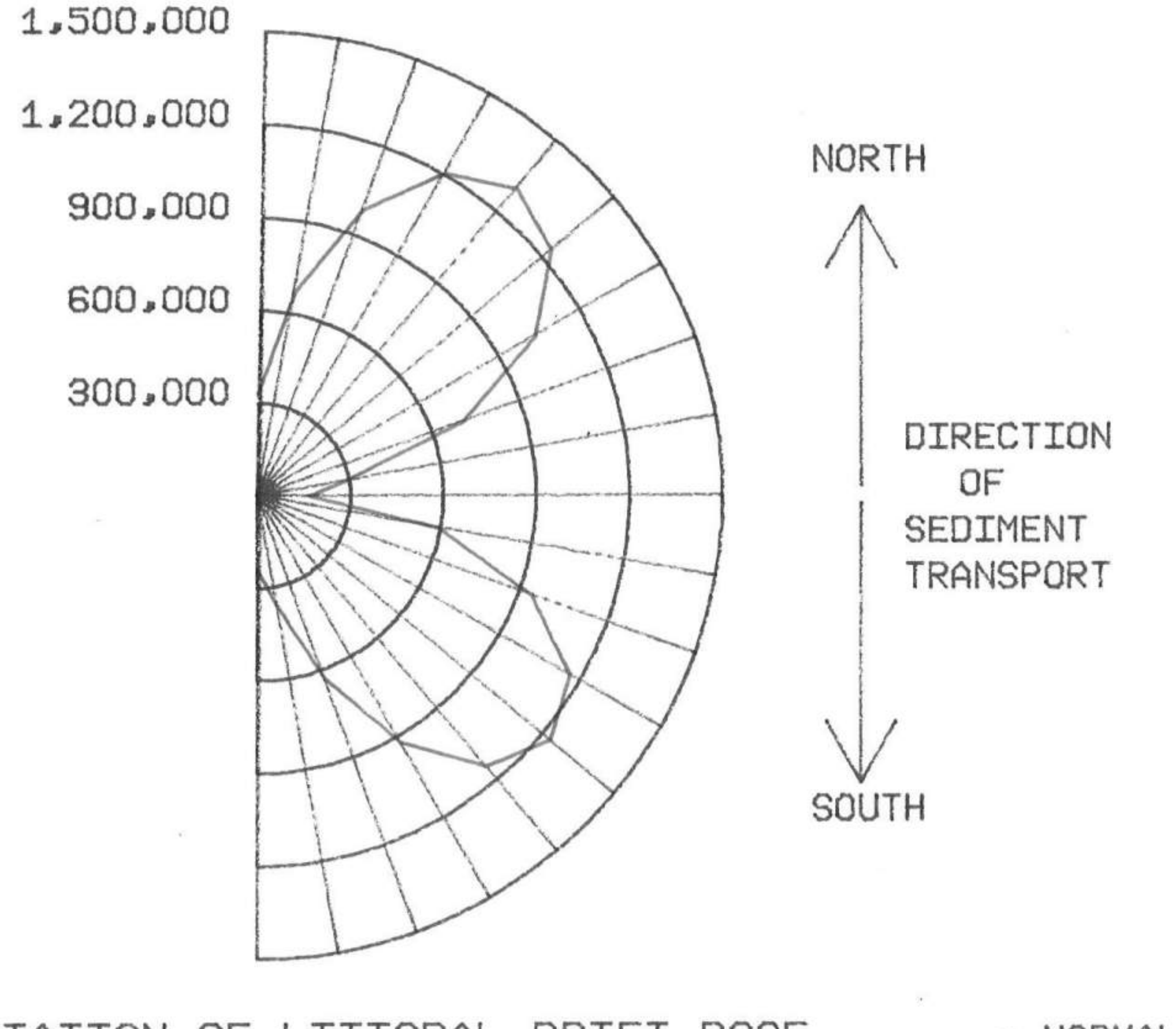
Littoral Drift Roses--Since beach planforms are constantly changing, it is advantageous to have a tool that predicts littoral drift under any orientation of the beach. The littoral drift rose (Walton and Dean, 1973) serves this purpose, by providing a convenient way to determine the littoral drift on a shoreline of arbitrary orientation for a given wave climate. Littoral drift roses were constructed for each of the WIS stations, using the previously described sediment transport model, by varying the beach normal orientation from due South to due North, and averaging the wave data over the twenty year history. The net littoral drift roses can be seen in Figures 18, 19, 20. The magnitude of the potential littoral drift at a site can be determined by overlaying a radial line on the littoral drift rose in the direction of the beach normal. Comparing the littoral drift roses, the magnitude of littoral drift decreases from stations 65 to 67.

The use of the littoral drift roses as a prediction tool is illustrated in the following example. Suppose the sediment transport was desired near the tip of Cape Henlopen, Delaware. The angle between the shoreline normal and due north is measured (clockwise is positive) as 45 degrees. Using the Station 65 littoral drift rose, the sediment transport is read from the curve as 1,300,000 yd^3/year to the north, which is likely to be too high. It should be remembered that the model is based on sediment transport induced by waves only and does not include the tidal effects caused by Delaware Bay. This probably accounts for the over-prediction at Cape Henlopen.

FIELD INSPECTIONS AND BATHYMETRIC SURVEYS

The beach at Fenwick Island has been inspected three times. In February, the beach was in its winter profile with a narrow berm. The day of the inspection was after a recent storm, so there was a one foot scarp at the base of some of the dunes.

A plot of the Absolute value of littoral drift versus beach normal orientation.



STATION 65 LITTORAL DRIFT ROSE

VOLUME IN CUBIC YDS PER YEAR
BASED ON A 20 YEAR AVERAGE
VERSUS BEACH NORMAL ORIENTATION

FIGURE 18

A plot of the absolute value of littoral drift versus beach normal orientation.

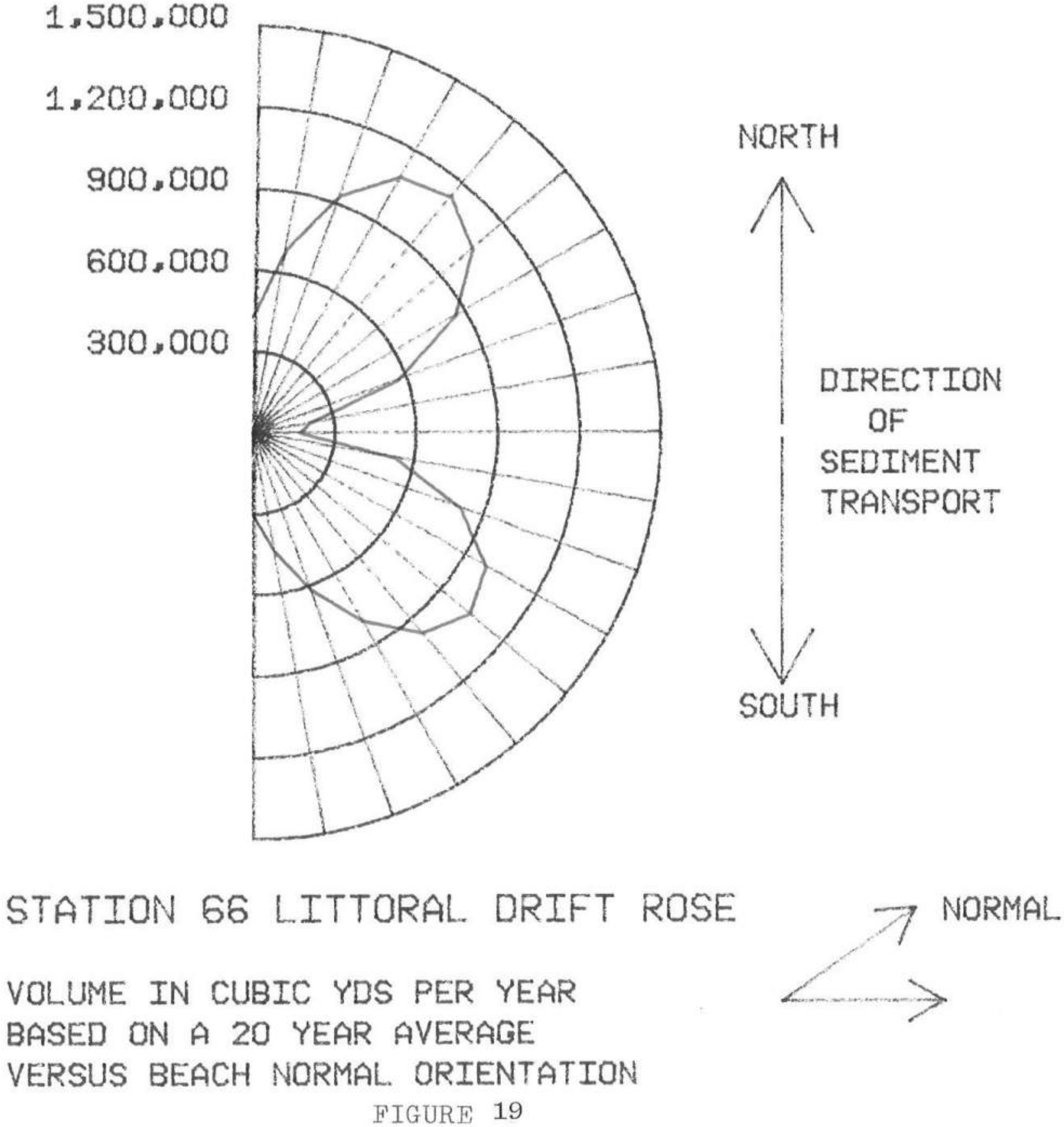


FIGURE 19

A plot of the absolute value of littoral drift versus beach normal orientation.

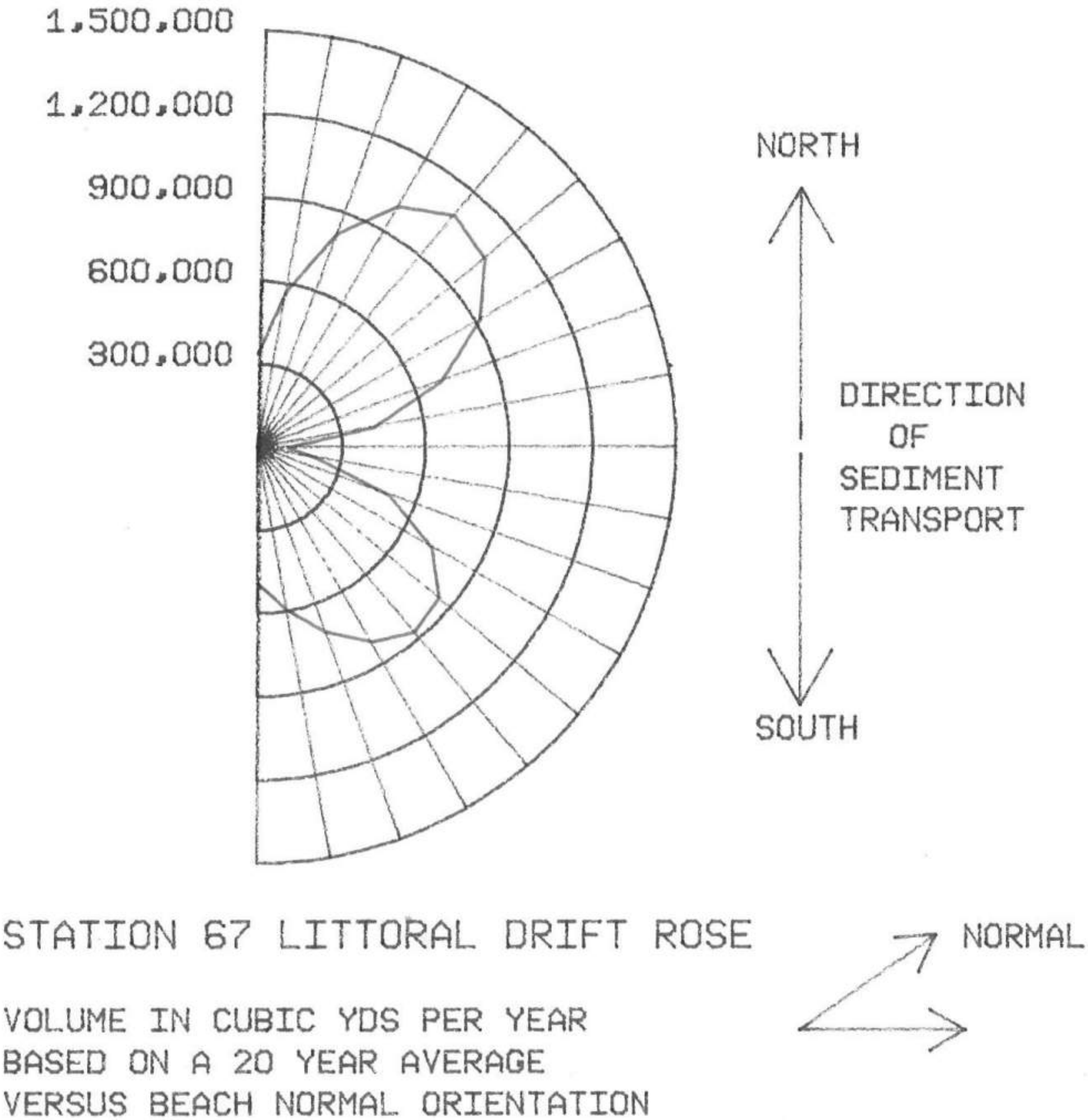


FIGURE 20

FIGURE 21. Beach Front at Fenwick Island - February, 1985

FIGURE 22. Beach Front at Fenwick Island - February, 1985

FIGURE 23. Fenwick Island on August 25, 1961
(Delaware State Highway Department)

Examining the dead dune grass on the face of the dune, it appeared that wave runup was occurring on the seaward face of the dune (Figure 21). In general the dunes were steep and narrow with sparse dune grass only on the back and top of the dunes. The dunes have migrated landward in some locations to the point where their movement has been stopped by the presence of trees or homes. Since the dunes cannot continue to migrate landward, they have narrowed significantly (see Figure 22 and 23). Figure 22 was taken in February 1985 while Figure 23 was taken in Aug. 1961 by the State Highway Department. The sand fences appear to be effective. A small decaying groin exists south of the Maryland-Delaware line. This groin was uncovered in the February storm, which indicates that the groin has little effect on the sediment transport in the area.

During the inspections in April and May, the beach was recovering from its winter state of denuded beach with a large offshore bar, as there was a larger volume of sand on the berm. The offshore bar was moving landward and at one point was attached to the beach. During the April inspection, it was noticed that the dunes had been breached in one spot during an early winter storm.

Two sand samples were taken for analysis. A sieve analysis indicated that there is a variation in sand grain diameter across the beach. The grain size at the high water level was larger than at the toe of the dune as expected. The mean grain diameter, d_m ,=0.41 mm (standard deviation = 1.5 mm) at the high water line. At the toe of the dune, the mean grain diameter was 0.30 mm (standard deviation is 1.4 mm).

The beach at Fenwick Island has been surveyed by DNREC personnel four times. The dates of the surveys are November 2,1984, February 15, March 29 and July 9,1985. The surveys consisted of fifteen profiles taken every five hundred feet along the baseline. The first part of the profiles were measured using standard surveying techniques (a theodolite, and a surveyor's rod). The beach was measured from the landward side of the dune to wading depth. The second part of the profiles were measured using a fathometer on a boat seaward of the breaker line. The boat was positioned using standard surveying techniques. The fathometer sounding was corrected for tidal and wave effects. The second part of the profile was measured from near the breaker line to approximately -35 foot contour.

Examples of the beach surveys are profiles 1,7 and 15, corresponding to DNREC profiles: 4+00S, 25+00N and 65+00N, which are shown as Figures 24, 25 and 26. The numbers on each profile indicate the survey date: (1)-November 11, 1984, (2)-February 15, 1985, (3)-March 29, 1985, and (4)-July 7, 1985. Profiles denoted with surveys 1 and 4 are very similar, as they are pre- and post-winter season. Survey profiles 2 and 3 show the winter denuding of the beach, with an offshore bar located between 300-500 feet offshore.

<u>Depth of Closure</u>--The depth of closure is the depth at which there is no active sediment transport. At this depth all beach

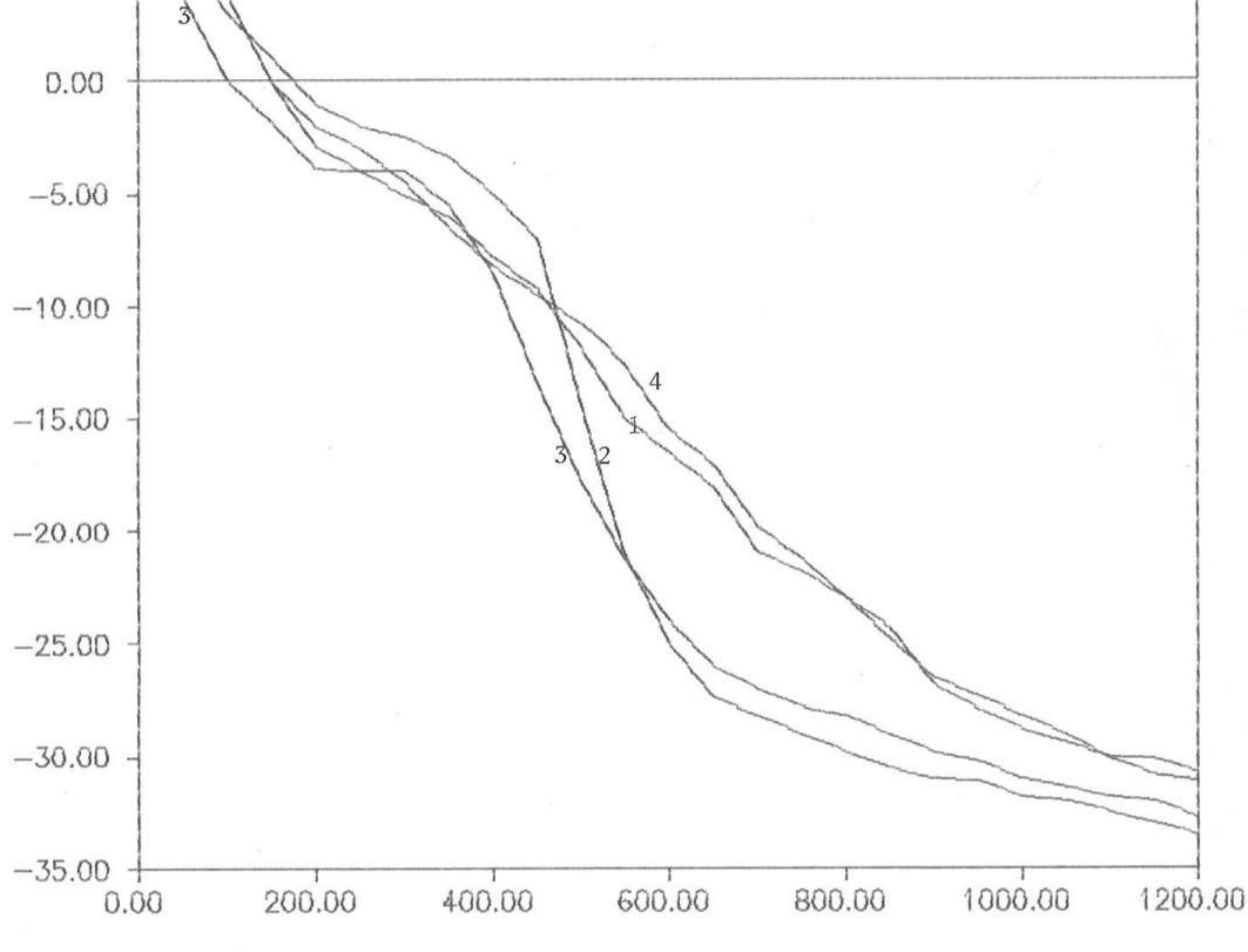

Profile 1
15.00
10.00
5.00
0.00
−5.00
−10.00
−15.00
−20.00
−25.00
−30.00
−35.00
0.00
200.00
400.00
600.00
800.00
1000.00
1200.00
1
2
3
4

Figure 24

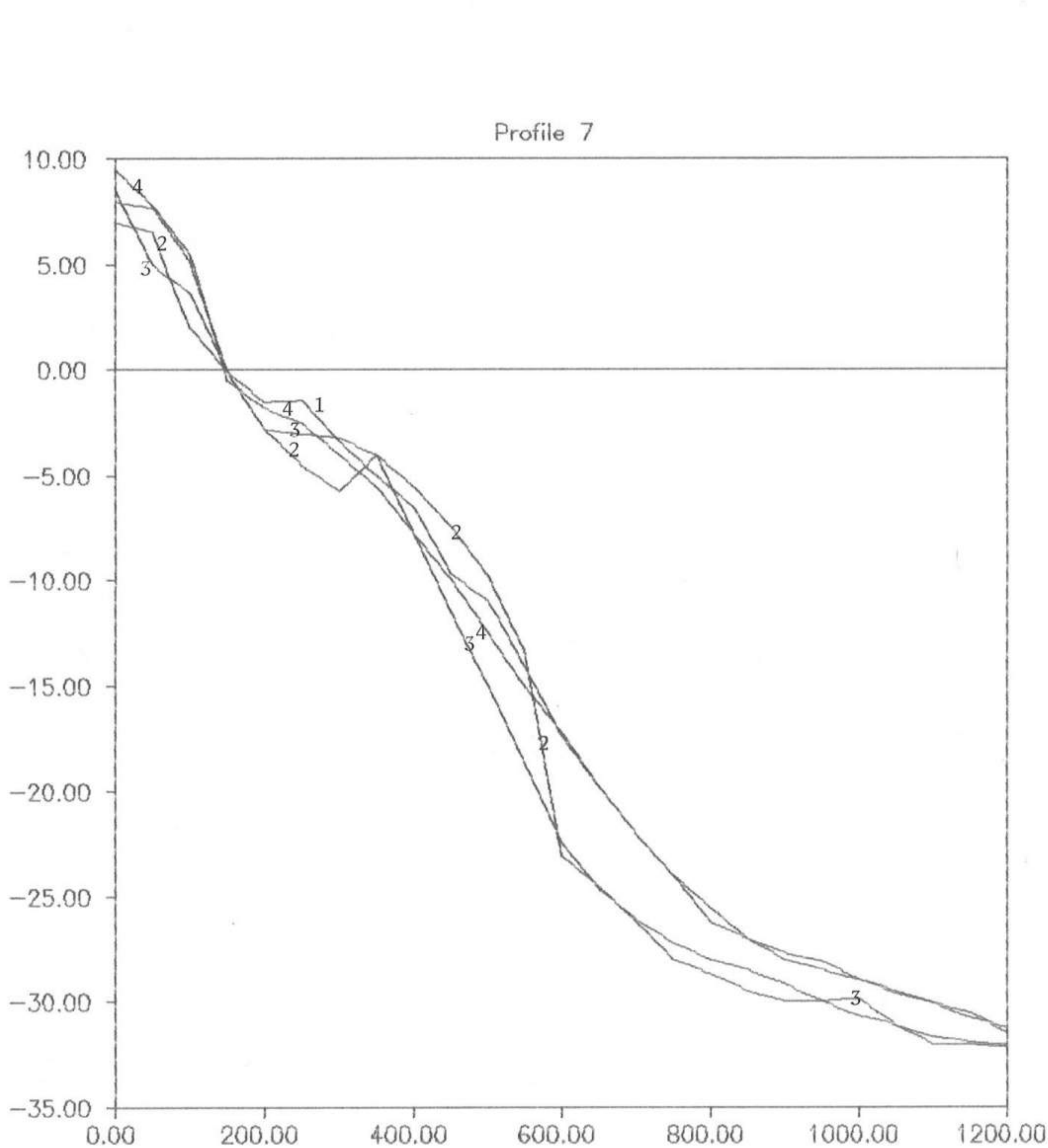

Profile 7
10.00
5.00
0.00
−5.00
−10.00
−15.00
−20.00
−25.00
−30.00
−35.00
0.00
200.00
400.00
600.00
800.00
1000.00
1200.00
1
2
3
4

Figure 25

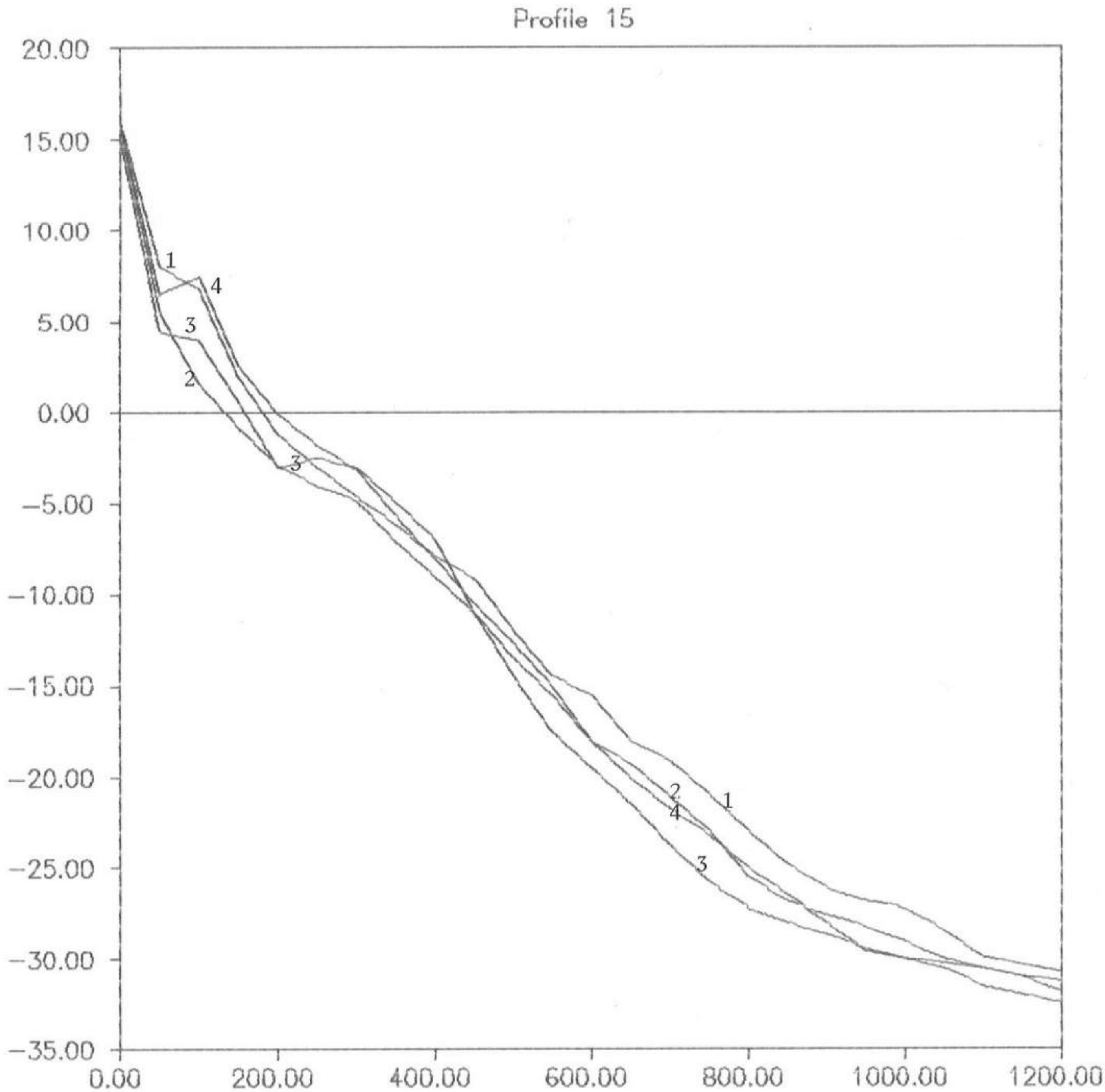

Profile 15
20.00
15.00
10.00
5.00
0.00
−5.00
−10.00
−15.00
−20.00
−25.00
−30.00
−35.00
0.00
200.00
400.00
600.00
800.00
1000.00
1200.00
1
2
3
4

Figure 26

profiles should coalesce. Accurately determining the depth of closure is important since the conservation of sand equation (Equation 17) is very sensitive to the depth of closure. This section deals with several methods for the estimation of the depth of closure.

The depth of closure should be calculable from repeated bathymetric surveys. Based on the beach profile data supplied by the DNREC, the depth of closure would appear to be greater than thirty five feet. Figure 27 is a plot of the variation of the end points over time for each profile. The end point is taken as the depth at one thousand nine hundred feet from the baseline. As can be seen in Figure 27, there are variations in the end points on the order of two feet. While it is possible that this variation could be natural, it is more likely that the technique currently being used by the DNREC for the offshore section of the beach profiles not accurate at these depths.

Therefore, empirical equations were used to estimate the limit of active sediment movement. Hallermeier (1978) suggests that the depth of closure is related to the extreme wave conditions. These wave conditions occur only twelve hours per year or the top 0.2% of the total number of waves. His depth of closure is given by

$$d = 2.28\, H_e - 68.5\, (H_e^2/gT_e^2) \quad (18)$$

where H_e is the extreme wave height and T_e is the period of the extreme wave. Using the WIS data as an estimate of the wave climate at Fenwick Island, the extreme wave height would be approximately three meters with a period of nine seconds. Recall that the wave height is given at a depth of thirty feet and not at breaking. The three meter wave would break at a height of 9.5 feet and yield a depth of closure of 19.8 feet.

Hallermeier (1983) provides another method of calculating the depth of closure. It is suggested that the depth of closure is based on both wave and soil conditions. The alternative depth of closure is given by

$$d = 2.9\, H\, (S-1)^{-0.5} \quad (19)$$

where H is the wave height of a representative wave and S is the specific gravity of the sand. With S=2.58 and using a ten foot representative wave the depth of closure would be twenty-three feet. During the March 1962 storm, fifteen foot waves were seen along the Delaware coastline. A fifteen foot wave, using Equation 19, would produce a depth of closure of thirty-five feet.

Weggel (1979) introduced a method of empirically determining the depth of closure from a beach profile. Weggel assumes the profile of the beach can be fitted by an exponential curve:

$$(h-h_o) = d\, e^{-\alpha x} \quad (20)$$

where x and h are the horizontal and vertical coordinates respectively, h_o is a datum adjustment factor that is determined

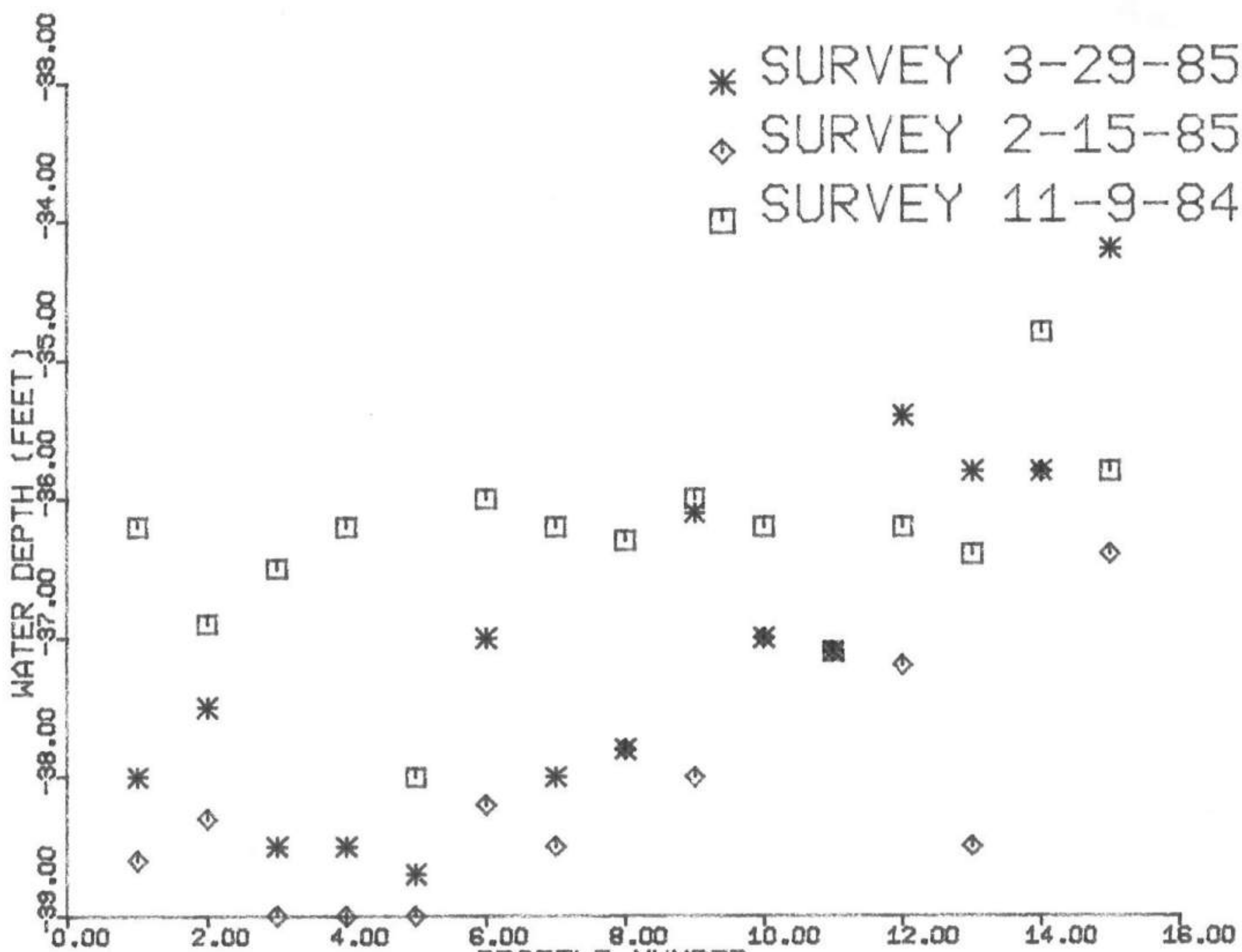


Figure 27. Variation in Water Depth 1,900 Offshore from Survey to Survey

by trial and error, d is the depth of closure, an empirical constant. Several attempts were made to fit the beach survey data according to Equation 20. The plotted curves indicated that the depth of closure was between forty and fifty feet. It is felt that this depth of closure is too large and that the beach profile data can not be described by an exponential fit.

Bruun (1954) and Dean (1977) have suggested that equilibrium beach profiles can be described by a power law:

$$h = A X^n \tag{21}$$

where A and n are empirical constants. Since no reasonable fit was attainable for the beach survey data, the beach at Fenwick Island is probably not in equilibrium.

Equations 18 and 19 suggest that the depth of closure for the Delaware coastline could not be thirty five feet. A depth of closure of 28 feet is likely to be more appropriate.

Empirical Eigenfunction Analysis of Beach Profiles--The empirical eigenfunction analysis is a statistical method of quantifying beach profile data. The method provides some empirical insight into the temporal variability of a beach and indicates the prevailing modes of on/offshore sediment transport. The method however does not indicate the ongoing physical processes as it is totally empirical.

The mathematical development of the empirical eigenfunction method is presented in Appendix II. The basic principles are that each beach profile is decomposed into a number of superimposable terms (eigenfunctions). The terms often can be interpreted physically. For the analysis of a beach profile which has been surveyed at different times during a year or longer, the first of these terms is the average beach profile. The next term is usually referred to the bar/berm function and typically measures the amount of change the profile undergoes from its winter to summer profile. Additional terms generally turn out to be relatively minor, so that the addition of two or three terms adequately describes the variation of the profile through time.

Another method of using the empirical eigenfunction method is to examine profiles along a beach, which have been surveyed at one time. The first eigenfunction in this case again gives the mean profile at that time for the beach. The second eigenfunction shows the major variability in the profiles along the beach.

If a spatial eigenfunction analysis is performed on the difference between two surveys, the mean beach profile is subtracted out. The first eigenfunction in this case corresponds to the bar/berm function. Dick and Dalrymple (1984) show that the difference between a summer profile and a winter profile provides nearly as much information on the bar-berm function as does a series of profiles taken over the course of a year.

In Figures 28, 29 and 30, the first two temporal eigenfunctions for profiles 1, 7 and 15 (4+00S, 25+00N and 65+00N) are shown. Note that in these and the subsequent figures, only the first 1200 ft. of the profiles are used in order to emphasize the nearshore profile and to avoid the biases induced by offshore survey errors. The mean profiles and the bar/berm function for the three profiles are very similar. The bar/berm function reflects the deflation of the beach and the building of the bar, but more strongly shows the large winter increase in the offshore depth 400-1000 feet offshore. In fact, surveys 2 and 3 indicate a net loss of material from the survey area. This may be a result of survey error.

Figures 31, 32, 33 and 34 show the first two spatial eigenfunctions for each of the four surveys. The first and the last figure show that the mean profile (or first eigenfunction) is approximately the same for both surveys, with the major longshore variability occurring on the dry beach and the region 400-1000 feet offshore. (This region of variability in the offshore is further offshore than the location of overlap between the wading surveys and the boat surveys, where often due to lack of data, the profiles were interpolated.) Surveys 2 and 3 show the existence of an offshore bar between 300 and 500 feet offshore. The second eigenfunction shows that the beach at the south end is lower at the baseline and the region 400-1000 feet offshore. The fact that the second eigenfunction is nearly zero at the mean water line indicates that the beach is relatively straight.

The first difference eigenfunction, for the difference between the first survey and the third survey (which is a measure of the difference between the fuller summer profiles and the winter profiles), is shown in Figure 35. The difference eigenfunction has the same characteristics as the temporal eigenfunctions as anticipated; however, again, the offshore deepening 400-1200 feet offshore dominates the change between the survey in November and that in March.

DELAWARE'S NODAL POINT

The Atlantic nodal point is the location along the Delaware Atlantic shoreline where the net alongshore sediment transport is zero. To the north of this location, the transport is northward, while to the south the net drift is southerly. Since there is no net sediment transport at a nodal point, the area in the vicinity of the nodal point is an area of higher erosion.

The importance of the location of the nodal point lies in the efficacy of certain coastal structures. Groins, for example, are effective when there is a large gross (sum of northerly and southerly) littoral transport along the shoreline. At a nodal point in the net littoral drift, a groin system may not be very effective, as the beach there is a source of sand material; there

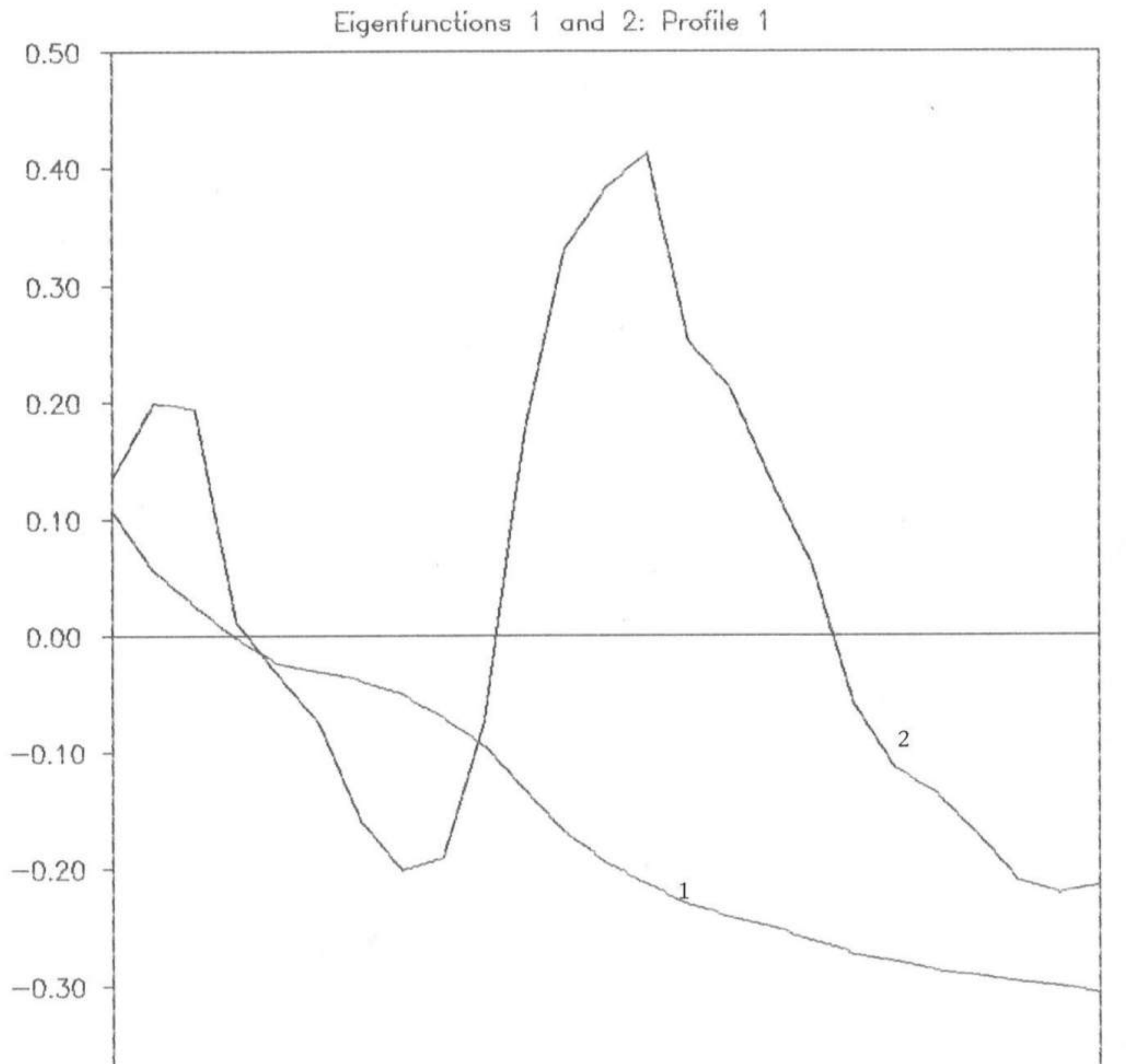

Eigenfunctions 1 and 2: Profile 1
0.50
0.40
0.30
0.20
0.10
0.00
−0.10
−0.20
−0.30
−0.40
0.00
4.00
8.00
12.00
16.00
20.00
24.00
1
2

Figure 28

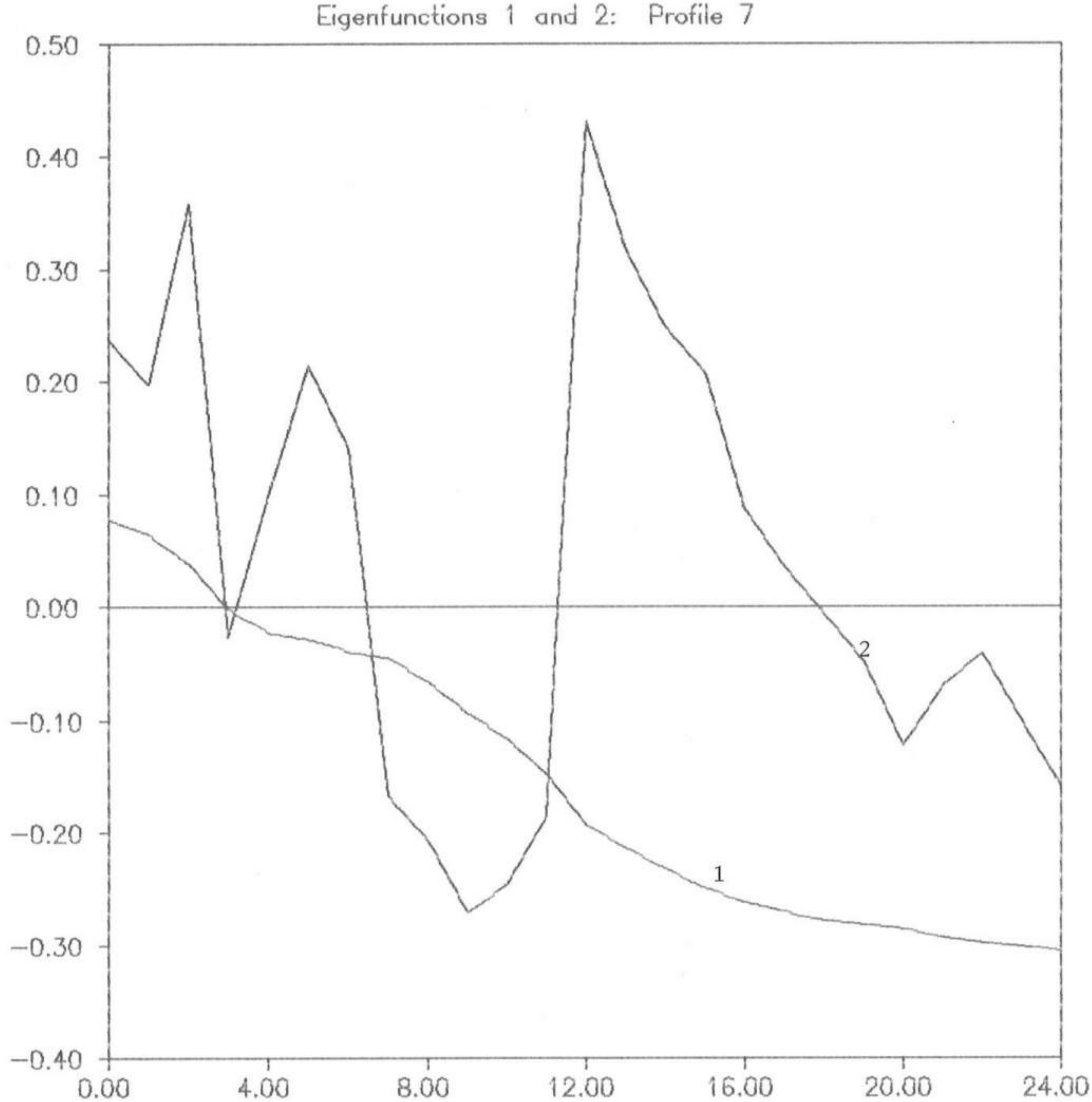
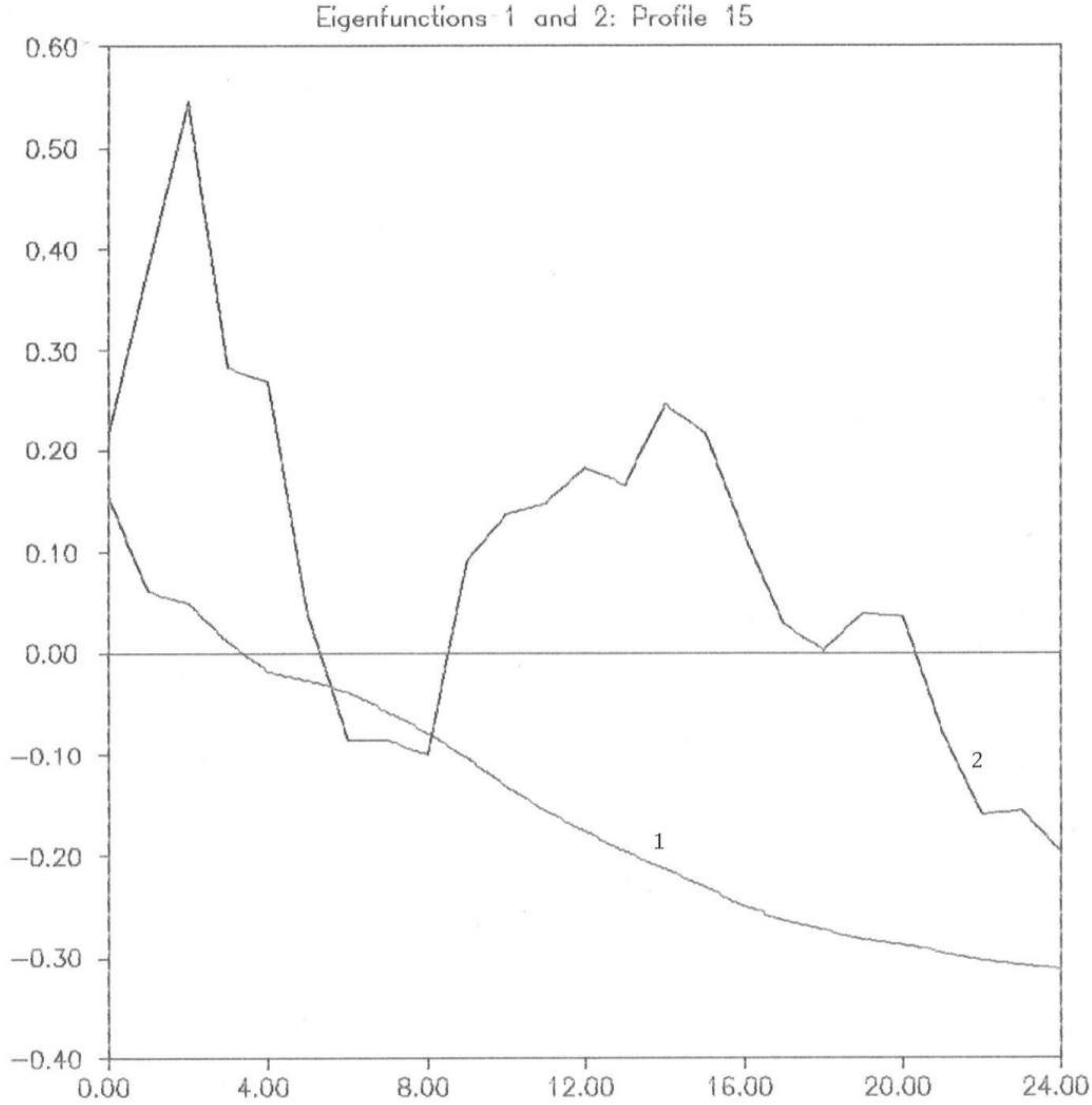
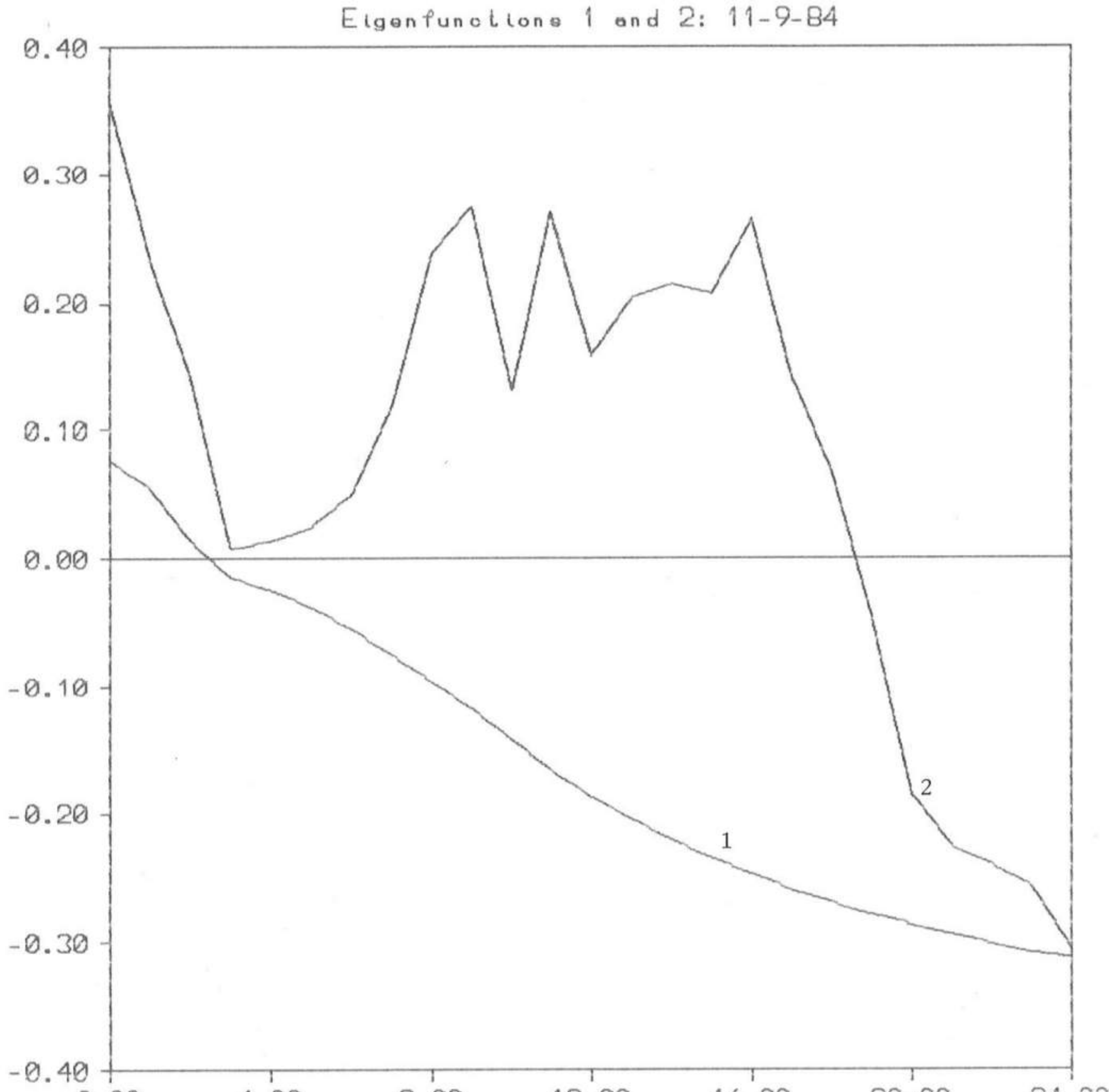

Eigenfunctions 1 and 2: Profile 7
0.50
0.40
0.30
0.20
0.10
0.00
−0.10
−0.20
−0.30
−0.40
0.00
4.00
8.00
12.00
16.00
20.00
24.00
1
2

Figure 29



Eigenfunctions 1 and 2: Profile 15
0.60
0.50
0.40
0.30
0.20
0.10
0.00
−0.10
−0.20
−0.30
−0.40
0.00
4.00
8.00
12.00
16.00
20.00
24.00
1
2

Figure 30



Eigenfunctions 1 and 2: 11-9-84
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
0.00
4.00
8.00
12.00
16.00
20.00
24.00
1
2

Figure 31

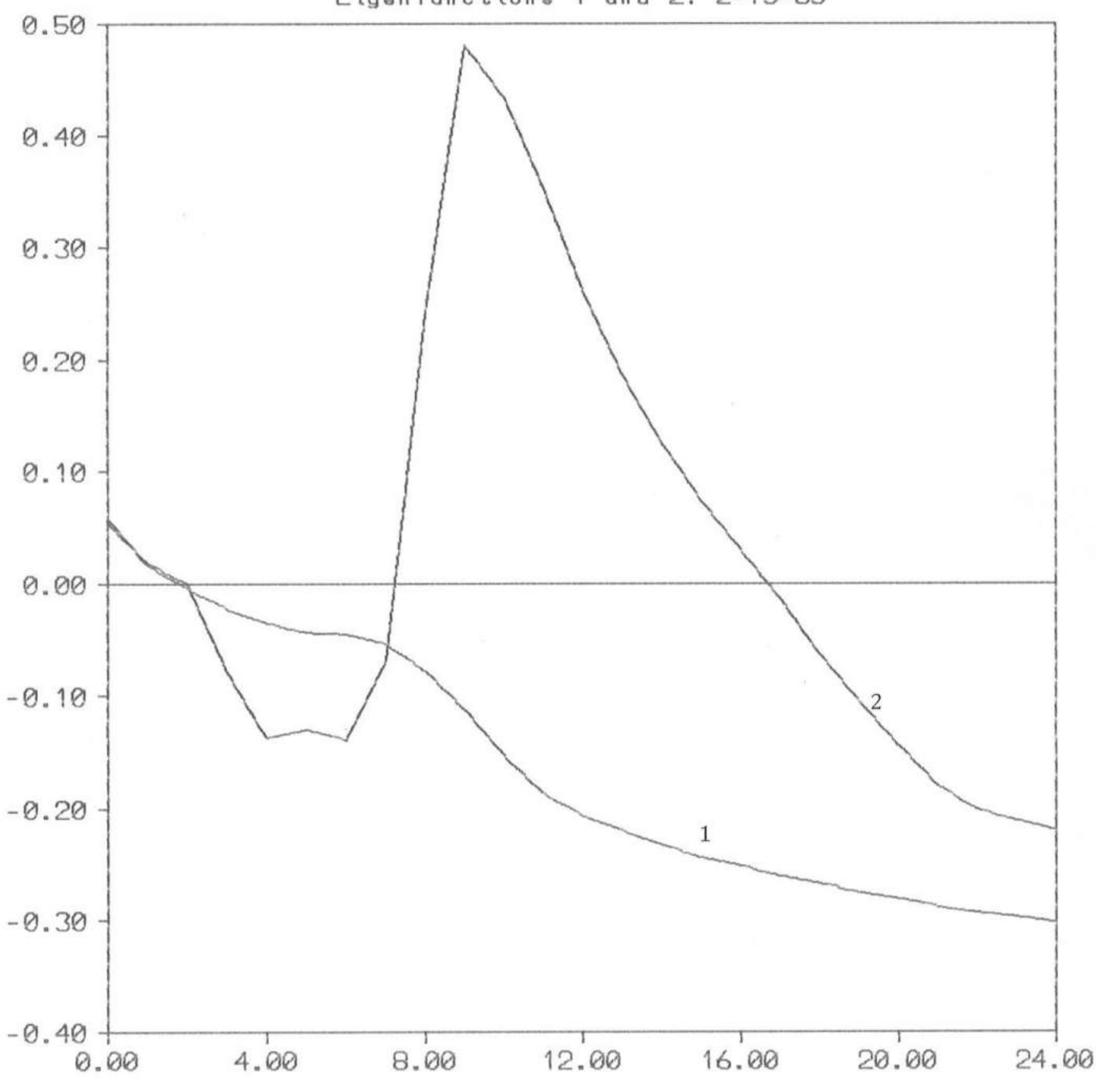
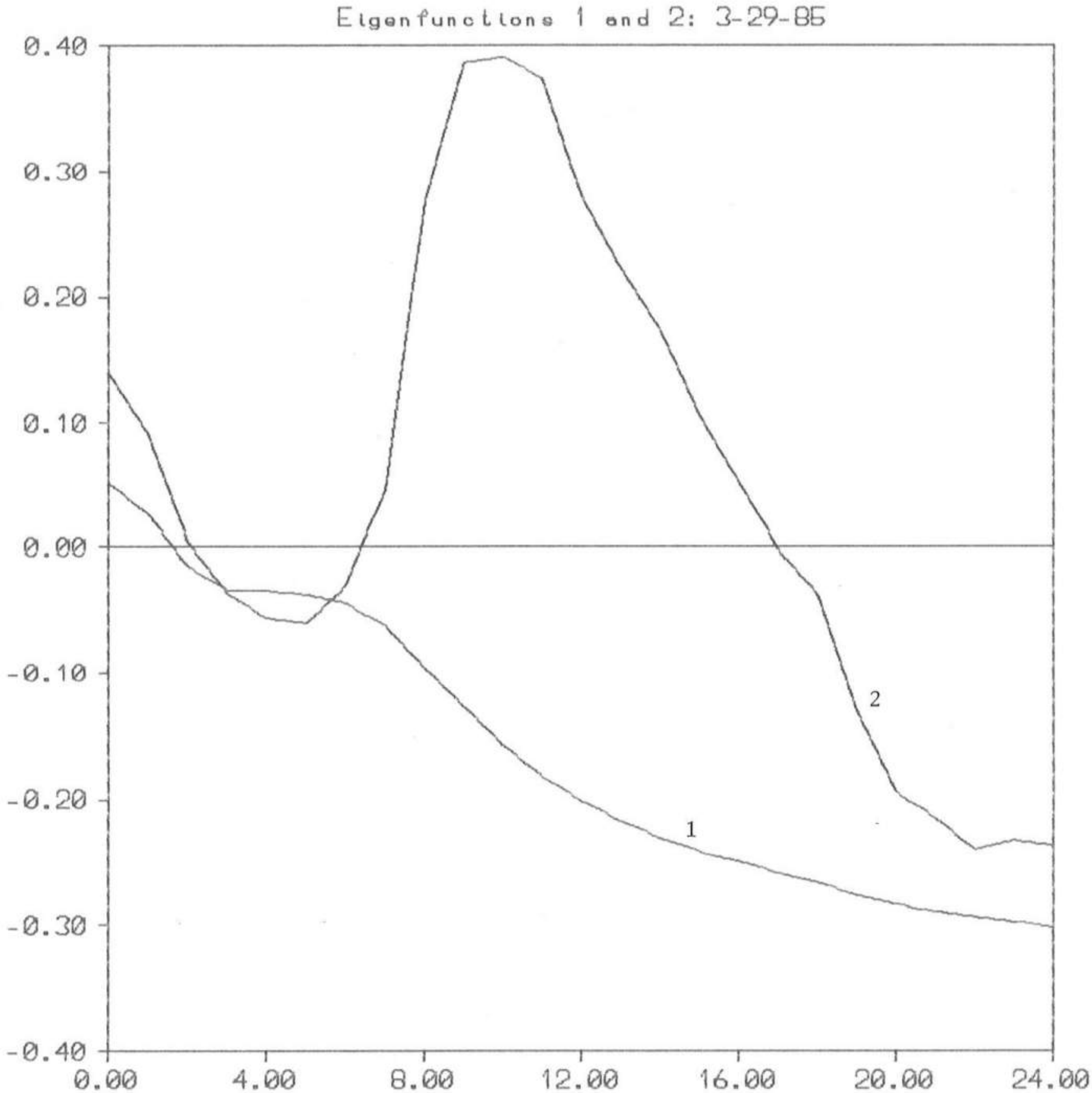

Eigenfunctions 1 and 2: 2-16-86
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
0.00
4.00
8.00
12.00
16.00
20.00
24.00
1
2

Figure 32



Eigenfunctions 1 and 2: 3-29-85
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
0.00
4.00
8.00
12.00
16.00
20.00
24.00
1
2

Figure 33

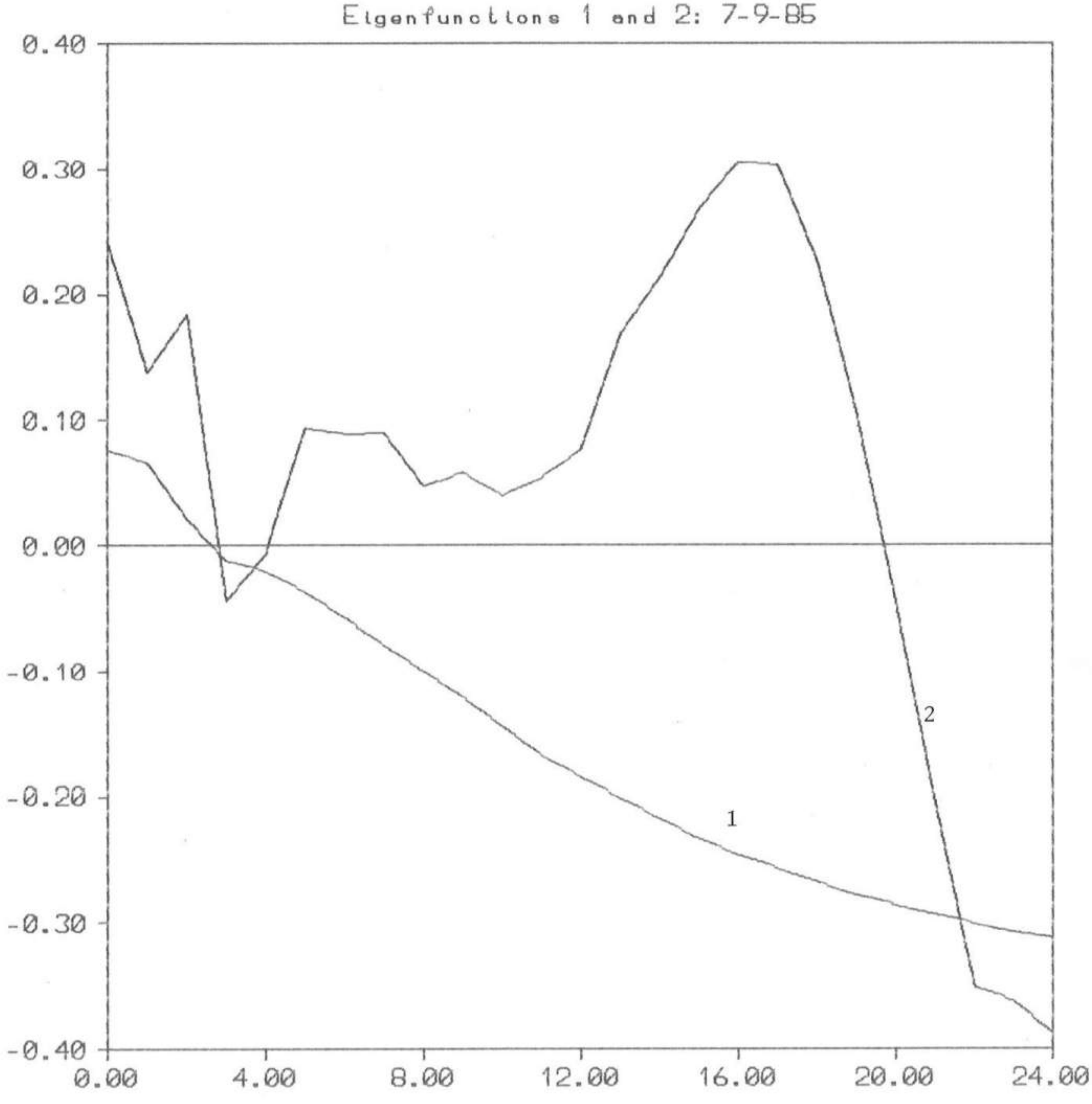

Eigenfunctions 1 and 2: 7-9-85
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
0.00
4.00
8.00
12.00
16.00
20.00
24.00
1
2

Figure 34

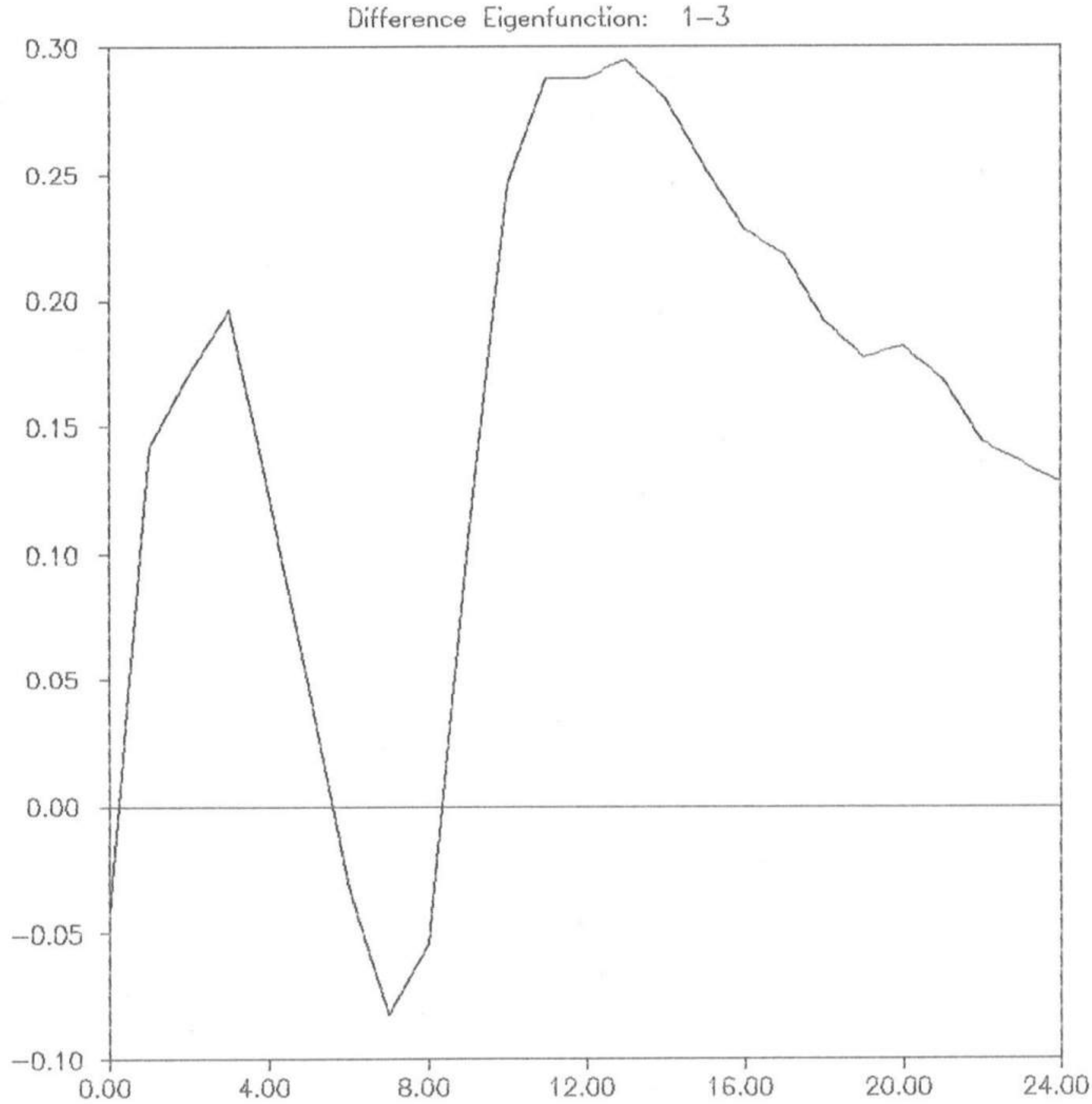

Difference Eigenfunction: 1–3
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
0.00
4.00
8.00
12.00
16.00
20.00
24.00

Figure 35

is no sand supplied to the area to be trapped by the groins.

The sediment transport at any location on a beach is very dependent on the orientation of the shoreline. A sudden change in beach orientation is a indication of a possible node. A nodal location is expected near the Delaware/Maryland border, as one can see readily on a map that the Delaware shoreline faces more northerly than does the Maryland shoreline. The orientation of the beach at Fenwick Island changes by five degrees, indicating that the Fenwick Island area is a candidate for being the nodal point.

Over the past twenty years, there have been many estimates of the location of the nodal point on the Delaware-Maryland coast. The Corps of Engineers (1968,1972) have stated that the nodal point is near Bethany Beach. In their study of the shoreline processes at Bethany Beach, Dick and Dalrymple (1983) concluded that the nodal point was not at Bethany Beach and that the nodal point was closer to the Delaware-Maryland state line.

Using the WIS data, the location of the nodal point was calculated in two ways. First, the values of sediment transport in Table 4 were used to estimate the mean and standard deviation of the nodal location. Second, the littoral drift roses were used to obtain beach normal orientations where no littoral drift would occur.

The sediment transport rates in Table 4 which are determined for points along the Atlantic shoreline were used to determine the nodal point by fitting a function to the spatially varying sediment transport rates and determining where the function was equal to zero. Since there is a large change in beach normal orientations, Station 67 was split into two sections. The first (northern) section is 2.7 nautical miles long with an orientation of 8.5 degrees. The second (southern) section is 7.3 nautical miles long with an orientation of 13.5 degrees. Sediment transport values were calculated for each of these two sections with Station 67 wave data as input. The values of sediment transport for each section were assumed to be representative of the entire section and were placed at the midpoint of each section for computational reasons. A third order polynomial was fitted through the four points using a Lagrange polynomial:

$$f(X) = \frac{(X-X_2)(X-X_3)(X-X_4)\ Q_1}{(X_1-X_2)(X_1-X_3)(X_1-X_4)} + \frac{(X-X_1)(X-X_3)(X-X_4)\ Q_2}{(X_2-X_1)(X_2-X_3)(X_2-X_4)} + \frac{(X-X_1)(X-X_2)(X-X_4)\ Q_3}{(X_3-X_1)(X_3-X_2)(X_3-X_4)} + \frac{(X-X_1)(X-X_2)(X-X_3)\ Q_4}{(X_4-X_1)(X_4-X_2)(X_4-X_3)} \tag{22}$$

where X_n is the location of the nth data point, and Q_n is the sediment transport at X_n. The locations of the data points were measured with respect to the southern end of Station 67. The nodal point corresponds to the location where Equation 22 is equal to zero. The location of the zero is found by the Newton-Raphson technique. The results are shown in Table 5.

TABLE 5

LOCATION OF NODAL POINT FROM SEDIMENT TRANSPORT CALCULATIONS

Year	Nodal Point (Miles from the southern end of St 67.)
1956	19.4
1957	11.1
1958	7.7
1959	3.3
1960	18.1
1961	22.0
1962	24.5
1963	18.6
1964	23.7
1965	21.9
1966	11.7
1967	23.7
1968	11.0
1969	23.0
1970	11.0
1971	10.1
1972	21.3
1973	----
1974	----
1975	17.7
Based on 20 year averages from Table 2	18.8
Mean(18 years)	17.1
Standard Deviation	6.2

Note: In 1973 and 1974, all three stations, including the division of Station 67 into two areas, indicated sediment transport was to the north. As a result, no nodal calculation was possible.

A linear interpolation between Stations 65 and 66 and Stations 66 and 67 was performed yielding similar results to Table 5. According to Table 5, the nodal point is located approximately three miles south of Indian River Inlet. More importantly, the large standard deviation shows the extreme annual variability in the location of the nodal point.

Using the littoral drift roses, the angle of shoreline normal orientation corresponding to zero sediment transport was determined. This was accomplished by a linear interpolation of the two nearest data points. The shoreline normal orientations representing zero littoral drift are 93.6, 94.5, 86.7 degrees for Stations 65, 66, 67 respectively. The angles are measured from due south in a counter clockwise direction.

As a first attempt, U.S.G.S. quadrangles were used to obtain locations of zero drift. Since these maps did not provide sufficient detail, the only confident result obtained was that the nodal point was not in Maryland.

An aerial photograph of the entire Delaware coastline (Corps of Engineers, 1972) was then used to determine the nodal location as it provided more detail than the quadrangle maps. Bethany Beach was found to have a beach normal orientation of 82.2 degrees. The area from the northern end of Fenwick Island State Park to Middlesex Shores exhibits a beach normal angle of 86.8 degrees. According to the Station 66 littoral drift rose, neither of the two locations are nodal points because Station 66 requires a beach normal angle of 94.5 degrees. If the area of wave climate validity for Station 67 is extended three miles northward , then York beach would be the nodal point. Station 67 requires a beach normal orientation of 86.7 degrees. (The error in the angle measurements is 0.1 degrees.)

By varying the shoreline in the calculation of the littoral drift roses, the littoral drift technique would appear to be more accurate than the location calculated from the sediment transport potentials. The term potentials is used since the values have appreciable error in them due to the assumption of straight and parallel contours.

Nodal Point Summary--In order to get an overall view of the location and movement of the nodal point, Table 6 has been constructed. Table 6 shows that between 1843-1929 Bethany Beach was a good estimate of the location of the nodal point. Between 1929 and 1954, the nodal point apparently migrated to the south. The most recent indications are that the nodal point is in the vicinity of York Beach and Fenwick Island State Park. The estimates from the WIS data, based on sediment transport volumes, can be discounted on the basis of other data. The sediment transport volumes do indicate the location of the nodal point varies widely from year to year.

TABLE 6

SUMMARY OF NODAL ESTIMATES

Years	Basis	Location	Reference
1843-1929	Bathymetric Surveys	Bethany Beach	Corps, 1968
1929-1954	Shoreline Position	South of Bethany	DSHD, 1956
1929-1954	Bathymetric Surveys	South of Bethany	Corps, 1968
1929-1954	" "	York Beach	Dalrymple et al. (1976)
1954-1964	Bathymetric Surveys	South of Bethany	Corps, 1968
1964-1982	" "	York Beach (FISP)	COER,Inc.(1982)
1956-1975	WIS/transport	South of IRI	This study
1956-1975	WIS/drift rose	York Beach	This study

Abbreviations:	Corps	U.S.Army Corps of Engineers
	DSHP	Delaware State Highway Department
	FISP	Fenwick Island State Park
	IRI	Indian River Inlet

RELATIVE SEA LEVEL RISE AND FUTURE SHORELINE RECESSION

The level of the sea with respect to land is slowly rising. Local land subsidence, warming of the ocean and the melting of glaciers and the polar ice caps are a few reasons for this relative sea level rise (RSLR). Though this sea level change is very gradual, its effects can be quite alarming.

The National Ocean Survey has been monitoring the changes in sea level for many years. On the Delaware-Maryland coastline, there are tide gages at Lewes Breakwater, Indian River Inlet, and at the Ocean City fishing pier. Data provided by the National Ocean Survey from each of these gages was used to estimate the sea level rise. The Ocean City data was not used since the period over which the gage had been less than ten years. There were also gaps in the data where the gage had failed. Based on the other two gages, the RSLR at Fenwick Island is 2.1 mm/year.

Contributing to the RSLR is Delaware's current and ongoing subsidence. Holdahl and Morrison (1974) indicate the land at Fenwick Island is subsiding at a rate of 2.6 mm/year. Their figures are based on the releveling of bench marks. The total relative sea level rise at Fenwick Island is therefore 4.7 mm/year. Over a fifty year period, the sea level would rise 0.77 feet, at the current rate.

In order to estimate the effects of the relative sea level rise on shoreline recession, it is assumed that the beach at Fenwick Island is in an equilibrium profile. Bruun's Rule has been widely used for calculating beach recession based on an equilibrium profile

$$R = \frac{B \Delta s}{(h_d + d)} \qquad (23)$$

where d is the depth of closure, taken as 28 feet, and B is the width of the active littoral zone, about 1000 feet, h_d is the height of the dunes, say, 11 feet above Mean Sea Level, and is the change in relative sea level = 0.77 feet. Over the next fifty years, the shoreline should recede about 20 feet, simply due to the sea level rise. The ongoing recession, due to the proximity to the nodal point, is added to this sea level-induced recession, resulting in a prediction of just over 100 feet of recession in the next 50 years, in the absence of any erosion mitigation measures.

EROSION CONTROL MEASURES

There are a variety of approaches which can be used with the erosion problem at Fenwick Island. One approach is to do nothing, which involves no funds, yet will result in the loss of the beach front homes in the community due to storm damage within a few years. The other approaches involve coastal structures and a considerable cost.

The cost of coastal protection should be less than the benefits which accrue (such as the retention of the beach and property). As a guide to the value of the community, we can roughly estimate the value of the privately-held beach front property by using $250,000 for an average beach front lot (50 feet of beach front by 100 feet deep). This results in a total value of just the beachfront property in Fenwick Island on the order of $33,000,000. Since the majority of the structures in Fenwick Island are small older homes, the structures contribute less to the total value of the town than does the real estate on which they stand. This is not necessarily the case for some of the newer homes being built which cost several hundred thousand dollars. The costs to benefits ratio must be determined through a careful analysis, particularly when examining the public benefit of beach protection at Fenwick Island.

In the discussion below, estimated costs are provided for the various shoreline protection schemes. These costs are only for comparisons between schemes and are not to be used for actual cost estimation, as they are based on experience and not actual cost estimations for the various schemes. Further engineering design and cost estimations must be done to provide accurate costs.

Shoreline Protection--There are numerous coastal structures or other means available to ameliorate beach erosion. Several of these are listed with comments as to their appropriateness for Fenwick Island:

Beach nourishment--This involves the placement of new sand on the beach. Sources of sand can be offshore or onshore, provided that the size distribution of the new sand be compatible with the native sand (Dean, 1974). The placement of the sand also entails the rebuilding of the dunes, to provide storm protection for the beach front homes.

There are many ways to design a beach fill plan. For any plan, it is necessary to provide to a sufficient amount of "compatible" sand to provide for a certain number of year's worth of erosion. In general, the greater the volume of placed sand, the longer the beach fill will remain in place.

One plan could be to follow the lead of Ocean City and the State of Maryland. The beach fill designed for Ocean City calls for the placement of about

300,000 cubic yards of fill per mile at a cost of $2 million per mile. The beach profile would be widened by 100 feet at the elevation of 10 ft. above MLW. This plan could be carried out in conjunction with the Maryland fill project, with some attendant savings in mobilization costs for the offshore dredging. Total costs for nourishing 6000 ft. of Fenwick Island would be about $2,300,000.

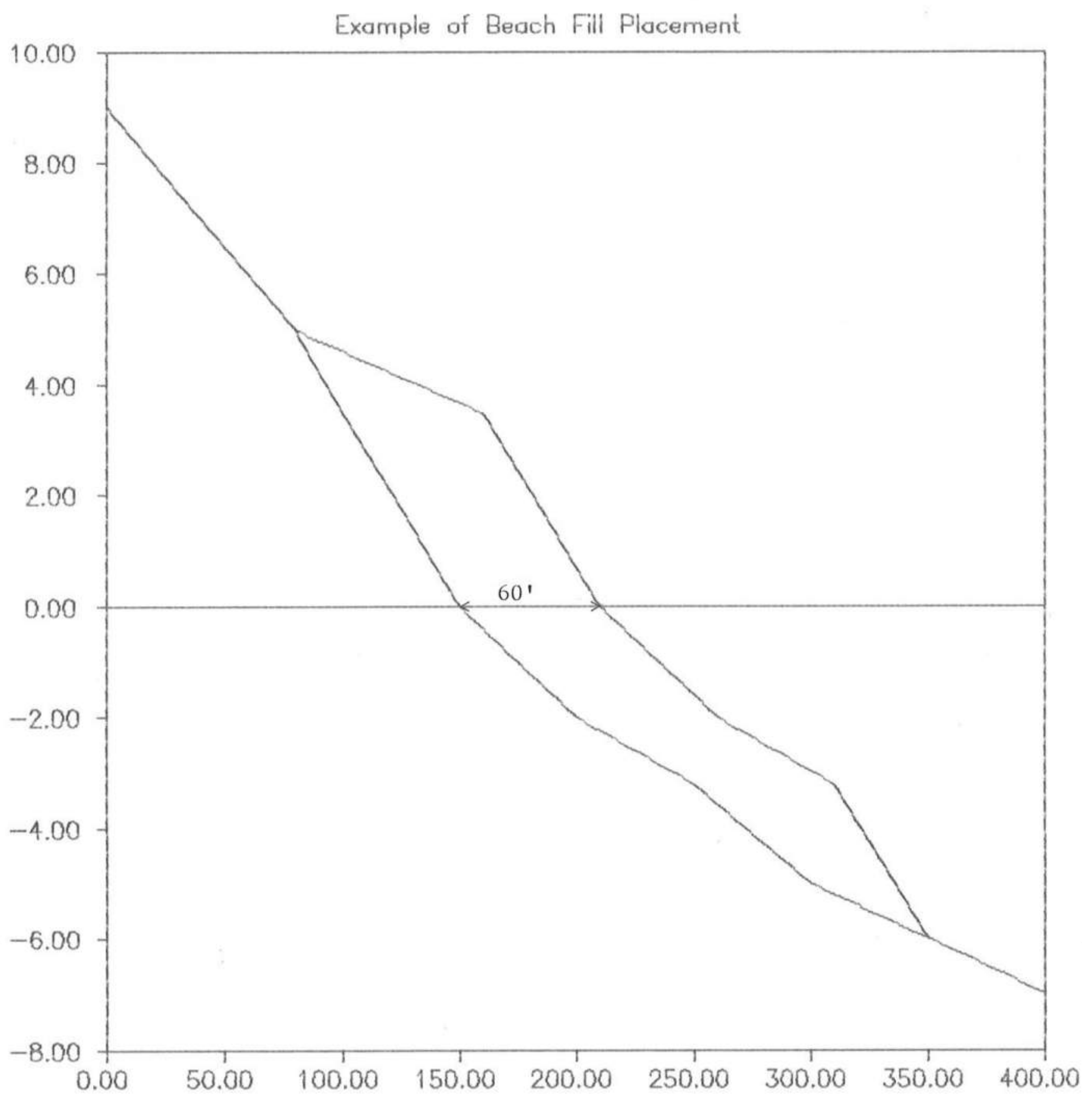
Smaller scale projects can also be carried out. As an example, if the beach were extended by 60 feet for the entire length of Fenwick Island, the volume of sand required would be approximately 112,000 cubic yards, less than half the amount for the larger scale Ocean City type project. See Figure 36 for a schematic of the fill placement. In the absence of major storms and with a conservative 3 ft/yr recession rate, this fill might be expected to last more than a decade (a factor of two is used to account for potential profile readjustment). The Corps of Engineers (1980) report an offshore borrow area in the vicinity of the Delaware-Maryland line which has an effective cost of $5.40/cubic yard. Allowing for inflation by assuming the total cost to be $8.00/ cubic yard, the total cost to nourish the beach would be $896,000.

Smaller fill projects would be less costly, but would last a correspondingly shorter period of time. One merit of a smaller project would be its use as an experimental prototype for a more costly larger project. By careful monitoring of the fate of the fill during several years, conclusions about the long-term fate of a major fill project could verified.

The disadvantage of beach fill is that the processes which caused the erosion are not altered. Therefore the beach will continue to erode, requiring additional fill on a regular basis. In the absence of inflation, the annual maintenance cost might be equivalent to 15% of the original cost. The time between maintenance filling will vary with the wave climate.

Groins with beach fill--Groin fields have been proven to be very effective in Delaware. Both Rehoboth Beach and Bethany Beach have been protected from erosion by groins. While the effectiveness of groins is likely to be less in Fenwick Island due to the proximity to the nodal point, the numerical modelling effort discussed in the next section indicates that deposition will occur at the groins.

Beach fill of the same magnitude as mentioned above should be utilized with the groins to reduce the downdrift erosion. As each groin is constructed the



Example of Beach Fill Placement
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
60'
0.00
50.00
100.00
150.00
200.00
250.00
300.00
350.00
400.00

Figure 36

neighboring beach should be filled.

The construction sequence of the groin field is probably more of a political matter than a coastal engineering one, due to the presence of the State line at the south end of the beach. The groins should be constructed starting at the south end of the community and then moving north. However, with the use of fill, the sequence is less important than with no fill.

The effective length of the groins should be comparable to those at Bethany Beach. This involves extending the groins out to the -6 contour, which is about the location of the winter storm bar. This contour is about 350' offshore of the survey base line. Care should be taken to ensure that the groins will not be flanked within the next 30-50 years, which entails anchoring the groins well into the dunes. The spacing of the groins should be about two and a half to three times their effective length, which results in about a 1000 ft. spacing. Six groins should be sufficient to span the community, although the most southerly groin might be made 50-100 ft. shorter so as to reduce the downdrift erosion. Possible locations of the groins are at the ends of the following streets: King, Houston, Essex, Bayard, West Virginia and Lighthouse Road.

The elevation of the groins should not be excessive, only several feet above the existing contour lines. (Either an additional 0.5 feet can be added to account for sea level rise or provision can be engineered now to allow for raising of the groins later.) Fill will therefore raise the beach elevation by about one foot over a portion of the beach profile. Less than about 50,000 cubic yards of fill will be necessary to fill the groin field.

With an estimate of $300,000 per groin, the total cost of groin field construction would be $1,800,000. Fill costs, about $900,000, must also be added.

The construction material used in the groins will not effect their performance in the short term. Wood or steel sheet pile groins have a much shorter lifetime than stone groins; therefore higher maintenance costs.

Annual maintenance costs associated with the groin fields will be much less than the beach fill plan, as the groins will retain the fill. A reasonable estimate might be 10% of the fill costs.

<u>Artificial Seaweed</u>--The efficacy of this means of retaining sand has not been shown in its applications in Delaware and it is not recommended for use.

Seawalls and Bulkheads--These measures are considered to be more of a last ditch effort. While a sturdy and continuous bulkhead along the entire beach may be required in the future to hold the beach front properties, it is not recommended at the present time, as bulkheads and seawalls tended to exacerbate the erosion problem. The beach fill recommended above, with the reconstruction of the dunes, should provide a softer form of the same protection.

Offshore Breakwaters--Probably the most effective means for shore protection for Fenwick Island would be offshore breakwaters which would reduce the wave energy which reaches the shoreline. Well-built offshore breakwaters would result in nearly total trapping of sand behind them. The disadvantage of these structures, beside the aesthetics, is the extremely high cost, which would likely require Federal assistance. Also, since they would work as effective littoral barriers, downdrift erosion would occur, requiring mitigation.

Shoreline Prediction Model Formulation--Groins rely on the presence of a littoral drift in order to be effective. The success of the groins at other Delaware coastal cities is due to the reasonably large net drift to the north. For Fenwick Island, which is near (and south of) the nodal point in the littoral drift, the question arises as to whether there is enough sand in motion to enable the groins to trap and hold sand. Recently, a shoreline model was developed by the Ocean Engineering Group that predicts shoreline position based on the variation in the alongshore sediment transport (Suh,1985). This model is used to predict the behavior of a single groin at Fenwick Island. The development of this model is presented in Appendix III for the interested reader.

For all of the results presented below, WIS data from Station 66 for 1964 was used as wave input for the model. This set of data most closely approximates the twenty year average of sediment transport at Fenwick Island.

The model is not of sufficient accuracy to provide a detailed prediction for the location of the equilibrium shore line after the installation of a groin; however, it is useful in predicting the presence of deposition and erosion. As can be seen in Figure 37, the groin has trapped a significant amount of sand on the north side, while there is the usual downdrift erosion, expected with a groin. The conclusion of this numerical study is that there is enough sand in motion during the course of an average year to support a groin field.

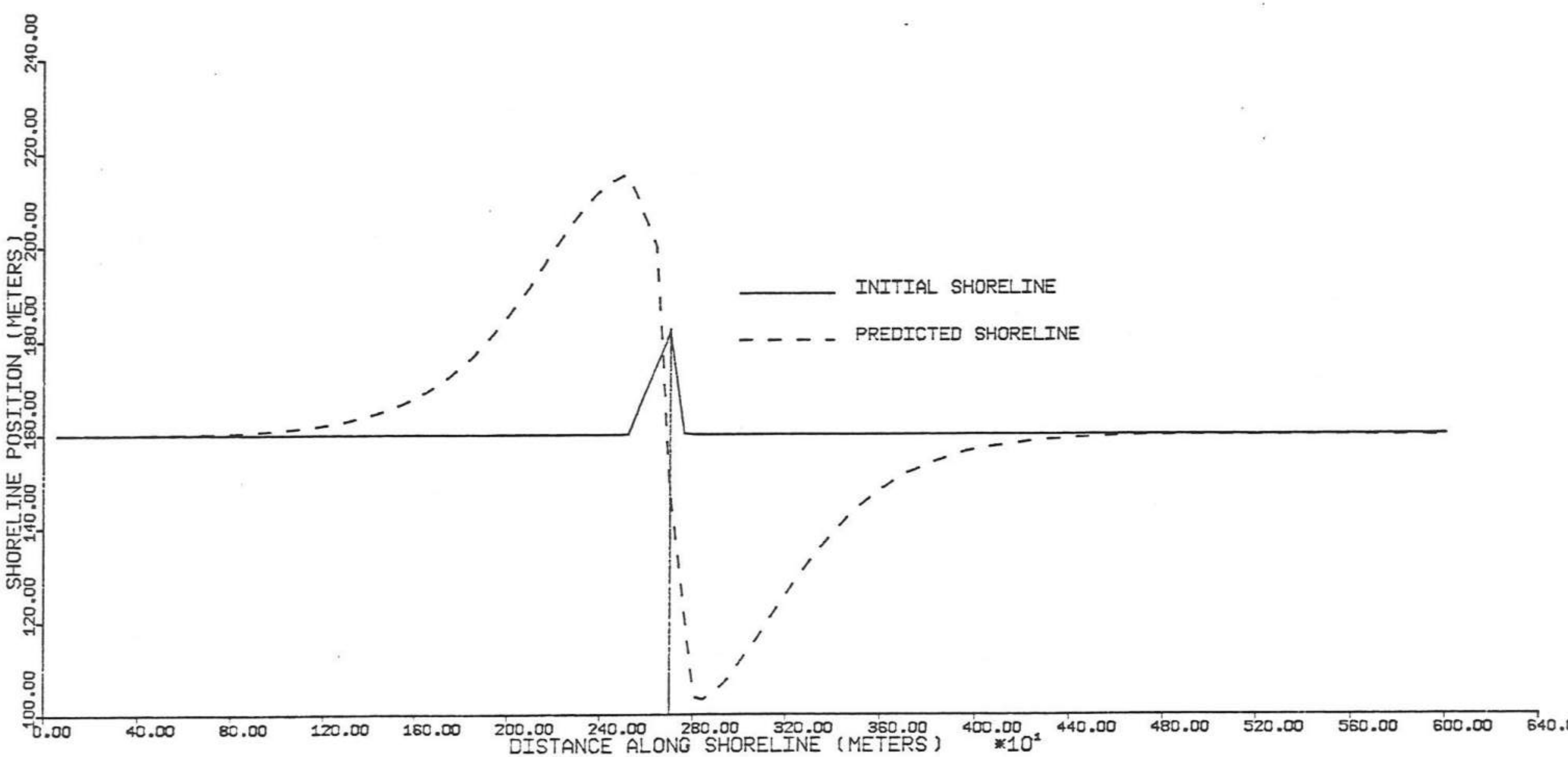


FIGURE 37. Initial and Predicted Shoreline Based on the Use of a Long Filled Groin Over One Year.

CONCLUSIONS AND RECOMMENDATIONS

The shore line at Fenwick Island has been receding historically at a rate of about 2 ft./year, based on aerial photographic analysis and beach profile measurements. This historical rate is probably lower than the present rate, particularly since the primary dune has nearly disappeared.

The Wave Information Study has provided a twenty year hindcast wave data set for the Delaware Atlantic shoreline. The wave climate clearly has a seasonal variation, with waves coming primarily from the Northeast in the Winter season, defined as January to March, while in the summer the waves arrive from the southeast, driving the littoral drift to the north.

Sediment transport calculations, using the WIS data, show that there is a very large annual variation in the littoral drift, corresponding to the differences in wave climate from year to year. The twenty year predicted littoral drift for Fenwick Island is 57,000 yds^3/yr, moving southward. Littoral drift roses which provide information for littoral drift for any arbitrary shoreline orientation were generated for the entire Delaware Atlantic shoreline using the WIS data.

Beach profile analysis, with empirical eigenfunction analysis, showed that the shoreline did not retreat during the period covered by the surveys. Seasonal variation of the mean water line was on the order of 75 feet. Most of the variability between surveys was the deflation of the dry beach and the region 400-100 feet offshore during the winter; however, this loss, measured in the second and third surveys, was recovered by the fourth survey.

The location along the Delaware shoreline of the littoral drift nodal point was sought through several means: the WIS data, beach profile analysis and the littoral drift roses. The best estimate of the littoral drift location, found by averaging the results, is in the vicinity of York Beach, north of Fenwick Island. The annual variability of the location of the nodal point is quite large, mirroring the variability in the sediment transport.

Several alternatives are recommended for erosion mitigation at Fenwick Island. The first is beach fill, which involves placing new sand on the beach. The procedure is aesthetically pleasing, involving no structures; however, it is only temporary, as the erosion processes will continue. Periodic maintenance filling would be required to keep the beach in place. A more durable solution is the use of groins coupled with beach fill. The groins would likely be successful, despite the proximity to the nodal point, due to the reasonably large amount of littoral transport in the area and the wide variation in the nodal location. Beach fill during groin construction would reduce the downdrift erosion problem caused by the usual sand trapping of the groins. Despite the higher initial costs of the groins, their lifetime of 50 years is far more than the duration of a beach fill project. Choosing either solution requires that further engineering studies be carried out to determine all relevant design factors and costs.

APPENDIX I. FORMULATION OF THE WIS DATA

The main problem the Corps of Engineers faced when creating WIS was in the transforming of wave conditions at one location to another location. It was found too costly to transform individual wave conditions in a wave by wave approach. Also, by transforming wave probability distributions, too much information was lost especially during rare events. Rare events should not be overlooked. The top five percent of days in a year account for 50 percent of the total erosional losses (Douglass, 1985), A new approach was developed to maximize computational efficiency without a major loss in information (Jensen, 1983). This was accomplished by assuming straight and parallel bottom contours, and making calculations only when a significant change occurred. See Figure A.1.

The sea wave spectrum is described by a discretized two dimensional (frequency and direction) distribution. In order to retain detailed information, a parametric representation of the energy spectrum was used. If the frequency and direction are assumed to be independent, then the energy spectrum can be written:

$$E_2\ (f,\Theta) = E_1(f)\ \Phi(\Theta) \qquad \text{(A.0)}$$

where $E_1(f)$ is a one-dimensional energy spectrum, which represents the distribution of wave energy with frequency, f, and $\Phi(\Theta)$ is a normalized directional spectrum. The one-dimensional spectrum (Kitaigordskii,1962) is:

$$E_1(f) = \begin{cases} \alpha\ g^2\ f^{-5}/(2\Pi)^4\ , & f> f_m \\ (a\ g^2\ f^{-5}/(2\Pi)^4\ )\ e^{(1-(f/f_m)4)}\ , & f<f_m \end{cases} \qquad \text{(A.1)}$$

where α and f_m are free parameters determined from the total .

wave energy,E, by:

$$E= \int E_1\ (f)\ df = \alpha\ \lambda\ g^2\ f_m{}^{-5} \qquad \text{(A.3)}$$

where λ , a constant, α and f_m are parameters contained in the Phase II, the transitional region, wave records. Jensen(1983) has shown that Equation A.1 gives good results during wave generation and that it accurately represents the local sea state. Since the angular variational characteristics significantly effect the longshore energy fluxes, it is necessary to account for these characteristics (Resio,1978). Jensen (1983) uses a $\cos^4(\Theta-\overline{\Theta})$ distribution to approximate the directional distribution, $\Phi(\Theta)$, where $\overline{\Theta}$ is the cental angle of the distribution.

$E_2(f,\Theta)$ describes the sea component of the total wave in terms of $\alpha, f_m, \overline{\Theta}$. Recall that the wave height is related to the wave energy as:

$$E = \rho g H^2/8 \tag{A.4}$$

where ρ is the density of water, g is gravity, and H is the wave height. The period of the spectral peak is defined as:

$$T_p = 1 / f_m \tag{A.5}$$

Using Equations A.3, A.4, A.5, the sea component of the total wave can be represented in terms of H, T_p, $\overline{\Theta}$.

The swell component of the wave is represented by a monochromatic unidirectional wave. This representation is used because the waves have travelled long distances from their generation areas. The wide band frequencies are assumed to be filtered out through the interaction with other waves. It is possible to have more than one wave train at a particular location, but it is suggested that only one wave train is necessary to fully describe the swell component(Jensen,1983). The swell component can be described in terms of H, T, θ. Figure A.1 shows a schematic representation of the inputs into the Phase III model. Figure A.2 shows a comparison between measured and hindcasted waves. Figure A.2 shows that the model used by the Corps of Engineers does simulate the actual wave conditions. The wave angles, Θ, and $\overline{\Theta}$, were calculated with respect to the average shoreline orientation of the station.

The WIS is limited since the analysis is based on the assumption of straight and parallel contours. Along the Delaware coastline, the assumption of straight and parallel contours is not valid due to the bathymetry close to shore and the offshore shoals. Even though the wave climate, as described by WIS is not entirely valid, it does represent the best available set of wave data. It is for this reason that the WIS data is used in this study.

An attempt was not made to create a wave refraction diagram for the Fenwick Island area. The complex bathymetry extends out to the Fenwick Shoals which is believed to have some effect on the wave climate at Fenwick Island. Since the study area would have to be extended out to the Fenwick Shoals, it became impossible run the wave refraction-diffraction programs due to limited computer memory.

Goldsmith et al (1975) prepared a wave refraction analysis for the Maryland-Virginia coastline which predicts areas of wave concentration for storm conditions. One of the areas of high wave concentration was at the Maryland-Delaware line.

APPENDIX II. EMPIRICAL EIGENFUNCTION ANALYSIS

For a series of K beach profiles each with I elevations, the depth at any location can be represented as h_{ik}. These K profiles can arise due to repeated surveying of the same profile with time or by taking a number of profiles along a beach at the same time. The empirical eigenfunction analysis seeks a representation of the profile as a series of orthogonal eigenfunctions, similar to the Fourier Series:

$$h_{ik} = \sum_{n} C_{nk} E_{ni} \tag{B.1}$$

where E_{ni} represents the n^{th} empirical eigenfunction at the i^{th} location and C_{nk} is the coefficient for the n^{th} eigenfunction and the k^{th} profile.

To obtain the coefficient, C_{nk}, we minimize the mean square error to the fit of the h_{ik}, where the error is defined as

$$\varepsilon_{ik} = h_{ik} - \sum_{n} C_{nk} E_{ni} \tag{B.2}$$

Therefore we minimize $\sum_{i} \varepsilon_{ik}^2$ with respect to C_{mk}, which yields

$$C_{mk} = \sum_{i} h_{ik} E_{mi} \tag{B.3}$$

after using the orthogonality condition: $\sum_{i} E_{ni} E_{mi} = \delta_{nm}$,

where δ_{nm} is the Kronecker delta function. The delta function is unity for n=m and is zero otherwise. Now the coefficients (sometimes called the weights, as they measure the contribution of each eigenfunction to the k^{th} profile) are known, but the eigenfunctions themselves are still unknown.

The variance in the beach profile data is the sum of the squares of the depths and it is defined as σ^2:

$$\sigma^2 = \frac{1}{IK} \sum_{k}^{K} \sum_{i}^{I} h_{ik}^2 \tag{B.4}$$

Substituting Eq. B.1 for h_{ik} and using the orthogonality property of the eigenfunctions, we obtain

$$\sigma^2 = \frac{1}{IK} \sum_{k}^{K} \sum_{n} C_{nk}^2 \tag{B.5}$$

The eigenfunctions are found by requiring that each eigenfunction explain as much of the total variance in the profile data as possible. The contribution to the total variance of the n^{th} eigenfunction is

$$(\sigma^2)_n = \sum_k C_{nk}^2 \qquad (B.6)$$

From the definition for C_{nk} ,(B.3), C_{nk} can be made as large as desired by having large values for the eigenfunction. Therefore to make the problem unique, we will constrain the eigenfunction to have a magnitude of unity. Now to find the eigenfunction, E_{ni}, we maximize the following expression with respect to E_{ni}.

$$(\sigma^2)_n - \lambda \left(\sum_i E_{ni}^2 - 1 \right) \qquad (B.7)$$

The λ is a Lagrange multiplier which is commonly introduced into minimization/maximization problems with constraints. Taking the derivative of Eq. B.7 with respect to E_{nm} and setting the result equal to zero, yields the following equation.

$$\sum_i a_{im} E_{ni} = \lambda E_{nm} \qquad (B.8)$$

where

$$a_{im} \equiv \frac{1}{IK} \sum_k h_{ik} h_{mk} \qquad (B.9)$$

Equation B.8 is an eigenvalue problem for E_{nm}; however, it contains I unknowns. Therefore we let m vary from 1 to I to find I equations for I unknowns. The resulting equations can be easily represented in matrix form:

$$\underline{A}\ \underline{e}_n = \lambda\ \underline{e}_n \qquad (B.10)$$

where $\underline{A}$ is the correlation matrix:

$$\underline{A} = \{ a_{ij} \}$$

and $$\underline{e}_n = \{ E_{ni} \}.$$

The eigenvalues and eigenvectors were calculated using International Mathematical and Statistical Library (IMSL) subroutine, EIGRS, which is available for PCs.

A useful property of this analysis technique is that the mean square of the data (the variance, σ^2) is equal to the sum of the eigenvalues which is also equal to the mean square of the coefficients of the eigenfunctions. This provides a useful check in the determination of the eigenfunctions, but also provides a measure of the amount of the total variance accounted for by each eigenfunction. Since the eigenfunctions were maximized such that they contribute as much as possible to the total variance of the data, only a few eigenfunctions are necessary to represent the data with minimal error.

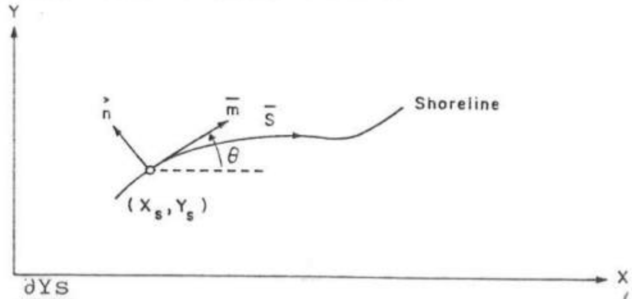
The eigenfunctions have physical significance, which depends on the nature of the profiles that they are describing. For example, for a single profile, which has been surveyed throughout a year, the first eigenfunction, corresponding to the largest eigenvalue, represents the average profile and is called the mean beach profile. The second eigenfunction is the berm/bar function. It has a maximum at the location of the summer berm and a minimum at the offshore winter bar. The third eigenfunction is the terrace function and it has maximum at the location of the low tide terrace (see Winant _et al_., 1975, for more details).

APPENDIX III. SHORELINE PREDICTION MODEL

The variation in sediment transport can occur from a change in breaker height or breaker angle. Using an orthogonal curvilinear coordinate system(Figure C.1), the following are defined:

$\overline{S}$ is the distance along a curved shoreline.
$\overline{m}$ is the unit tangent vector in the direction of increasing S.
$\overline{n}$ is the unit normal vector to the shoreline.
is the angle between the x axis and m

Figure C.1. Orthogonal Curvilinear Coordinate



$$\cos\theta = \frac{\partial Xs}{\partial s} \text{ and } \sin\theta = \frac{\partial Ys}{\partial s} \qquad (C.1)$$

$$\text{then } \overline{m} = \left(\frac{\partial Xs}{\partial s}, \frac{\partial Ys}{\partial s}\right) \qquad (C.2)$$

$$\text{and } \overline{n} = \left(-\frac{\partial Ys}{\partial s}, \frac{\partial Xs}{\partial s}\right). \qquad (C.3)$$

If (X_s, Y_s) moves perpendicular to the instantaneous shoreline, then the velocity$(\partial Xs/\partial t, \partial Ys/\partial t) = e\overline{n}$,where t is time, and e is the rate of shoreline movement and is calculated by a curvilinear version of the conservation of sand equation(Equation 17). Equating components of the velocity:

$$\frac{\partial Xs}{\partial t} = -e\frac{\partial Ys}{\partial s}$$

$$\frac{\partial Ys}{\partial t} = e\frac{\partial Xs}{\partial s} \qquad (C.4)$$

Substituting a curvilinear form of Equation 17 into Equation C.4 yields,

$$\frac{\partial Xs}{\partial t} = -\frac{1}{d}\frac{\partial Q}{\partial s}\frac{\partial Ys}{\partial s}$$

$$\frac{\partial Ys}{\partial t} = -\frac{1}{d}\frac{\partial Q}{\partial s}\frac{\partial Xs}{\partial s} \qquad (C.5)$$

Defining the complex variable $Z_s = X_s + iY_s$, it can be shown that Equation C.5 becomes

$$\frac{\partial Zs}{\partial t} = \frac{1}{d}\frac{\partial Q}{\partial s} e^{-i(\theta - \pi/2)} \qquad (C.6)$$

Equation C.6 is solved using a finite difference technique, Suh (1985).

The sediment transport at the i^{th} point,Q_i, is given by

$$Q_i = \Gamma\left(\frac{H_{b_i} + H_{b_{iH}}}{2}\right)^{5/2}\left[K_1 \sin(2\delta_{b_i}) - K_2 \cot\beta \cos\delta_{b_i}\left(\frac{H_{s_{i+i}} - H_{s_i}}{\Delta s}\right)\right]$$

where H_b = the breaker height at the i^{th} point

β = the beach slope(assumed constant)

$\Gamma = C/K_1$, where C is given by Equation 15

δ_{b_i} = breaker angle at i^{th} point

Δs = the distance between points

K_1 = 0.77 (Komar and Inman,1977)

K_2 = K_1 .

The breaker heights and angles used in the numerical model were determined by the same means as discussed for the sediment transport model above.

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